Basics of Turbulence II:

Some Aspect of Mixing and Scale Selection

P.H. Diamond

UC San Diego and SWIP

1st Chengdu Theory Festival
Aug. 2018

This research was supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, under Award Number DE-FG02-04ER54738 and CMTFO.

Outline

- Prelude: Thoughts on Mixing and Scale Selection
- Mixing in Pipes and Donuts:
  - Prandtl
  - Kadomtsev

  Simple, useful ideas
- Potential Vorticity: Not all mixing is bad...
- Inhomogeneous Mixing: Corrugations and Beyond
- Brief Discussion

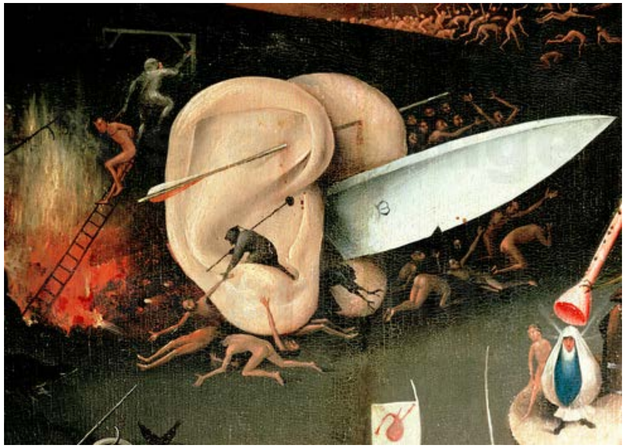
Prelude

- Why is plasma turbulence “hard”?
  - – 40+ years
  - – Modest ‘Re’

1) Broad dynamic range: $\rho_e \rightarrow \rho_i < l < L_p$ (mesoscopic structures galore)
2) Multi-scale bifurcations, bi-stability
3) $\mathrm{Ku} \sim 1 \; (\tilde{V}\tau_c/\Delta \equiv Ku)$ (coherent vs stochastic)
4) Boundary dynamics/dynamic boundaries
5) Dynamic phases

Bosch as Metaphor (After Kadomtsev)

“The Garden of Earthly Delights”, Hieronymous Bosch



- Most Problems: Scale Selection
- Classic example:
  - Mixing Length Estimate / “Rule” (Kadomtsev ‘66)
  - Still used for modelling

$$\partial\tilde{P} + \tilde{V}\cdot\underset{\uparrow}{\nabla\tilde{P}} = -\tilde{V}_r\, d\underset{\uparrow}{\langle P\rangle}/dr$$

$$\frac{\delta P}{P} \sim \frac{\Delta}{L_p}$$

N.B. $\frac{\delta P}{P} \sim \frac{\Delta}{L_p} \Leftrightarrow Ku \sim O(1)$

- What is Δ?
  - ρ_i
  - linear mode scale, which?
  - shearing modified scale
  - domain scale
  - ... what?

A Simpler Problem:

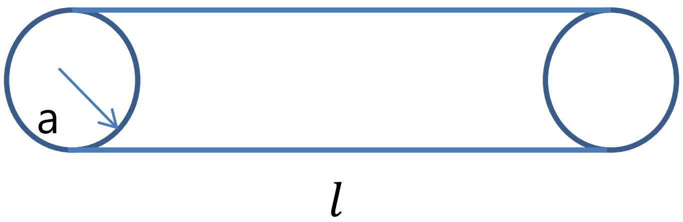
→ Drag in Turbulent Pipe Flow

- L. Prandtl 1932, et. seq

- Prototype for mixing length model

- Essence of confinement problem:
  - given device, sources; what profile is achieved?
  - $\tau_E = W/P_{in}$, How optimize W, stored energy
- Related problem: Pipe flow → drag ↔ momentum flux

ΔP → pressure drop



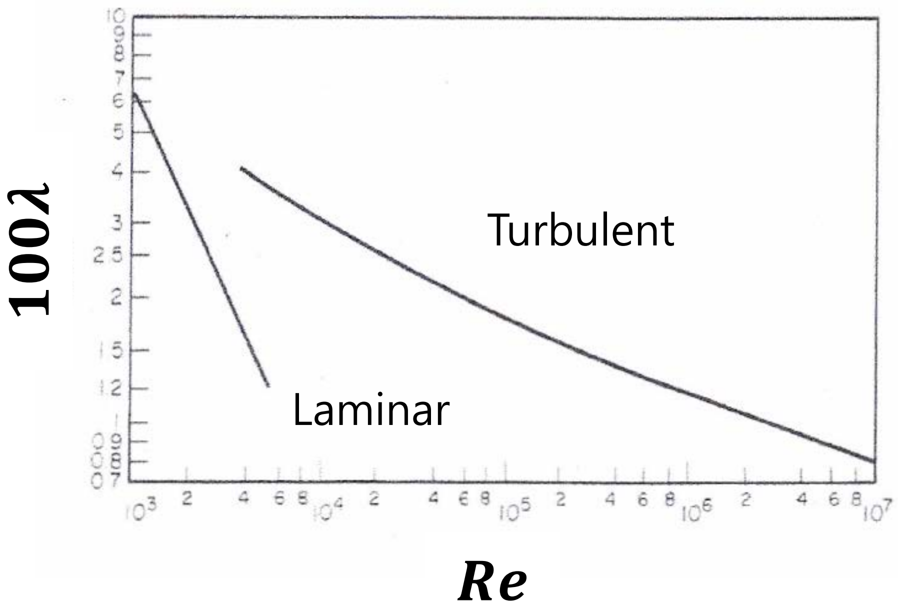
$$\Delta P \pi a^2 = \rho V_*^2 2\pi a l$$

→ friction velocity V_* ↔ u_{turb}

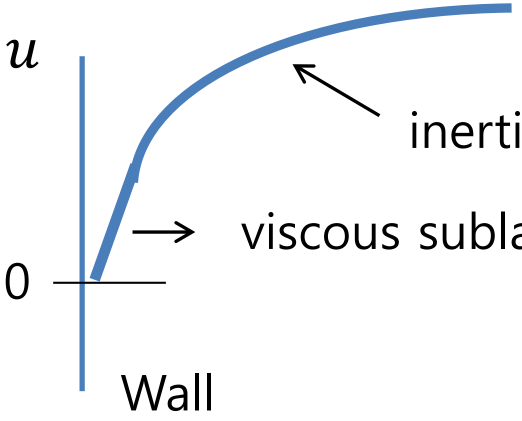
Balance: momentum transport to wall

(Reynolds stress) vs ΔP

➔ Flow velocity profile



$$\lambda = \frac{2a\Delta P/l}{1/2\rho u^2}$$



inertial sublayer → ~ logarithmic (~ universal)

→ viscous sublayer (linear)

- Problem: physics of ~ universal logarithmic profile?
- Universality → scale invariance

- Prandtl Mixing Length Theory (1932)
  - Wall stress = $\rho V_*^2 = -\rho \nu_T \, \partial u / \partial x$ or: $\frac{\partial u}{\partial x} \sim \frac{V_*}{x}$ ← Spatial counterpart of K41

    eddy viscosity

    Scale of velocity gradient?
  - Absence of characteristic scale →

turbulent transport model →

$$\nu_T \sim V_* x$$

$$u \sim V_* \ln(x/x_0)$$

$x \equiv$ mixing length, distance from wall

Analogy with kinetic theory …

$\nu_T = \nu \rightarrow x_0$, viscous layer → $x_0 = \nu / V_*$

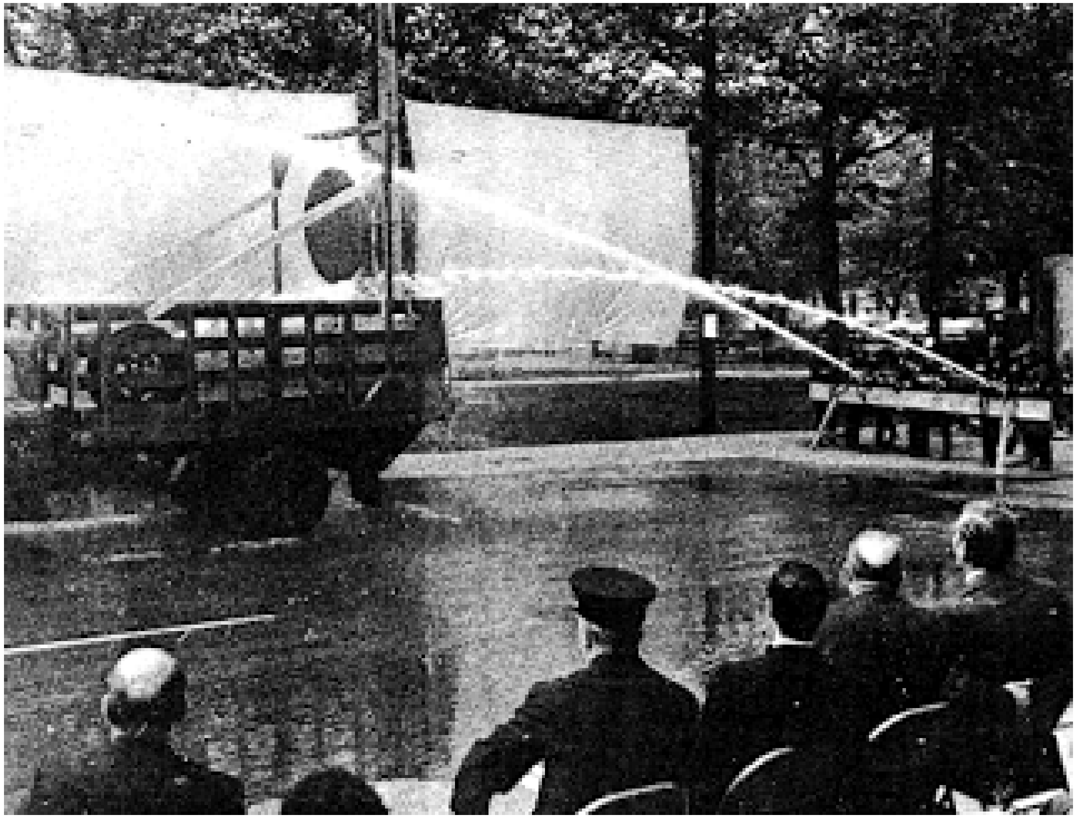
Some key elements:

- Momentum flux driven process $\Delta P \leftrightarrow V_*^2 \rightarrow \nu_T \partial u / \partial x$
- Turbulent diffusion model of transport - eddy viscosity
- Mixing length – scale selection

  ~ x → macroscopic, eddys span system $x_0 < x < a$

  → ~ flat profile – strong mixing

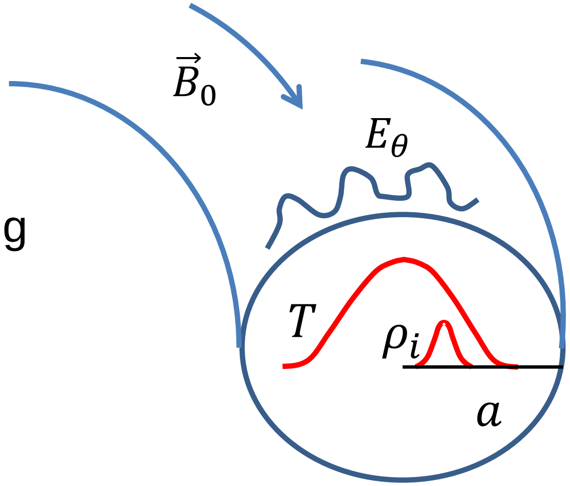
  $\nu_T = V_* x$
- Self-similarity → x ↔ no scale, within $[x_0, a]$
- Reduce drag by creation of buffer layer i.e. steeper gradient than inertial sublayer (by polymer) – enhanced confinement



Without vs With Polymers
Comparison → NYFD 1969

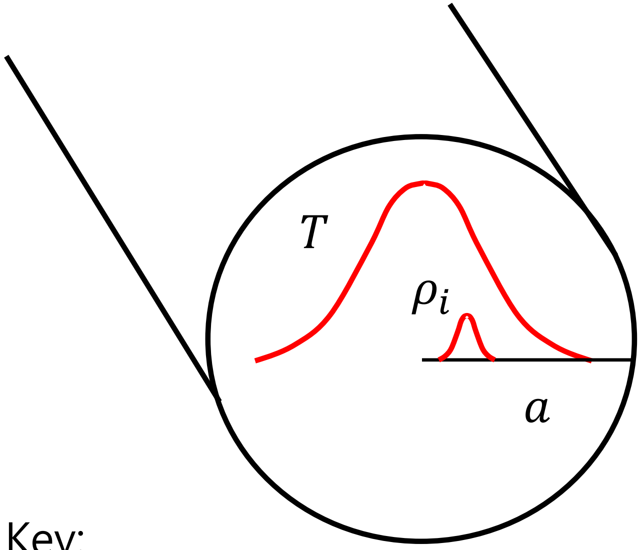
Confinement

Primer on Turbulence in Tokamaks I



- Strongly magnetized
  - – Quasi 2D cells, Low Rossby #
  - * – Localized by $\vec{k} \cdot \vec{B} = 0$ (resonance) - pinning
- $\vec{V}_\perp = +\frac{c}{B} \vec{E} \times \hat{z}, \quad \frac{V_\perp}{l\Omega_{ci}} \sim R_0 \ll 1$
- $\nabla T_e, \nabla T_i, \nabla n$ driven
- Akin to thermal convection with: g → magnetic curvature
- → $\mathrm{Re} \approx VL/\nu$ ill defined, not representative of dynamics
- Resembles wave turbulence, not high Re Navier-Stokes turbulence
- → $K \sim \tilde{V}\tau_c/\Delta \sim 1$ → *Kubo* # ≈ 1
- → Broad dynamic range, due electron and ion scales, i.e. a, ρ_i, ρ_e

Primer on Turbulence in Tokamaks II



Key:

2 scales:

$\rho \equiv$ gyro-radius

$a \equiv$ cross-section

$\rho_* \equiv \rho/a$ ➔ key ratio

$\rho_* \ll 1$

- Correlation scale ~ few ρ_i → "mixing length"(?!)
- Characteristic velocity $v_d \sim \rho_* c_s$
- Transport scaling: $D_{GB} \sim \rho\, V_d \sim \rho_*\, D_B$

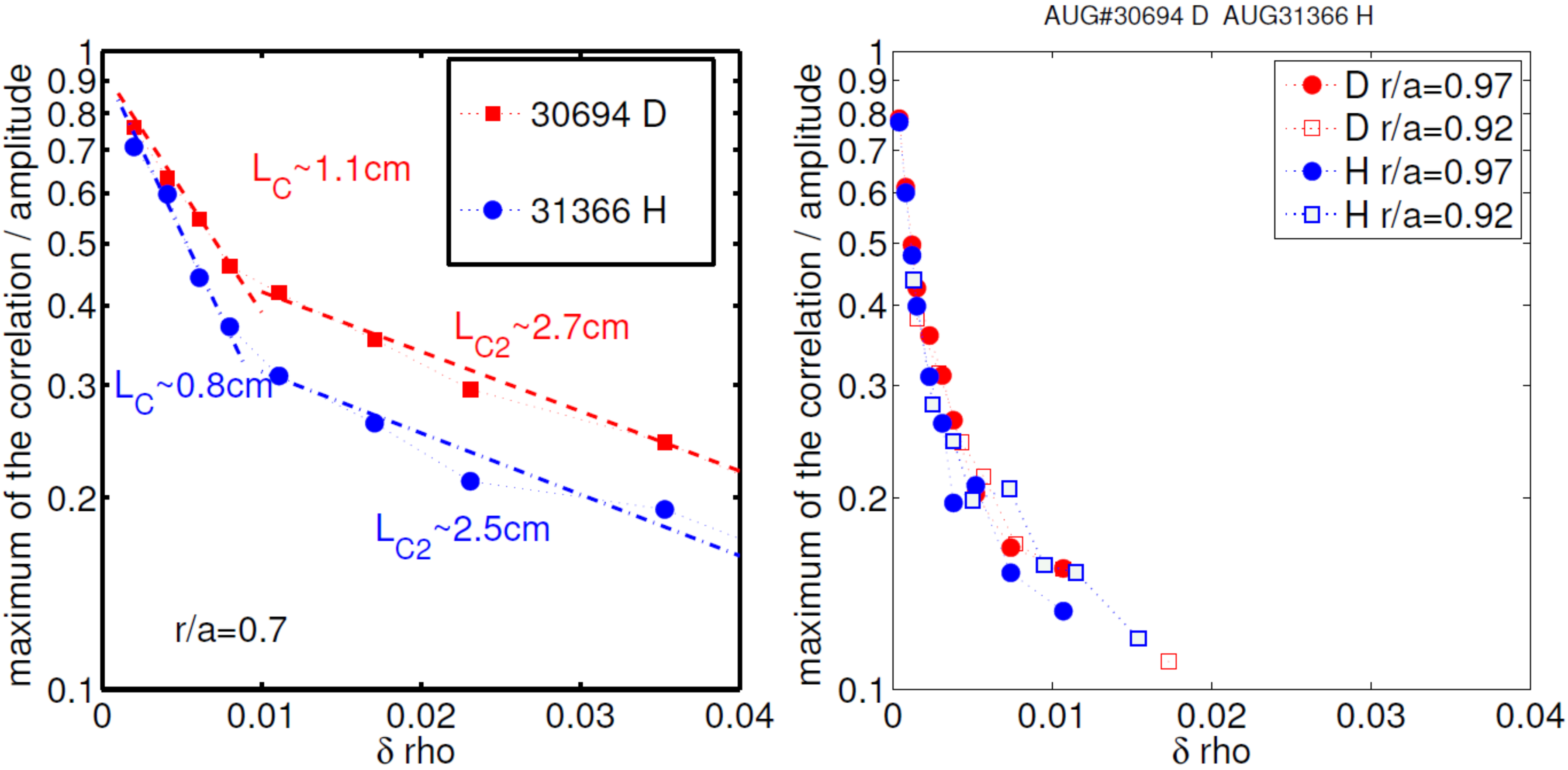
  $D_B \sim \rho\, c_s \sim T/B$
- i.e. Bigger is better! ➔ sets profile scale via heat balance (Why ITER is huge…)
- Reality: $D \sim \rho_*^{\alpha}\, D_B$, $\alpha < 1$ ➔ 'Gyro-Bohm breaking'
- 2 Scales, $\rho_* \ll 1$ ➔ key contrast to pipe flow

THE Question ↔ Scale Selection

- Pessimistic Expectation (from pipe flow):
  - $l \sim a$
  - $D \sim D_B$
- Hope (mode scales)
  - $l \sim \rho_i$
  - $D \sim D_{GB} \sim \rho_* D_B$
- Reality: $D \sim \rho_*^{\alpha}\, D_B, \qquad \alpha < 1$

Why? What physics competition set α?

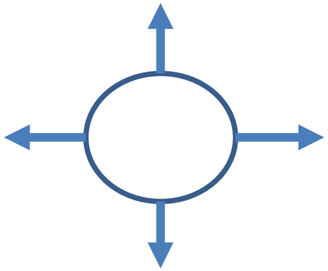
➔ Focus of a large part of this Festival



Correlation function (DBS) exhibits multiple scale behavior

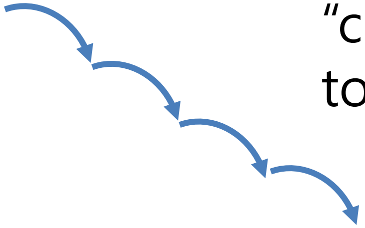
(Hennequin, et. al. 2015)

Players in Scale Selection



"wave emission scattering"

- Mesoscales: $\Delta_c < l < L_p$
- Transport Events: Enhanced Mixing
  - – Turbulence spreading
  - – Avalanching
  - – c.f. Hahm, Diamond 2018 (OV), Dif-Pradalier, this meeting



"correlated topplings"

- Zonal shears – regulation

c.f. Kosuga
this meeting

➔ Produced by PV mixing

Potential Vorticity

and Zonal Flows

➔ Not all mixing is bad...

➔ A different take on a familiar theme

Basic Aspects of PV Dynamics

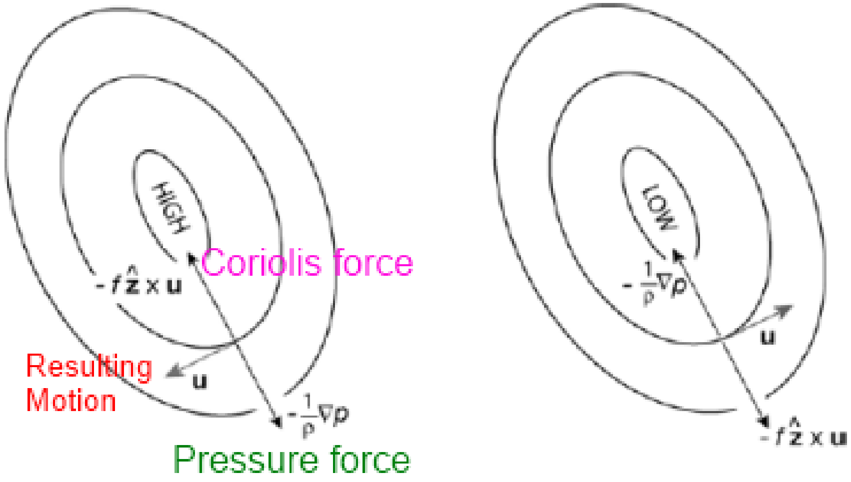
Geophysical fluids

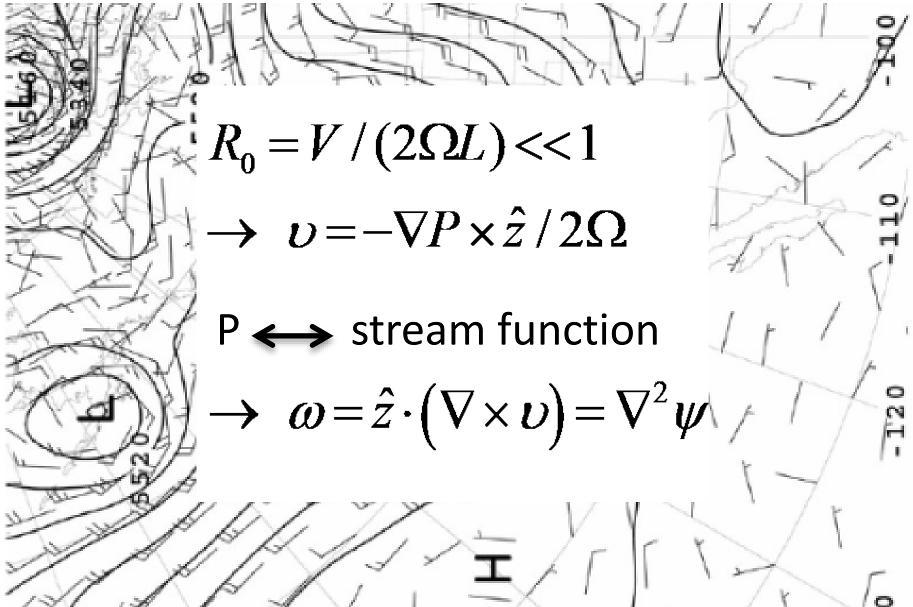
- Phenomena: weather, waves, large scale atmospheric and oceanic circulations, water circulation, jets…
- Geophysical fluid dynamics (GFD): low frequency ($\omega < \Omega$)

> *"We might say that the atmosphere is a musical instrument on which one can play many tunes. High notes are sound waves, low notes are long inertial waves, and nature is a musician more of the Beethoven than the Chopin type. He much prefers the low notes and only occasionally plays arpeggios in the treble and then only with a light hand." – J.G. Charney*

("Turing's Cathedral")

- Geostrophic motion: balance between the Coriolis force and pressure gradient





$$R_0 = V/(2\Omega L) << 1$$

$$\rightarrow \upsilon = -\nabla P \times \hat{z}/2\Omega$$

P ⟷ stream function

$$\rightarrow \omega = \hat{z}\cdot(\nabla\times\upsilon) = \nabla^2\psi$$

Kelvin's theorem – unifying principle

- Kelvin's circulation theorem for rotating system

$$\oint v\cdot dl = \int \left(\underbrace{\nabla\times v}_{\text{relative}} + \underbrace{2\Omega}_{\text{planetary}}\right)\cdot\hat{z}\,dS \equiv C \qquad \dot{C}=0$$

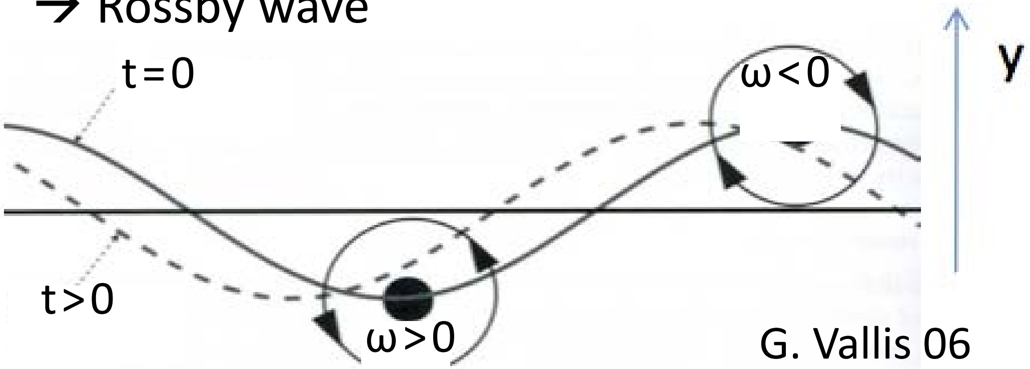
- Displacement on beta-plane

$$\dot{C}=0 \;\rightarrow\; \frac{d}{dt}\nabla^2\psi = -2\Omega\cos\theta\frac{d\theta}{dt} = -\underset{\beta=2\Omega\cos\theta_0/R_\oplus}{\beta} v_y$$

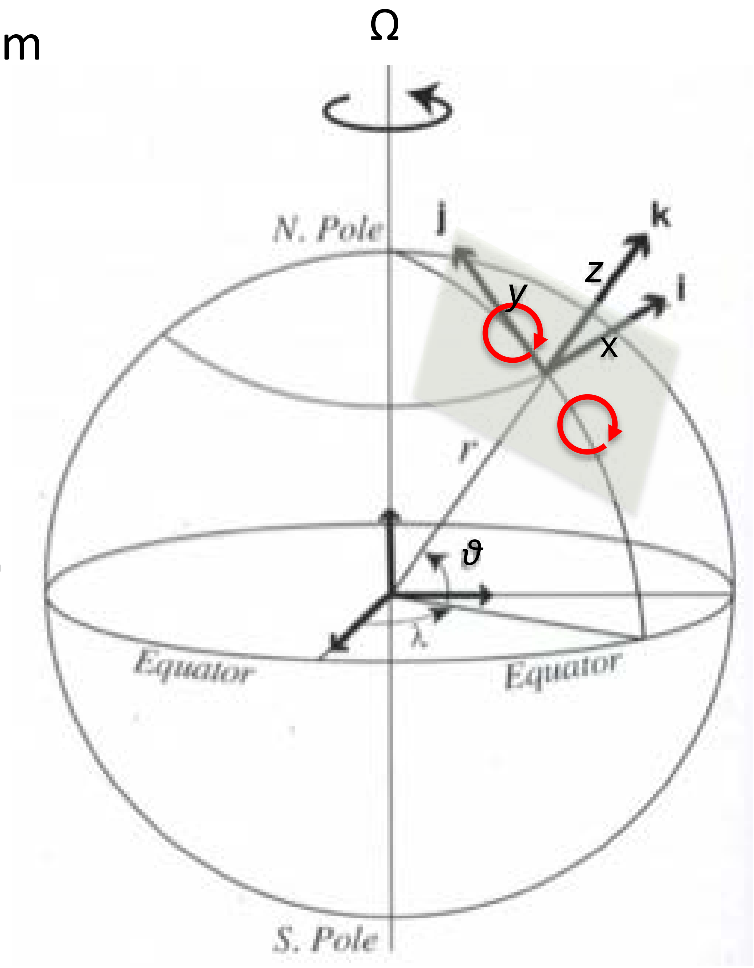
- Quasi-geostrophic eq

$$\boxed{\frac{d}{dt}\left(\nabla^2\psi+\beta y\right)=0 \quad \textbf{PV conservation}}$$

→ Rossby wave



G. Vallis 06



PV conservation $\frac{dq}{dt} = 0$

GFD: Quasi-geostrophic system	Plasma: Hasegawa-Wakatani system
$q = \nabla^2 \psi + \beta y$ relative vorticity planetary vorticity	$q = n - \nabla^2 \phi$ density (guiding center) ion vorticity (polarization)
Physics: $\Delta y \rightarrow \Delta\left(\nabla^2 \psi\right) \rightarrow$ZF	Physics: $\Delta r \rightarrow \Delta n \rightarrow \Delta\left(\nabla^2 \phi\right) \rightarrow$ZF!

(branching)

- Charney-Hasewgawa-Mima equation

$$n = n_0 + \tilde{n}$$

$$\tilde{n} \sim \frac{e\tilde{\phi}}{T}$$

H-W → H-M: $$\frac{1}{\omega_{ci}} \frac{\partial}{\partial t}\left(\nabla^2 \phi - \rho_s^{-2} \phi\right) - \frac{1}{L_n} \frac{\partial}{\partial y} \phi + \frac{\rho_s}{L_n} J(\phi, \nabla^2 \phi) = 0$$

Q-G: $$\frac{\partial}{\partial t}\left(\nabla^2 \psi - L_d^{-2} \psi\right) + \beta \frac{\partial}{\partial x} \psi + J(\psi, \nabla^2 \psi) = 0$$

PV Transport

- Zonal flows are generated by nonlinear interactions/mixing and transport.
- In x space, zonal flows are driven by Reynolds stress

$$\frac{\partial}{\partial t}\langle v_x\rangle = -\frac{\partial}{\partial y}\langle \tilde{v}_x\tilde{v}_y\rangle - \mu\langle v_x\rangle$$

Taylor's Identity

$$\langle \tilde{v}_y\tilde{q}\rangle = -\frac{\partial}{\partial y}\langle \tilde{v}_x\tilde{v}_y\rangle$$

→ **PV flux fundamental to zonal flow formation**

- Inhomogeneous PV mixing, not momentum mixing (dq/dt=0)

→ up-gradient momentum transport (negative-viscosity) not an enigma

- Reynolds stresses intimately linked to wave propagation

but: $\langle \tilde{v}_x\tilde{v}_y\rangle \rightarrow \sum_{\underline{k}} k_x k_y \left|\hat{\phi}_k\right|^2$

{ Wave-mixing, transport duality

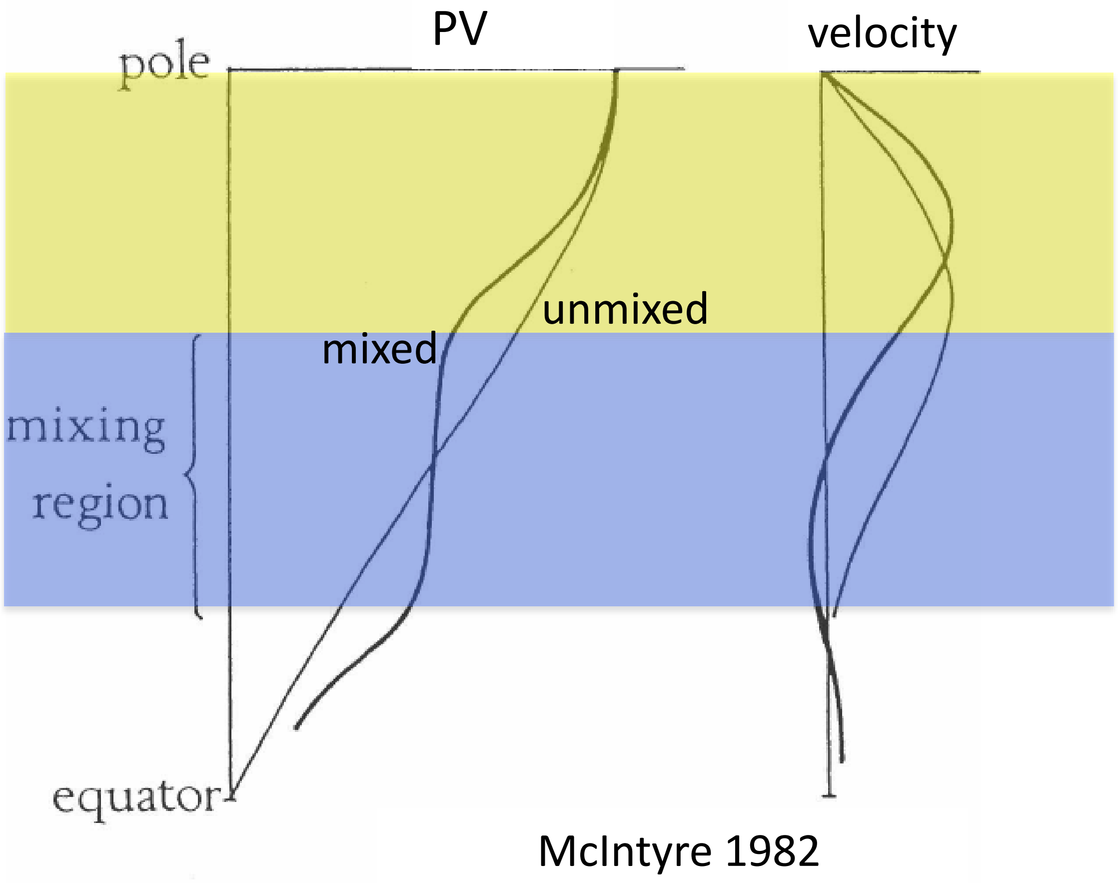
$$v_{gy} = \frac{2k_x k_y \beta}{(k^2)^2} \quad , \quad S_y = v_{gy}\varepsilon$$

c.f. Review: O.D. Gurcan, P.D.; J. Phys. A (2015)
real space emphasis

How make a ZF? → Inhomogeneous PV mixing

- **PV mixing is the fundamental mechanism for zonal flow formation**

$$\delta(\mathrm{PV}) \rightarrow \delta(\nabla^2 \psi) \rightarrow \delta(\psi) \rightarrow \upsilon = \nabla \times \psi$$



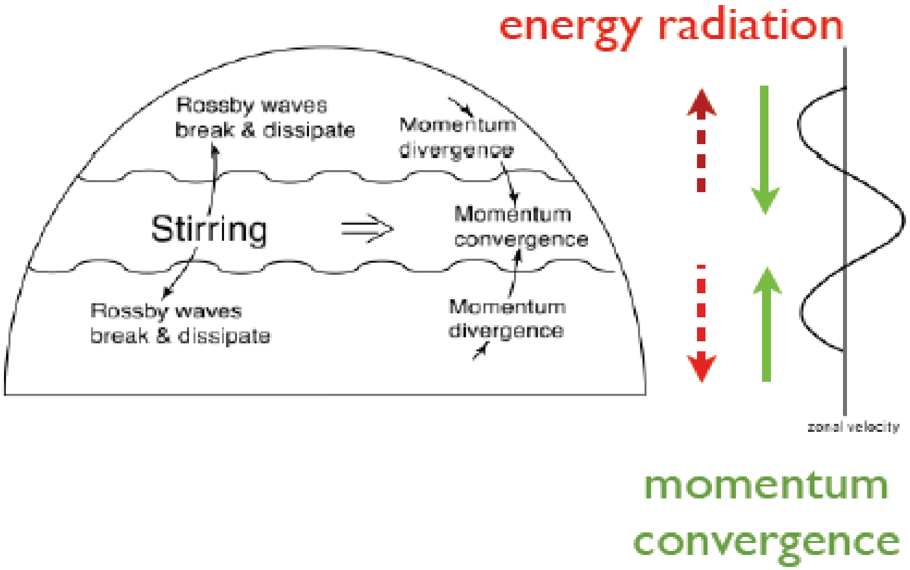
McIntyre 1982
Dritschel & McIntyre 2008

→ PV Mixing ⇔ Wave Propagation

→ How do Zonal Flow Form?

Simple Example: Zonally Averaged Mid-Latitude Circulation

- classic GFD example: Rossby waves + Zonal flow (c.f. Vallis '07, Held '01)
- Key Physics:



Rossby Wave:

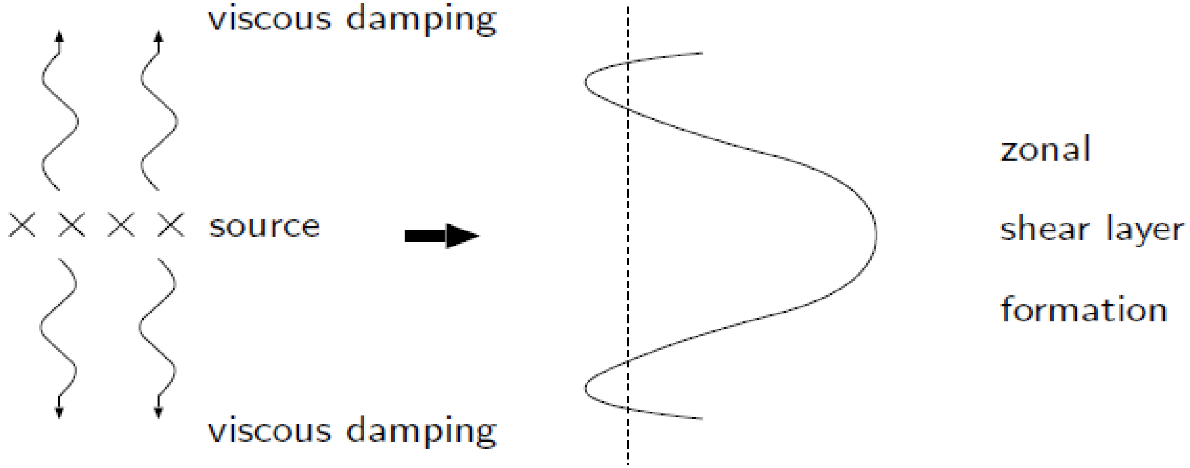
$$\omega_k = -\frac{\beta k_x}{k_\perp^2}$$

$$v_{gy} = 2\beta \frac{k_x k_y}{(k_\perp^2)^2}, \ \langle \tilde{v}_y \tilde{v}_x \rangle = \sum_{\vec{k}} -k_x k_y \left|\hat{\phi}_{\vec{k}}\right|^2$$

$\therefore v_{gy} v_{phy} < 0$ → Backward wave!

➔ Momentum convergence at stirring location

- …"the central result that a rapidly rotating flow, when stirred in a localized region, will converge angular momentum into this region." (I. Held, '01)
- Outgoing waves $\Rightarrow$ incoming wave momentum flux



- Local Flow Direction (northern hemisphere):
  - eastward in source region
  - westward in sink region
  - set by $\beta > 0$
  - Some similarity to spinodal decomposition phenomena → Both 'negative diffusion' phenomena

Minimum Enstrophy Relaxation

→ Principle for ~ 2D Relaxation?

→ How Represent PV mixing?
Non-perturbative?

Foundation: Dual Cascade

- 2D turbulence conservation of energy and potential enstrophy

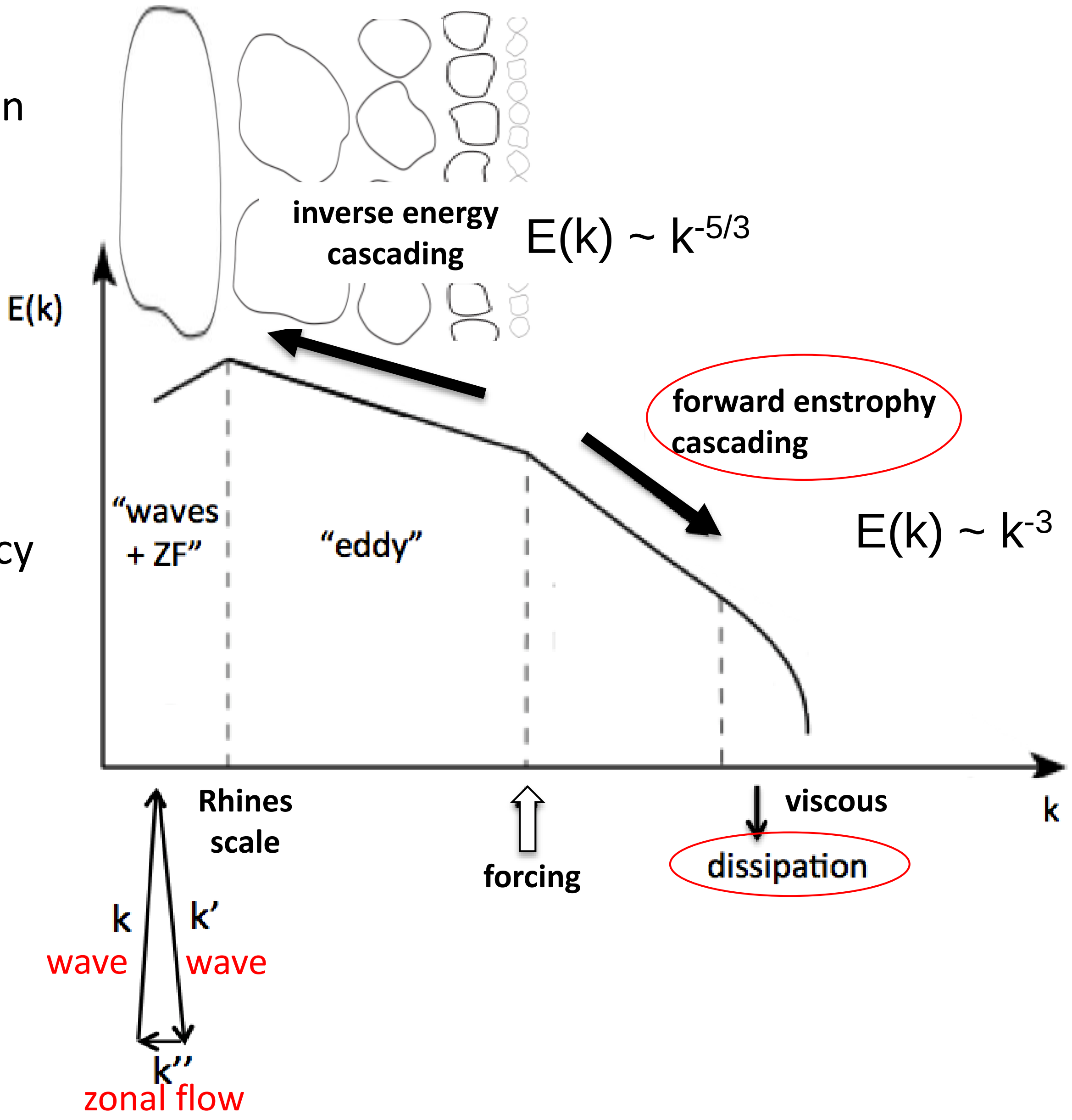
→ dual cascade (Kraichnan)

- When eddy turnover rate and Rossby wave frequency mismatch are comparable

$$\frac{\partial \omega}{\partial t} + \vec{u} \cdot \nabla \omega + \beta \upsilon = 0$$

$$\frac{U}{LT} \quad \boxed{\frac{U^2}{L^2}} \quad \boxed{\beta U}$$

→ Rhines scale $L_R \sim \sqrt{\frac{U}{\beta}}$



- Upshot : Minimum Enstrophy State

  (Bretherton and Haidvogel, 1976)

  -- idea : final state

  -- potential enstrophy forward cascades to viscous dissipation

  -- kinetic energy inverse cascades (drag?!)

-

-- calculate macrostate by minimizing potential enstrophy Ω subject to conservation of kinetic energy E, i.e.

$$\delta(\Omega + \mu E) = 0$$

[n.b. can include topography]

→ “Minimum Enstrophy Theory”

A Natural Question:

How exploit relaxation theory in dynamics?
(with P.-C. Hsu, S.M. Tobias)

Further Non-perturbative Approach for Flow!

- PV mixing in space is essential in ZF generation.

Taylor identity: $\left\langle \tilde{\upsilon}_y \nabla^2 \tilde{\phi} \right\rangle = -\partial_y \left\langle \tilde{\upsilon}_y \tilde{\upsilon}_x \right\rangle$

vorticity flux Reynolds force

Key:
How represent inhomogeneous PV mixing

General structure of PV flux?
→relaxation principles!

most treatment of ZF:
-- perturbation theory
-- modulational instability
(test shear + gas of waves)
~ linear theory based

-> physics of evolved PV mixing?
-> something more general?

non-perturb model: use selective decay principle

What form must the PV flux have so as to dissipate enstrophy while conserving energy?

Using selective decay for flux

analogy ⟷

dual cascade

	minimum enstrophy relaxation **(Bretherton & Haidvogel 1976)**	**Taylor relaxation** **(J.B. Taylor, 1974)**
turbulence	2D hydro	3D MHD
conserved quantity (constraint)	total kinetic energy	global magnetic helicity
dissipated quantity (minimized)	fluctuation potential enstrophy	magnetic energy
final state	minimum enstrophy state flow structure emergent	Taylor state force free B field configuration
structural approach	$\frac{\partial}{\partial t}\Omega < 0 \Rightarrow \Gamma_E \Rightarrow \Gamma_q$	$\frac{\partial}{\partial t}E_M < 0 \Rightarrow \Gamma_H$ (Boozer, '86)

- flux? what can be said about dynamics?

→ structural approach (this work): *What form must the PV flux have so as to dissipate enstrophy while conserving energy?*

General principle based on general physical ideas → useful for dynamical model

PV flux

→ PV conservation

mean field PV: $$\frac{\partial\langle q\rangle}{\partial t}+\partial_y\underbrace{\langle \upsilon_y q\rangle}_{}=\nu_0\partial_y^2\langle q\rangle$$

Γ_q : mean field PV flux

Key Point:
form of PV flux Γ$_q$ which dissipates enstrophy & conserves energy

selective decay

→ energy conserved $$E=\int\frac{\left(\partial_y\langle\phi\rangle\right)^2}{2}$$

$$\frac{\partial E}{\partial t}=\int\langle\phi\rangle\partial_y\Gamma_q=-\int\partial_y\langle\phi\rangle\Gamma_q \qquad \Rightarrow\enclose{circle}{\Gamma_q}=\frac{\partial_y\enclose{circle}{\Gamma_E}}{\partial_y\langle\phi\rangle}$$

→ enstrophy minimized $$\Omega=\int\frac{\langle q\rangle^2}{2}$$

$$\frac{\partial\Omega}{\partial t}=-\int\langle q\rangle\partial_y\Gamma_q=-\int\partial_y\left(\frac{\partial_y\langle q\rangle}{\partial_y\langle\phi\rangle}\right)\Gamma_E$$

$$\frac{\partial\Omega}{\partial t}<0\Rightarrow\enclose{circle}{\Gamma_E}=\mu\partial_y\left(\frac{\partial_y\langle q\rangle}{\partial_y\langle\phi\rangle}\right)$$

parameter TBD

$\langle\upsilon_x\rangle$

$$\Rightarrow\boxed{\Gamma_q=\frac{1}{\partial_y\langle\phi\rangle}\partial_y\left[\mu\partial_y\left(\frac{\partial_y\langle q\rangle}{\partial_y\langle\phi\rangle}\right)\right]}$$

general form of PV flux

Structure of PV flux

$$\Gamma_q = \frac{1}{\langle v_x \rangle}\partial_y\left[\mu\,\partial_y\left(\frac{\partial_y\langle q\rangle}{\langle v_x\rangle}\right)\right] = \frac{1}{\langle v_x \rangle}\partial_y\left[\mu\left(\underbrace{\frac{\langle q\rangle\partial_y\langle q\rangle}{\langle v_x\rangle^2}}+\underbrace{\frac{\partial_y^2\langle q\rangle}{\langle v_x\rangle}}\right)\right]$$

transport parameter calculated by perturbation theory, numerics…

drift and hyper diffusion of PV

<--> usual story : Fick's diffusion

relaxed state:

Homogenization of $\frac{\partial_y\langle q\rangle}{\langle v_x\rangle}$

characteristic scale $\ell_c \equiv \sqrt{\left|\frac{\langle v_x\rangle}{\partial_y\langle q\rangle}\right|}$

$\ell > \ell_c$: zonal flow growth

$\ell < \ell_c$: zonal flow damping (hyper viscosity-dominated)

Rhines scale $L_R \sim \sqrt{\frac{U}{\beta}}$

$\ell > L_R$: wave-dominated

$\ell < L_R$: eddy-dominated

What sets the “minimum enstrophy”

- Decay drives relaxation. The relaxation rate can be derived by linear perturbation theory about the minimum enstrophy state

$$\left.\begin{aligned} \langle q \rangle &= q_m(y) + \delta q(y,t) \\ \langle \phi \rangle &= \phi_m(y) + \delta\phi(y,t) \\ \partial_y q_m &= \lambda \partial_y \phi_m \\ \delta q(y,t) &= \delta q_0 \exp(-\gamma_{rel} t - i\omega t + iky) \end{aligned}\right\} \quad \begin{aligned} \gamma_{rel} &= \mu\left(\frac{k^4 + 4\lambda k^2 + 3\lambda^2}{\langle \upsilon_x \rangle^2} - \frac{8q_m^2(k^2+\lambda)}{\langle \upsilon_x \rangle^4}\right) \\ \omega_k &= \mu\left(-\frac{4q_m k^3 + 10 q_m k\lambda}{\langle \upsilon_x \rangle^3} - \frac{8q_m^3 k}{\langle \upsilon_x \rangle^5}\right) \end{aligned}$$

>0

relaxation

- The condition of relaxation (modes are damped):

$$\gamma_{rel} > 0 \;\Rightarrow\; k^2 > \frac{8q_m^2}{\langle \upsilon_x \rangle^2} - 3\lambda \;\Rightarrow\; \boxed{\frac{8q_m^2}{\langle \upsilon_x \rangle^2} > 3\lambda}$$

→ Relates q_m^2 with ZF and scale factor

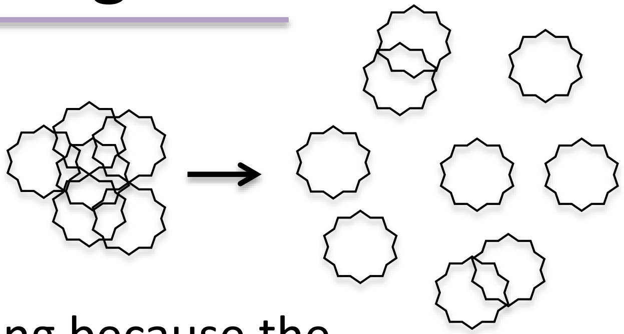
$$\Rightarrow \langle \upsilon_x \rangle^2 < \frac{3\lambda}{8q_m^2}$$

ZF can’t grow arbitrarily large

$$\Rightarrow 8\, q_m^2 > \langle \upsilon_x \rangle^2\, 3\lambda$$

the ‘minimum enstrophy’ of relaxation, related to scale

Role of turbulence spreading



- Turbulence spreading: tendency of turbulence to self-scatter and entrain stable regime

- Turbulence spreading is closely related to PV mixing because the transport/mixing of turbulence intensity has influence on Reynolds stresses and so on flow dynamics.

- PV mixing is closely related to turbulence spreading

$$\frac{\partial E}{\partial t} = \int \langle \phi \rangle \partial_y \Gamma_q = -\int \partial_y \langle \phi \rangle \Gamma_q \qquad \Rightarrow \boxed{\Gamma_q} = \frac{\boxed{\partial_y \Gamma_E}}{\partial_y \langle \phi \rangle}$$

condition of
energy conservation

→the gradient of $\partial_y\langle q\rangle/\langle v_x\rangle$, drives spreading

→ the spreading flux vanishes when $\partial_y\langle q\rangle/\langle v_x\rangle$ is homogenized

Inhomogeneous Mixing

→ Formation of corrugations, layering, etc

→ Focus: Stratified Fluid

See also: Dif-Pradalier Lecture

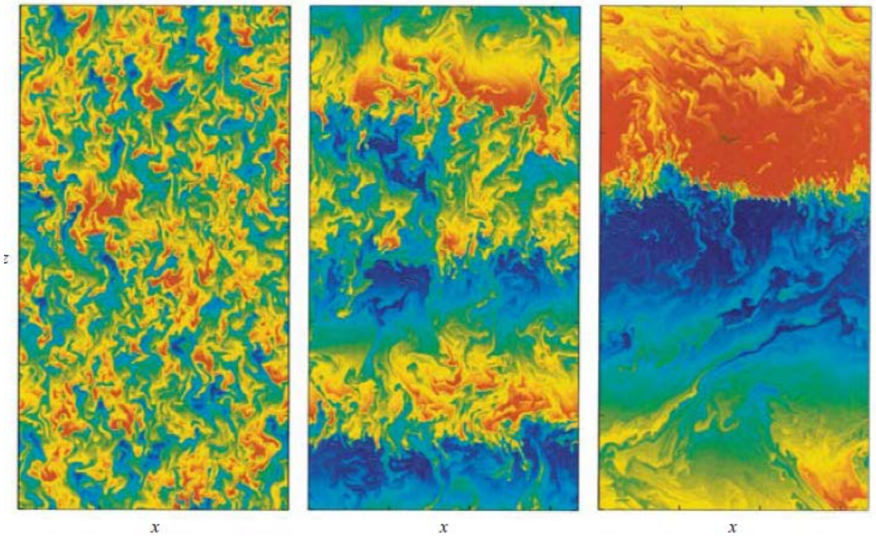
Weixin Guo

Robin Heinonen

Posters

Inhomogeneous Mixing

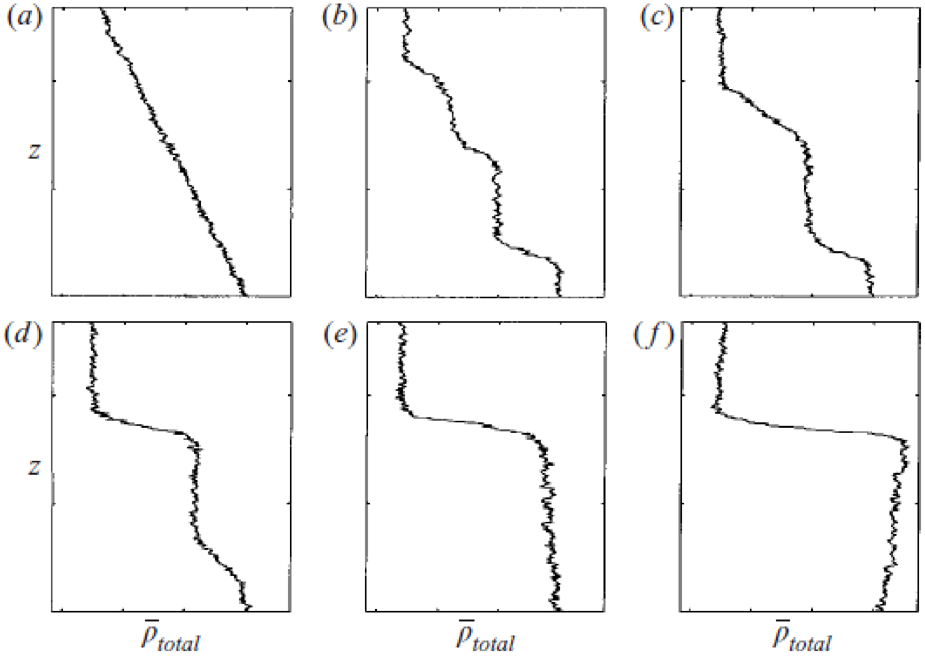
Example: Thermohaline Layer Simulation (Radko, 2003)



Sharp interface formed
colors → salt concentration

Modulation → Corrugations
→ Mergers → ”Barrier”

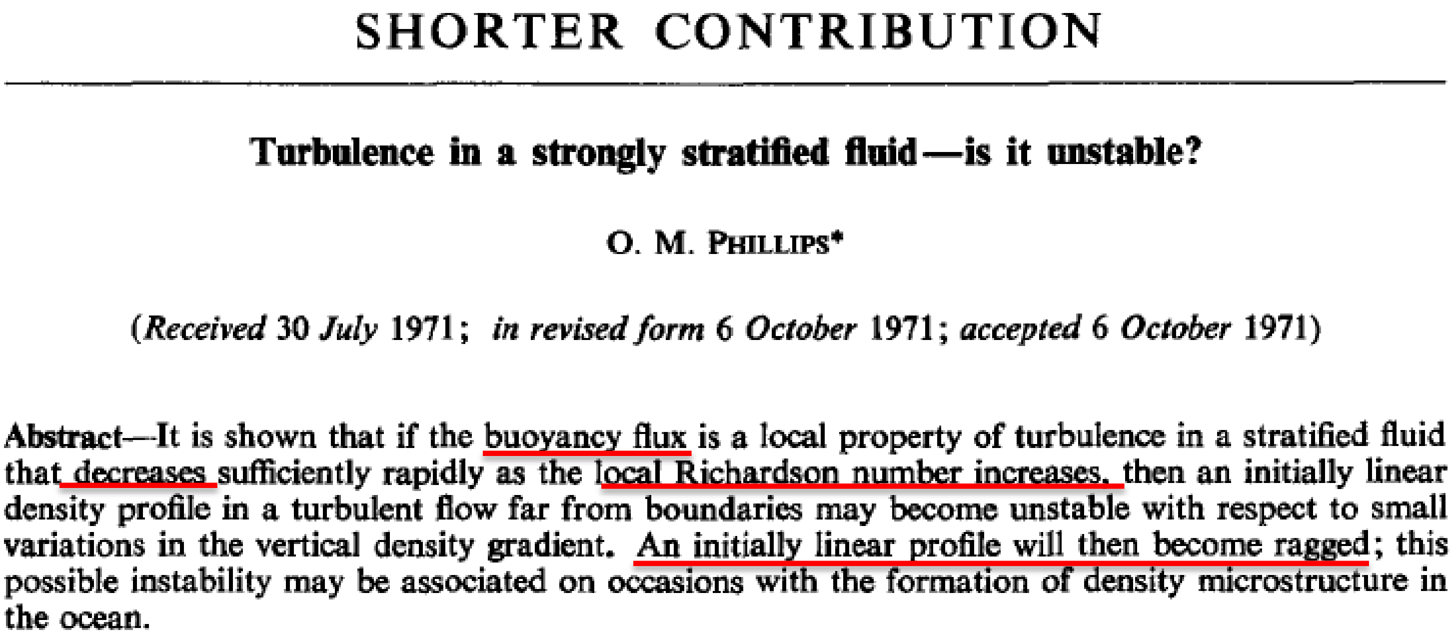
Corrugated Profile



Single layer

Corrugations formed, followed
by ‘condensation’ to single layer
→ Merger events

- Inhomogeneous Mixing → Corrugations? How? → Bistable Modulations
- Cf Phillips'72:



SHORTER CONTRIBUTION

Turbulence in a strongly stratified fluid—is it unstable?

O. M. PHILLIPS*

(*Received* 30 *July* 1971; *in revised form* 6 *October* 1971; *accepted* 6 *October* 1971)

Abstract—It is shown that if the buoyancy flux is a local property of turbulence in a stratified fluid that decreases sufficiently rapidly as the local Richardson number increases, then an initially linear density profile in a turbulent flow far from boundaries may become unstable with respect to small variations in the vertical density gradient. An initially linear profile will then become ragged; this possible instability may be associated on occasions with the formation of density microstructure in the ocean.

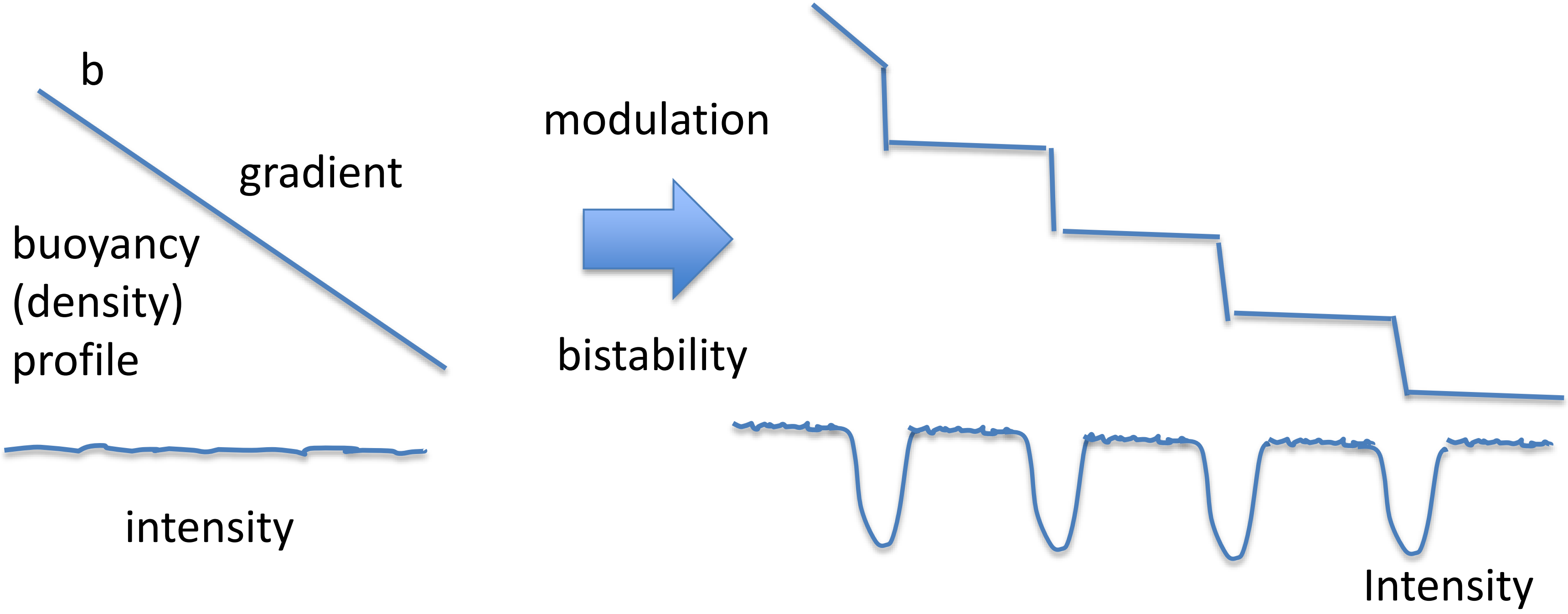
- Instability of mean + turbulence field requiring:

$\delta\Gamma_b/\delta Ri < 0$ → flux dropping with increased gradient

$\Gamma_b = -D_b \nabla b, Ri = g\nabla b/v'^2$ (Richardson #)

- Obvious similarity to transport bifurcation, but now a sequence of layers...

Mechanism: Inhomogeneous Mixing



b

gradient

buoyancy
(density)
profile

intensity

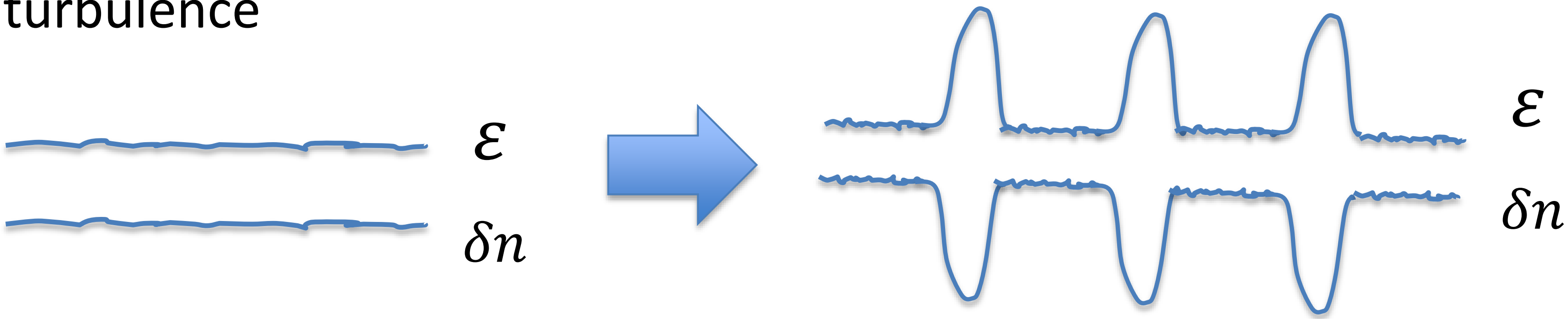
modulation

bistability

Intensity

Resembles: Caviton train for Langmuir turbulence

$$\delta n/n \approx -\varepsilon$$



ε

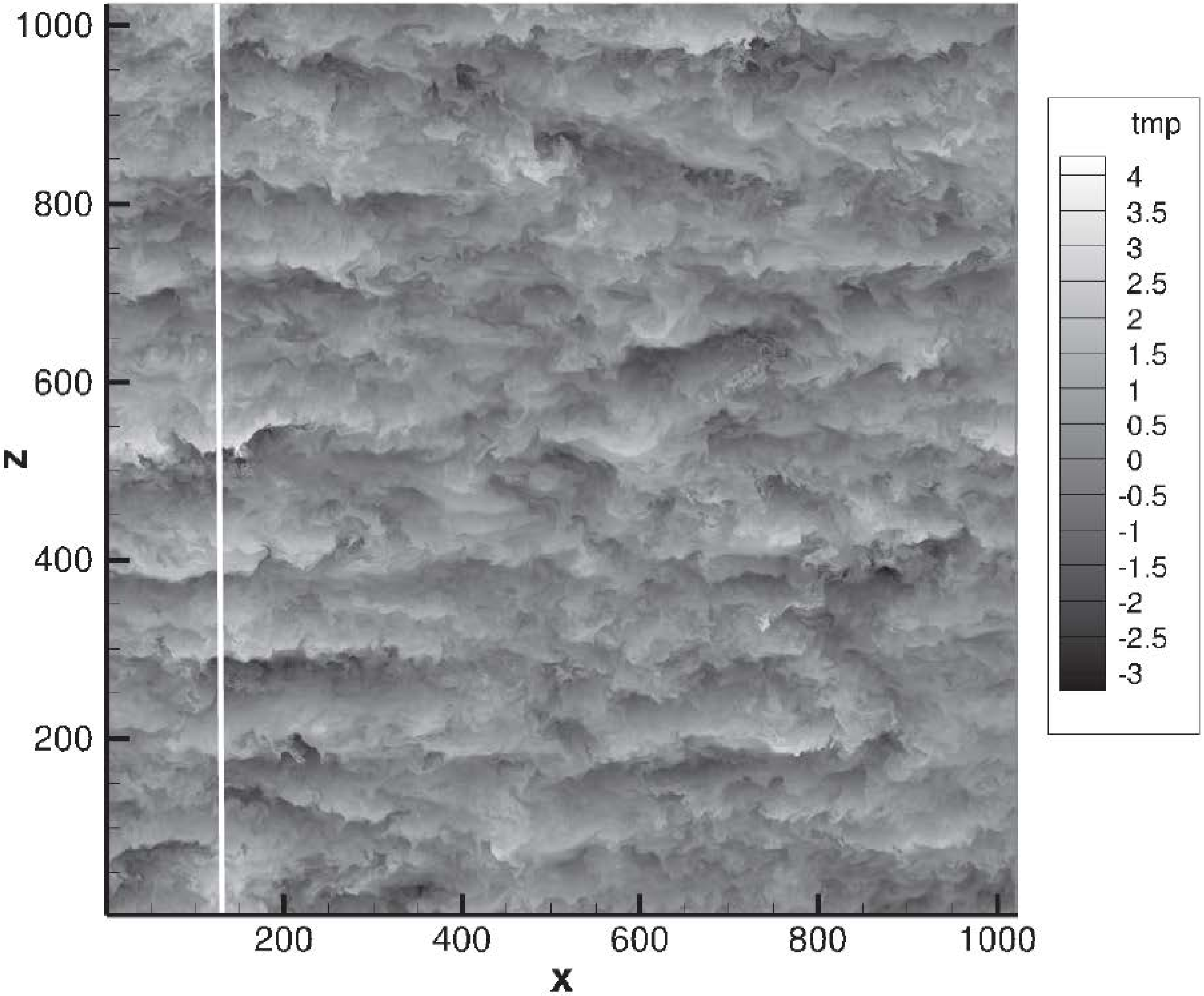
δn

ε

δn

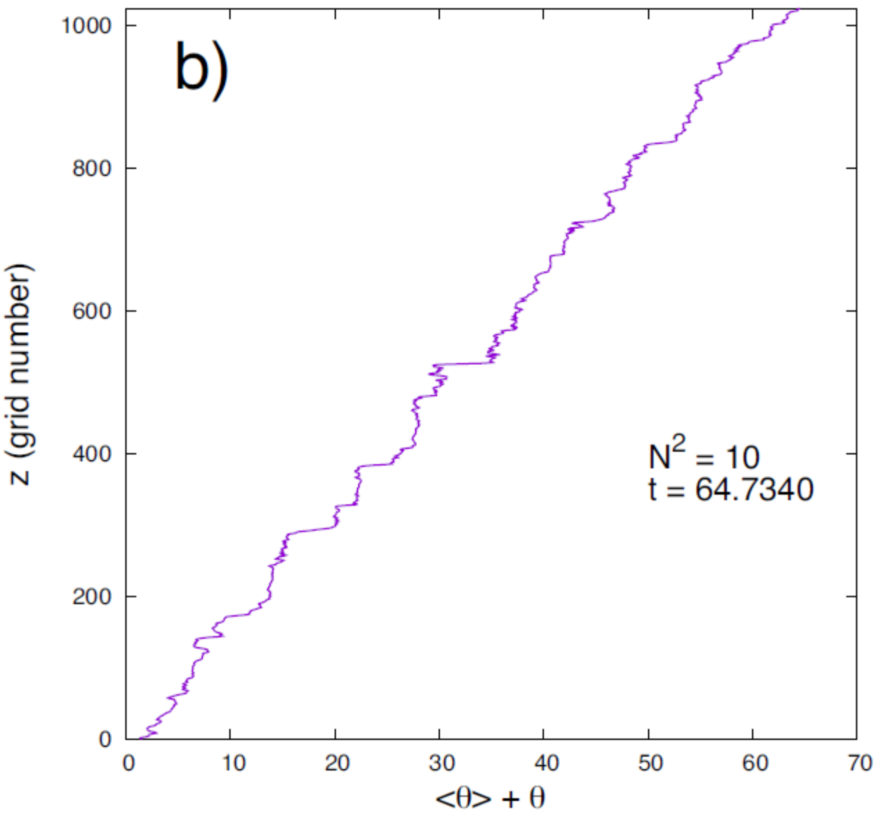
Corrugated Layering

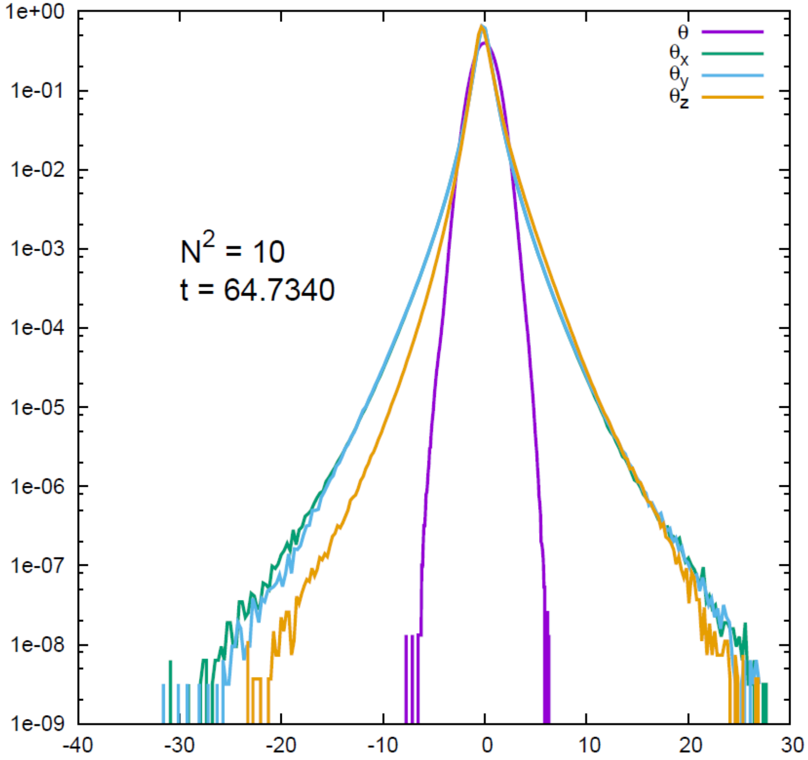
Kimura, et. al.



Contours of temperature fluctuations

Kimura, et. al.





Corrugated (total) temperature profile

PDFs of $T, \nabla T$.
θ_z is skewed.

- Is there a “simple model” encapsulating these ideas
- N. Balmforth, et al 1998 → corrugated profile in stirred, stable stratified turbulence (c.f. A. Ashourvan, P.D.; ’17, ‘18 for drift waves)
- Idea
  - bistable modulation
  - kinetic energy, mean density evolution
  - $D \sim \tilde{V}\, l_{mix} \sim (\varepsilon)^{1/2}\; l_{mix}$
  - l_{mix} → key

- What is the Mixing Length (l_{mix})?

- Stratified fluid: buoyance frequency ($\sim (g/L_\rho)^{1/2}$)

- $\frac{V(l)}{l} \sim N$ → l_{oz} Ozmidov scale $\boxed{Ku(l_{oz}) \to 1}$

~ small "stratified" scale

buoyancy production

$$\sim \frac{V^3}{l} \sim g\langle V \delta b \rangle \rightarrow \frac{1}{l_{oz}} \sim \left(\frac{\partial_z b}{\varepsilon}\right)^{1/2}$$

turbulent dissipation

- $\frac{1}{l_{mix}^2} \sim \frac{1}{l_f^2} \sim \frac{1}{l_{oz}^2}$

system →
2 scales, intrinsically

- So:

$$l_{mix}^2 = \frac{l_f^2 l_{oz}^2}{l_{oz}^2 + l_f^2}$$

$\rightarrow l_{oz}^2$

$l_{oz}^2 \ll l_f^2$

steep $\partial_z b$

$\sim (\varepsilon/\partial_z b)^{1/2}$

→ Feedback loop emerges, as l_{mix} drops with steepening $\partial_z b$

→ some resemblance to flux limited transport models

$$D = \varepsilon^{1/2}\, l_{mix}$$

$$\varepsilon = \langle \tilde{V}^2 \rangle$$

$$\frac{1}{l_{mix}^2} = \frac{1}{l_f^2} + \frac{1}{l_{oz}^2}$$

- Model

$$\partial_t b = \partial_z (D \partial_z b)$$

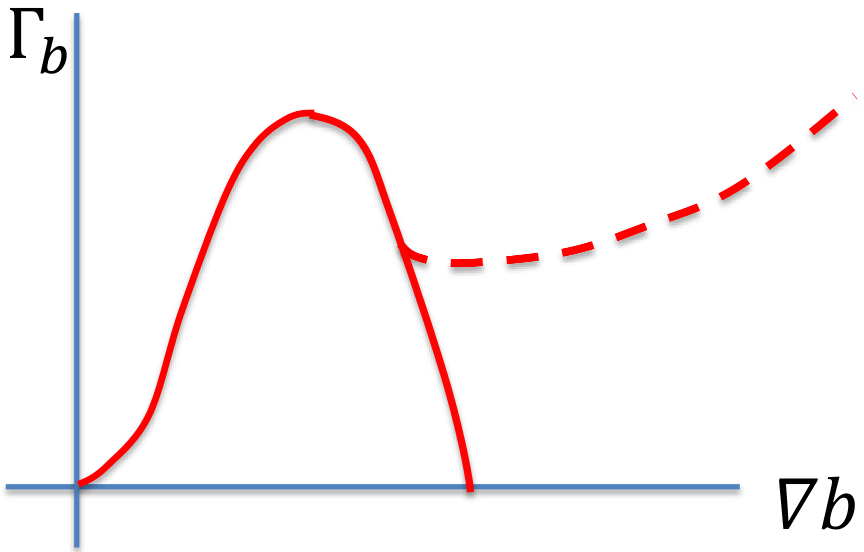
$$\partial_t \varepsilon = \overset{\text{spreading}}{\swarrow}\, \partial_z D \partial_z \varepsilon \; - \overset{\text{production}}{\swarrow}\, l\, \varepsilon^{\frac{1}{2}}\, \partial_z b \; - \underset{\nwarrow \text{ dissipation}}{\frac{\varepsilon^{\frac{3}{2}}}{l}} + F \longleftarrow \text{external forcing}$$

Energetics:

$$\partial_t \int [\varepsilon - zb] = 0 + \text{source, sink}$$

- <u>Some observations</u>
  - No molecular diffusion branch (“neoclassical H-mode”) Steep $\partial_z b$ balanced by dissipation, as l reduced
  - Step layer set by turbulence spreading (N.B. interesting lesson for case when D_{neo} feeble – i.e. particles)
  - Forcing acts to initiate fluctuations, but production by gradient ($\sim \partial_z b$) is the main driver
  - Gradient-fluctuation energy balance is crucial
  - Can explore stability of initial uniform $e, \partial_z b$ field → akin modulation problem

- The physics: Negative Diffusion



"H-mode" like branch
(i.e. residual collisional diffusion)
need not be input
- often feeble residual diffusion
- gradient regulated by spreading

- Instability driven by local transport bifurcation

- $\delta\Gamma_b/\delta\nabla b < 0$

➔ 'negative diffusion'

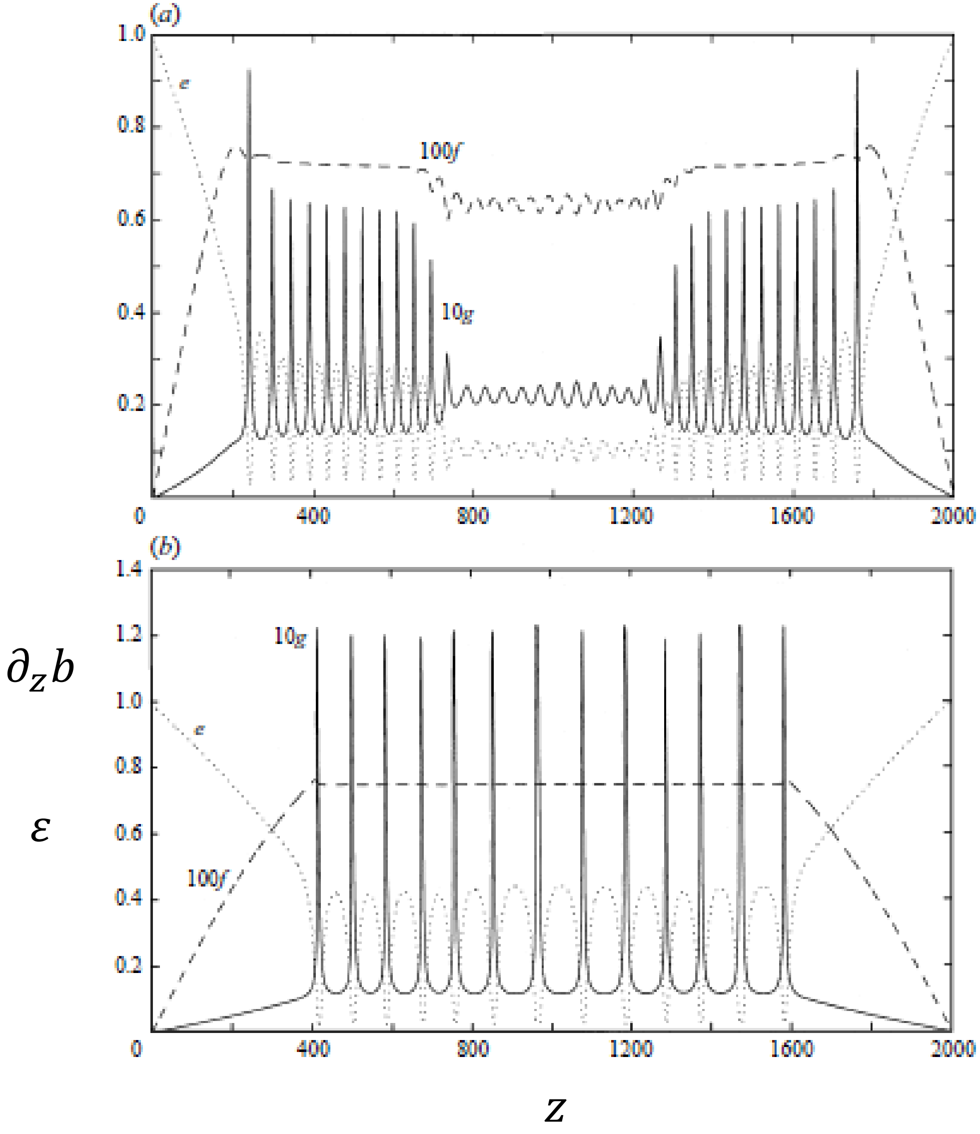
Negative slope
Unstable branch

- Feedback loop $\Gamma_b \downarrow \rightarrow \nabla b \uparrow \rightarrow I \downarrow \rightarrow \Gamma_{\mathrm{b}} \downarrow$

Critical element:
$l \rightarrow$ mixing length

Brings a new wrinkle: bi-stable mixing length models

- Some Results

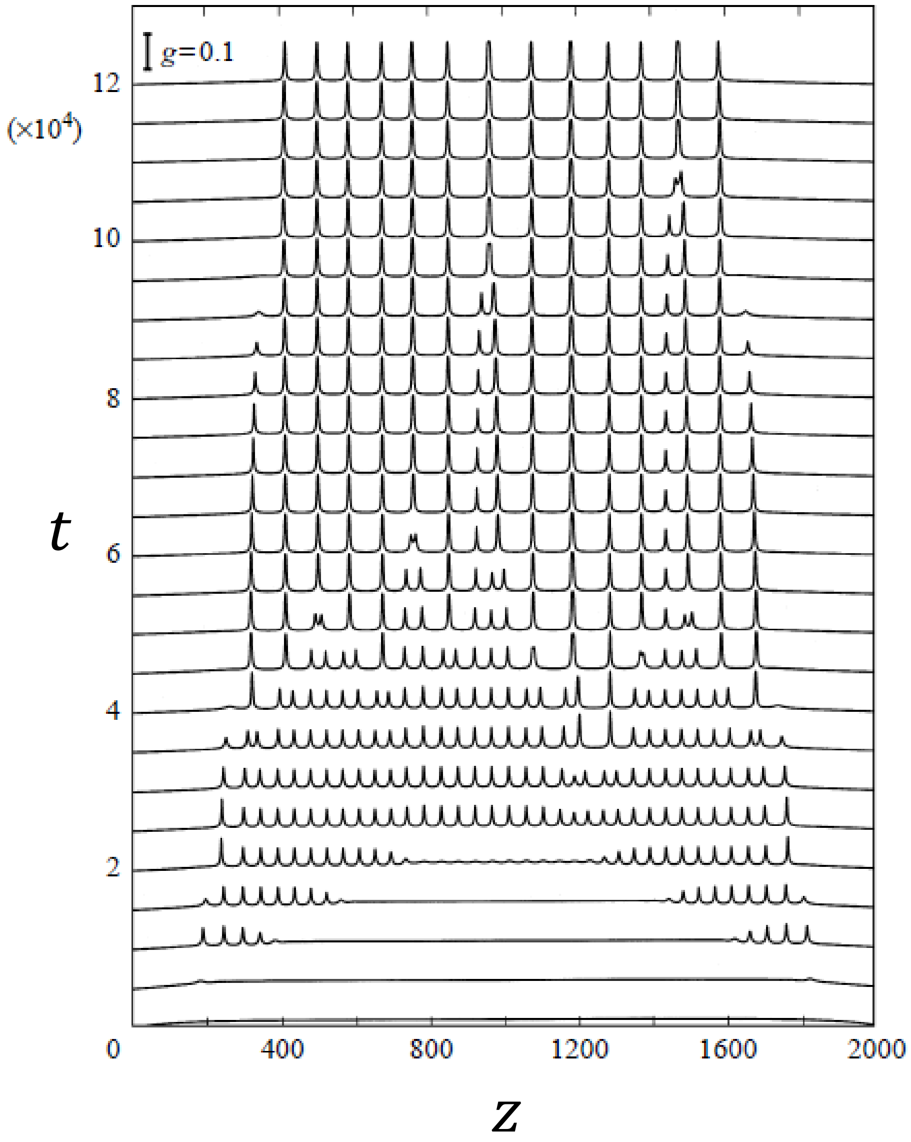


Plot of $\partial_z b$ (solid) and ε (dotted) at early time. Buoyancy flux is dashed → near constant in core (not flux driven)

Later time → more like expected "corrugation". Some condensation into larger scale structures has occurred.

- <u>Time Evolution</u>

$\partial_z b\ (\times 10^4)$



- Time progression shows merger process – akin bubble competition for steps
- Suggests trend to merger into fewer, larger steps
- Relaxation description in terms of merger process!? i.e. population evolution
- Predict/control position of final large step?

Inhomogeneous Mixing - Summary

- Highly relevant to MFE confinement
- Theory already extended to simple drift wave systems
- Bistable inhomogeneous mixing significantly extend concept of "modulational instability"
- Natural synthesis of:
  - modulational instability
  - transport bifurcation
- Defines a new, mesoscopic state

Discussion

- “Choppy Profiles” – TFTR?
- L-mode hysteresis in Q vs $\tilde{I}$, Q vs ∇T ?
- Will turbulence spreading saturate ZF for $\nu \rightarrow 0$?
- Statistical distribution of l_{mix} ?

Shameless Advertising:

- "Mesoscopic Transport Events and the Breakdown of Fick's Law for Turbulent Fluxes"

  T.S. Hahm, P.H. Diamond

  - in press, J. Kor. Phys. Soc., 50th Ann. Special Issue
  - preprint available

- This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Science, under Award Number DE-FG02-04ER54738