Competition of Mean Perpendicular and Parallel Flows in a Linear Device

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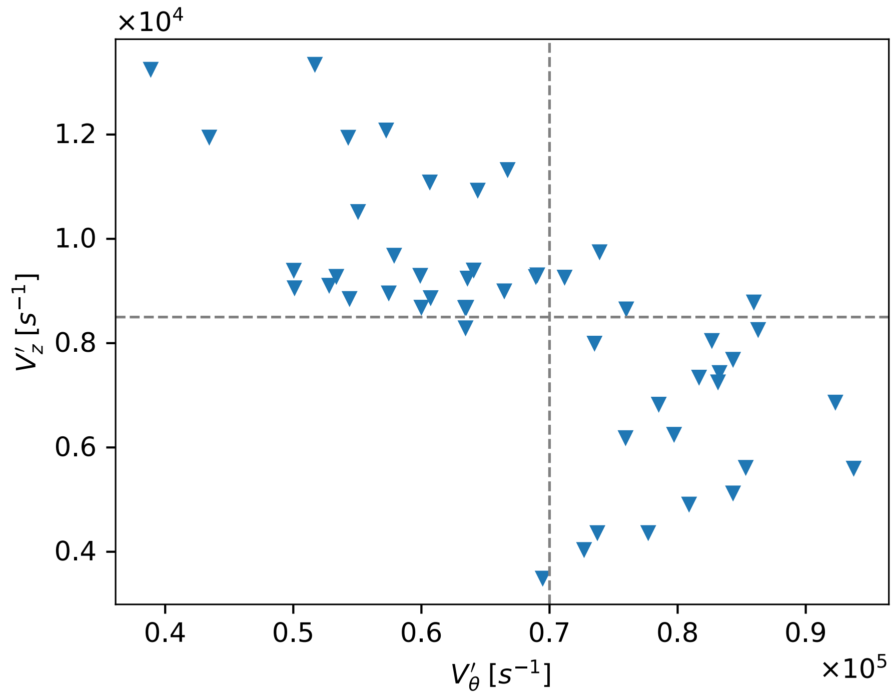
Background

- ***Intrinsic*** axial and azimuthal flows observed in linear device (CSDX)
- Increase B → scan mean flows-$V_\perp$ and $V_\parallel$
- ***Dynamical*** competition between mean perpendicular and parallel flows
- [See George Tynan's talk earlier]

  – ***Dynamical***: $V_\perp$ and $V_\parallel$ exchange energy with the background turbulence, and each other.
  → Energy balance between $V_\perp$ and $V_\parallel$
  → ***Tradeoff*** between $V_\perp$ and $V_\parallel$

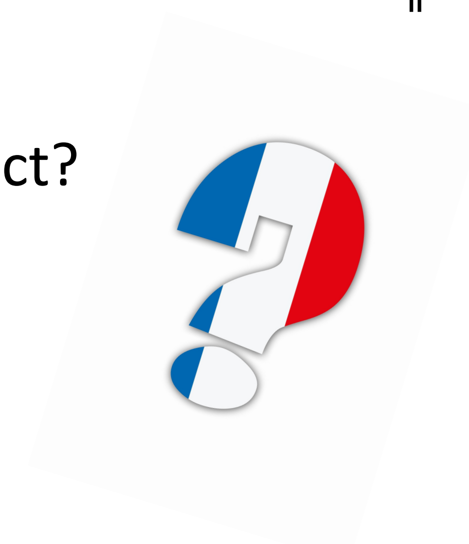


Key Questions and Why

- What's the coupling between ***mean*** perpendicular and parallel flows ($V_\perp$ and $V_\parallel$)?
  - How do they interact?
    → How do they compete for energy from turbulence?
  - Can we have a reduced model of the coupling between $V_\perp$ and $V_\parallel$?
- Why should we care?
  - Linear device (CSDX) studies suggest apportionment of turbulence energy between $V_\perp$ and $V_\parallel$
  - Relevant to L-H transition
    - Both $V_\perp'$ and $V_\parallel$ increase, during transition.
    - The coupling of the two is relevant to transition threshold and dynamics.

Outline of the Rest

- Current status of model
- Exploration of $V_\perp$ and $V_\parallel$ competition
  - ***Turbulent energy branching*** between $V_\parallel$ and $V_\perp$
  - Reynolds power ratio $P_\parallel^R / P_\perp^R$ decreases as $V_\perp$ increases
    → tradeoff between $V_\perp$ and $V_\parallel$
  - $P_\parallel^R / P_\perp^R$ maximum occurs when $|\nabla V_\parallel|$ is below the PSFI (parallel shear flow instability) threshold → saturation of intrinsic $V_\parallel$
- Wave-flow resonance effects
  - Are shear suppression "rules" always correct?
  - $V_\perp'$ weakens resonance
    → ***flow shear enhances instability***
  - Implication for zonal flow dynamics



Current status of model

- Conventional wisdom of $V_\perp \rightarrow V_\parallel$ coupling:
  - $V_\perp'$ breaks the symmetry in $k_\parallel$, but requires finite magnetic shear
  - ***Not applicable*** in linear device (straight magnetic field)
- $V_\parallel \rightarrow V_\perp$ coupling via parallel compression:
  - 3D coupled drift-ion acoustic wave system [Wang et al, PPCF 2012]
  - Coupling between fluctuating PV and parallel compression $\langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle$ ***breaks PV conservation***

    → Sink/source for fluctuating potential enstrophy density

    → Zonal flow generation

Section II: Exploration of $V_\perp$-$V_\parallel$ Coupling

- Goal: study ***how extrinsic flows affect Reynolds powers***

  → generation of intrinsic flows

  → **turbulent energy branching** between intrinsic $V_\perp$ and $V_\parallel$

- Analogous to increment study

- **Hasegawa-Wakatani** drift wave

  → near adiabatic electron:

  $\tilde{n} = (1 - i\delta)\phi, \delta \ll 1$

- Slab geometry

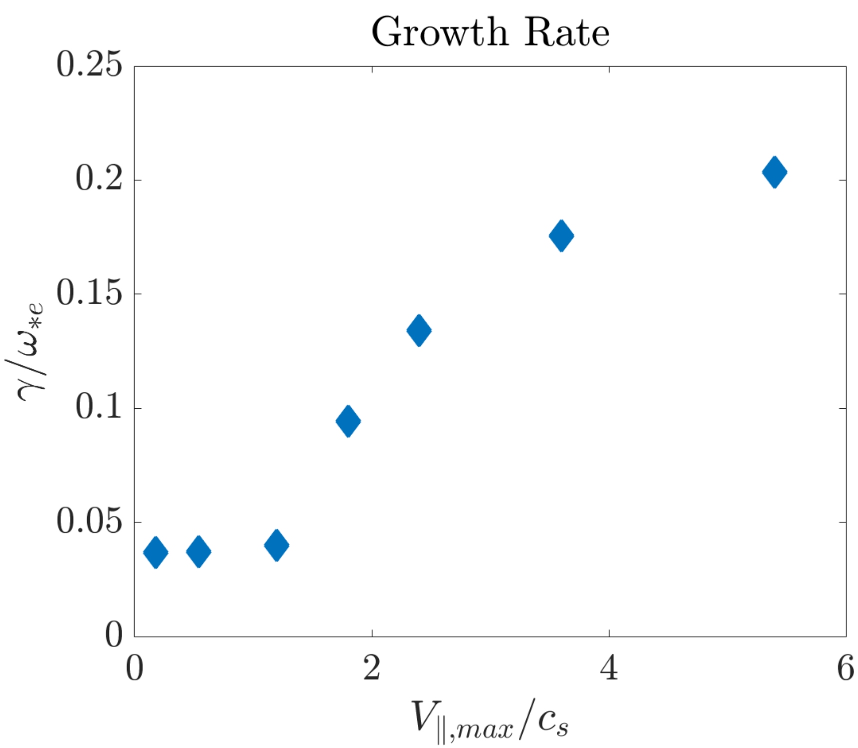
$$\frac{D}{Dt}\tilde{n} + \tilde{v}_r \frac{\nabla n_0}{n_0} + \nabla_\parallel \tilde{v}_\parallel = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi}),$$

$$\frac{D}{Dt}\nabla_\perp^2 \tilde{\phi} + \tilde{v}_r V_\perp'' = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi}),$$

$$\frac{D}{Dt}\tilde{v}_\parallel + \tilde{v}_r V_\parallel' = \nabla_\parallel \tilde{n},$$

Bottom Line: ∇n_0 is the Primary Instability Drive

- Other potential drives:
  - $V_\perp''$ → Kelvin-Helmholtz instability
  - $\nabla V_\parallel$ → Parallel shear flow instability
- KH is not important
  - $V_\perp''$ drive weaker than ∇n_0 drive, i.e. $|k_y \rho_s^2 V_\perp''| \ll \omega_{*e}$
- ***PSFI stable*** in CSDX



Definition: Reynolds Power

- Mean flow evolution is driven by Reynolds power

$$\frac{1}{2}\frac{\partial |V_\parallel|^2}{\partial t} \sim -\frac{\partial}{\partial x}\langle \tilde{v}_x \tilde{v}_\parallel \rangle V_\parallel$$

  - Parallel Reynolds power of a single eigenmode

$$P_\parallel^R = \int_0^{L_x} dx \left[-\frac{\partial}{\partial x}\left(\tilde{v}_{x,k}^* \tilde{v}_{\parallel,k}\right)\right] V_\parallel$$

  - Perpendicular Reynolds power of a single eigenmode

$$P_\perp^R = \int_0^{L_x} dx \left[-\frac{\partial}{\partial x}\left(\tilde{v}_{x,k}^* \tilde{v}_{y,k}\right)\right] V_\perp$$

- Effects of extrinsic $V_\parallel$ and $V_\perp$ on the ratio $P_\parallel^R / P_\perp^R$ are studied

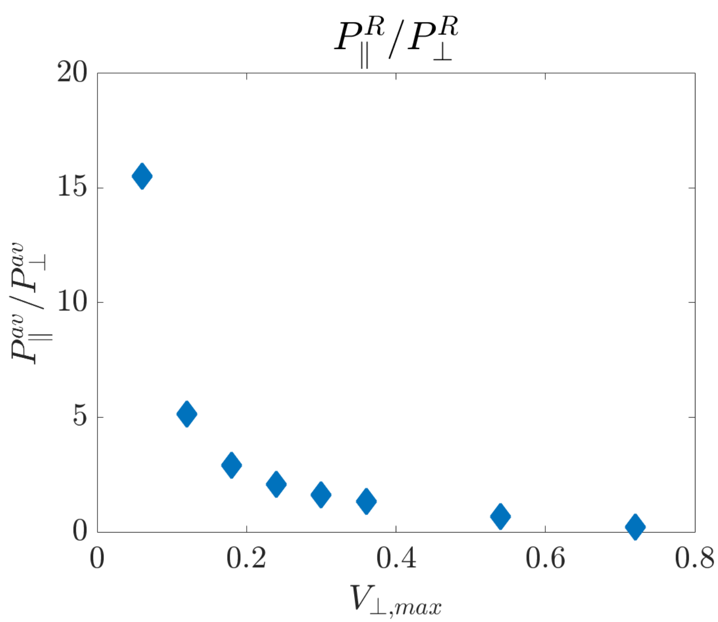
Coupling of $V_\perp$ and $V_\parallel$ ↔ Ratio of Reynolds Powers

- Ratio $P^R_\parallel / P^R_\perp$ decreases with $V_\perp$

→ Energy branching of $V_\parallel$ reduced

→ $V_\perp$ reduces generation of $V_\parallel$

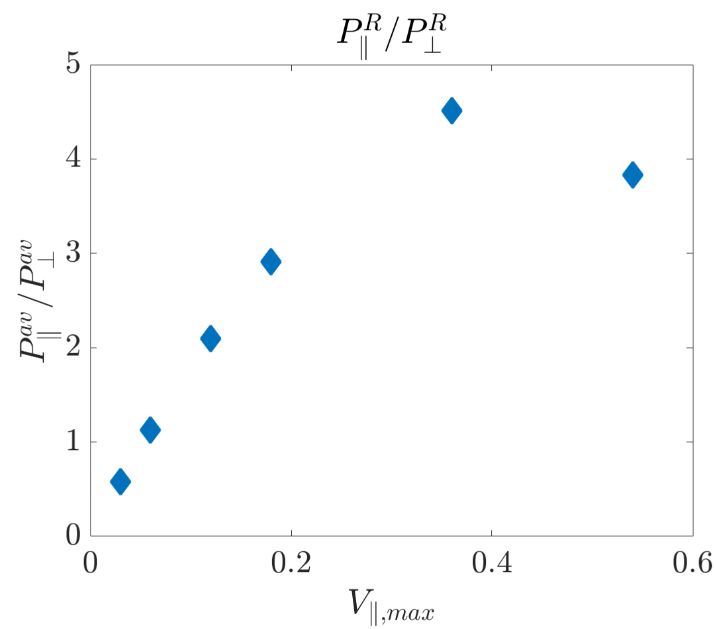
→ ***Competition*** between $V_\perp$ and $V_\parallel$



- Increase $V_\parallel$ → $P^R_\parallel / P^R_\perp$ turnover ***before*** $\nabla V_\parallel$ hits PSFI threshold

→ Max energy branching of $V_\parallel$ below PSFI threshold

→ $V_\parallel$ saturates ***below*** PSFI threshold



Section III: Revisiting Wave-Flow Resonance

[Li & Diamond, manuscript in preparation]

- ***Are conventional shear suppression "rules" always correct?***
  - $E \times B$ flow shear suppresses instability ← Is it correct with resonance?
  - Wave-flow resonance effect is often overlooked, though was mentioned in past works.
- Findings:
  - Wave-flow resonance stabilizes drift wave instability
  - Perpendicular flow shear weakens the resonance, and thus ***destabilizes*** the instability
- Implications for ***zonal flow saturation***:
  - ***Collisionless*** zonal flow saturation (without involving tertiary instabilities, such as KH) set by resonance, $D_\rho \sim \left(\omega_k - k_y V_\perp\right)^{-2}$

Wave-flow resonance

- Resonance: $\omega_k - k_y V_\perp - k_\parallel V_\parallel$
  $|k_\parallel|/k_y \ll 1$ → Resonance dominated by $\omega_k - k_y V_\perp$
- Hasegawa-Wakatani drift wave model, with extrinsic $V_\perp$

$$\frac{D}{Dt}\tilde{n} + \tilde{v}_r \frac{\nabla n_0}{n_0} = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi}),$$

$$\frac{D}{Dt}\nabla_\perp^2 \tilde{\phi} + \tilde{v}_r V_\perp'' = D_\parallel \nabla_\parallel^2 (\tilde{n} - \tilde{\phi})$$

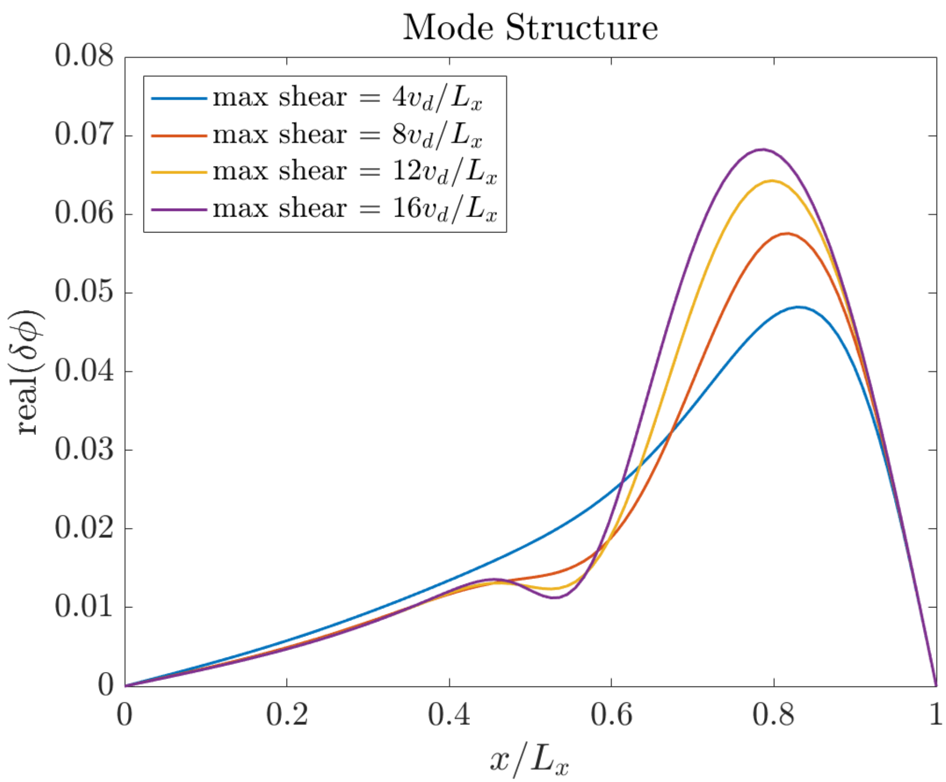
- **KH drive negligible** → Drift wave instability dominant
  - Near adiabatic electron: $\tilde{n} = (1 - i\delta)\phi$, $\delta \ll 1$
  - $\delta = (\omega_{*e} - \omega_k + k_y V_\perp)/k_\parallel^2 D_\parallel^2 = \nu_{ei}(\omega_{*e} - \omega_k + k_y V_\perp)/k_\parallel^2 v_{The}^2$

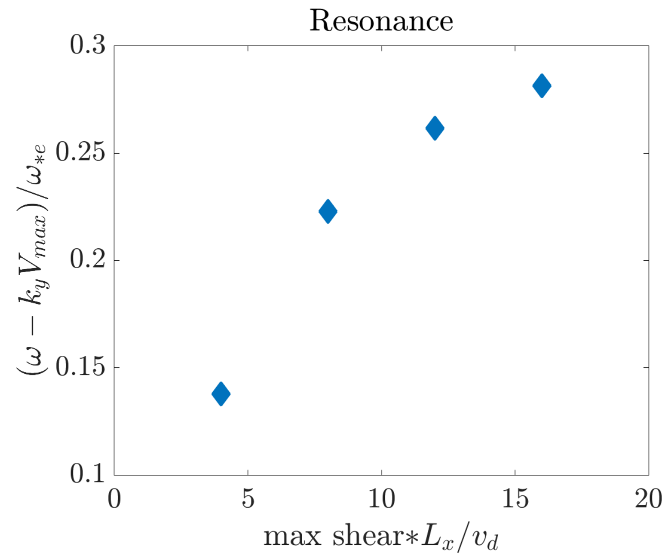
- Resonance reduces the eigenmode scale → Suppresses instability

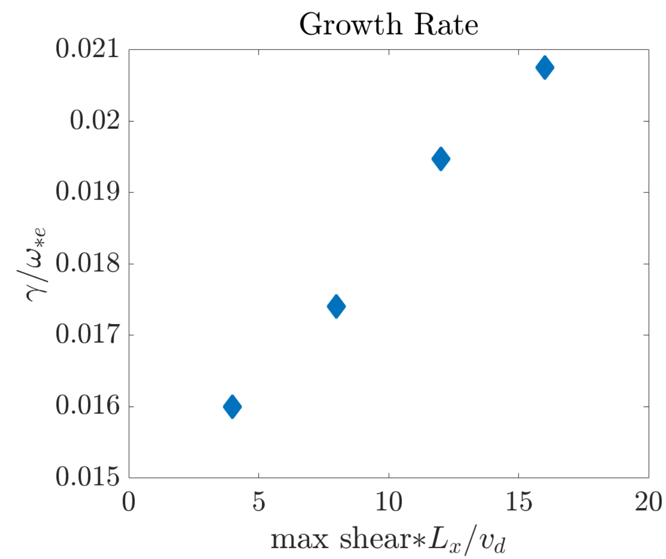
(Width of eigenmode)

Perpendicular flow shear ***destabilizes*** turbulence

- Mean perpendicular flow shear increases mode scale L_m/ρ_s
  → Weakens resonance
  → Enhances instability
- KH drive **negligible** compared to ∇n_0







Implications for Zonal Flow Saturation

- Connection to collisionless saturation of ZF
- Zonal flow evolution → Mean enstrophy equation:

$$\frac{\partial}{\partial t}\int dr \frac{\langle\rho\rangle^2}{2} = \int dr \langle\tilde{v}_r\tilde{\rho}\rangle \frac{d\langle\rho\rangle}{dr} - \nu_i \int dr \langle\rho\rangle^2 + \cdots$$

- Vorticity ($\rho \equiv \nabla_\perp^2 \phi$) flux: $\langle\tilde{v}_r\tilde{\rho}\rangle = -D_\rho \frac{d\langle\rho\rangle}{dr} + \Gamma_\rho^{Res}$ → Conserves enstrophy between mean flow and fluctuations

$$\frac{\partial}{\partial t}\int dr \frac{\langle\rho\rangle^2}{2} = -\int dr D_\rho \left(\frac{d\langle\rho\rangle}{dr}\right)^2 + \int dr \Gamma_\rho^{Res} \frac{d\langle\rho\rangle}{dr} - \nu_i \int dr \langle\rho\rangle^2 + \cdots$$

- $\nu_i \to 0$ → Dimits shift regime → ***Resonance saturates ZF, w/o KH***
  - Collisionless damping by turbulent viscosity: $d\langle\rho\rangle/dr \sim \Gamma_\rho^{Res}/D_\rho$
  - Resonance sets D_ρ → ZF saturation

$$\Gamma_\rho^{Res} = \sum_k k_y c_s^2 |\phi_k|^2 \left[\frac{\gamma_k \omega_{*e} + \alpha_n(\omega_{*e} - \omega_k + k_y V_\perp)}{|\omega_k - k_y V_\perp + i\alpha_n|^2} - \frac{|\gamma_k|\omega_{*e}}{|\omega_k - k_y V_\perp|^2}\right], D_\rho = \sum_k k_y^2 c_s^2 |\phi_k|^2 \frac{|\gamma_k|}{|\omega_k - k_y V_\perp|^2}$$

Summary

- CSDX experiments suggest **energy apportionment** between mean $V_\perp$ and $V_\parallel$
- Reynolds power ratio $P_\parallel^R / P_\perp^R$ changes in response to external flow increment
  - $P_\parallel^R / P_\perp^R$ decreases with $V_\perp$ → tradeoff between $V_\perp$ and $V_\parallel$
  - $P_\parallel^R / P_\perp^R$ maximum occurs ***before*** $\nabla V_\parallel$ hits PSFI threshold
- Testing misconceptions of shearing effects on stability
  - Wave-flow resonance suppresses instability
  - $V_\perp'$ weakens resonance → $V_\perp'$ ***enhances*** instability
  - Resonance produces turbulent viscosity
    → collisionless saturation of ZF, **without** involving tertiary instabilities

Backup

Details on Acoustic Coupling

- $V_\parallel \rightarrow V_\perp$ coupling via parallel compression:
  - 3D coupled drift-ion acoustic wave system [Wang et al, PPCF 2012]
  - Coupling between fluctuating PV and parallel compression $\langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle$ ***breaks PV conservation***
    → Sink/source for fluctuating potential enstrophy density
    → Zonal flow generation
  - Perpendicular flow dynamics:

PV diffusion

$$\frac{\partial}{\partial t}\left[V_\perp - L_n \left\langle \frac{\tilde{q}^2}{2} \right\rangle\right] \sim -\nu_i V_\perp + L_n \left[\frac{\partial}{\partial r}\left\langle \tilde{v}_x \frac{\tilde{q}^2}{2}\right\rangle + \mu \langle (\nabla \tilde{q})^2 \rangle - \langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle\right]$$

collisional damping

$$\langle \tilde{q} \nabla_\parallel \tilde{v}_\parallel \rangle \sim -\sum_k \frac{|\Delta \omega_k|}{\omega_k^2} k_\parallel^2 |\phi_k|^2 < 0$$

Stationary Zonal Flow Profile

- Turbulent viscosity set by resonance:

$$D_\rho = \sum_k k_y^2 c_s^2 |\phi_k|^2 \frac{|\gamma_k|}{\left|\omega_k - k_y V_\perp\right|^2} \sim \sum_k \frac{k_y^2 \rho_s^2 c_s^2}{k_\parallel^2 D_\parallel} \frac{k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2}{1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2} |\phi_k|^2$$

- Residual vorticity flux:

$$\Gamma_\rho^{Res} = \sum_k k_y c_s^2 |\phi_k|^2 \left[\frac{\gamma_k \omega_{*e} + \alpha_n (\omega_{*e} - \omega_k + k_y V_\perp)}{\left|\omega_k - k_y V_\perp + i\alpha_n\right|^2} - \frac{|\gamma_k| \omega_{*e}}{\left|\omega_k - k_y V_\perp\right|^2} \right],$$

- Reynolds force (i.e. net production) = 0
  → Stationary flow profile:

$$\langle v_y \rangle'' = \langle \rho \rangle' = \frac{\Gamma_\rho^{Res}}{D_\rho} \sim -\frac{k_y^2 \rho_s^2 c_s^2}{(k_\parallel^2 D_\parallel)^2} \frac{1}{L_n^3} \frac{1}{(1 + k_y^2 \rho_s^2 + L_m^{-2} \rho_s^2)^2}.$$

Resonance and Instability Related to Mode Scale

- Eigenmode equation with resonant effect:

$$(\omega_k - k_y V_\perp + i\gamma_k)\rho_s^2 \partial_x^2 \phi = \left[(1 + k_y^2 \rho_s^2 - i\delta)(\omega_k - k_y V_\perp + i\gamma_k) - \omega_{*e}\right]\phi$$

- Mode scale: $L_m^{-2}\rho_s^2 \equiv \rho_s^2 \int_0^{L_x} dx |\partial_x \phi|^2 \ / \int_0^{L_x} dx |\phi|^2$

- Results:

$$|\omega_k - k_y \langle v_y \rangle|_{min} \cong \frac{\omega_{*e}}{1 + k_y^2 \rho_s^2 + L_m^{-2}\rho_s^2},$$

$k_y^2 \rho_s^2 + L_m^{-2}\rho_s^2$ ⟷ Effectively, $k_\perp^2 \rho_s^2$

$$\gamma_k \cong \frac{\omega_{*e}^2}{k_\parallel^2 D_\parallel} \frac{k_y^2 \rho_s^2 + L_m^{-2}\rho_s^2}{(1 + k_y^2 \rho_s^2 + L_m^{-2}\rho_s^2)^3}.$$

- Strong resonance

$$\gamma_k \ll \omega_k - k_y V_\perp \ll \omega_{*e}$$

⟷

- Eigenmode peaks ($L_m^{-2}\rho_s^2$ increases) as resonance becomes stronger
- Resonance suppresses drift wave instability

Analogy to Landau Damping Absorption

	Langmuir Turbulence Collapse	Collisionless ZF Saturation
Players	Plasmon-Langmuir wave and ion-acoustic wave (caviton)	Drift wave and zonal flow
Final State	(Nearly) empty cavity	Dimits state zonal flow dominant
Free energy source	Langmuir turbulence driver	$\nabla n, \nabla T$ drives
Resonance effect	Landau damping as cavity collapses	Absorption by $D_\rho \sim (\omega_k - k_y V_\perp)^{-2}$
Other saturation mechanisms	Ion-acoustic radiation from empty cavity	Kelvin-Helmholtz relaxation

Revisit predator-prey model

- Resonance induces collisionless saturation through D_ρ, apart from KH:

$$\gamma_k = \text{linear instability } (\gamma_L) + \text{resonance absorption } \gamma_R \sim \gamma_R(\omega_k - k_y V_\perp)$$

⇩

Analogy to ion-acoustic absorption during collapse of Langmuir waves

- Revisit predator-prey model with resonance effect
  → Mechanism for collisionless damping, without KH