Modelling Axial Flow Generation and Profile Evolution in the CSDX Linear Device

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What are we doing?

Main Goals:

1. Investigate the relation between the parallel and perpendicular flow dynamics in a coupled drift-ion acoustic wave plasma.

2. Model the profiles evolution and the fluctuations measurements in the CSDX linear device.

3. Investigate how the CSDX axial flows are generated and their relation to azimuthal shear and density gradient.

Along the side:

1. Relate the mean profile evolution and the variations of the transport fluxes to variations of the mean plasma density, via the adiabaticity parameter.

2. Interpretation of the corresponding variations in view of a zonal flow production/turbulence enhancement perspective.

How is this related to tokamaks?

1. Intrinsic rotation is essential for plasma stability, specially in large scale devices where methods of external axial momentum input are not efficient.

2. The relation between the axial flow shear and the edge pressure gradients in CSDX, reminds us of the well known Rice scaling in tokamaks.

3. Tokamak L-H transitions are associated with a nonlinear energy transfer to the mean flows via the Reynolds stress. The current model captures the energy transfer between flows and fluctuations, in both parallel and perpendicular directions. It can thus be used to understand tokamak physics.

Why do we care?

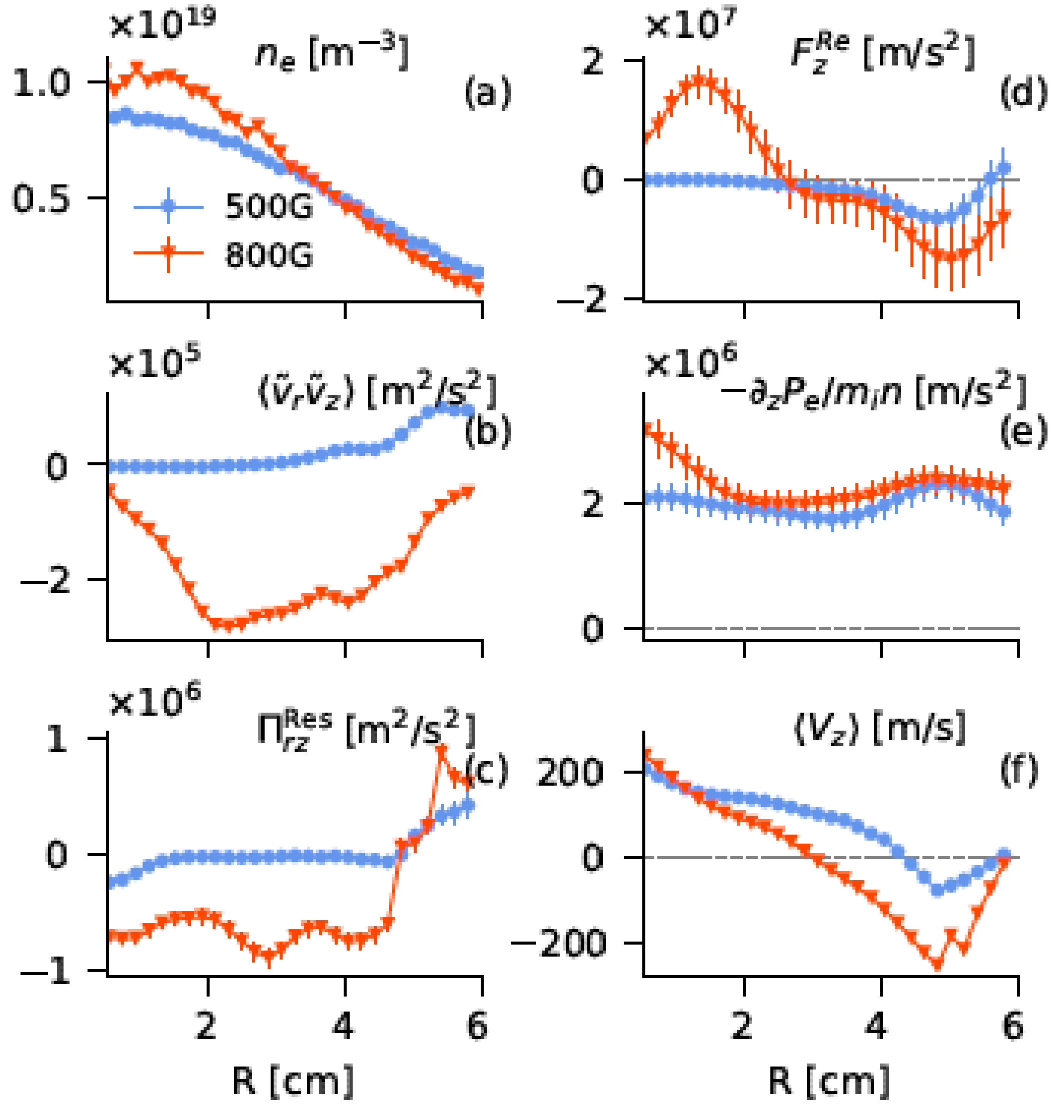
1. The reduced model presents an opportunity to study profile evolution in terms of kinetic energies v_z^2 ,v_y^2 (Rice scaling) and turbulent energy ε.

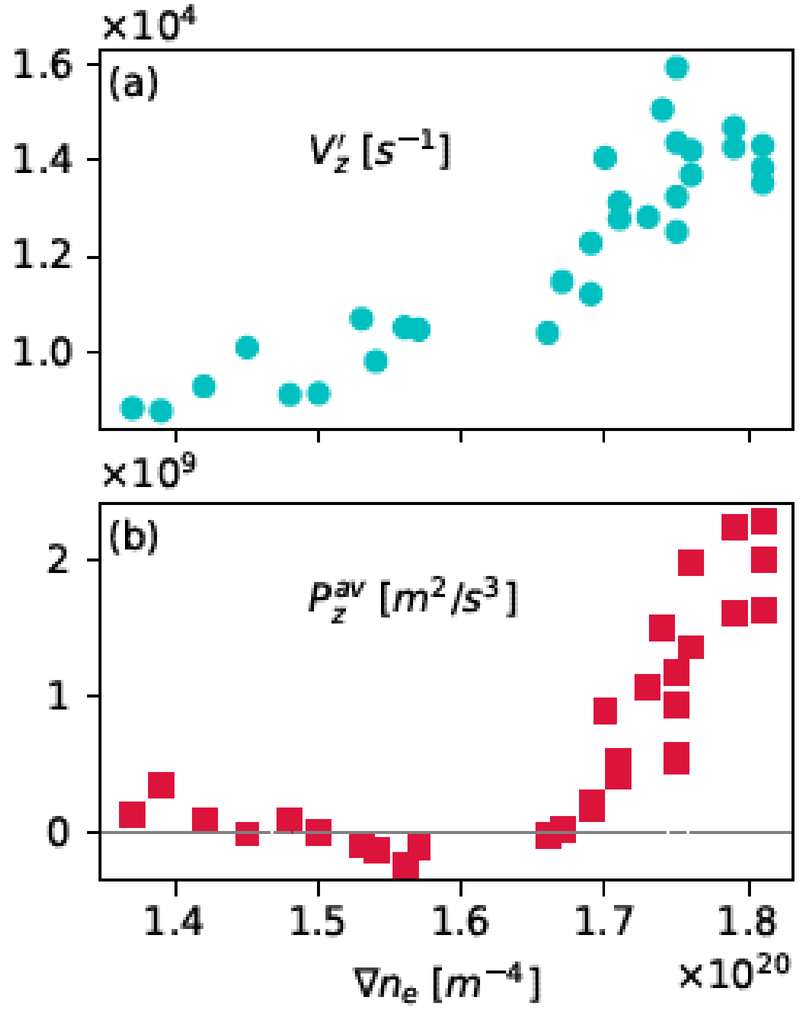
2. The model relates variations of the parallel and perpendicular Reynolds stresses: $\langle \tilde{v}_x \tilde{v}_z \rangle$ and $\langle \tilde{v}_x \tilde{v}_y \rangle$, to the gradients of n, v_z and v_y. All of these quantities are experimentally measurable in CSDX.

3. It adds to what we already know about DWs and ZFs, by introducing a similar physical relation between DWs and axial flows.

4. The model allows us to understand the coupling/competition relation between the parallel and the perpendicular flow dynamics.

Experimental Results (R. Hong in preparation)





Experimental Results (R. Hong in preparation)

As B is raised:

1. Density gradient increases.

2. Parallel stress increases in magnitude.

3. Residual parallel stress and Reynolds force increase significantly.

4. Radial shear of v_z increases.

5. Axial flow shear increases with the density gradient (proportional to azimuthal shearing) $\Longrightarrow$ coupling between parallel and perpendicular flow shear.

Why another reduced model?

1. CSDX plasma is a multiscale system $\Longrightarrow$ a reduced model is a necessary and appropriate intermediary between the experiments and the full blown simulations.

2. Reduced models are essential to formulate the right questions that guide the experiment and interpret the experimentally acting brute forces.

3. Reduced models facilitate understanding the physics behind the experimental feedback loops between the mean profiles and the fluctuation intensities.

4. Reduced models have a relatively low computational cost.

What is new in this model?

1. Self-consistent treatment of the coupling relation between the parallel and perpendicular flow dynamics.

2. The acoustic coupling breaks conservation of PV $\Longrightarrow$ Need to formulate a new conserved energy. This new energy is formulated in terms of both parallel and perpendicular kinetic energies, as well as the plasma internal energy.

3. The model introduces an experimental measure of the correlator $\langle k_m k_z \rangle$ that relates the parallel residual stress proportional to the density gradient.

Formulation of the model

Hasegawa-Wakatani model
+
Parallel compression term

$$
\left\{
\begin{aligned}
&\frac{d\tilde{n}}{dt} + \mathbf{v}_E.\nabla\langle n\rangle + n_0\nabla_z\tilde{v}_z = -\frac{v_{th}^2}{\nu_{ei}}\nabla_z^2(\tilde{\phi}-\tilde{n}) + D_0\nabla_\perp^2\tilde{n} + \{\tilde{n},\tilde{\phi}\}\\
&\frac{d\nabla_\perp^2\tilde{\phi}}{dt} + \mathbf{v}_E.\nabla\langle\nabla_\perp^2\phi\rangle = -\frac{v_{th}^2}{\nu_{ei}}\nabla_z^2\tilde{\phi} + \mu_0\nabla_\perp^4\tilde{\phi} - \nu_{in}\bar{v}_y + \{\nabla_\perp^2\tilde{\phi},\tilde{\phi}\}\\
&\frac{d\tilde{v}_z}{dt} + \mathbf{v}_E.\nabla\langle v_z\rangle = -C_s^2\nabla_z\tilde{n} + \nu_0\nabla_\perp^2\tilde{v}_z + \{\tilde{v}_z,\tilde{\phi}\}
\end{aligned}
\right.
$$

Parallel compression $\Longrightarrow$ PV conservation is broken

$\Longrightarrow$ define a new conserved energy ε:

$$\langle\varepsilon\rangle = \int_0^{L_z} dz\int_0^{L_y} dy\,\varepsilon(x) = \frac{\langle\tilde{n}^2 + (\nabla_\perp\tilde{\phi})^2 + \tilde{v}_z^2\rangle}{2}$$

Model Equations

Diffusion

Coupling terms

Production

$$\frac{\partial \varepsilon}{\partial t} - \boxed{\partial_x(\varepsilon^{1/2} l_{mix} \partial_x \varepsilon)} = \boxed{-\Gamma \frac{d\bar{n}}{dx} - \langle \tilde{v}_x \tilde{v}_y \rangle \frac{d\bar{v}_y}{dx} - \langle \tilde{v}_x \tilde{v}_z \rangle \frac{d\bar{v}_z}{dx}} - \boxed{\frac{\varepsilon^{3/2}}{l_{mix}} - \frac{v_{th}^2}{\nu_{ei}} \int [\partial_z(\tilde{\varphi} - \tilde{n})]^2} - \langle \tilde{n} \tilde{v}_z \rangle + \boxed{P}$$

Dissipation

$$\frac{\partial \bar{n}}{\partial t} = -\frac{\partial}{\partial x}\langle \tilde{v}_x \tilde{n} \rangle + D_c \frac{\partial^2 \bar{n}}{\partial x^2} + S_n$$

$$\frac{\partial \bar{v}_z}{\partial t} = -\frac{\partial}{\partial x}\langle \tilde{v}_x \tilde{v}_z \rangle + \nu_{c,\parallel} \frac{\partial^2 \bar{v}_z}{\partial x^2} + S_{v_z}$$

$$\frac{\partial \bar{v}_y}{\partial t} = -\frac{\partial}{\partial x}\langle \tilde{v}_x \tilde{v}_y \rangle + \nu_{c,\perp} \frac{\partial^2 \bar{v}_y}{\partial x^2} - \nu_{in} \bar{v}_y - \nu_{ii} \bar{v}_y$$

<u>We need expressions for:</u>

1. Particle Flux
2. Vorticity flux and Reynolds work
3. Parallel Reynolds stress
4. Mixing length

1- The Quasi Linear form of the Particle Flux:

$$\Gamma = \sum_m \alpha \frac{v_d - \omega^r / k_m}{|\omega / k_m + i\alpha / k_m|^2} |\delta\phi^2|$$

In the adiabatic limit, $\omega \cong \omega^r = \dfrac{\omega^*}{1 + k_\perp^2 \rho_s^2}$ $\quad v_d = \rho_s C_s / L_n$ and the particle flux is:

$$\Gamma = -D.\frac{1}{n_0}\frac{d\bar{n}}{dx} \qquad \text{with} \qquad D = \frac{\langle \delta v_x^2 \rangle}{\alpha} \approx \frac{f\varepsilon}{\alpha}$$

Here the adiabaticity parameter is: $\alpha = \dfrac{k_z^2 v_{th}^2}{\nu_{ei}} >> 1$

2- Vorticity Flux:

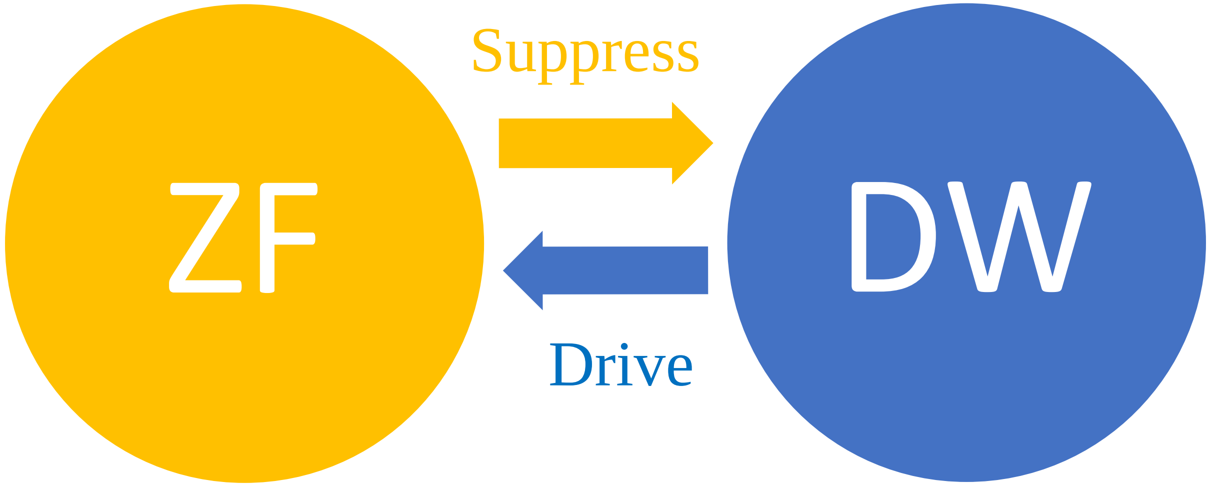
$$\left\langle \tilde{v}_x \nabla^2 \varphi \right\rangle = -\chi_y \frac{d^2 v_y}{dx^2} + \Pi_{xz}^{res}$$

$$\left\langle \tilde{v}_x \nabla^2 \varphi \right\rangle = -fl_{mix} \frac{d^2 v_y}{dx^2} + \frac{fl_{mix}\sqrt{\varepsilon}\,\omega_{ci}}{L_n}$$

Reynolds power rate:

$$\left\langle \tilde{v}_x \tilde{v}_y \right\rangle \frac{d\bar{v}_y}{dx} = \bar{v}_y[-\chi_y \frac{d^2 \bar{v}_y}{dx^2} + \Pi_{xy}^{res}]$$

$$\left\langle \tilde{v}_x \tilde{v}_y \right\rangle \frac{d\bar{v}_y}{dx} = \bar{v}_y[-fl_{mix}\sqrt{\varepsilon} \frac{d^2 \bar{v}_y}{dx^2} + \frac{fl_{mix}\sqrt{\varepsilon}\,\omega_{ci}}{L_n}]$$



3- Parallel Reynolds Stress

a) Quasi Linear Expression:

$$\langle \tilde{v}_x \tilde{v}_z \rangle = -f l_{mix} \sqrt{\varepsilon} \frac{dv_z}{dx} + \langle k_m k_z \rangle \rho_s C_s^3 [\frac{l_{mix}}{\sqrt{\varepsilon}} + \frac{\rho_s^2 k_\perp^2}{\alpha}]$$

b) Empirical form: In analogy with pipe flows, we write

$$\frac{d\tilde{v}_z}{dt} = -C_s^2 \nabla_z \left[\frac{e\tilde{\phi}}{T_e} + \frac{\tilde{p}_e}{p_e} \right] - \tilde{v}_x \frac{d\bar{v}_z}{dx} \longrightarrow \tilde{v}_z^{res} = \frac{\sigma_{VT} C_s^2 \tau_c}{L_{||}} . \left(\frac{-l_{mix}}{\bar{n}} \frac{d\bar{n}}{dx} \right)$$

$$\tilde{v}_z = \underbrace{-l_{mix}\frac{d\bar{v}_z}{dx}}_{\text{Turb. diffusivity}} + \underbrace{\frac{\sigma_{VT} C_s^2 \tau_c}{L_{||}} . \left(\frac{-l_{mix}}{\bar{n}}\frac{dn}{dx}\right)}_{\text{Coupling with perp. direction which generates a residual stress}}$$

σ_{VT} is introduced as a constant that bears the cross phase between radial and axial velocity fluctuations

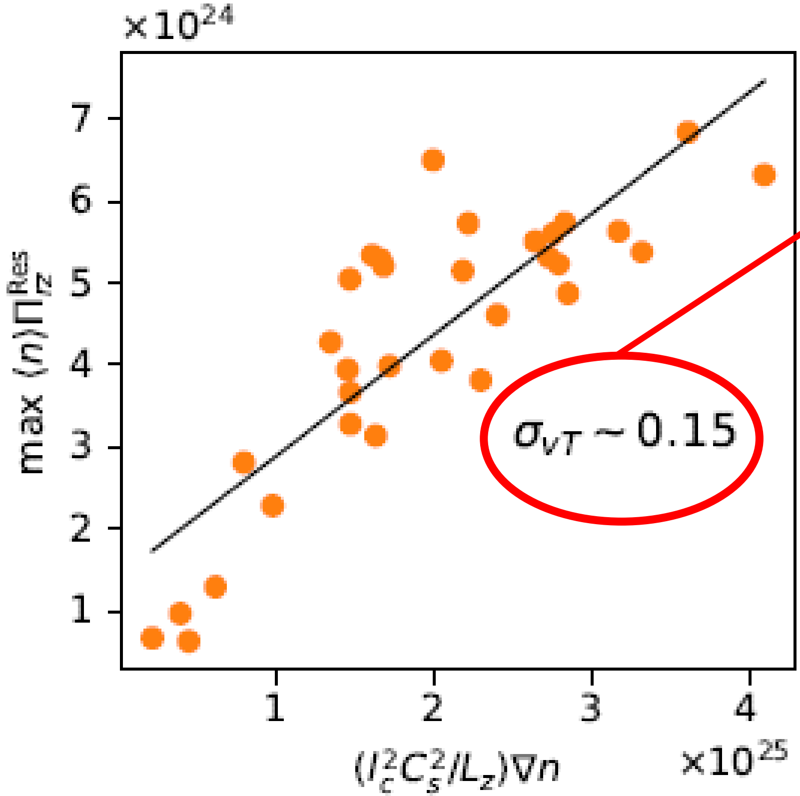
- The parallel Reynolds stress is then:

$$\langle \tilde{v}_x \tilde{v}_z \rangle = -\chi_z \frac{d\bar{v}_z}{dx} - \frac{\sigma_{VT} C_s^2 \langle l_{mix}^2 \rangle}{L_{||}} . \frac{\nabla \bar{n}}{\bar{n}}$$

- σ_{VT} is an empirically testable constant of the correlator $<k_m k_z>$, since both $L_{//}$ and $<k_\perp^2>$ can be determined experimentally:

$$\boxed{\sigma_{VT} = \frac{\langle k_m k_z \rangle}{\langle k_\perp^2 \rangle^{1/2} / L_{||}}}$$

$$\Pi_{xz}^{res} = -\sigma_{VT} C_s^2 l_{mix}^2 / L_{//} L_n$$



$L_{//}$=150 cm, C_s~3.5 km/s, l_{mix}~1cm, $\nabla n = L_n$

1) Density gradient is responsible of generating the axial flow (the parallel residual stress) via the relation:

$$\nabla \bar{v}_z = \frac{\sigma_{VT} C_s^2 l_{mix}}{L_\parallel \bar{n} \langle \delta v_x^2 \rangle^{1/2}} \nabla \bar{n}$$

2) Since ∇v_y also depends on ∇n , we can write:

$$\frac{d}{dx} \nabla \bar{v}_y = \frac{\langle \delta v_x^2 \rangle^{1/2} \omega_{ci} L_\parallel}{\sigma_{VT} C_s^2 l_{mix}} \nabla \bar{v}_z$$

Parallel and Perpendicular Coupling

4-Mixing length

$$l_{mix}^2 = \frac{l_0^2}{\left[1 + \left(k_m \bar{v}_y' + k_z \bar{v}_z'\right)^2 \tau_c^2\right]} = l_0^2 \left[1 + \left(\frac{\bar{v}_y'}{l_0} + \frac{\bar{v}_z'}{L_\parallel}\right)^2 \frac{l_0^2}{f\varepsilon}\right]^{-1}$$

- Mixing length is inversely proportional to the shear: v_y' and v_z'. This allows for a closed feedback loop to form for the energy, thereby enhancing the mean energy.
- $k_m=1/l_0$ and $k_z=1/L_{//}$
- $\tau_c = l_0/\langle\delta v_x^2\rangle^{1/2} = l_0/(f l_{mix})^{1/2}$
- In CSDX, azimuthal shear is greater than the axial shear

Simplification by slaving

- For fast correlation time $\tau_c < \tau_{conf}$, and when parallel energy transfer is much smaller than perpendicular transfer, we simplify the model by dropping v_z equation and slaving ε to the density and the azimuthal shear:

$$\frac{\partial \bar{n}}{\partial t} = -\frac{\partial}{\partial x}\langle \tilde{v}_x \tilde{n} \rangle + D_c \frac{\partial^2 \bar{n}}{\partial x^2} + S_n$$

$$\frac{\partial \bar{v}_y}{\partial t} = -\frac{\partial}{\partial x}\langle \tilde{v}_x \tilde{v}_y \rangle + \nu_{c,\perp} \frac{\partial^2 \bar{v}_y}{\partial x^2} - \nu_{in}(\bar{v}_y - \bar{v}_n) - \nu_{ii}\bar{v}_y$$

Improved model from Hinton et al. (1993) where fluctuations are treated as an *ad hoc* constant.

$$\Gamma = -\frac{\varepsilon f^2}{\alpha}\frac{d \ln n}{dx} = -D\frac{d \ln n}{dx}$$

$$\Pi = -f\sqrt{\varepsilon} l_{mix} \frac{d^2 \bar{v}_y}{dx^2} + \frac{f l_{mix} \sqrt{\varepsilon} \omega_{ci}}{L_n}$$

$$\varepsilon = -\rho_s^2 \left(\frac{d\bar{v}_y}{dx}\right)^2 + \frac{l_0^2}{4}\left[\frac{f^2}{\alpha}\left(\frac{dn}{dx}\right)^2 + \sqrt{\Delta}\right]^2$$

Where $\Delta = f(n, \nabla v_y)$

Transition from adiabatic to hydrodynamic response

General Expression for the particle flux:

$$\Gamma = -\Big[\frac{(\alpha + |\gamma_m|)}{|\omega + i\alpha|^2}\frac{d\ln n}{dx} + \frac{\alpha\omega^r}{k_m\rho_s C_s|\omega + i\alpha|^2}\Big]\langle\delta v_x^2\rangle$$

General Expression for the vorticity flux:

$$\Pi = \sum_m -\frac{k_m^2\rho_s^2 C_s^2|\gamma_m|}{|\omega|^2}|\tilde{\phi}^2|\frac{d^2\bar{v}_y}{dx^2} + 2Re\Big[\frac{k_m\rho_s C_s\alpha}{\omega}\left(\frac{\omega^\star - \omega}{\omega + i\alpha}\right)|\tilde{\phi}|^2\Big]$$

Adiabatic limit

$$\alpha >> 1$$

$$\omega \cong \omega^*$$

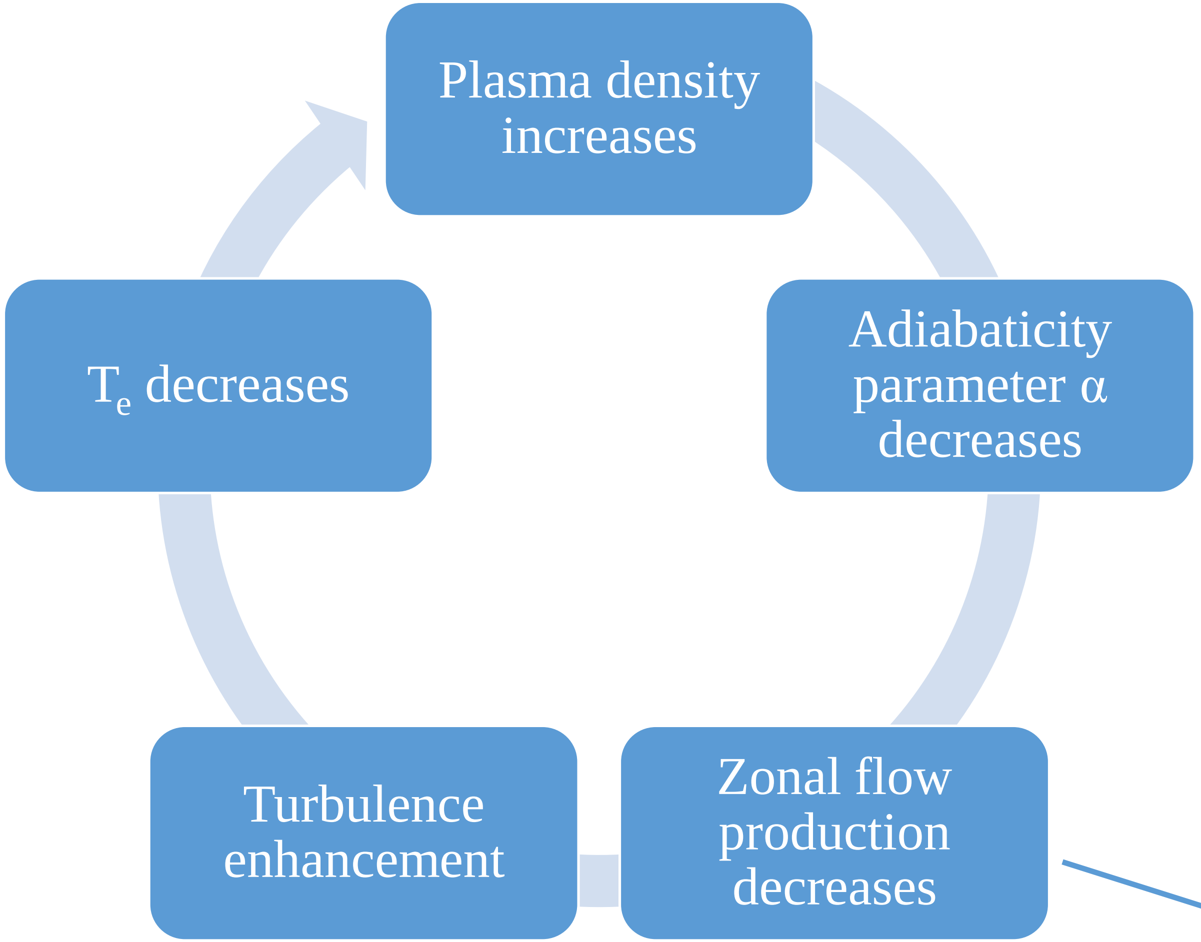
$$|\gamma| \cong 1/\alpha \approx 0$$

Fluxes are:

$$\Gamma \simeq -(\varepsilon/\alpha)\nabla n$$

$$\chi_y \simeq \varepsilon/\alpha$$

$$\Pi^{res} \simeq -(\omega_{ci}\varepsilon/\alpha)\nabla n$$



Hydrodynamic limit

$$\alpha << 1$$

$$\omega \cong |\gamma| \cong \sqrt{\omega^* \alpha}$$

Fluxes are:

$$\Gamma \simeq -\varepsilon\sqrt{|\nabla n|/\alpha}$$

$$\chi_y \simeq \varepsilon/\sqrt{\alpha|\nabla n|}$$

$$\Pi^{res} \simeq -\omega_{ci}\varepsilon\sqrt{\alpha/|\nabla n|}$$

$$\frac{d^2\bar{v}_y}{dx^2} = \frac{\Pi^{res}}{\chi_y}$$

Future Work

1. Numerical investigation of the 3-field model and the corresponding paralle to perpendicular coupling.

2. Numerical investigation of the 2-field model.

3. Numerical investigation of the transition from an adiabatic to a hydrodynamic plasma regime