UC San Diego

Competition of Mean Perpendicular and Parallel Flows in a Linear Device

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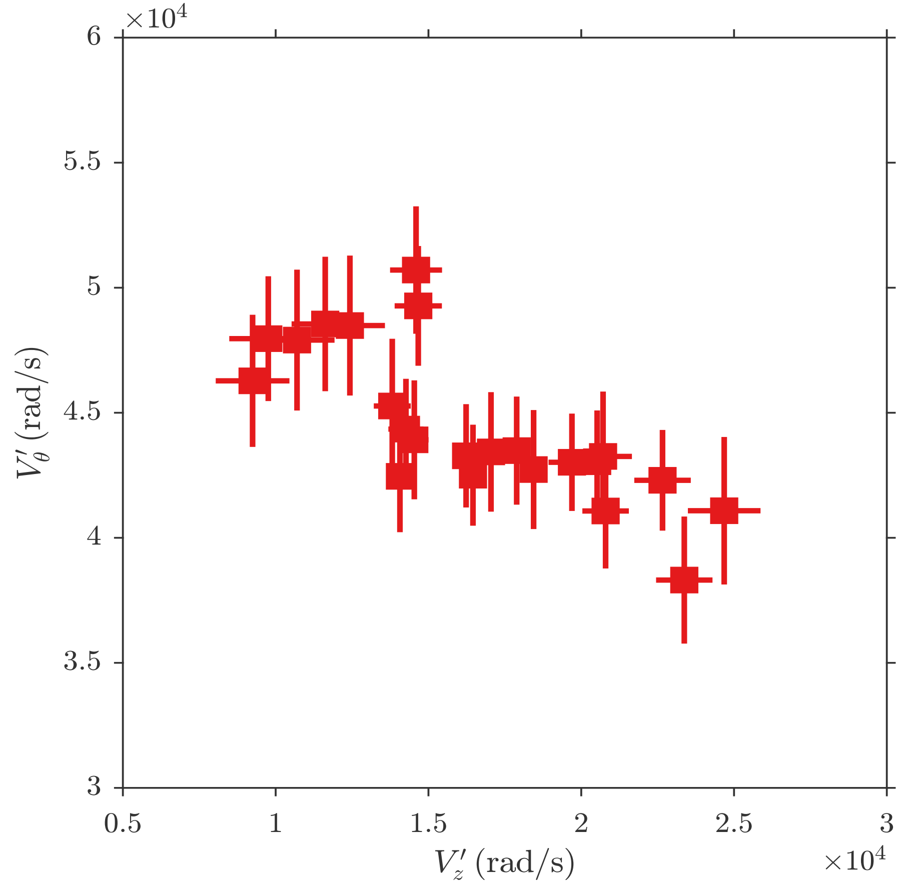
Background

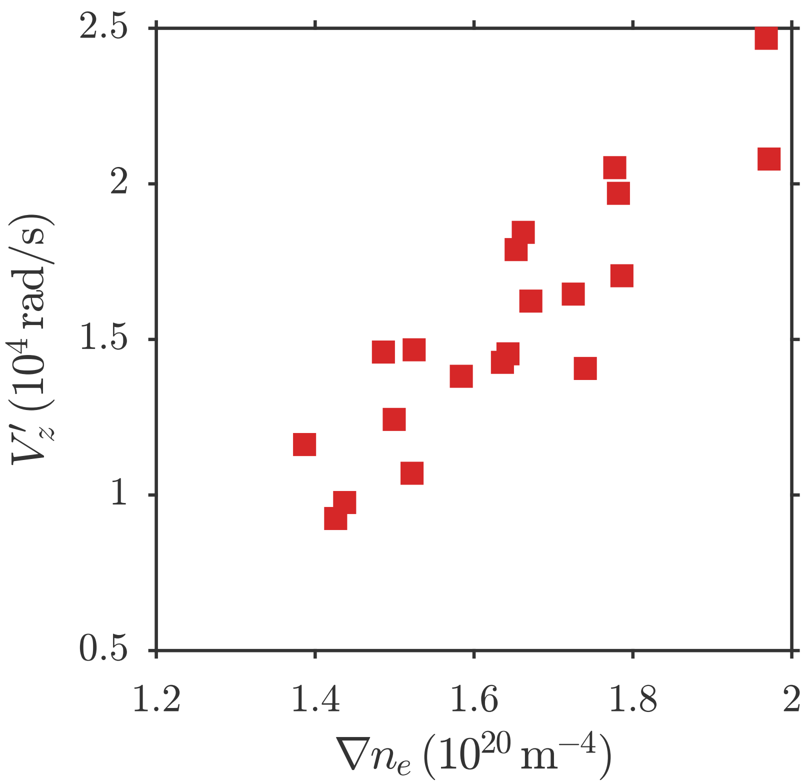
- ***Intrinsic*** axial flows observed in linear device (CSDX)
- Linear device studies suggest ***dynamical*** competition between mean perpendicular and parallel flows

  – ***Dynamical***: $V_\perp$ and $V_\parallel$ exchange energy with the background turbulence, and each other.
  → Energy balance between $V_\perp$ and $V_\parallel$
  → ***Tradeoff*** between $V_\perp$ and $V_\parallel$

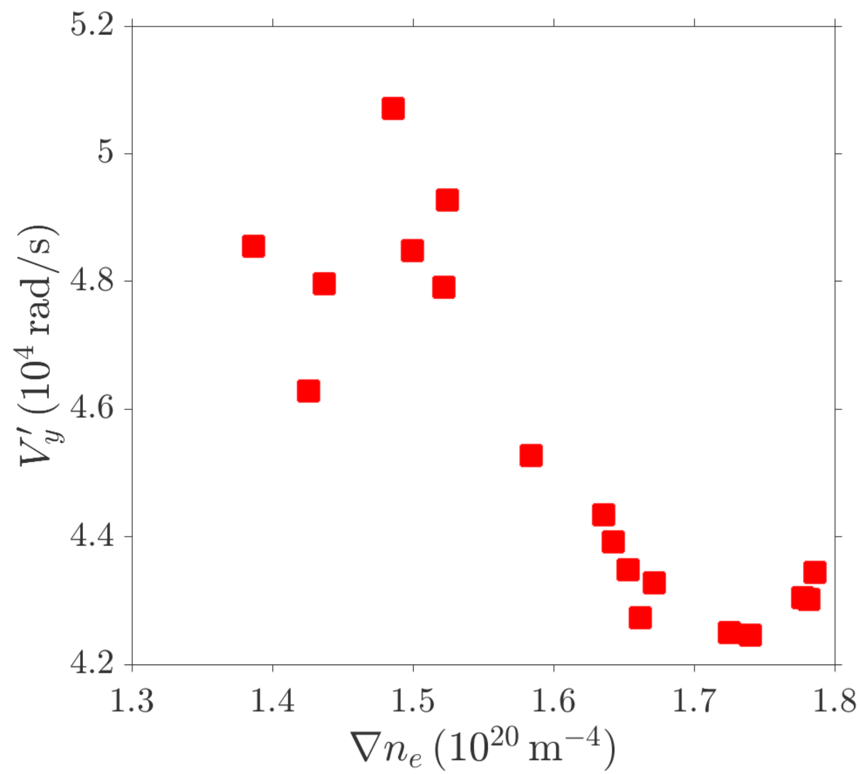


Experimental observations: $V'_\perp$ and $V'_\parallel$

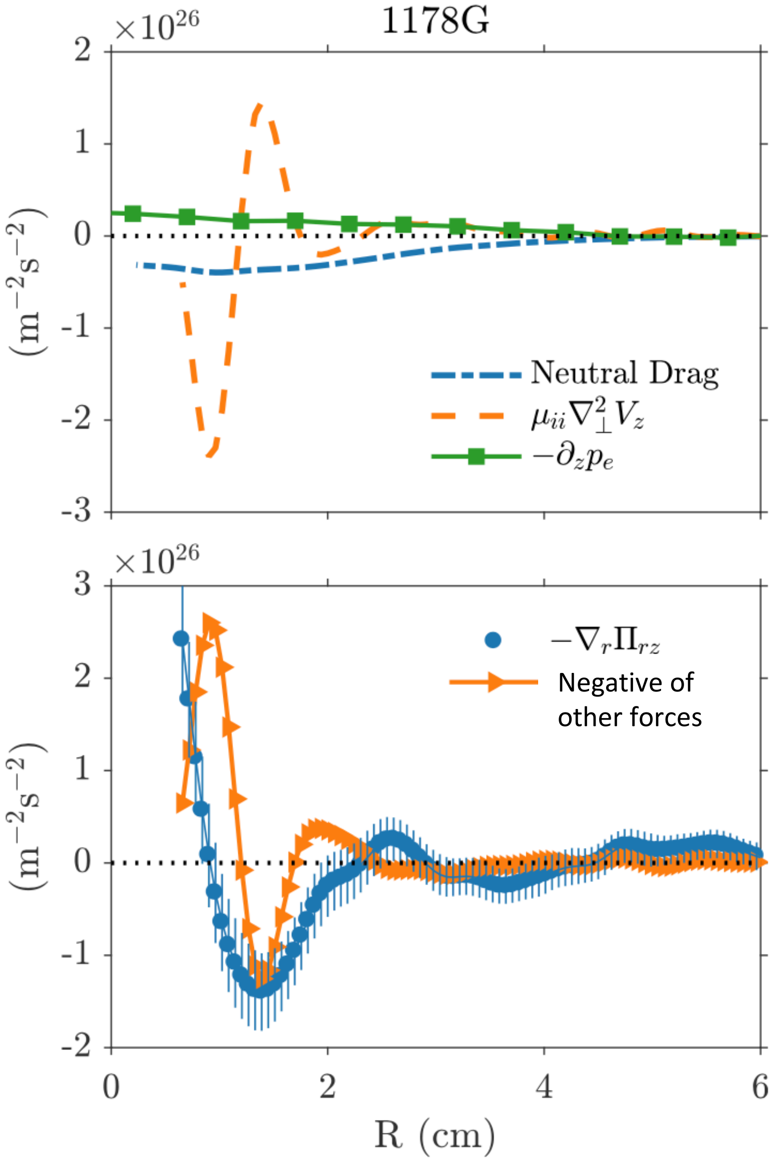
- $V'_\parallel$ scaling with ∇n_0
  - Analogy to ***Rice-type scaling***: $\Delta V_\parallel \propto \nabla T$ [Rice et al, PRL, 2011]



- $V'_\perp$ scaling with ∇n_0
  - Tradeoff between $V'_\perp$ and $V'_\parallel$
  - $V'_\perp$ saturation by $V'_\parallel$

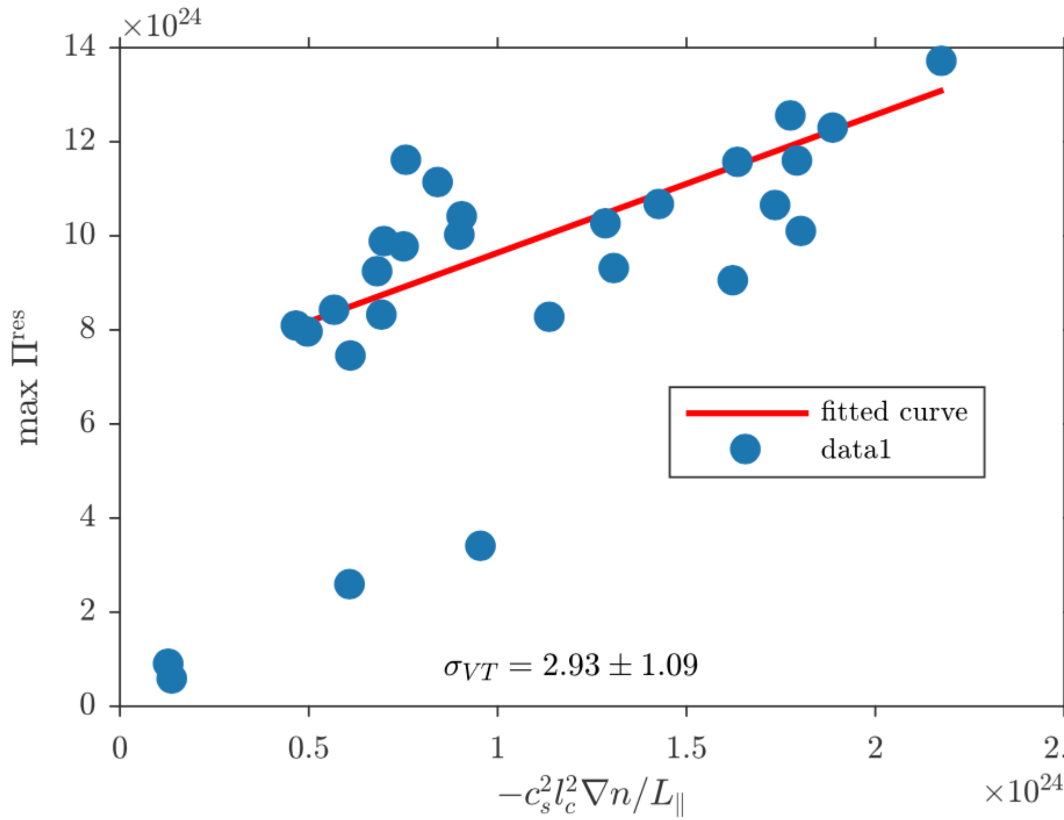


Measurements: Parallel Reynolds Stress $\langle \tilde{v}_r \tilde{v}_\parallel \rangle$



|Reynolds force| ≫ |axial pressure gradient|
→ **$V_\parallel$ driven by turbulence** → $V_\parallel' \sim \nabla n_0$

$\Pi_\parallel^{Res}$ scaling with ∇n_0



Outline of the Rest

- Introduction
  - Key questions and why
  - Current status of model
- Exploration of coupling
  - Study turbulent energy branching between $V_\parallel$ and $V_\perp$
  - Reynolds power ratio $P_\parallel^R/P_\perp^R$ decreases as $V_\perp$ increases → tradeoff between $V_\perp$ and $V_\parallel$
  - $P_\parallel^R/P_\perp^R$ maximum occurs when $\left|\nabla V_\parallel\right|$ is below the PSFI (parallel shear flow instability) threshold → saturation of intrinsic $V_\parallel$
- Are shear suppression "rules" correct?
  - Revisiting the resonance effect
  - Wave-flow resonance suppresses instability
  - $V_\perp'$ weakens resonance → enhances instability
  - Implication for zonal flow dynamics

Key Questions and Why

- What's the coupling between ***mean*** perpendicular and parallel flows ($V_\perp$ and $V_\parallel$)?
  - How do they interact? → How do they compete for energy from the background turbulence?
  - How does $V_\parallel$ affect the production and saturation of intrinsic $V_\perp$?
- Why should we care?
  - Relevant to L-H transition
    - Both $V_\perp'$ and $V_\parallel$ increase, during transition.
    - The coupling of the two is relevant to transition threshold and dynamics.
  - Linear device (CSDX) studies suggest competition between $V_\perp$ and $V_\parallel$

Why linear device?

- Relevance: zero magnetic shear ← Enhanced-confinement states (H-mode) favor low magnetic shear.
- Self-generated, sheared $V_\perp$ (zonal flow) observed, which regulates the drift wave turbulence.
- ***Intrinsic*** $V_\parallel$ observed: driven by drift wave turbulence (∇n_0) via turbulent Reynolds work, i.e. $-\partial_r \langle \tilde{v}_r \tilde{v}_\parallel \rangle V_\parallel$.
  → New in linear device (zero magnetic shear). New mechanism for $V_\parallel$ generation proposed. [Li et al, PoP 2016 & 2017]
- Advantage of CSDX: ***unique measurements of parallel Reynolds stress*** $\langle \tilde{v}_r \tilde{v}_\parallel \rangle$ ***and Reynolds power*** $(-\partial_r \langle \tilde{v}_r \tilde{v}_\parallel \rangle V_\parallel)$
  → Not achieved in tokamak cores or other linear devices.

Current status of model

- Conventional wisdom of $V_\perp \to V_\parallel$ coupling:
  - $V_\perp'$ breaks the symmetry in $k_\parallel$, but requires finite magnetic shear
  - ***Not applicable*** in linear device (straight magnetic field)
- $V_\parallel \to V_\perp$ coupling:
  - 3D coupled drift-ion acoustic wave system [Wang et al, PPCF 2012]
  - Coupling between fluctuating PV and parallel compression $\langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle$ ***breaks PV conservation***
    → Sink/source for fluctuating potential enstrophy density
    → Zonal flow generation
  - Perpendicular flow dynamics:

PV diffusion

$$\frac{\partial}{\partial t}\left[V_\perp - L_n \left\langle \frac{\tilde{q}^2}{2} \right\rangle\right] \sim -\nu_i V_\perp + L_n \left[\frac{\partial}{\partial r}\left\langle \tilde{v}_x \frac{\tilde{q}^2}{2} \right\rangle + \mu \langle (\nabla \tilde{q})^2 \rangle - \langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle \right]$$

collisional damping

$$\langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle \sim -\sum_k \frac{|\Delta\omega_k|}{\omega_k^2} k_\parallel^2 |\phi_k|^2 < 0$$

Section II: Exploration of $V_\perp$-$V_\parallel$ Coupling

- Goal: study ***how extrinsic flows affect Reynolds powers***

  → generation of intrinsic flows

  → **turbulent energy branching**

  between intrinsic $V_\perp$ and $V_\parallel$

- Analogy to biasing experiments

- Hasegawa-Wakatani drift wave

  → near adiabatic electron:

  $$\tilde{n} = (1 - i\delta)\phi, \delta \ll 1$$

  $$\frac{D}{Dt}\tilde{n} + \tilde{v}_r \frac{\nabla n_0}{n_0} + \nabla_\parallel \tilde{v}_\parallel = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi}),$$

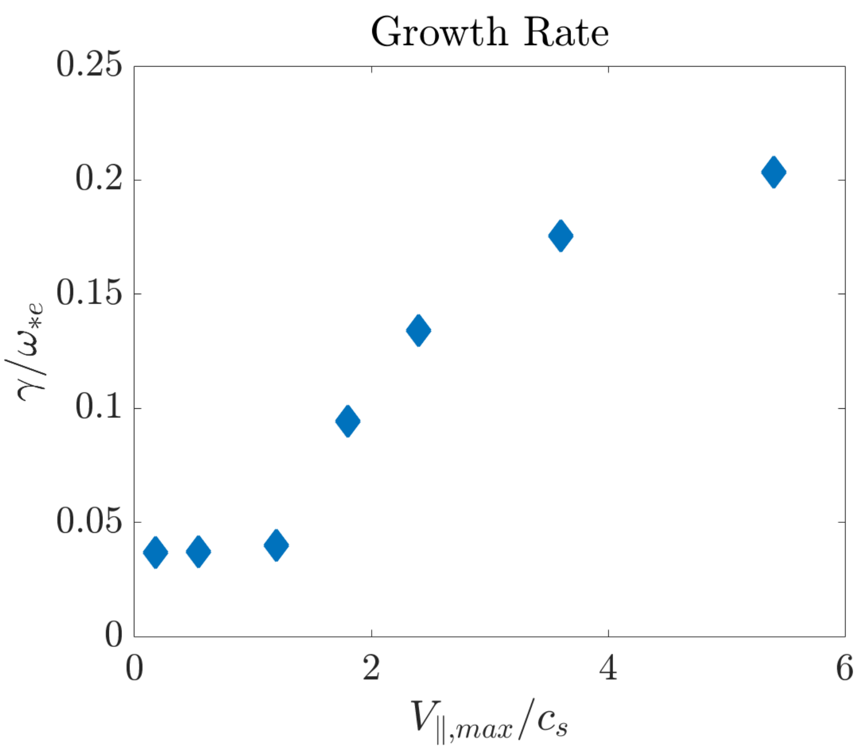
  $$\frac{D}{Dt}\nabla_\perp^2 \tilde{\phi} + \tilde{v}_r V_\perp'' = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi}),$$

  $$\frac{D}{Dt}\tilde{v}_\parallel + \tilde{v}_r V_\parallel' = \nabla_\parallel \tilde{n},$$

- Prescribed flows vary in x direction:
  $V_\perp = V_\perp^{max} \sin[q_x(x - L_x/2)]$; $V_\parallel = -V_\parallel^{max} \sin[q_x(x - L_x/2)]$
- Fourier decomposition in y, z directions:
  $\tilde{f} = \sum_k f_k(x)\, e^{i(k_y y + k_\parallel z)} e^{-i(\omega_k + i\gamma_k)t}$, where $\tilde{f} = \tilde{n}, \tilde{v}_\parallel, \tilde{\phi}$
- Solve for growth rate, frequency, and eigenmode function $\phi_k(x)$ for **drift wave instability** (∇n_0 driven) with prescribed $V_\perp$ and $V_\parallel$

Bottom Line: ∇n_0 is the Primary Instability Drive

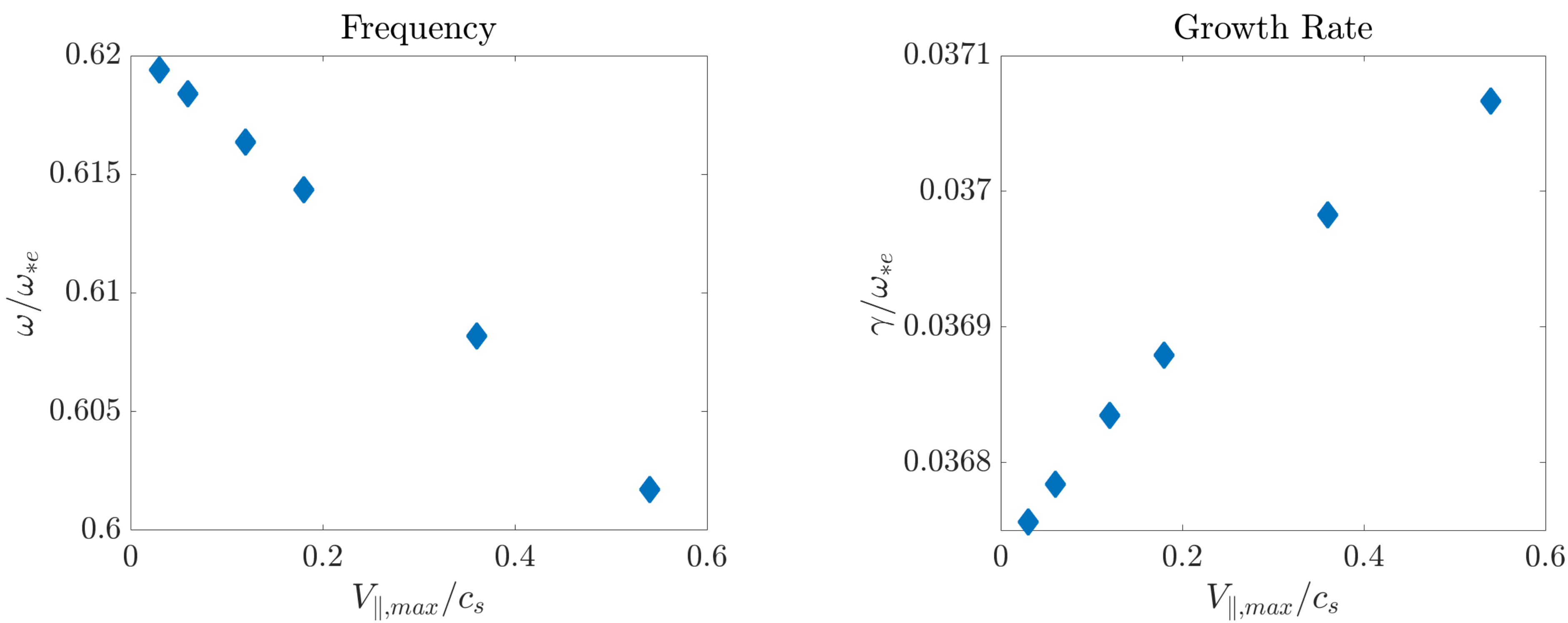
- Other potential drives:
  - $V_\perp''$ → Kelvin-Helmholtz instability
  - $\nabla V_\parallel$ → Parallel shear flow instability



- KH is not important
  - $V_\perp''$ drive weaker than ∇n_0 drive, i.e. $|k_y \rho_s^2 V_\perp''| \ll \omega_{*e}$
  - $V_\perp$ affects the drift wave instability via wave-flow resonance $\omega_k - k_y V_\perp$ (see Section III)
- ***PSFI stable*** in CSDX

$\nabla V_\parallel$ has little effect on drift wave instability

- Influence drift wave instability via frequency shift
- $\gamma_k \sim \omega_{*e} - \omega_k \sim \frac{k_\perp^2 \rho_s^2}{1+k_\perp^2 \rho_s^2}\,\omega_{*e} + \frac{k_\theta k_\parallel \rho_s c_s V_\parallel'}{\omega_{*e}}$



Definition: Reynolds Power

- Mean flow evolution is powered by Reynolds power

$$\frac{1}{2}\frac{\partial\left|V_{\parallel}\right|^{2}}{\partial t} \sim -\frac{\partial}{\partial x}\langle\tilde{v}_x\tilde{v}_{\parallel}\rangle V_{\parallel}$$

  - Parallel Reynolds power of a single eigenmode

$$P_{\parallel}^{R} = \int_0^{L_x} dx\left[-\frac{\partial}{\partial x}\left(\tilde{v}_{x,k}^{*}\tilde{v}_{\parallel,k}\right)\right]V_{\parallel}$$

  - Perpendicular Reynolds power of a single eigenmode

$$P_{\perp}^{R} = \int_0^{L_x} dx\left[-\frac{\partial}{\partial x}\left(\tilde{v}_{x,k}^{*}\tilde{v}_{y,k}\right)\right]V_{\perp}$$

- Effects of extrinsic $V_{\parallel}$ and $V_{\perp}$ on the ratio $P_{\parallel}^{R}/P_{\perp}^{R}$ are studied

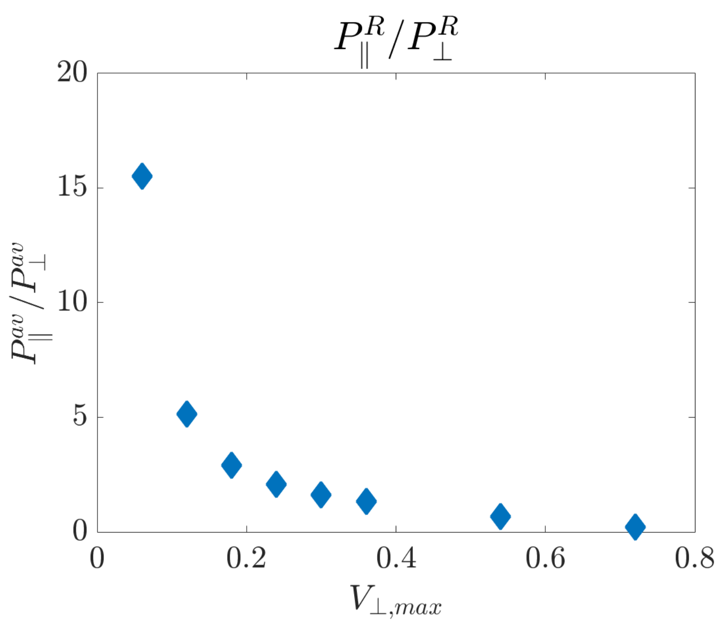
Coupling of $V_\perp$ and $V_\parallel$ ↔ Ratio of Reynolds Powers

- Ratio $P^R_\parallel / P^R_\perp$ decreases with $V_\perp$

  → Energy branching of $V_\parallel$ reduced

  → $V_\perp$ reduces generation of $V_\parallel$

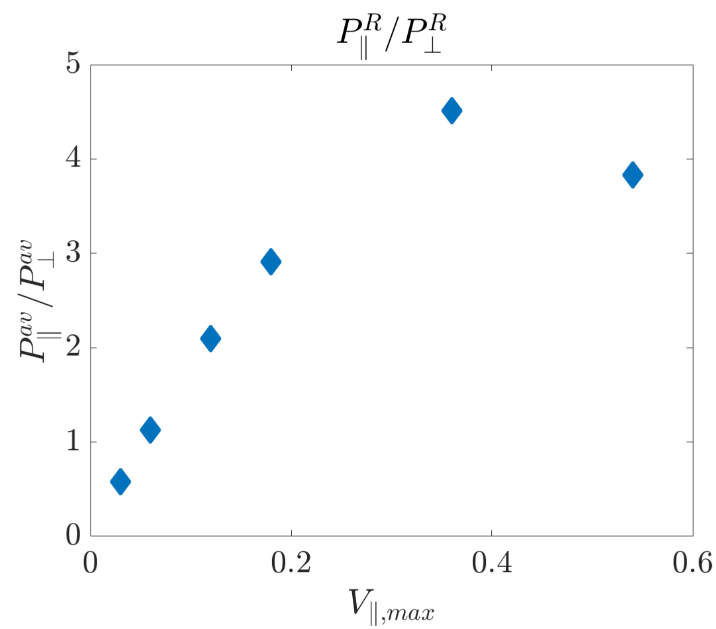
  → ***Competition*** between $V_\perp$ and $V_\parallel$



- Increase $V_\parallel$ → $P^R_\parallel / P^R_\perp$ turnover ***before*** $\nabla V_\parallel$ hits PSFI threshold

  → Max energy branching of $V_\parallel$ below PSFI threshold

  → $V_\parallel$ saturates ***below*** PSFI threshold



Reduced model developed to study the coupling → See poster **43** on Thursday afternoon

Section III: Revisiting Shearing Effects

- ***Are conventional shear suppression "rules" correct?***
- Aim to test well known (mis)conceptions about shearing effects on stability
- Conventional wisdoms:
  - $E \times B$ flow shear suppresses instability ← Is it correct?
  - Wave-flow resonance effect is often overlooked, though was mentioned in past works.
- Findings:
  - Explore linear instability, using ***fixed extrinsic flows***
  - Wave-flow resonance stabilizes drift wave instability
  - Perpendicular flow shear weakens the resonance, and thus ***destabilizes*** the instability
- Implications for zonal flow generation and saturation:
  - Revisit predator-prey model with resonance effects
    → Mechanism for ***collisionless*** zonal flow damping (without involving tertiary instability, such as KH)

Wave-flow resonance

- Resonance: $\omega_k - k_y V_\perp - k_\parallel V_\parallel$
  $|k_\parallel|/k_y \ll 1$ → Resonance dominated by $\omega_k - k_y V_\perp$
- Hasegawa-Wakatani drift wave model, with extrinsic $V_\perp$

$$\frac{D}{Dt}\tilde{n} + \tilde{v}_r \frac{\nabla n_0}{n_0} = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi}),$$

$$\frac{D}{Dt}\nabla_\perp^2 \tilde{\phi} + \tilde{v}_r V_\perp'' = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi})$$

- **KH drive negligible** → ∇n_0 driven instability
  - Near adiabatic electron: $\tilde{n} = (1 - i\delta)\phi$, $\delta \ll 1$
  - $\delta = (\omega_{*e} - \omega_k + k_y V_\perp)/k_\parallel^2 D_\parallel^2 = \nu_{ei}(\omega_{*e} - \omega_k + k_y V_\perp)/k_\parallel^2 v_{The}^2$
- In the limit of strong resonance, i.e. $\gamma_k \ll \omega_k - k_y V_\perp \ll \omega_{*e}$,
  $\delta \to \nu_{ei}\omega_{*e}/k_\parallel^2 v_{The}^2$
- Resonance affects the eigenmode scale → Influence instability

Resonance and Instability Related to Mode Scale

- Eigenmode equation with resonant effect:

$$(\omega_k - k_y V_\perp + i\gamma_k)\rho_s^2 \partial_x^2 \phi = \left[(1 + k_y^2 \rho_s^2 - i\delta)(\omega_k - k_y V_\perp + i\gamma_k) - \omega_{*e}\right]\phi$$

- Mode scale defined as $L_m^{-2}\rho_s^2 \equiv \rho_s^2 \int_0^{L_x} dx|\partial_x \phi|^2 \ / \int_0^{L_x} dx|\phi|^2$
- Results:

$$\omega_k - k_y V_\perp = \frac{\omega_{*e}(1 + k_y^2\rho_s^2 + L_m^{-2}\rho_s^2)}{(1 + k_y^2\rho_s^2 + L_m^{-2}\rho_s^2)^2 + \delta^2},$$

Effectively, $k_\perp^2 \rho_s^2$

$$\gamma_k = \frac{\delta(\omega_k - k_y V_\perp)}{1 + k_y^2\rho_s^2 + L_m^{-2}\rho_s^2} = \frac{\delta\omega_{*e}}{(1 + k_y^2\rho_s^2 + L_m^{-2}\rho_s^2)^2 + \delta^2}.$$

- In the limit of strong resonance

$$\gamma_k \ll \omega_k - k_y V_\perp \ll \omega_{*e}$$

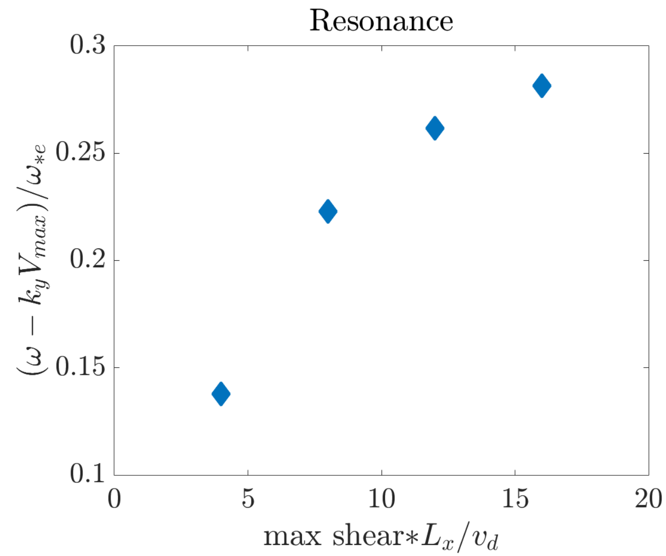
$$\omega_k - k_y V_\perp \sim \omega_{*e} L_m^2/\rho_s^2$$

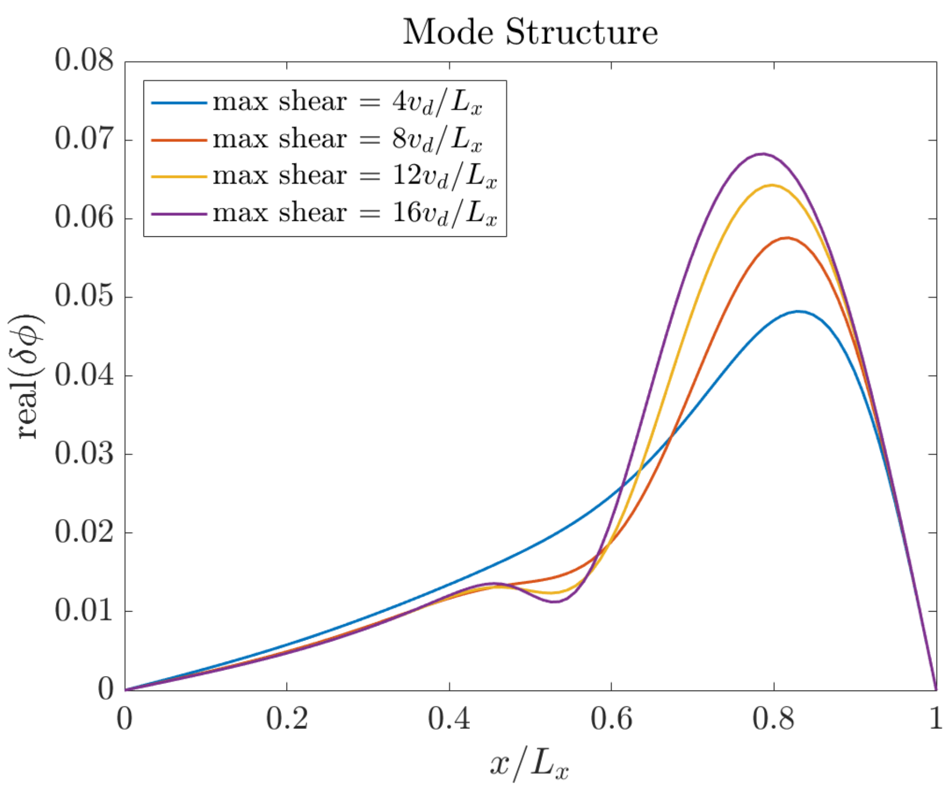
$$\gamma_k \sim \delta(\omega_k - k_y V_\perp)L_m^2 \sim \delta\omega_{*e} L_m^4/\rho_s^4$$

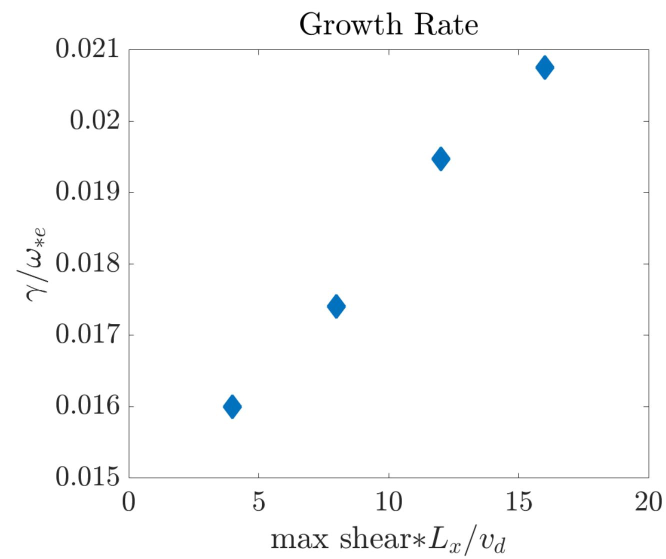
- Eigenmode peaks ($L_m^{-2}\rho_s^2$ increases) as resonance becomes stronger
- Resonance suppresses drift wave instability

Perpendicular flow shear ***destabilizes*** turbulence

- Mean perpendicular flow shear increases mode scale L_m/ρ_s
  → Weakens resonance
  → Enhances instability
- KH drive **negligible** compared to ∇n_0







Implications for Zonal Flow Dynamics

- Connection to collisionless damping of ZF
- Zonal flow evolution → Mean enstrophy equation:

$$\frac{\partial}{\partial t}\int dr\frac{\langle\rho\rangle^2}{2} = \int dr\langle\tilde{v}_r\tilde{\rho}\rangle\frac{d\langle\rho\rangle}{dr} - \nu_i\int dr\langle\rho\rangle^2 + \cdots$$

- Vorticity ($\rho \equiv \nabla_\perp^2\phi$) flux: $\langle\tilde{v}_r\tilde{\rho}\rangle = -D_\rho\frac{d\langle\rho\rangle}{dr} + \Gamma_\rho^{Res}$ → Conserves enstrophy between mean flow and fluctuations

$$\frac{\partial}{\partial t}\int dr\frac{\langle\rho\rangle^2}{2} = -\int drD_\rho\left(\frac{d\langle\rho\rangle}{dr}\right)^2 + \int dr\Gamma_\rho^{Res}\frac{d\langle\rho\rangle}{dr} - \nu_i\int dr\langle\rho\rangle^2 + \cdots$$

- $\nu_i \to 0$ → Dimits shift regime → Resonance gives collisionless damping
  - Collisionless damping by turbulent viscosity: $d\langle\rho\rangle/dr \sim \Gamma_\rho^{Res}/D_\rho$
  - Resonance sets D_ρ → ZF damping

$$\Gamma_\rho^{Res} = k_y c_s^2|\phi_k|^2\left[\frac{\gamma_k\omega_{*e} + \alpha_n(\omega_{*e} - \omega_k + k_yV_\perp)}{\left|\omega_k - k_yV_\perp + i\alpha_n\right|^2} - \frac{|\gamma_k|\omega_{*e}}{\left|\omega_k - k_yV_\perp\right|^2}\right],\quad D_\rho = k_y^2c_s^2|\phi_k|^2\frac{|\gamma_k|}{\left|\omega_k - k_yV_\perp\right|^2}$$

Collisionless ZF damping by vorticity flux resonance

- Resonance replaces need for KH:

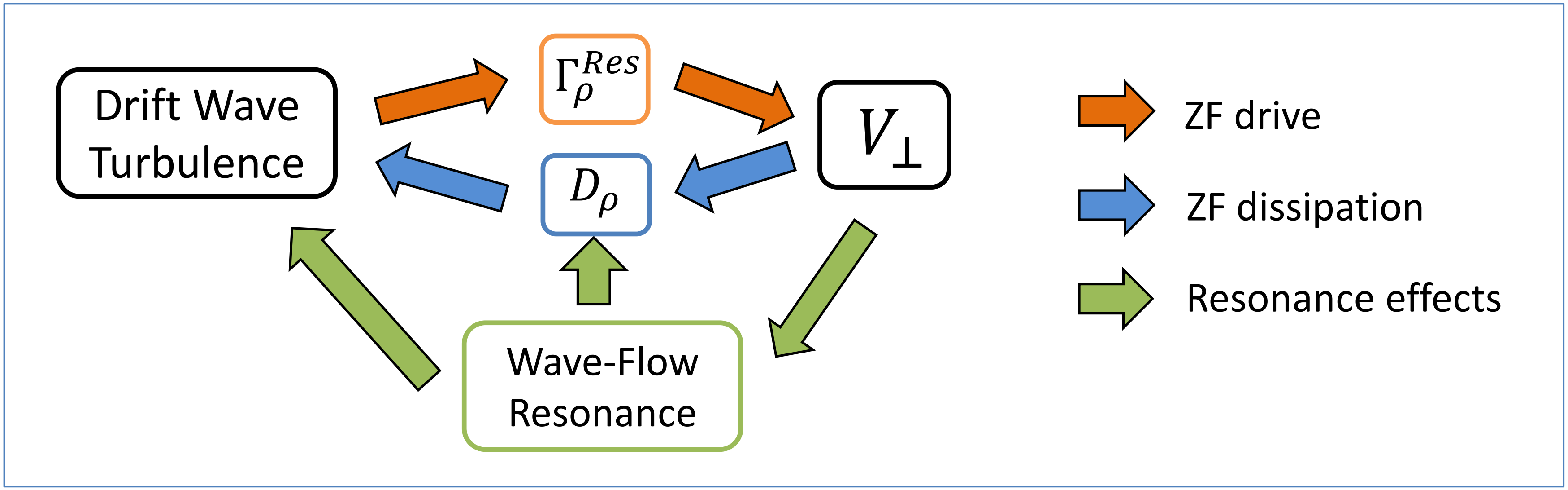
$$\gamma_k = \text{linear instability } (\gamma_L) + \text{resonance absorption } \gamma_R \sim \gamma_R(\omega_k - k_y V_\perp)$$

⇩

Analogy to ion-acoustic absorption during collapse of Langmuir waves

- Resonance induces collisionless damping through D_ρ

- Revisit predator-prey model with resonance effect
  → Mechanism for collisionless damping, without KH



Summary

- Experimental observations suggest **competition** between mean $V_\perp$ and $V_\parallel$
- Reynolds power ratio $P_\parallel^R / P_\perp^R$ changes with prescribed extrinsic mean flows
  - $P_\parallel^R / P_\perp^R$ decreases with $V_\perp$ → tradeoff between $V_\perp$ and $V_\parallel$
  - $P_\parallel^R / P_\perp^R$ maximum occurs ***before*** $\nabla V_\parallel$ hits PSFI threshold
- Testing misconceptions of shearing effects on stability
  - Wave-flow resonance suppresses instability
  - $V_\perp'$ weakens resonance → $V_\perp'$ ***enhances*** instability →
  - Resonance produces turbulent viscosity
→ **collisionless** damping of ZF, without involving KH
  - Suggest revisit predator-prey model with resonance effects
→ mechanism for collisionless ZF damping, without tertiary instability