A Model for Axial Flow and Profile Evolution in the CSDX Linear Device

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Outline

- Introduction:

What are we doing, why do we care and how is this related to tokamaks?

-Intrinsic rotation is essential for plasma stability. Therefore investigating parallel flow dynamics is essential to achieve a refined confinement state.

-Relation between sheared axial flow $\bar{v}_z'$ and plasma edge pressure gradient $\nabla p \sim \nabla n$ a.k.a Rice scaling.

- Experimental Observations:
  Coupling and competition between parallel and perpendicular flow dynamics

- Formulation of the Model:
  From Hasegawa-Wakatani equations to a mean $\bar{n}, \bar{v}_y, \bar{v}_z$ and turbulent energy ε system.

- Something new:
  a 2-field model $\bar{n}, \bar{v}_y$

1-What are we doing ?

- Investigate what generates axial and zonal flows in CSDX, a linear device characterized by a constant magnetic field *B*, and what controls the saturation mechanism of the plasma mean profiles.

- Model profiles evolution and fluctuation measurements in CSDX using a reduced model that addresses and tests the physical understanding of coupled flow dynamics in the collisional DW plasma.

- The model is testable by CSDX unique measurements and capabilities.

In the same spirit of Poster # 14, J. Li et al, Wednesday

2-How is useful to tokamaks?

- Parallel flows are important for tokamak stability.

- L-H transitions are associated with nonlinear energy transfer to mean flows via Reynolds stresses. The current model captures the coupled energy transfer in both parallel and perpendicular directions between fluctuations and flows.

3-Why do we care?

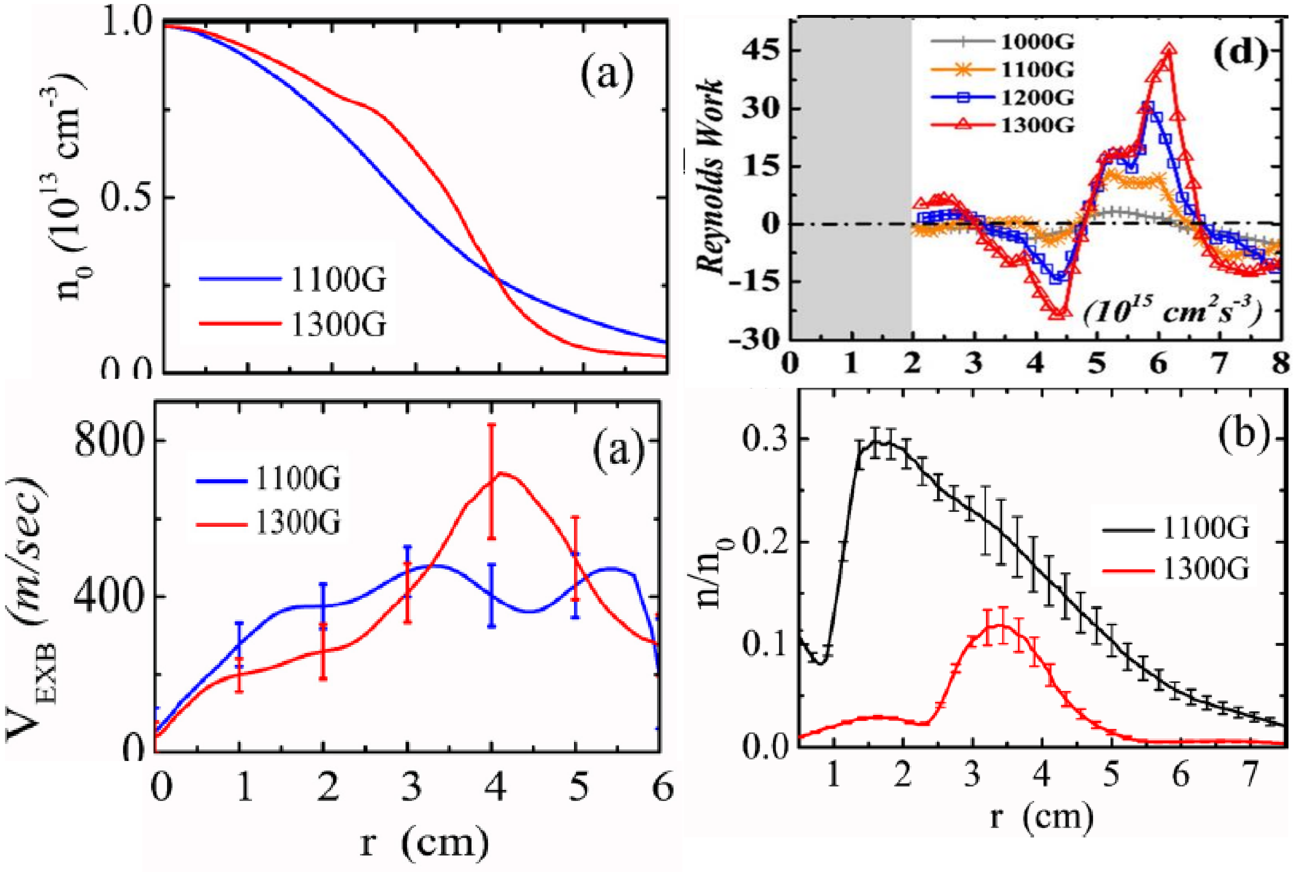
- The model presents an opportunity to study confinement and profile evolution in terms of turbulent energy ε, parallel and perpendicular kinetic energy $V_{\parallel}^2, V_{\perp}^2 (\Longrightarrow$ relation with Rice scaling).

- CSDX presents a unique opportunity to measure fluctuation stresses $\langle \tilde{v}_x \tilde{v}_y \rangle$ and $\langle \tilde{v}_x \tilde{v}_z \rangle$, and relate their variations to mean profile variations.

- Turbulence transport scaling properties observed in CSDX are similar to those observed in tokamaks.

- CSDX observations suggest a correlation between perpendicular turbulence reduction and parallel flow acceleration

4-Experimental Observations: Old



L. Cui *et al*, POP **22**, 050704 (2015)
L. Cui *et al*, POP **23**, 055704 (2016)

<u>Perpendicular flow:</u>

- Steepening of density profile.

+

- Development of a radial velocity shear.

+

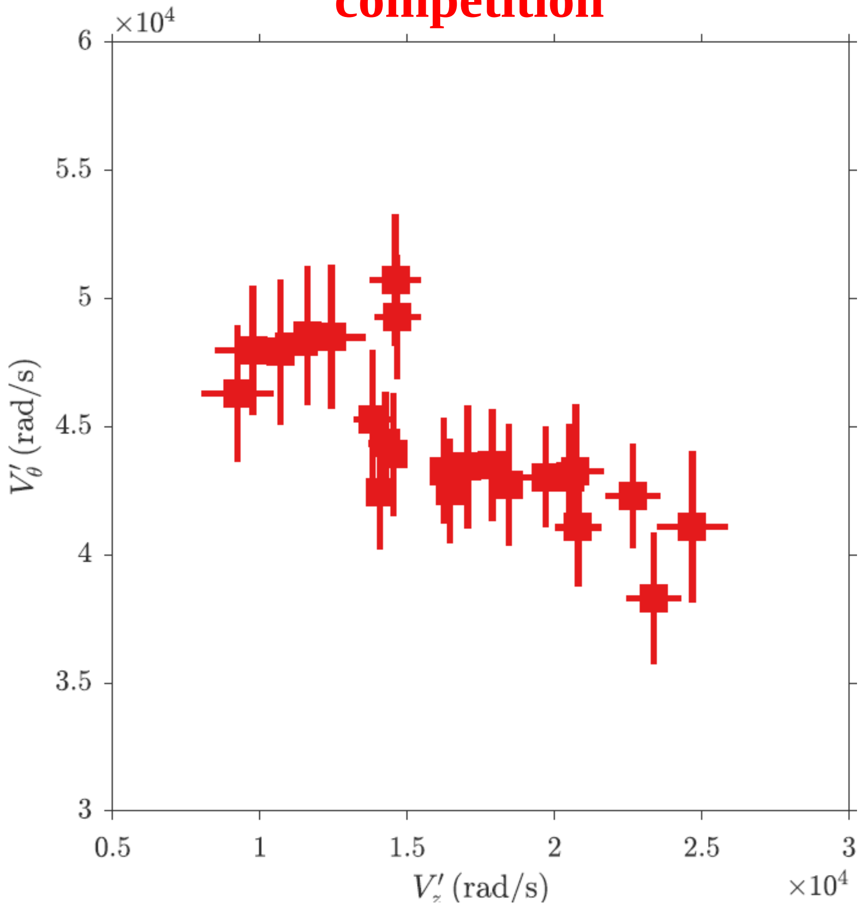
- Decrease in turbulence level.

=

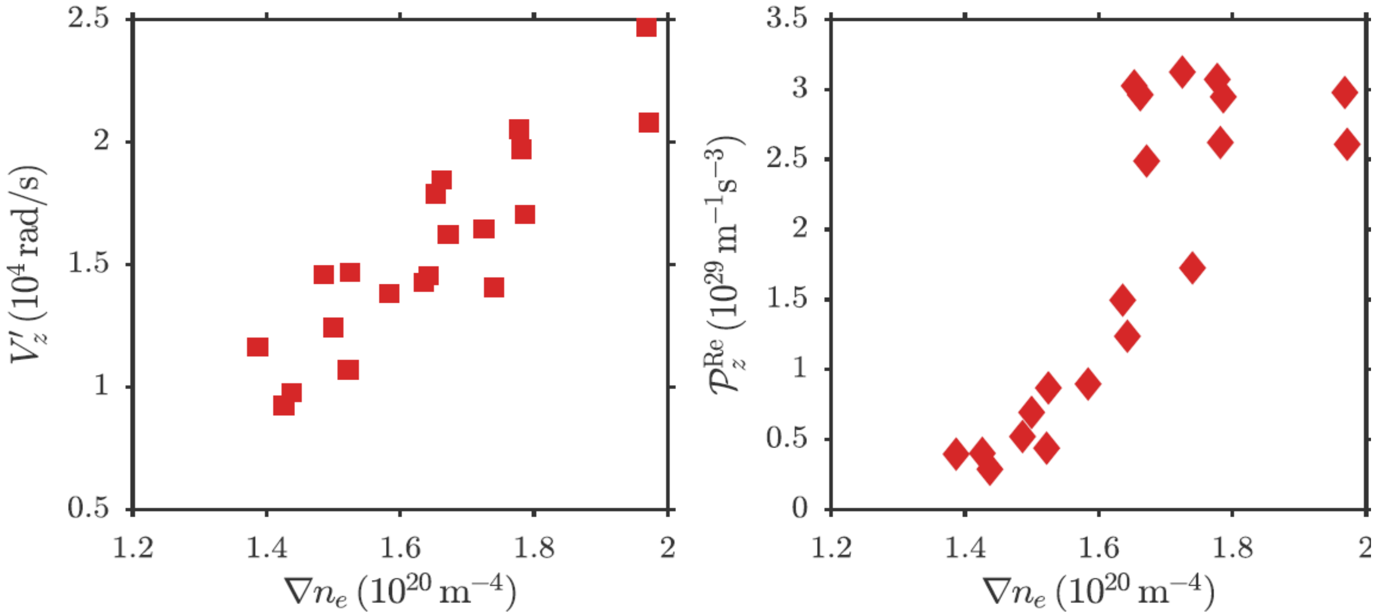
Transition to a refined state.

4-Experimental Observations: New

Parallel and perp. competition



Relation to Rice scaling



Parallel Flow

- Increase in the axial velocity shear $\bar{v}_z'$ associated with a steepening of the density profile and an increase in azimuthal velocity shear $\bar{v}_y'$

R. Hong – in preparation (2017)

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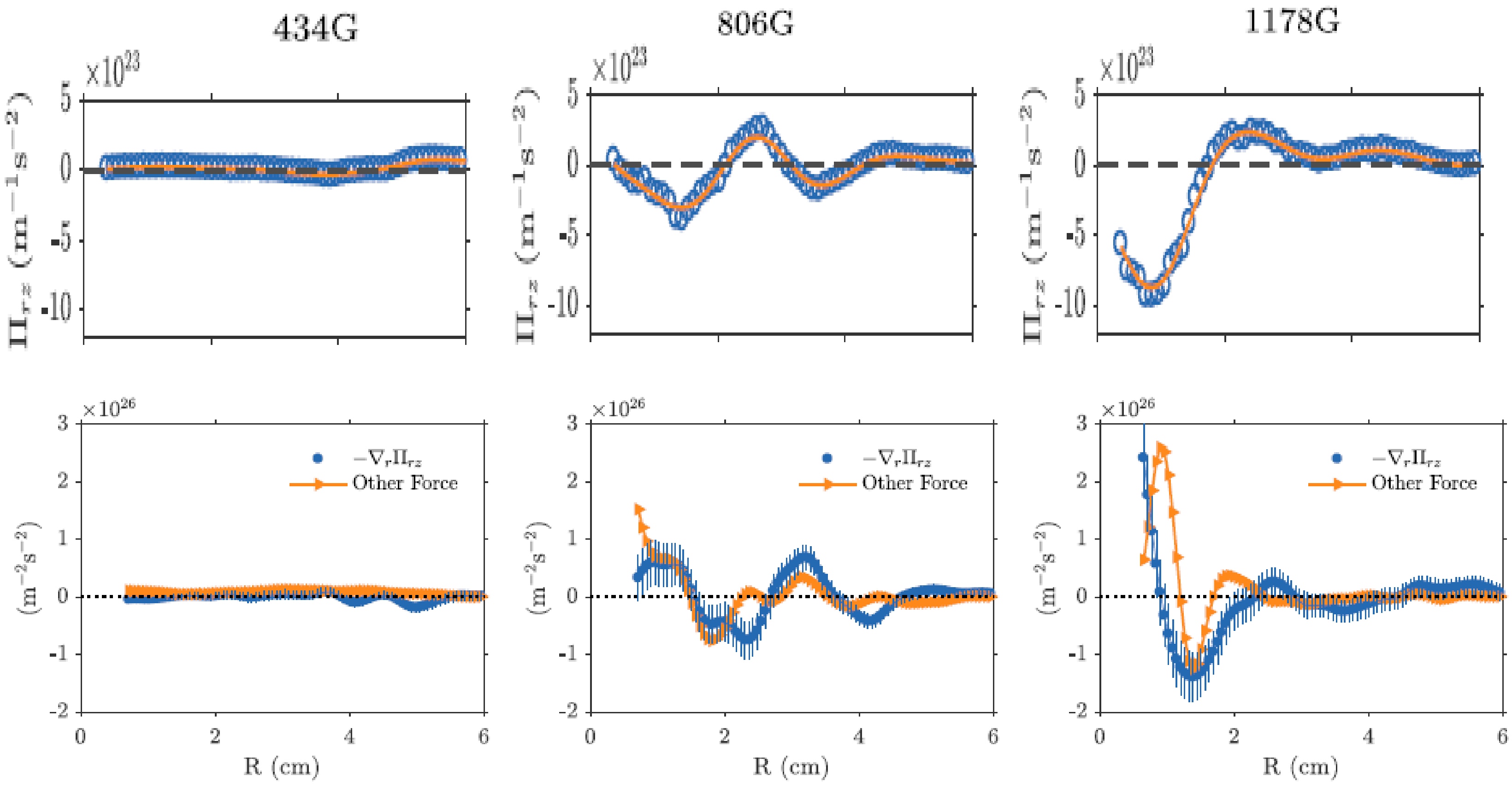


Here other forces include the neutrals drag, the pressure gradient drop and ion-ion collisional viscosity

Parallel Flow

- Build up of parallel $\langle \tilde{v}_x \tilde{v}_z \rangle$ Reynolds stress
- Increase in magnitude of the parallel Reynolds power rate leading to acceleration of the axial flow.

$$P_{\mathrm{Re},z} = -\partial_x [\bar{n} \langle \tilde{v}_x \tilde{v}_z \rangle] \bar{v}_z$$

5-Why a reduced model?

1. CSDX plasma is a multiscale system $\Longrightarrow$ a reduced model is an appropriate and necessary intermediary between the experiments and the large scale computations.

2. Reduced models are necessary to formulate questions that guide the experiment, and interpret the experimentally acting brute forces.

3. A reduced model facilitates understanding the physics behind the experimental feedback loops between the mean profiles and the fluctuation intensities.

4. A reduced model has a relatively low computational cost.

6-What is new in this model?

- Treating the coupling and competition between parallel and perpendicular flow dynamics.

- Conventional intrinsic rotation models adopt a fixed azimuthal velocity shear (constant $\bar{V}_{E\times B}$'). In this model, the shear evolves in time.

- Parallel coupling term, relating DWs and IAWs breaks PV conservation. Alternatively, the model explicitly conserves turbulent energy. Here energy is formulated in terms of turbulent energy ε, and mean fields: $\bar{n}, \bar{v}_y, \bar{v}_z$

- Residual stresses in both parallel and perpendicular Reynolds stress expressions are density gradient dependent.

Formulation of the Model

Primitive Hasegawa-Wakatani equations.
+
r-dep. axial velocity

$$\begin{cases} \dfrac{d\tilde{n}}{dt} + \dfrac{\tilde{v}_E.\nabla\langle n_0\rangle}{n_0} + \boxed{\nabla_{\parallel}\tilde{v}_z} = -\dfrac{T_e}{m_e\nu_{ei}}\nabla_{\parallel}^2(\tilde{\Phi} - \tilde{n}) + D_0\nabla_{\perp}^2\tilde{n} \\ \dfrac{d\nabla_{\perp}^2\tilde{\phi}}{dt} + \tilde{v}_E.\nabla\langle\nabla_{\perp}^2\phi\rangle = -\dfrac{T_e}{m_e\nu_{ei}}\nabla_{\parallel}^2(\tilde{\phi} - \tilde{n}) + \mu_0\nabla_{\perp}^4\tilde{\phi} \\ \dfrac{d\tilde{v}_z}{dt} + \boxed{\tilde{v}_E.\nabla\langle v_z\rangle} = -\dfrac{T_e}{n_0 M}\nabla_{\parallel}\tilde{n} + \nu_0\nabla_{\perp}^2\tilde{v}_z \end{cases}$$

Define the zonal average of turbulent energy:

$$\langle\varepsilon\rangle = \int_0^{L_{//}} dz \int_0^{2\pi} \varepsilon(r)d\theta = \int dz \int \varepsilon(x)dy = \frac{\langle \tilde{n}^2 + (\nabla_{\perp}\tilde{\varphi})^2 + \tilde{v}_z^2 \rangle}{2}$$

Model Equations

Diffusion | Coupling to mean profiles | Dissipation terms | Forcing

$$\frac{\partial \varepsilon}{\partial t} - \partial_x(\varepsilon^{1/2} l_{mix} \partial_x \varepsilon) = -\Gamma_n \frac{d\bar{n}}{dx} - \langle \tilde{v}_x \tilde{v}_y \rangle \frac{d\bar{v}_y}{dx} - \langle \tilde{v}_x \tilde{v}_z \rangle \frac{d\bar{v}_z}{dx} - \frac{\varepsilon^{3/2}}{l_{mix}} - C\int [\partial_z(\tilde{\varphi} - \tilde{n})]^2 dz + F - \eta_{//} \tilde{J}_{//}^2$$

Turb. Energy equation
+
Mean flow equations:

$$\frac{\partial \bar{n}}{\partial t} = -\frac{\partial}{\partial x}\langle \tilde{n}\tilde{v}_x \rangle + D_0 + Source/Sink$$

$$\frac{\partial \bar{v}_z}{\partial t} = -\frac{\partial}{\partial x}\langle \tilde{v}_x \tilde{v}_z \rangle + \mu_{0//}\frac{\partial^2 \bar{v}_z}{\partial x^2} - C_s^2 \frac{\partial \bar{n}}{\partial x}$$

$$\frac{\partial \bar{v}_y}{\partial t} = -\frac{\partial}{\partial x}\langle \tilde{v}_x \tilde{v}_y \rangle + \mu_{0\perp}\frac{\partial^2 \bar{v}_y}{\partial x^2} - \nu_{in}(\bar{v}_y - \bar{v}_n) - \nu_{ii}\bar{v}_y$$

<u>Closure of the system requires expressions for:</u>

- The particle flux Γ_n
- The parallel Reynolds stress $\langle \tilde{v}_x \tilde{v}_z \rangle$
- The Reynolds stress $\langle \tilde{v}_x \tilde{v}_y \rangle$
  And the respective derivatives.
- Expression for the mixing length l_{mix}

Energy conservation

- Define total energy: $E_{tot} = \varepsilon + \frac{\bar{n}^2 + (\nabla_\perp \bar{\varphi})^2 + \bar{v}_z^2}{2} \Rightarrow \left\langle \frac{dE_{tot}}{dt} \right\rangle = \left\langle \frac{d\varepsilon}{dt} \right\rangle + \frac{1}{2}\int \bar{v}_z^2 + \bar{n}^2 + (\nabla_\perp \bar{\varphi})^2 = -\frac{\varepsilon^{3/2}}{l_{mix}} + F$

Flux Expressions

- <u>QL expression for particle flux:</u>

$$\Gamma_n = \langle \tilde{n}\tilde{v}_x \rangle = \sum_m [-\frac{\nu_{ei} k_m^2 \rho_s^2 C_s^2}{k_z^2 v_{th}^2} \frac{dn}{dx} - \frac{\nu_{ei} k_m^2 \rho_s C_s}{k_z^2 v_{th}^2} \frac{\omega_m^r}{k_m}] \langle \tilde{\varphi}^2 \rangle$$

Diffusion Coefficient= $(\nu_{ei} f \ / \ k_z^2 \ v_{th}^2)\ \varepsilon$

Note that:

$\langle \tilde{v}_x^2 \rangle = k_m^2 \rho_s^2 C_s^2 \langle \tilde{\varphi}^2 \rangle = f\varepsilon$

where f is the fraction of the turbulent energy in the radial direction

Vorticity flux

Reynolds power rate

$$\langle \tilde{v}_x \nabla_\perp^2 \varphi \rangle = -\chi_y \frac{d^2 \bar{v}_y}{dx^2} + \Pi_{xy}^{res}$$

$$\langle \tilde{v}_x \nabla_\perp^2 \varphi \rangle = -f\sqrt{\varepsilon} l_{mix} \frac{d^2 \bar{v}_y}{dx^2} + \frac{f l_{mix} \sqrt{\varepsilon} C_s}{\rho_s L_n}$$

$$\langle \tilde{v}_x \tilde{v}_y \rangle \frac{d\bar{v}_y}{dx} = \bar{v}_y [-\chi_y \frac{d^2 \bar{v}_y}{dx^2} + \Pi_{xy}^{res}]$$

$$\langle \tilde{v}_x \tilde{v}_y \rangle \frac{d\bar{v}_y}{dx} = \bar{v}_y [-f\sqrt{\varepsilon} l_{mix} \frac{d^2 \bar{v}_y}{dx^2} + \frac{f l_{mix} \sqrt{\varepsilon} C_s}{\rho_s L_n}]$$

Story of the shearing that destroys eddies and drives mean and zonal flows by non-linear energy transfer

$$\langle \tilde{v}_x \nabla_\perp^2 \varphi \rangle = -\frac{d}{dx}(\chi_y \frac{d\bar{v}_y}{dx}) + \frac{d\chi_y}{dx} \frac{d\bar{v}_y}{dx} + \Pi_{xy}^{res}$$

Persists when flow and shear are equal to zero.

Viscous diffusion of v_y

Flow convection at $V_{conv} = d\chi_y / dx$

Parallel Reynolds Stress - 1

- Parallel flows are driven by parallel residual stresses $\Pi_{//}^{res}(\nabla n, \nabla T)$.
- Conventional parallel symmetry breaking mechanisms require a magnetic shear to create a residual stress and a correlator $\Pi_{//}^{res} \sim \langle k_m k_z \rangle$.
- A novel symmetry breaking mechanism relating perpendicular DWs to generation of parallel flows was recently developed.

1-Modulational Expression of parallel Reynolds stress:

$$\langle \tilde{v}_x \tilde{v}_z \rangle = \sum_m -\chi_z \frac{d\bar{v}_z}{dx} + \Pi_{xz}^{res}$$

$$\langle \tilde{v}_x \tilde{v}_z \rangle = \sum_m -\frac{k_m^2 \rho_s^2 C_s^2 |\gamma_m|}{|\omega|^2} \langle \tilde{\varphi}^2 \rangle \frac{d\bar{v}_z}{dx} + \boxed{k_m k_z} \rho_s C_s^3 [\frac{|\gamma_m|}{|\omega|^2} + \frac{\nu_{ei}(\omega^* - \omega_m)}{\omega k_z^2 v_{th}^2}] \langle \tilde{\varphi}^2 \rangle$$

Li *et al*, POP **23**, 052311 (2016)

Parallel Reynolds Stress - 2

2-QL expression:

$$\langle \tilde{v}_x \tilde{v}_z \rangle = \sum_m -\chi_z \frac{d\bar{v}_z}{dx} + \Pi_{xz}^{res}$$

$$\langle \tilde{v}_x \tilde{v}_z \rangle = -l_{mix} f \sqrt{\varepsilon} \frac{d\bar{v}_z}{dx} + \sum k_m k_z \rho_s C_s^3 [\frac{l_{mix}}{\sqrt{\varepsilon}} + \frac{\nu_{ei} \rho_s^2 k_\perp^2}{k_z^2 v_{th}^2}] \langle \tilde{\varphi}^2 \rangle$$

3-Empirical form: In analogy with turbulence in pipe flows, we write:

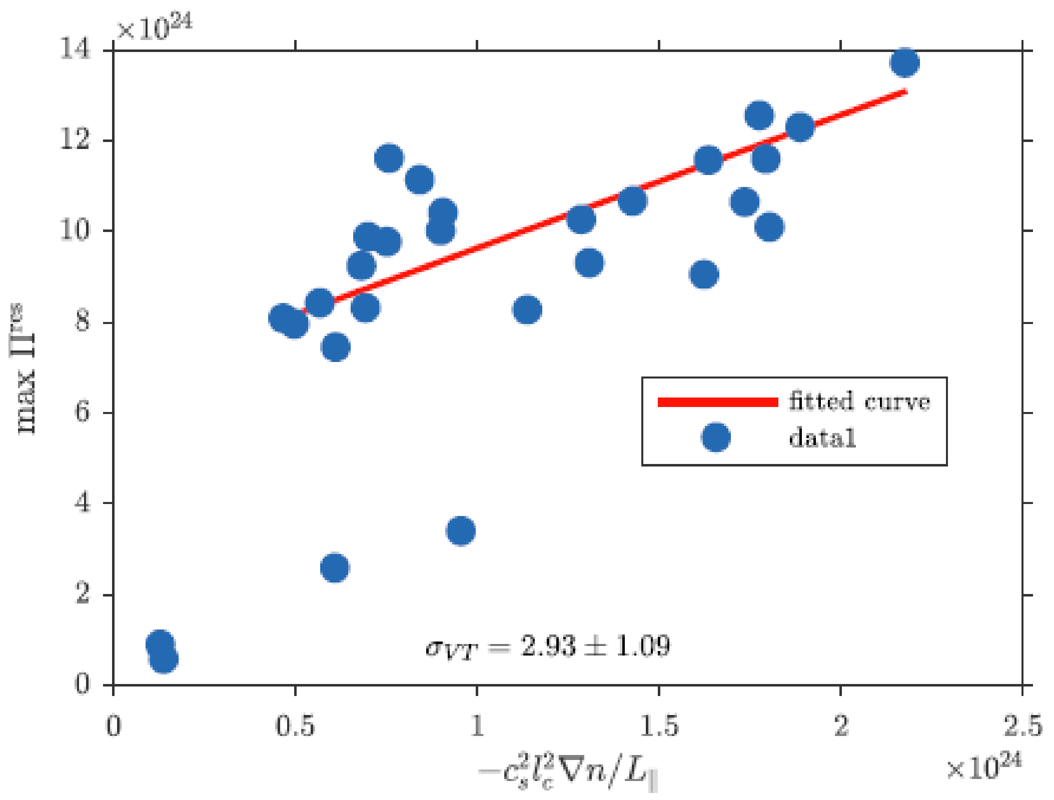
$$\tilde{v}_z = -l_{mix} \frac{d\bar{v}_z}{dx} + \frac{\sigma_{VT} C_s^2 \tau_c}{L_\parallel} . \left(\frac{-l_{mix}}{\bar{n}} \frac{dn}{dx} \right)$$

Turb. diffusivity

Coupling with perp. direction which generates a residual stress

σ_{VT} is introduced as a constant that bears the cross phase between radial and axial velocity fluctuations

<u>Preliminary Results</u> R. Hong – in preparation (2017)



$L_{//}$=150 cm, C_s~3.5 km/s, l_{mix}~1cm, $\nabla n = L_n$

- In CSDX, the residual parallel stress is measured by subtracting the diffusive part from the total stress, and fitting the data with the experimentally retrieved correlation length $\langle l_{mix}^2 \rangle$:

$$\Pi_{xz}^{res} = -\sigma_{VT} C_s^2 \langle l_{mix}^2 \rangle / L_{\|} L_n$$

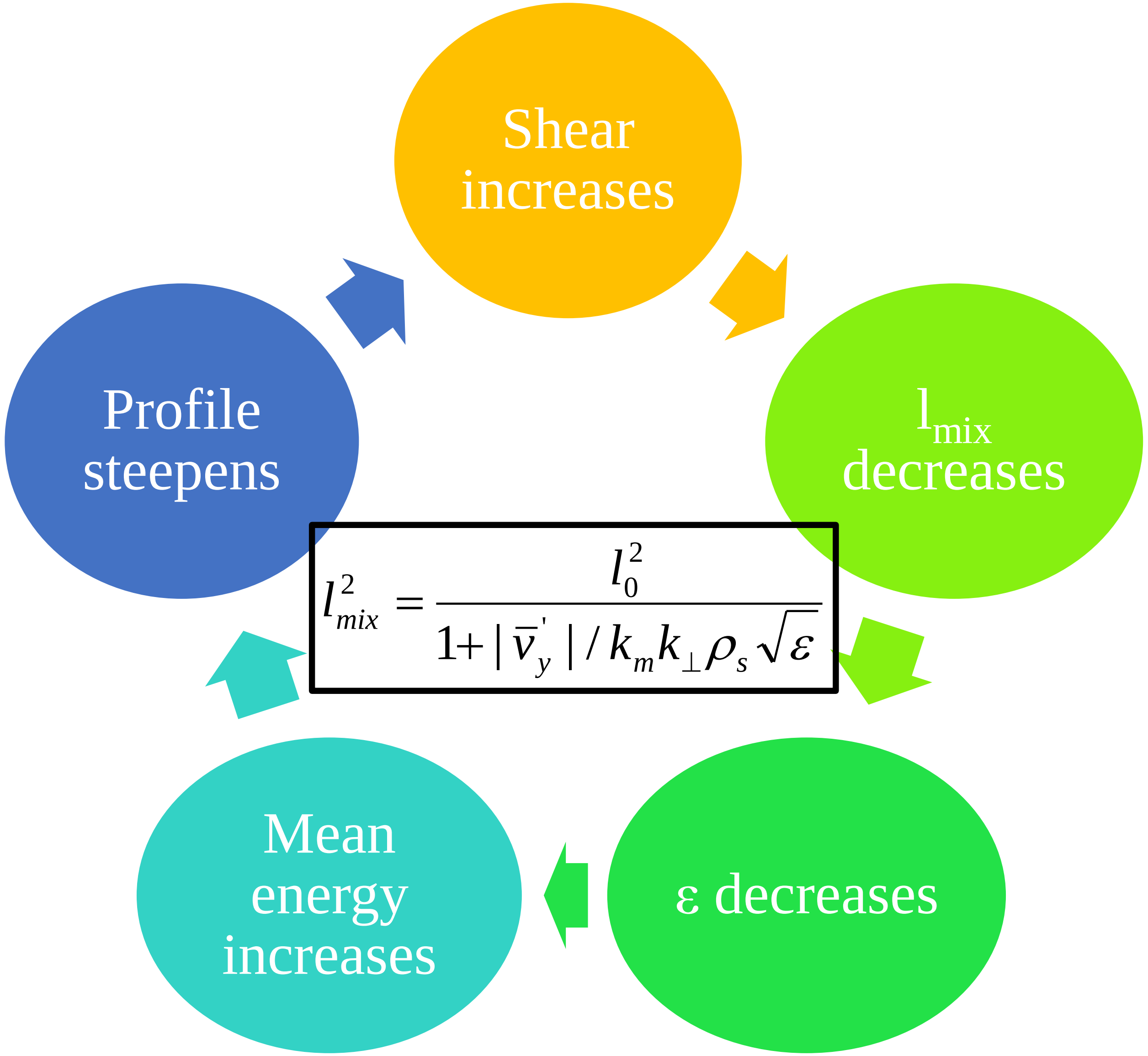
$$\frac{\langle \tilde{v}_x \tilde{v}_y \rangle}{\langle \tilde{v}_x \tilde{v}_z \rangle} \sim \sigma_{VT}^{-1}$$

$\Longrightarrow$ this empirical constant is an indication of how important the parallel/perpendicular coupling is.

- One can use the empirical value of σ_{VT} to calculate the exact value of the $\langle k_m k_z \rangle$ correlator.

Mixing length l_{mix} ⇔ Feedback loops

Shearing effects on mixing length generate a feedback loop



$$l_{mix}^2 = \frac{l_0^2}{1 + |\bar{v}_y'| / k_m k_\perp \rho_s \sqrt{\varepsilon}}$$

What is the mixing scale length l_0 really equal to?

Measurements of l_0 from density fluctuations :

$B(G)$	800	900	1000	1200	1300
$\rho_i(cm)$	0.44	0.39	0.35	0.29	0.27
$\rho_s(cm)$	1.40	1.24	1.12	0.93	0.86
$L_n^{-1}(cm^{-1})$	0.53	0.55	0.6	0.62	0.5
$\bar{k}_r(cm^{-1})$	0.33	0.33	0.37	0.32	0.34
$(\bar{k}_r^2)^{1/2}(cm^{-1})$	0.30	0.30	0.48	0.38	0.31
$1/[2.3\rho_s^{0.6}L_n^{0.3}]$	0.29	0.32	0.34	0.39	0.37

$$l_0 = \left[(\bar{k}_r^2)^{1/2} \right]^{-1} = 2.3\rho_s^{0.6} L_n^{0.3}$$

- **Concrete illustration that l_{mix} is a hybrid of ρ_s and L_n, in contrast to a simple dependence on ρ_s or L_n.**
- **Suggestion of a turbulent diffusion with $D \sim D_B\, \rho_s^{0.6}$**

Simplification by slaving

- For a fast turbulent correlation time $\tau_c < \tau_{conf}$ and v_z=0, the model is simplified to a 2-field model:

$$\frac{\partial \bar{n}}{\partial t} = -\frac{\partial}{\partial x}\langle \tilde{n}\tilde{v}_x \rangle + D_0 + Source/Sink$$

$$\frac{\partial \bar{v}_y}{\partial t} = -\frac{\partial}{\partial x}\langle \tilde{v}_x\tilde{v}_y \rangle + \mu_{0\perp}\frac{\partial^2 \bar{v}_y}{\partial x^2} - \nu_{in}(\bar{v}_y - \bar{v}_n) - \nu_{ii}\bar{v}_y$$

- With $\Gamma_n = \frac{\nu_{ei}\langle \delta v_x^2 \rangle}{k_z^2 v_{th,e}^2}\frac{d(n/n_0)}{dx} = \frac{\nu_{ei} f \varepsilon}{k_z^2 v_{th,e}^2 L_n}$ and $-\frac{\partial \langle \tilde{v}_x \tilde{v}_y \rangle}{\partial x} = -l_{mix}\sqrt{\varepsilon} f\left(\frac{d^2 \bar{v}_y}{dx^2} - \frac{\omega_{ci}}{L_n}\right)$

- Expression for ε is obtained from solving the intensity equation:

$$\left(\varepsilon + \frac{d\bar{v}_y/dx\sqrt{\varepsilon}}{k_m k_\perp \rho_s}\right)^{1/2}\frac{Af}{L_n^2} + f l_0 \bar{v}_y\left[\frac{d^2\bar{v}_y}{dx^2} - \frac{\omega_{ci}}{L_n}\right] - \frac{\left(\varepsilon + \frac{d\bar{v}_y/dx\sqrt{\varepsilon}}{k_m k_\perp \rho_s}\right)}{l_0} = 0$$

Improved from Hinton et al. which imposes a fluctuation intensity level

Future work

- Numerical implementation using appropriate boundary conditions and exact expressions for sources and diffusion terms.

- Develop a 3 field model: mean n, mean v_z and mean v_y, by adding the axial flow.

- Understand perpendicular and parallel axial flow competition.