Studies of Turbulence-driven FLOWs:

a) $V_\perp$, $V_\parallel$ Competition in a Tube

b) Revisiting Zonal Flow Saturation

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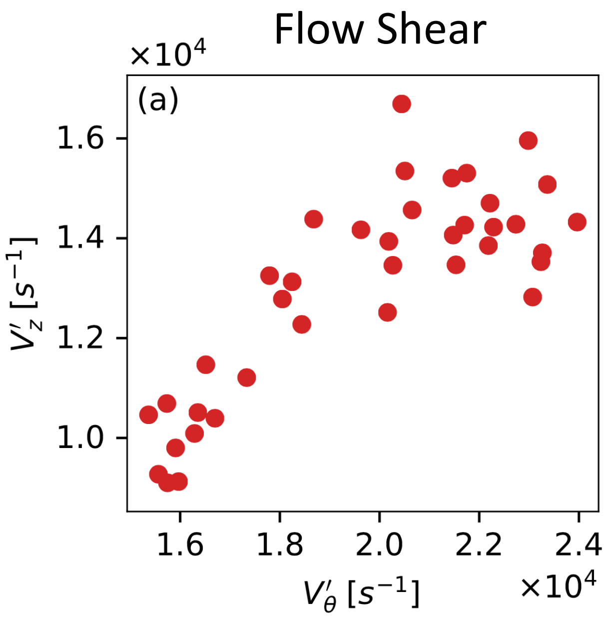
Outline

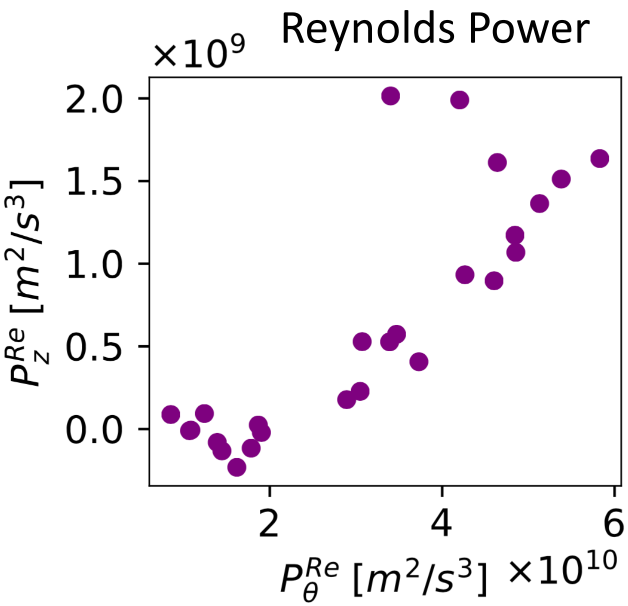
- Background: turbulence driven $V_\perp$ and $V_\parallel$ observed in CSDX
- Questions: How do they interact? How do they saturate?
- Increment study of $V_\perp, V_\parallel$ competition
  → Analogous to perturbation experiments
- Zonal flow saturation by wave-flow resonance
  - Wave-flow resonance effects on linear stability
    → ***Flow shear enhances instability via resonance***
  - Collisionless ZF saturation by resonance
    → ***Derives*** mesoscopic ZF scale, i.e. $L_{ZF} \sim \sqrt{\rho_s L_n}$
    → Extended predator-prey model, compared to old model

Background

- ***Intrinsic*** axial and azimuthal flows observed in linear device (CSDX)
  - Increase B → scans mean flows-both $V_\perp$ and $V_\parallel$
- ***Dynamical*** competition between perpendicular and parallel flows
- $V_\perp$ and $V_\parallel$ exchange energy with the turbulence, and each other.
  → Study energy apportionment between $V_\perp$ and $V_\parallel$
  → Tradeoff between $V_\perp$ and $V_\parallel$



Flow Shear



Reynolds Power

Key Questions

- What's the coupling between ***mean*** perpendicular and parallel flows ($V_\perp$ and $V_\parallel$)?
  - How do they compete for energy from turbulence?
- How/Why do flows saturate, especially in ***collisionless*** regime?
- Why?
  - Linear device (CSDX) studies suggest apportionment of turbulence energy between $V_\perp$ and $V_\parallel$
  - Relevant to L-H transition
    - Both $V_\perp'$ and $V_\parallel'$ increase, during transition.
    - The coupling of the two is relevant to transition threshold and dynamics.

Current status of coupling model

- Conventional wisdom of $V_\perp \rightarrow V_\parallel$ coupling:
  - $V_\perp'$ breaks the symmetry in $k_\parallel$, but requires finite magnetic shear
  - ***Not applicable*** in linear device (straight magnetic field)
- $V_\parallel \rightarrow V_\perp$ coupling via parallel compression:
  - 3D coupled drift-ion acoustic wave system [Wang et al, PPCF 2012]
  - Coupling between fluctuating PV and parallel compression $\langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle$ ***breaks PV conservation***

    → Sink/source for fluctuating potential enstrophy density

    → Zonal flow generation

$V_\perp$ and $V_\parallel$ competition

- Increment study of $V_\perp$ and $V_\parallel$ effects on Reynolds powers
  - ***Turbulent energy branching*** between $V_\parallel$ and $V_\perp$
  - Reynolds power ratio $P_\parallel^R/P_\perp^R$ decreases as $V_\perp$ increases
    → tradeoff between $V_\perp$ and $V_\parallel$
  - $P_\parallel^R/P_\perp^R$ maximum occurs when $\left|\nabla V_\parallel\right|$ is below the PSFI (parallel shear flow instability) threshold
    → saturation of intrinsic $V_\parallel$

Exploration of $V_\perp$-$V_\parallel$ Coupling

- Goal: ***How do extrinsic flows affect powers?***

  → **Turbulent energy branching** between intrinsic $V_\perp$ and $V_\parallel$

  → How does $V_\perp$ affect intrinsic $V_\parallel$ generation?

- Analogous to perturbation experiments, i.e. fix one flow and increase the other through external momentum source

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- Collisional drift wave

  → near adiabatic electron:

  $\tilde{n} = (1 - i\delta)\phi, \delta \ll 1$

- Slab geometry

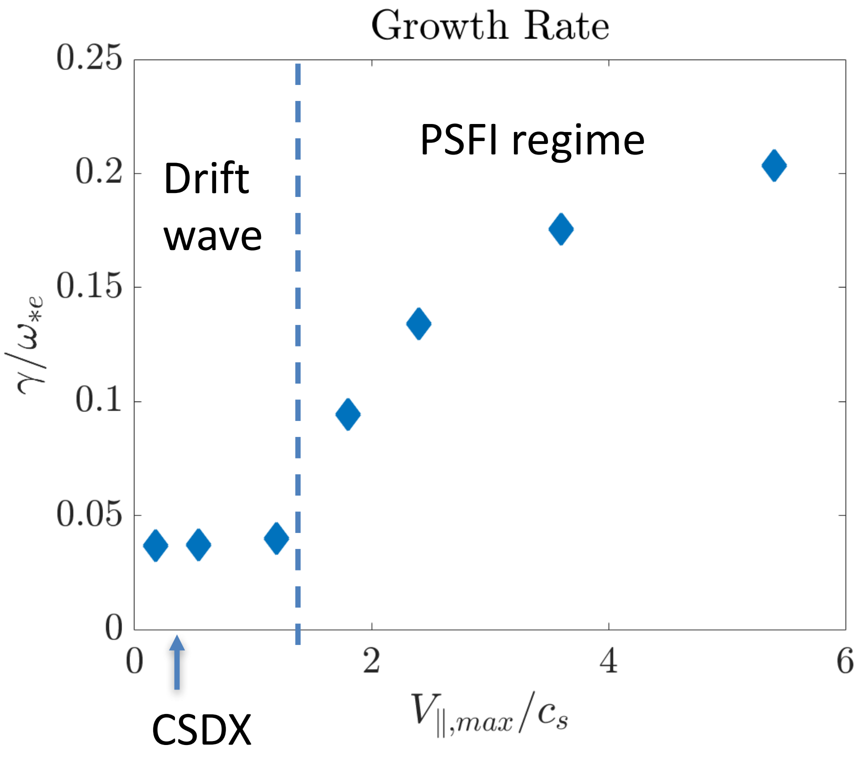
$$\frac{D}{Dt}\tilde{n} + \tilde{v}_r\frac{\nabla n_0}{n_0} + \nabla_\parallel \tilde{v}_\parallel = D_\parallel \nabla_\parallel^2(\tilde{n} - \tilde{\phi}),$$

$$\frac{D}{Dt}\nabla_\perp^2\tilde{\phi} + \tilde{v}_r V_\perp'' = D_\parallel \nabla_\parallel^2(\tilde{n} - \tilde{\phi}),$$

$$\frac{D}{Dt}\tilde{v}_\parallel + \tilde{v}_r V_\parallel' = \nabla_\parallel \tilde{n},$$

∇n_0 is the Primary Instability Drive

- Other potential drives:
  - $V_\perp''$ → Kelvin-Helmholtz instability
  - $\nabla V_\parallel$ → Parallel shear flow instability



- KH is not important
  - $V_\perp''$ drive weaker than ∇n_0 drive, i.e. $|k_y \rho_s^2 V_\perp''| \ll \omega_{*e}$

- $\nabla V_\parallel$ in CSDX is well below the PSFI linear threshold

→ ***PSFI stable*** in CSDX

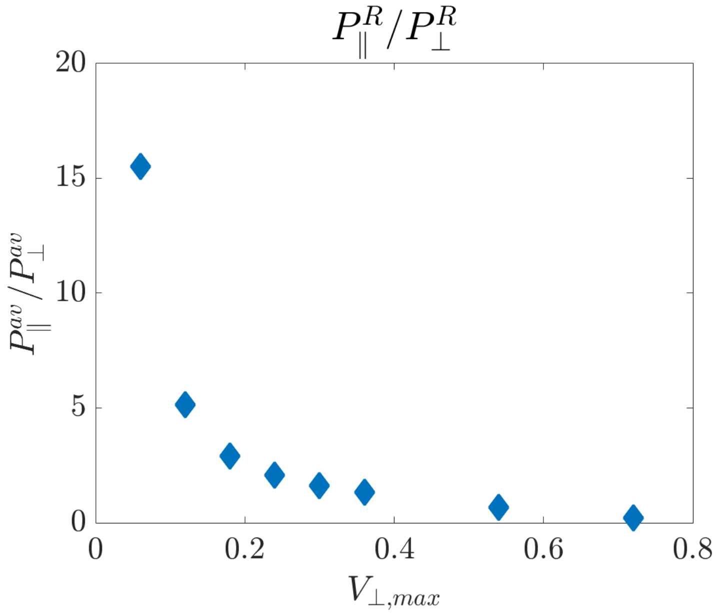
Coupling of $V_\perp$ and $V_\parallel$ ↔ Ratio of Reynolds Powers

- Ratio $P_\parallel^R/P_\perp^R$ decreases with $V_\perp$

→ Energy branching of $V_\parallel$ reduced

→ $V_\perp$ reduces generation of $V_\parallel$

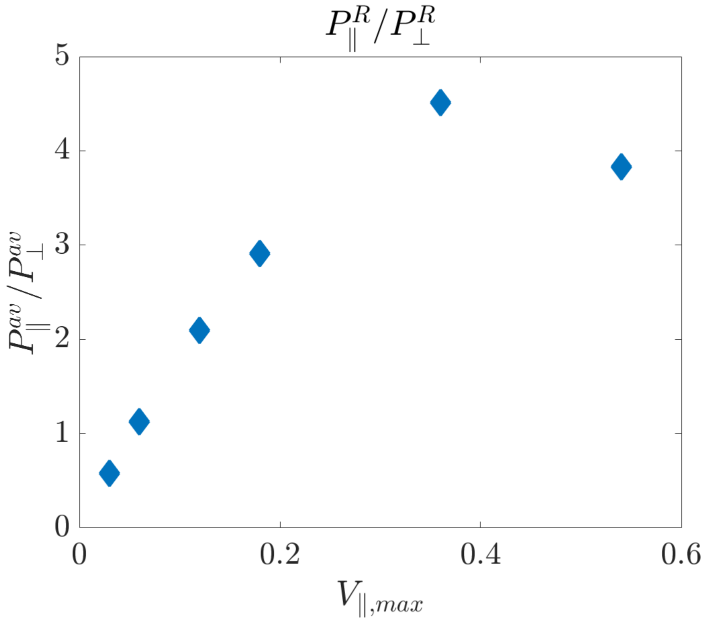
→ Suggest ***competition*** between $V_\perp$ and $V_\parallel$



- Increase $V_\parallel$ → $P_\parallel^R/P_\perp^R$ turnover ***before*** $\nabla V_\parallel$ hits PSFI threshold

→ Max energy branching of $V_\parallel$ below PSFI threshold

→ Suggest $V_\parallel$ saturates ***below*** PSFI threshold



Partial Summary 1

- CSDX experiments suggest **energy apportionment** between mean $V_\perp$ and $V_\parallel$
- Increment study on Reynolds power ratio $P_\parallel^R / P_\perp^R$
  - Analogous to perturbation study
  - $P_\parallel^R / P_\perp^R$ decreases with $V_\perp$ → tradeoff between $V_\perp$ and $V_\parallel$
  - $P_\parallel^R / P_\perp^R$ maximum occurs ***before*** $\nabla V_\parallel$ hits PSFI threshold

Collisionless zonal flow saturation

- Wave-flow resonance prominent in linear device (CSDX)
  - Enters turbulence regulation, both linearly and nonlinearly
  - Flow shear is not the exclusive control parameter
- Resonance suppresses linear instability by wave absorption
  - Are shear suppression “rules” correct?
  - $V_\perp'$ weakens resonance → ***flow shear enhances instability via resonance***

Collisionless zonal flow saturation (cont'd)

- Collisionless Zonal flow saturation by resonant PV mixing
  - Model of resonant PV mixing
  - Resonant diffusion of vorticity saturates zonal flow in collisionless regime
  - Incorporated in an extended predator-prey model
  - Drift wave mixes PV at zonal flow shear below that for KH/tertiary excitation

Wave-flow resonance effect on linear stability

- Resonance: $\omega_k - k_y V_\perp - k_\parallel V_\parallel$
  $|k_\parallel|/k_y \ll 1$ → Resonance set by $\omega_k - k_y V_\perp$

- Hasegawa-Wakatani drift wave model, with extrinsic $V_\perp$

$$\left(\frac{d}{dt}+\tilde{\mathbf{v}}_E\cdot\nabla\right)\tilde{n}+\tilde{v}_x\frac{\nabla n_0}{n_0}=D_\parallel\nabla_\parallel^2(\tilde{n}-\tilde{\phi})+D_c\nabla^2\tilde{n},$$

$$\left(\frac{d}{dt}+\tilde{\mathbf{v}}_E\cdot\nabla\right)\tilde{\rho}+\tilde{v}_x\langle\rho\rangle'=D_\parallel\nabla_\parallel^2(\tilde{n}-\tilde{\phi})+\chi_c\nabla^2\tilde{\rho},$$

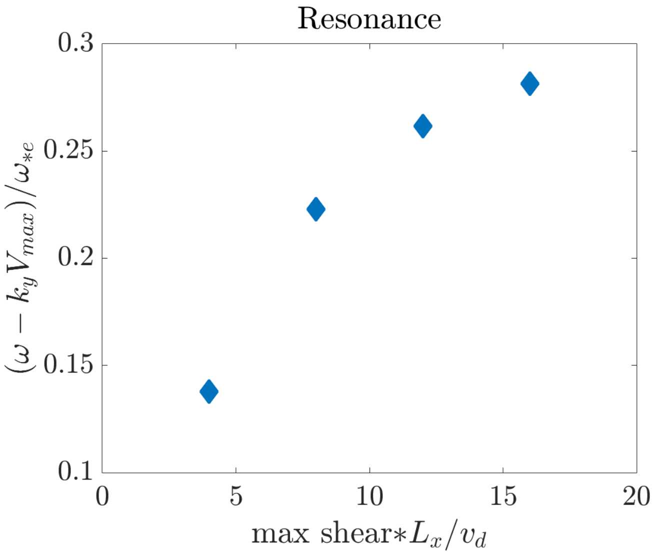
- **KH drive negligible**, i.e. $\left|k_y \rho_s^2 \langle v_y \rangle''\right| \ll \omega_{*e}$ → Drift wave instability dominant
  - Near adiabatic electron: $\tilde{n} = (1 - i\delta)\phi$, $\delta \ll 1$
  - $\delta = (\omega_{*e} - \omega_k + k_y V_\perp)/k_\parallel^2 D_\parallel^2 = \nu_{ei}(\omega_{*e} - \omega_k + k_y V_\perp)/k_\parallel^2 v_{The}^2$

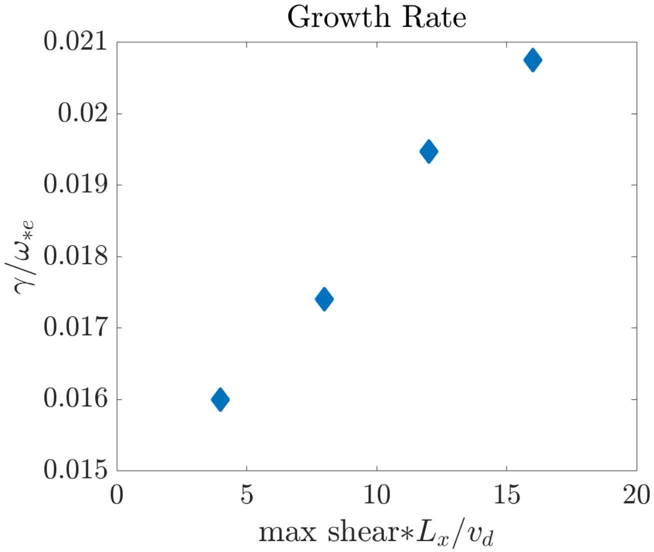
- Resonance reduces the eigenmode scale → Suppresses instability

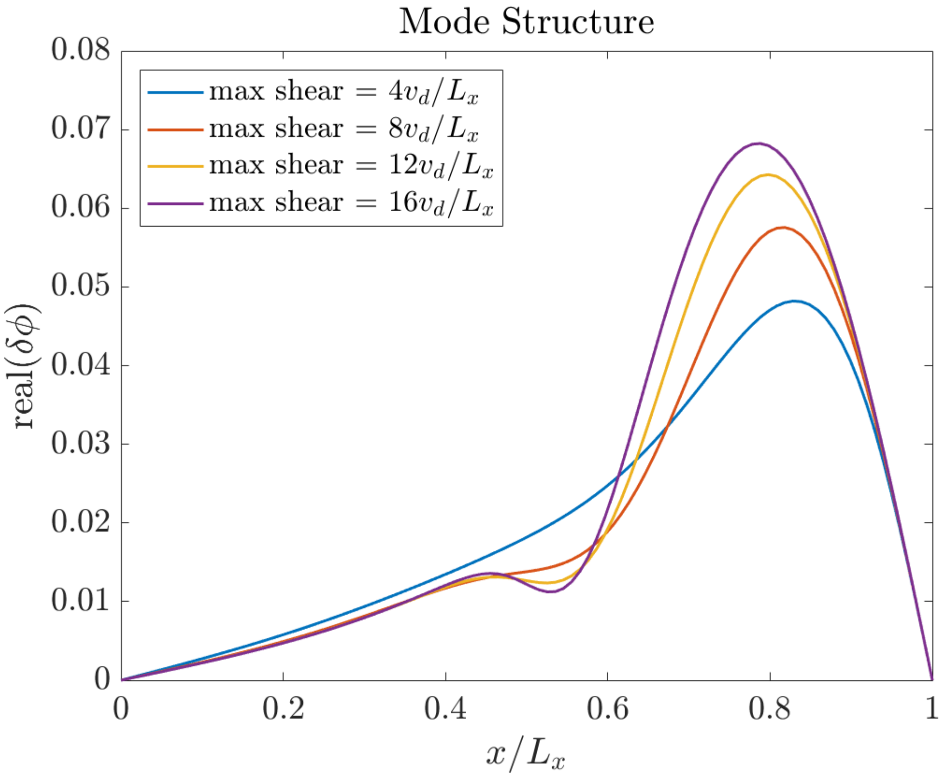
(Width of eigenmode)

Perpendicular flow shear ***destabilizes*** turbulence

- Mean perpendicular flow shear increases mode scale L_m/ρ_s
  → Weakens resonance
  → Enhances instability
- KH drive **negligible** compared to ∇n_0







Zonal Flow Saturation: Motivation

- Why?
  - – Crucial to understand Dimits state physics
  - → Collisionless zonal flow saturation, i.e. collisional damping $\rightarrow 0$
- Tertiary instability does not work
  - – Severely damped by magnetic shear
  - – Observed mean flow shear is always below the threshold for tertiary instability excitation

Nonlinear Model: ***Resonant*** PV Mixing

- Density:

$$\frac{\partial}{\partial t}\langle n\rangle = \frac{\partial}{\partial x}D_{n,\mathrm{turb}}\frac{\partial}{\partial x}\langle n\rangle + D_c\nabla^2\langle n\rangle,$$

- Vorticity:

$$\frac{\partial}{\partial t}\langle\rho\rangle = \frac{\partial}{\partial x}\left[(D_{n,\mathrm{turb}} - D_q^{\mathrm{res}})\frac{\partial}{\partial x}\langle n\rangle + D_q^{\mathrm{res}}\frac{\partial}{\partial x}\langle\rho\rangle\right] - \mu_c\langle\rho\rangle - \mu_{NL}\langle\rho\rangle + \chi_c\nabla^2\langle\rho\rangle,$$

- PE:

$$\frac{\partial}{\partial t}\Omega = D_\Omega\frac{\partial}{\partial x}\Omega + D_q^{\mathrm{res}}\left[\frac{\partial}{\partial x}(\langle n\rangle - \langle\rho\rangle)\right]^2 - \varepsilon_c\Omega^{3/2} + \gamma_L\Omega.$$

PE = Potential Enstrophy, i.e. $\Omega \equiv \langle\tilde{\rho}^2\rangle$

- $\mu_{NL} = \mu_{NL}(\langle v_y\rangle)$: nonlinear damping rate driven by tertiary mode ←

Irrelevant to most cases we have encountered

- D_c, μ_c, χ_c: collisional particle diffusivity, flow damping, vorticity diffusivity → vanishing in collisionless regime

Resonant PV diffusion

- PV flux → turbulent PV diffusion: $\langle \tilde{v}_x \tilde{q} \rangle = -D_{q,turb} \frac{\partial}{\partial x} \langle q \rangle$

↓

$D_{q,turb}$ = Resonant + Non-resonant

- Resonant PV diffusivity:

$$D_q^{res} = \sum_k |\tilde{v}_x|^2 \pi \delta\left(\omega_k - k_y V_\perp\right) \sim \sum_k \tau_{c,k} k_y^2 \rho_s^2 c_x^2 |\phi_k|^2$$

$$\tau_{c,k} \sim \left[|v_{g,y} - v_{ph,y}| \Delta k_y + v_{g,x} \Delta k_x \right]^{-1}$$

- Non-resonant PV diffusivity:

$$D_q^{\text{non-res}} = \sum_{\omega_k \neq k_y \langle v_y \rangle} k_y^2 \rho_s^2 c_s^2 |\phi_k|^2 \frac{|\gamma_k|}{|\omega_k - k_y \langle v_y \rangle|^2} \sim \sum_{\omega_k \neq k_y \langle v_y \rangle} \frac{k_y^2 \rho_s^2 c_s^2}{k_\parallel^2 D_\parallel} \frac{k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2}{1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2} |\phi_k|^2$$

Resonant diffusivity exceeds non-resonant part:

$$D_q^{non} / D_q^{non-res} \sim \tau_{c,k} k_\parallel^2 v_{The}^2 / \nu_{ei} \gg 1$$

Collisionless saturation by resonant diffusion of vorticity

- Zonal flow evolution ← Mean enstrophy equation:

$$\frac{\partial}{\partial t}\int dr\frac{\langle\rho\rangle^2}{2}=\int dr\langle\tilde{v}_r\tilde{\rho}\rangle\frac{d\langle\rho\rangle}{dr}-\nu_i\int dr\langle\rho\rangle^2+\cdots$$

Vorticity ($\rho\equiv\nabla_\perp^2\phi$) flux:

$$\langle\tilde{v}_r\tilde{\rho}\rangle=-D_q^{res}\frac{d\langle\rho\rangle}{dr}+\Gamma_\rho^{Res}$$

Conserves enstrophy between mean flow and fluctuations

$$\frac{\partial}{\partial t}\int dr\frac{\langle\rho\rangle^2}{2}=-\int drD_q^{res}\left(\frac{d\langle\rho\rangle}{dr}\right)^2+\int dr\Gamma_\rho^{Res}\frac{d\langle\rho\rangle}{dr}-\nu_i\int dr\langle\rho\rangle^2+\cdots$$

- $\nu_i\to 0$ → Dimits shift regime → ***Resonant diffusion saturates ZF***
- Collisionless damping by turbulent viscosity: $d\langle\rho\rangle/dr\sim\Gamma_\rho^{Res}/D_q^{res}$
  - Resonant vorticity diffusivity D_q^{res} → ZF saturation

Mesoscopic stationary zonal flow

- Balance vorticity flux: $\langle \tilde{v}_x \tilde{\rho} \rangle = -D_q^{res} \dfrac{d\langle \rho \rangle}{dx} + \Gamma_\rho^{Res} = 0$

  → $\langle v_y \rangle'' = d\langle \rho \rangle / dx \sim \Gamma_\rho^{Res} / D_q^{res}$

  - Vorticity flux driven by ∇n: $\Gamma_\rho^{Res} = \left(D_{n,\mathrm{turb}} - D_q^{\mathrm{res}}\right) \dfrac{\partial}{\partial x} \langle n \rangle$
  - Resonant PV diffusivity:

    $D_q^{res} = \sum_k \tau_{c,k} k_y^2 \rho_s^2 c_x^2 |\phi_k|^2$ with $\tau_{c,k} \sim \left[\left(v_{g,y} - v_{ph,y}\right)\Delta k_y + v_{g,x} \Delta k_x\right]^{-1}$

- Stationary flow:

$$\langle v_y \rangle'' = \langle \rho \rangle' = \left(1 - \frac{D_{n,\mathrm{turb}}}{D_q^{\mathrm{res}}}\right) \frac{\partial \langle n \rangle}{\partial x} \sim -\frac{c_s}{\rho_s L_n} \left(1 - \frac{1}{\tau_{c_k} k_\parallel^2 D_\parallel} \frac{k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2}{1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2}\right)$$

→ **Zonal flow scale**: $L_{ZF} \sim \sqrt{\rho_s L_n}$ → $\rho_s \ll L_{ZF} \ll L_n$

This ***derives*** the standard ordering, which is just invoked, in ad hoc way.

L_m: radial mode scale of drift wave eigenmode, regulated by resonance

Extended Predator-Prey Model

- Mean flow energy:

$$\frac{L_{ZF}^2}{2}\frac{dV''^2}{dt} = \underbrace{\boxed{\underbrace{\alpha_1|V''|E}_{\text{Production by residual vorticity flux}} - \overbrace{\alpha_2 V''^2 E}^{\text{Resonant diffusion of vorticity}}}}_{\text{new}} - \underbrace{\gamma_{NL} V''^2}_{\text{Nonlinear damping by tertiary modes}} - \overbrace{\mu_c V''^2}^{\text{Collisional Damping}}.$$

L_{ZF}: zonal flow profile scale, $\rho_s \ll L_{ZF} \ll L_n$

- Turbulence energy (PE):

$$\frac{dE}{dt} = -\alpha_1|V''|E + \alpha_2 V''^2 E - \underbrace{\varepsilon_c E^{3/2}}_{\text{Forward cascade of PE}} + \underbrace{\gamma_L E}_{\text{Linear instability}}.$$

Turbulence and flow states

- Compare by regime:

Regime	Collisionless	Weak Collisional	Strong Collisional
Collisional Damping Strength	$\mu_c \ll \alpha_2 E$	$\alpha_2 E \ll \mu_c \ll 4\gamma_L \alpha_1^2/\varepsilon_c^2$	$\mu_c \gg 4\gamma_L \alpha_1^2/\varepsilon_c^2$
Flow State	$\frac{\alpha_1}{\alpha_2}$	$\frac{\alpha_1 \gamma_L^2}{\mu_c \varepsilon_c^2}$	$\frac{\gamma_L}{\alpha_1}$
Turbulence Energy	$\frac{\gamma_L^2}{\varepsilon_c^2}$	$\frac{\gamma_L^2}{\varepsilon_c^2}$	$\frac{\gamma_L \mu_c}{\alpha_1^2}$

- Collisionless = collisional damping/viscosity → 0
- Collisionless saturation compared to usual collisional damping:
  - Turbulence energy determined by linear stability and small scale dissipation → Different from usual models, where turbulence energy ~ flow damping
  - Flow state basically independent of stability drive → There can be flows in nearly marginal turbulence

Analogy to Landau Damping Absorption in Langmuir Turbulence

	Langmuir Turbulence Collapse	**Collisionless ZF Saturation**
Primary player	Plasmon-Langmuir wave	Drift wave turbulence
Secondary player	Ion- acoustic wave (caviton)	Zonal flow
Free energy source	Langmuir turbulence driver	$\nabla n, \nabla T$ drives
Final State	(Nearly) empty cavity	Saturated zonal flow and *residual* turbulence
Resonance	Landau damping	Resonant wave absorption
Other damping effects	Ion-acoustic radiation	Kelvin-Helmholtz relaxation

- Landau damping: flattens PDF (negative slope) in phase space
- Resonant PV mixing: homogenizes mean PV in real space

Partial Summary 2

- Resonance effects on linear stability
  - Wave-flow resonance suppresses instability
  - $V_\perp'$ weakens resonance → $V_\perp'$ ***enhances*** instability via resonance
- Resonant diffusion of vorticity saturates zonal flow in collisionless regime
  - Resonant PV mixing ← resonant diffusion of PV
  - Model shows that stationary zonal flow scale is mesoscopic, i.e. $\rho_s \ll L_{ZF} \ll L_n$, since $L_{ZF} \sim \sqrt{\rho_s L_n}$
  - Extended predator-prey model
    → turbulence energy $\sim \gamma_L^2 / \varepsilon_c^2$ not $\sim \gamma_L$
  - Flow independent of turbulence level/drive
    → flow in marginal turbulence