What is the mechanism for the emergence and sustainment of staircase structure?

Weixin Guo[1], P. H. Diamond[2], Arash Ashourvan[3]

1. J-TEXT Lab, HUST
2. CASS and Dept. of Physics, UCSD
3. PPPL, New Jersey; GA

*This research was supported by the U. S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, under Award Number DE-FG02-04ER54738

*This project was also supported by Graduate School, HUST and NSFC Grant Nos. 11675059 and 11305071

Gratitude

- **Mentors: P. H. Diamond and Lu Wang**
- **Teachers and fans during courses: Plasma Physics+Fluid Dynamics** 
- **Friends---in academic and daily life.**
  - ✓ **A. Ashourvan, Jiacong Li, Xiang Fan and R. J. Hajjar;**
  - ✓ **Qiming Hu, Jie Chen and Huiqian Wang.**

Acknowledgements slide

Outline

I. Research

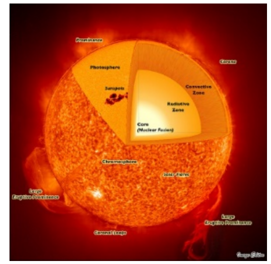
- **Background**
  - **Patterns formation in Drift wave-Zonal flow turb.**
    - **→Staircase vs other patterns**
  - **Mechanism: feedback loops**
    - **→Rhines vs Shearing**
- **Formation of staircase is sensitivity to**
  - ✓ **Parameters**
  - ✓ **B.C.**
  - ✓ **Initial condition**
- **Conclusions so far and furture pursuing**

II. Experiences (live communication)

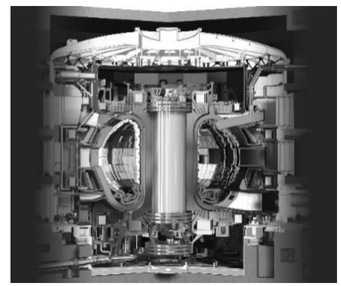
Suggestions on living in UCSD for the first time

Background

- Self-organizing, non-equilibrium nonlinear systems, formation of **patterns** is a common feature.
  - ✓ Stratification layers
  - ✓ Quasi-periodic flow
  - ✓ Staircase
  - ✓ E×B zonal flow (ZF) shear
  - ✓ ……



- ZF: regulate turbulent transport
  trigger L-H and ITBs



- Closing the feedback loop when **predators meet the preys**

Free energy source
∇P

Drift wave turbulence →Radial transport (bad for confinement)

Regulate by shear

Generation: Reynold stress

Zonal flow →No radial transport (bad for confinement)

"Ecology" between ZF and DW turb

What is the spatial structure?

- **Different spatial scale**
  - ✓ **Drift wave, micro-scale** ρ
  - ✓ **Mean flow, macro-scale** $L_n \sim a$
  - ✓ **Zonal flow, meso-scale** $\sqrt{\rho L_n}$

- ρ : Larmor radius
- $L_n = -n/\nabla n$: density scale length

- **Predicting turbulence and transport in states evolving from saturated instability is the goal**

→**Turbulent diffusivity** ***D*** **scaling with** ρ/a **is important in fusion!**

$$D = D_{\text{Bohm}}\left(\frac{\rho}{a}\right)^{\alpha}, 0 < \alpha < 1. \quad \begin{cases} \alpha = 0,\ Bohm\ scaling\ (Bad) \\ \alpha = 1,\ Gyro - Bohm\ scaling\ (Good) \end{cases}$$

- **The ecology of feedback should have some effects on scaling of spatial structure**

$$|\delta\varphi| \sim l_{mix} \sim \frac{1}{1 + v'_{E\times B}}$$

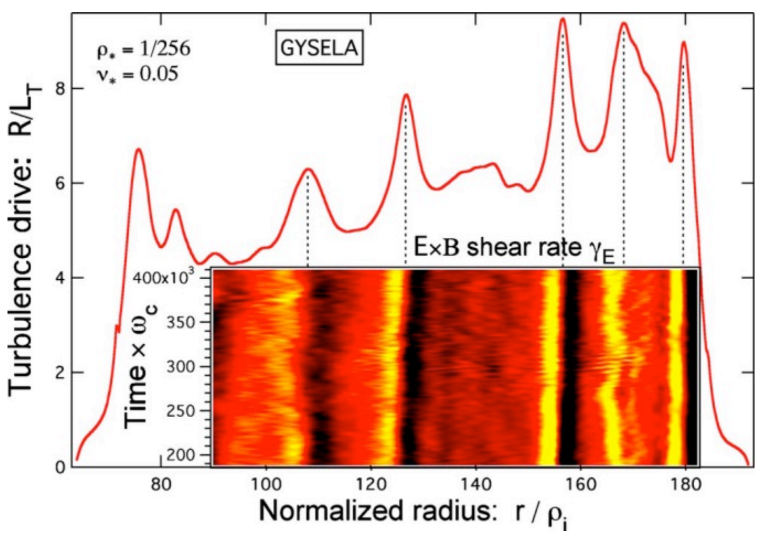
Staircase pattern in drift wave

- ✓ In drift wave, the E×B staircase (meso-scale).

  Q: E×B staircase $\stackrel{?}{=}$ Zonal flow

  A: E×B staircase is primarily produced by the zonal flow generation, and enhanced by the inphase mean flow variation.

- ✓ Quasi-regular *(spatial)* and long-lived *(temporal)* E×B flows with temperature corrugations coexist are observed numerically + experimentally



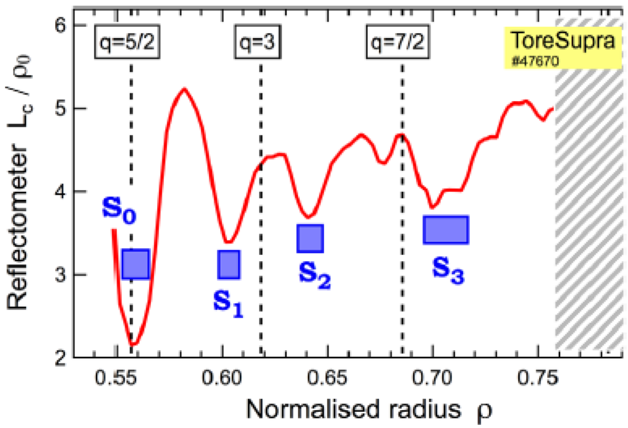


FIG. 3 (color online). The reflectometer coherence length plotted against radius shows clear experimental evidence of a staircase at locations S_1, S_2 and S_3, possibly also at S_0.

[G. Dif-pradalier, *et al*, Physical Rev. E **82**, 025401 (2010)]
[G. Dif-pradalier, *et al*, , Phys. Rev. Lett. **114,** 085004 (2015).]

Note: Coherent quasi-periodic shearing pattern !!!

Beauty of staircase pattern

- ✓ E×B staircase patterning → segregating regions where avalanching is dominant from regions where zonal flow concentrates (forming the staircase microbarriers)

"A natural and dynamic means for the simultaneous existence of these two antagonistic trends"

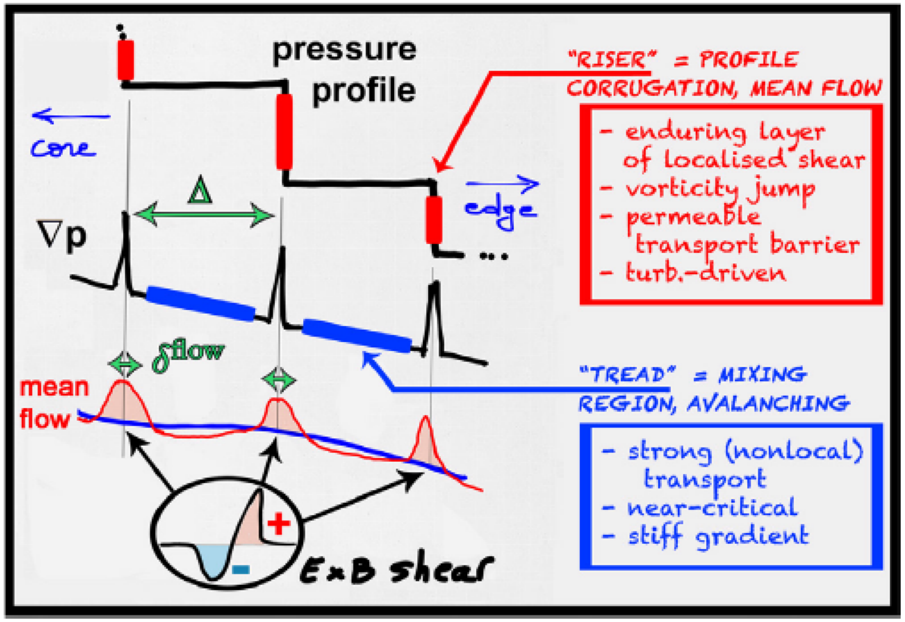


Figure 1. The E × B staircase, schematic view.

- **These zonal mean flows, of typical radial extent**

$$\delta^{\text{flow}} \sim 10\,\rho_s$$

- **The most probable step size of avalanches**

$$\Delta^{\text{stat}} \sim 40\,\rho_s$$

- **∇T corrugations temporally-averaged over 0.53 ms between 1030 a/c_s and 1343 a/c_s**

[G. Dif-Pradalier et al Nucl. Fusion 57 (2017) 066026]

Develop models!

Mechanism for the emergence and sustainment of staircase structure?

Mechanism for the formation and sustainment of staircase structure from H-W equation

- **Reduced models that self-consistently relate variations in mean plasma fields to fluctuation intensity !!!** [A. Ashourvan and P. H. Diamond, PoP 2017, PRE 2016]
  - ✓ **lower computational cost, supply essential understanding……..**
- **Two popular drift wave models : Hasegawa–Mima (H–M) and Hasegawa–Wakatani (H–W) model** [Hasegawa A and Mima K 1978 PoF 21 87] [Hasegawa A and Wakatani M 1983 PRL. 50 682]

(1) H-M equation: adiabatic and collisionless limit

✓ **2D fluid, H-M → potential vorticity (PV) conservation along fluid trajectories.**

$$q = n - u$$

n : normalized density
u : normalized vorticity

(2) H-W equation: collisional

✓ **Usually neglect the dynamical coupling along the magnetic field line → 2D system describing the collisional drift wave instability driven turbulence, which conserves the energy and PE**

(PE: potential enstrophy $\varepsilon = q^2/2$ **)**

✓ **Conservation of PE leads to the spontaneous generation of ZF by turbulence (Reynolds stress)**

Derivation details about H-W equation and physical understanding

- In strong magnetic field $B = B_z \vec{e}_z$, assume cold ion, the ion force balance Eq.

$$\frac{d\vec{u}}{dt} = -\frac{e}{M}\nabla\varphi + \frac{e}{M}\vec{u}\times\vec{B} - \cancel{\nabla P_i} - \nabla\Pi + \cancel{F}, \qquad \nabla\Pi = \mu\nabla^2\vec{u}$$

- Treat the drift velocity order by order → the lowest order is the ExB drift and the first order is the polarization drift and drift caused by viscosity

$$\mathbf{u}_0 = \mathbf{u}_E = -\nabla\phi\times\frac{\mathbf{B}}{B_0^2} \qquad \mathbf{u}_1 = \mathbf{u}_p + \mathbf{u}_{visc} = -\frac{1}{\omega_{ci}B_0}\frac{d\nabla\phi}{dt} - \frac{\mu}{\omega_{ci}B_0}\nabla^2(\nabla\phi)$$

- From the divergence free condition $\nabla\cdot\vec{J} = 0 \; with \; \vec{J} = \vec{J}_\perp + \vec{J}_\parallel$

$$\vec{J}_\perp = n|e|\vec{u}_1$$

$$\nabla_\parallel\phi + \eta\vec{J}_\parallel - \frac{1}{en_0}\nabla_\parallel p_e = 0$$

- Finally, the equation for evolution of $\nabla_\perp^2\phi$

$$\rho_s^2\frac{d}{dt}\nabla^2\phi = -D_\parallel\nabla_\parallel^2\left(\phi - \frac{n}{n_0}\right) + \nu\nabla^2\nabla^2\phi$$

Derivation details about H-W equation and physical understanding

- In the force balance equation for electron $\nabla \Pi = 0$ and treat $m_e n_e \frac{d\vec{u}}{dt} = 0$, but keeps electron-ion friction

$$\mathbf{F}_{e\|} = -m_e n_e \nu_{ei} \mathbf{u}_{e\|} = \frac{m_e \nu_{ei}}{e} \mathbf{J}_{\|}$$

- From the Ohm's law

$$\nabla_{\|}\phi + \eta \vec{J}_{\|} - \frac{1}{en_0}\nabla_{\|} p_e = 0$$

and the electron continuity equation

$$\frac{dn}{dt} + n\nabla_{\|} \cdot \vec{u}_{e,\|} = 0$$

$$\Rightarrow \quad \frac{d}{dt}n - D_0 \nabla^2 n = -D_{\|}\nabla_{\|}^2\left(\phi - \frac{n}{n_0}\right)$$

$$+ \quad \rho_s^2 \frac{d}{dt}\nabla^2\phi = -D_{\|}\nabla_{\|}^2\left(\phi - \frac{n}{n_0}\right) + \nu\nabla^2\nabla^2\phi$$

H-W equations !!!!

Derivation details about H-W equation and physical understanding

- When $D_{\parallel}k_{\parallel}^2/\omega \gg 1 \rightarrow \frac{\delta n}{n_0} = \frac{e\delta\phi}{T_e}$ i.e., electron is adiabatic. In the adiabatic and collisionless limit ⟹ reduce to the Hasegawa-Mima (H-M) equation

$$\frac{d}{dt}(\phi - \rho_s^2 \nabla^2 \phi) + v_* \partial_y \phi = 0$$

→ PV is conserved

✓ H–M model and the H–W) model are often used to investigate drift wave–ZF system

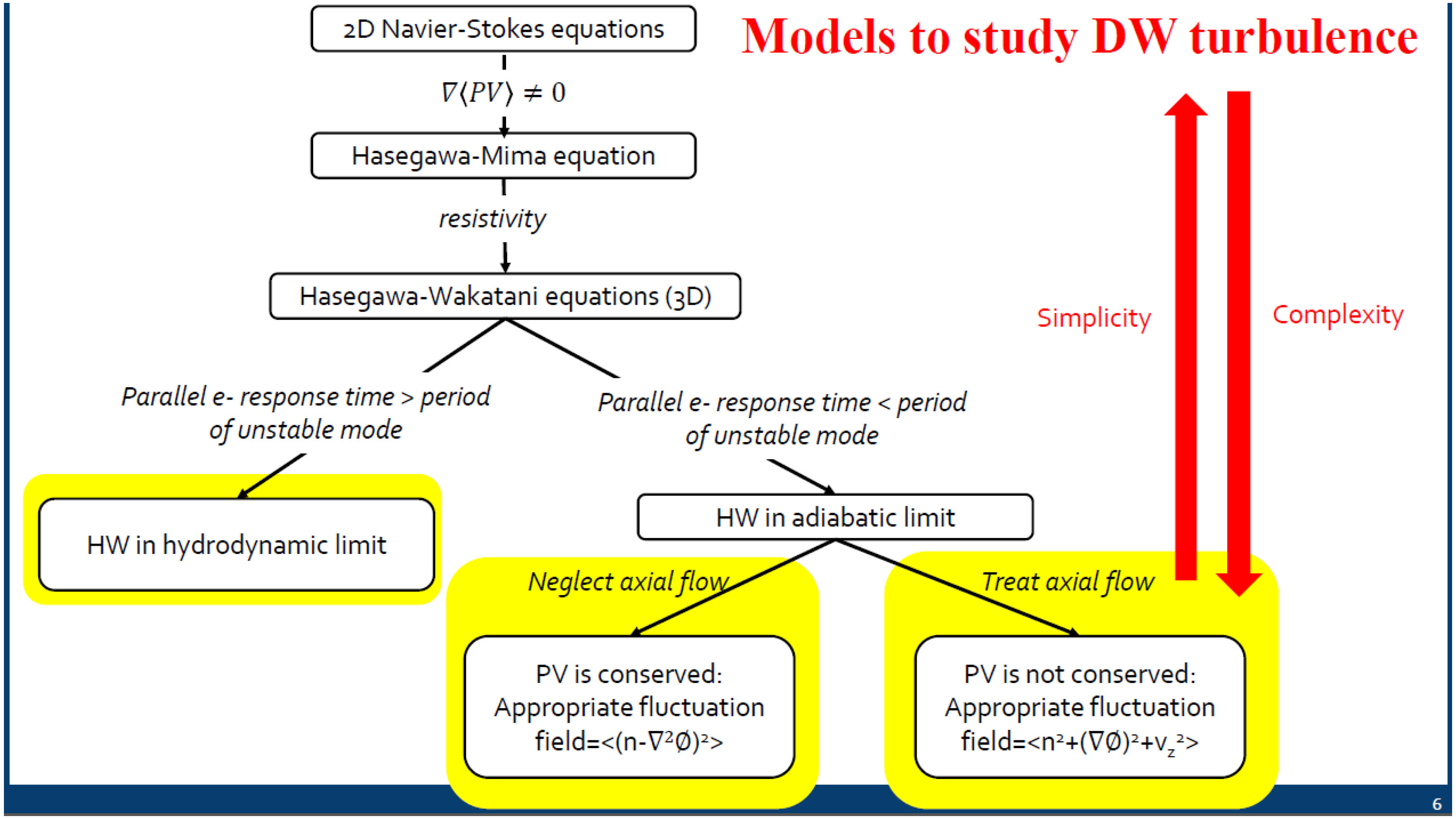
✓ H-W equation can be used to explain the generation of zonal-flow under the use of Taylor identity $\partial\langle v_r v_y\rangle/\partial r = \langle v_r \nabla_{\perp}^2 \phi\rangle$

$$\partial\langle v_y\rangle/\partial t = -\langle v_r \nabla_{\perp}^2 \phi\rangle - \nu\langle v_y\rangle$$

✓ The developing of this reduced model were widely investigated by professor Diamond

Development of the reduced model

- **Examples: (1) works done recently by doctor Rima Hajjar**



- **Examples: (2) works done recently by Jiacong Li**
  - ✓ **How electron DW generate the intrinsic** $v_{\|}$ **when** $s=0$
  - ✓ **The interaction between zonal flow and** $v_{\|}$

Bulid the reduced model to understand the mechanism of staircase based on H-W equation

- **The vorticity equation and the continuity equation**

$$\begin{cases} \frac{d}{dt}(\nabla^2\varphi) = \eta\nabla_\parallel^2(\log N - \varphi) + \mu_c\nabla_\perp^4\varphi, \\ \frac{d}{dt}\log N = \eta\nabla_\parallel^2(\log N - \varphi) + D_c\nabla_\perp^2 \log N. \end{cases} \qquad u \equiv \nabla_\perp^2\varphi,$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}_E.\nabla, \quad \mathbf{v}_E = \hat{z}\times\nabla\varphi. \quad u = \langle u\rangle + \delta u(x,y,z,t) \quad \mathbf{v} = \langle \mathrm{v}\rangle\hat{y} + \delta\mathbf{v}(x,y,z,t),$$

- **Quasi-linear equations for the mean density n and vorticity u**

$$\partial_t\langle u\rangle + \partial_x\langle\delta v_x\delta u\rangle - \mu_c\nabla_\perp^2\langle u\rangle = 0, \quad (11) \qquad \langle\delta v_x\delta u\rangle = (\chi - D_n)\partial_x\langle n\rangle - \chi\partial_x^2\langle \mathrm{v}\rangle$$

$$\partial_t\langle n\rangle + \partial_x\langle\delta v_x\delta n\rangle - D_c\nabla_\perp^2\langle n\rangle = 0. \quad (12) \qquad \langle\delta v_x\delta n\rangle = -D_n\partial_x\langle n\rangle + V_{\mathrm{pinch}},$$

- **The fluctuation equations are obtained as**

$$(\partial_t + \langle \mathrm{v}\rangle\partial_y)\delta n + \{\delta\varphi,\delta n\} + \delta v_x\partial_x\langle n\rangle - \mu_c\nabla_\perp^2\delta n = \eta\nabla_\parallel^2(\delta n - \delta\varphi),$$

$$(\partial_t + \langle \mathrm{v}\rangle\partial_y)\delta u + \{\delta\varphi,\delta u\} + \delta v_x\partial_x^2\langle \mathrm{v}\rangle - D_c\nabla_\perp^2\delta u = \eta\nabla_\parallel^2(\delta n - \delta\varphi).$$

- **Fluctuation PE** $\varepsilon = \langle\delta q^2\rangle/2$.**are obtained as**

$$\partial_t\langle\delta q^2\rangle + \partial_x\langle\delta v_x\delta q^2\rangle = -\langle\delta v_x\delta q\rangle\partial_x\langle q\rangle - \mu_\varepsilon\langle|\nabla_\perp\delta q|^2\rangle + P, \qquad \langle\delta v_x\delta q\rangle = -\chi\partial_x\langle q\rangle,$$

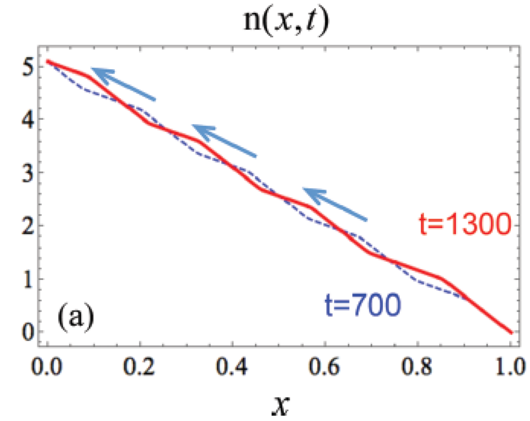
Bulid the reduced model to understand the mechanism of staircase based on H-W equation

- **Finally: mean density n, vorticity u, + turbulent potential enstrophy ε**

$$\partial_t n = \partial_x D_n \partial_x n + D_c \partial_x^2 n, \qquad (1)$$

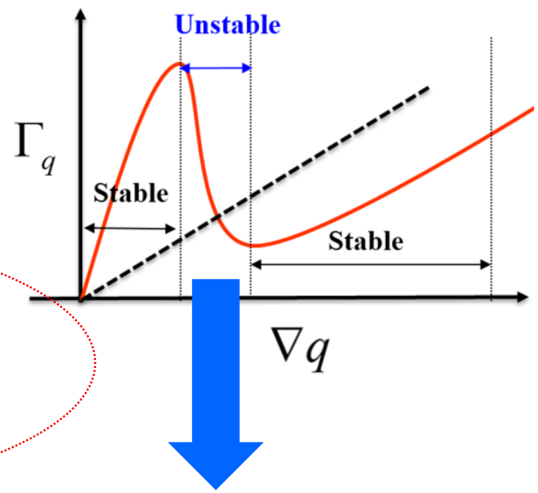
$$\partial_t u = \partial_x (D_n - \chi) \partial_x n + \chi \partial_x^2 u + \mu_c \partial_x^2 u, \qquad (2)$$

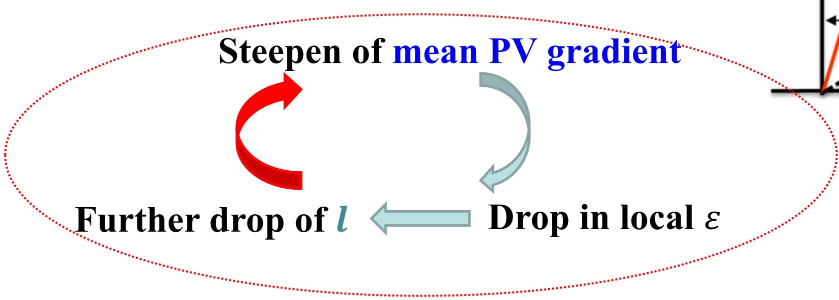
$$\partial_t \varepsilon = \partial_x D_\varepsilon \partial_x \varepsilon + \chi [\partial_x (n-u)]^2 - \varepsilon_c^{-1} \varepsilon^{3/2} + \mathscr{P}. \qquad (3)$$

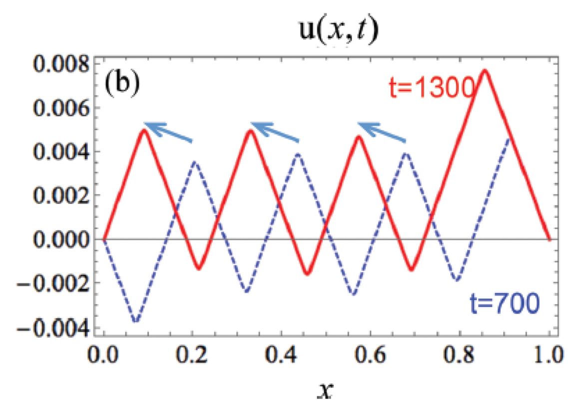


- $D_n \approx l^2 \frac{\varepsilon}{\alpha} \quad \chi(x) = c_\chi l^2 \frac{\varepsilon}{\alpha} \quad D_\varepsilon(x) \cong \beta l^2 \varepsilon^{1/2}$

$$l \rightarrow l_{mix} = \frac{l_0}{\left(1 + l_0^2 \left[\partial_x (n-u)\right]^2 / \varepsilon\right)^{\kappa/2}}$$







- **The positive feedback loop in the negative diffusion region drives the instabilities which lead to nonlinear feature formation in the mean profile. i.e., formation of density staircase and vorticity corrugation**

Further story

- Existence of the density staircase and vorticity corrugation is depends on the parameter and B.C.

  What kills the nonlinear features?
  →Scan the parameters.
  →Change the B.C.

- Is there exist other new feedback loop?

  →Shearing feedback loop

- Does the staircase structure can still be sustained or not if the external shear (macro-scale) exist in the plasma?

  →Modify the model
  →Run the modified model
  →Flux driven

Parameter dependence of staircase !!

Parameters \ Variation	Number of jumps
Suppression parameter $\kappa \neq 2$	**Reduce and kill**
Increase flow viscosity	Reduce
Increase particle viscosity	Reduce
Increase PE turbulence spreading multiplier β	Reduce
Increase Initial Density gradient	Firstly increase and then reduce
Increase production rate	Increase
Increase initial mean vorticity	Decrease

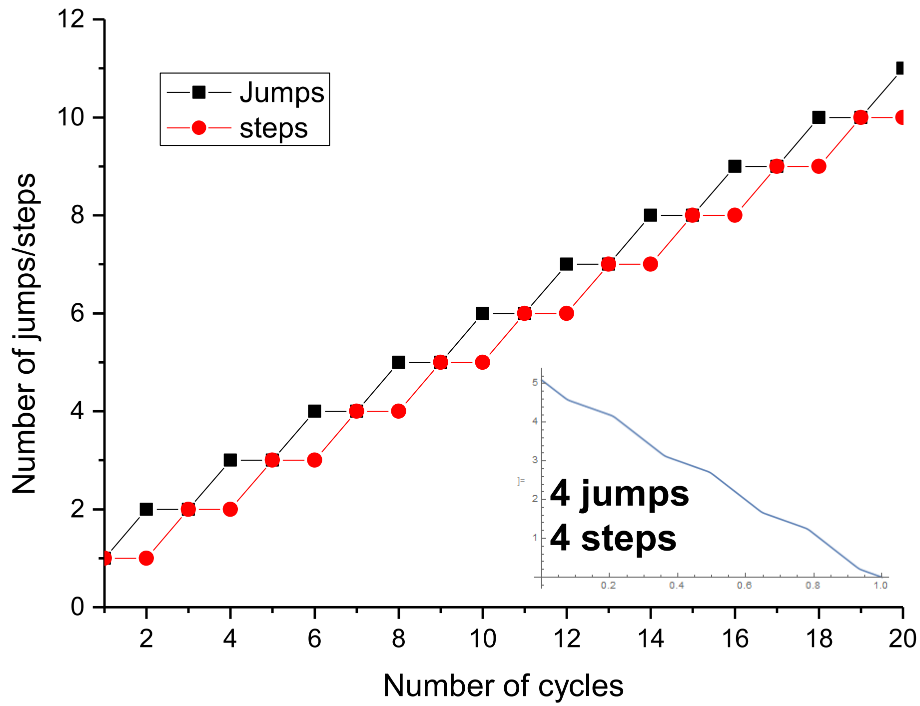
- **Vorticity corrugation disappear ⟹ Kill the staircase!**
- **Then, if we give a seed corrugation structure by changing the B.C. condition, what will happen ?????**

$$u(x, t=0) = 0; \quad u(0,t) = u(1,t) = 0. \implies u = a\sin(n\pi x) + b$$

a---background mean zonal shear;
b---constant mean shear

(1)Rhines: initial u = a sin($n\pi x$) + b
(a) a=1, b= 0.001, scan n

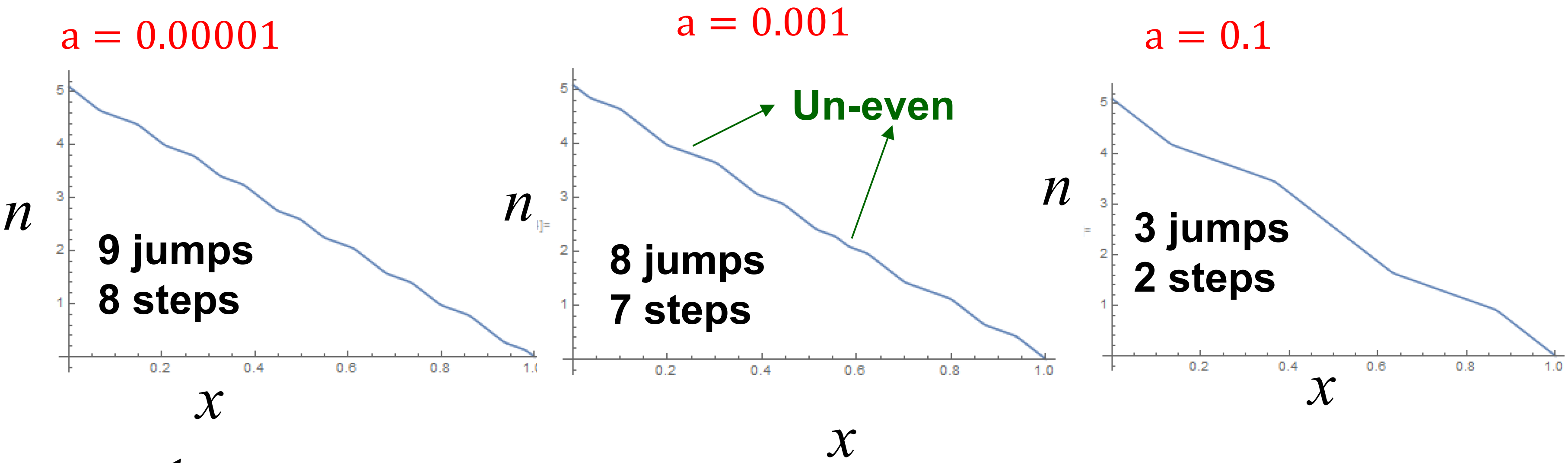
	Number of jumps N_{jump}	Number of steps N_{step}
$\sin(\pi x)$ +0.001	1	1
$\sin(2\pi x)$ +0.001	2	1
$\sin(3\pi x)$ +0.001	2	2
$\sin(4\pi x)$ +0.001	3	2
$\sin(5\pi x)$ +0.001	3	3
$\sin(6\pi x)$ +0.001	4	3
$\sin(7\pi x)$ +0.001	4	4
$\sin(8\pi x)$ +0.001	5	4
$\sin(9\pi x)$ +0.001	5	5
$\sin(10\pi x)$ +0.001	6	5
$\sin(11\pi x)$ +0.001	6	6
$\sin(12\pi x)$ +0.001	7	6



- **Specially, when n=7, Same to the wavelength in A&D paper**
- **What's more, also three stages!**

Microscale instabilities → NL mesoscale stru. → Merger → Migration

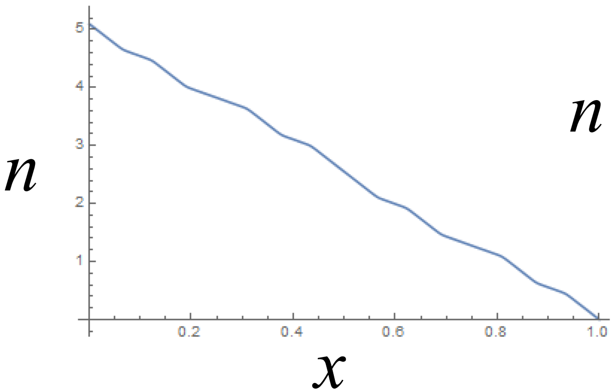
(1) Rhines: initial $u = a\sin(n\pi x) + b$
(b) n =4, b= 0.001, scan a



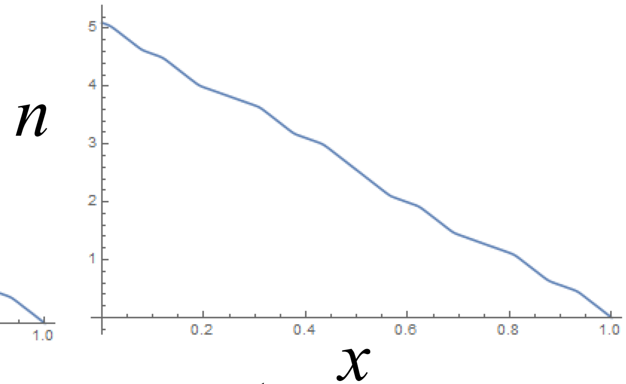
- a ↗ ⟹ both N_{jump} and N_{step} ↙
- Un-even staircase and be evident when a~b， which suggests the interplay between mean zonal shear and mean constant shear.
- Stronger zonal background shear a will suppress the drift wave and finally kills the staircase

(1) Rhines: initial $u = a\sin(n\pi x) + b$
(c) n = 4, a = 0.01, scan b

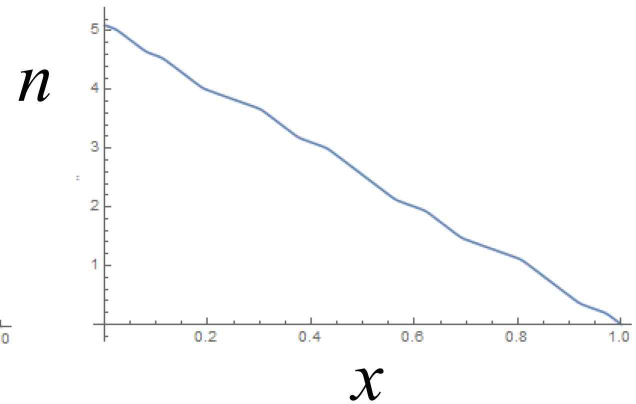
b = 0.0001



b = 0.01



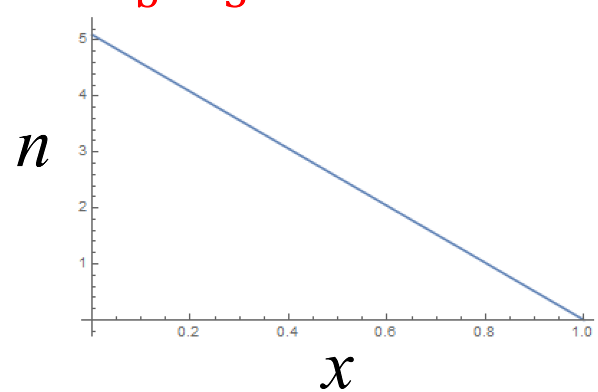
b = 1



- **Different with a, When b≤1, b ↗ ⟹ do not have the significant effects on staircase structure. It indicates that the density staircase structure is determined by the zonal mean shear.**

- **Both stronger mean zonal background shear a and mean constant shear b will finally kill the density staircase by suppressing the drift wave**

b = 3



(2)Shearing feedback loop

- **In the case of mean poloidal shears, the following form of mixing length is suggested** [H. Biglari, P. H. Diamond, and P. W. Terry, 2(1), 1–4 PoP 1990]

$$l_{mix}^2 = \frac{l_0^2}{\left[1 + \left(\bar{v}_y'\right)^2 \tau_c^2\right]^{\kappa}}$$

- l_0: the mixing scale without v_y
- τ_c: fluctuation correlation time
- $\bar{v}_y'$: poloidal shear rate

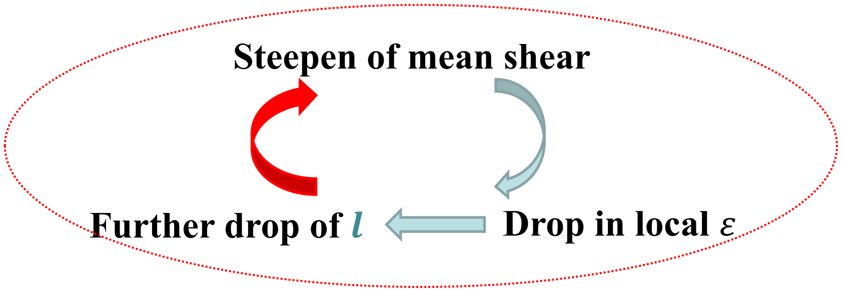
- **The correlation time is given** [R. J. Hajjar, P. H. Diamond, G. R. Tynan, PoP **25**, 022301 (2018)]

$$\tau_c = \left[\frac{\left(\bar{v}_y'\right)^2 \varepsilon}{l_0^2}\right]^{-1/4}$$

- ε: turbulent PE

- **Then, we have the new mixing lengthen and new feedback loop as**

$$l_{mix} = \frac{l_0}{\left[1 + \frac{1}{l_0}\frac{u}{\sqrt{\varepsilon}}\right]^{\kappa/2}}$$



- Note: $u = \bar{v}_y'$

(2)Shearing feedback loop

- **While, for the same parameters and same B.C. condition as in** *Ashourvan and Diamond, PoP 2017, PRE 2016*, **the staircase can not be reproduced !!!**

- **Rhines vs Shearing**

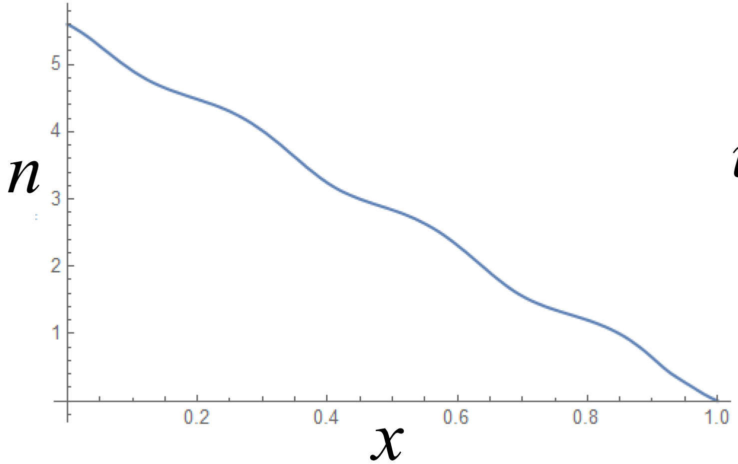
  $\partial_x(n-u)$ **vs** u **!!!!**

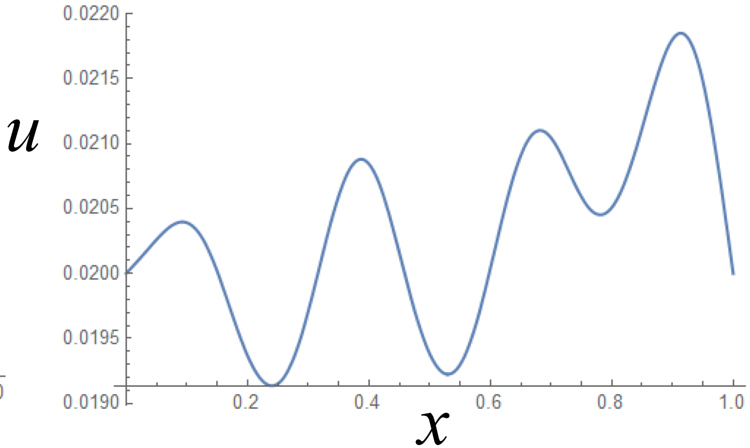
- **Only keep the density gradient**

- **Then, change the B.C. to** $\mathrm{u} = \mathrm{asin}(n\pi x) + \mathrm{b}$ **to give a zonal mean shear**

Shearing feedback loop: κ =2, initial u = $0.02\sin(n\pi x)$ +0.01

- ✓ **Lower D_c , μ_c and ε_c vs Rhines, increase the B.C. for turbulent PE,**
  $\varepsilon(x, t=0) = \varepsilon(0,t) = \varepsilon(1,t) = \varepsilon_i$ **=0.2048; n=7**





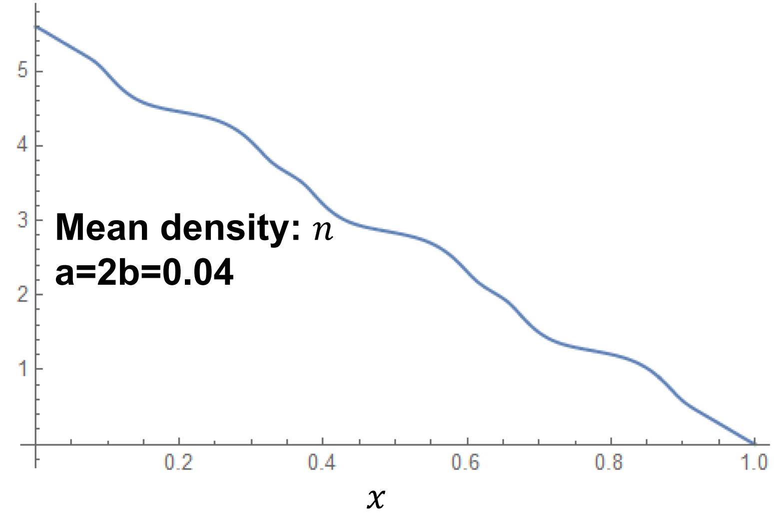
- ✓ **Scan initial density gradient** $n(x, t=0) = -g_i x$, **and period n**

	Number of jumps formed at t=1
Increase $g_i \in [1,6]$ with n=4	✓ Firstly increase and then decrease ✓ Widen the the initial density region for generating staircase by increasing the smallest g_i
Increase n∈ [1,10] with g_i=5.6	✓ Firstly increase and then decrease

Shearing feedback loop: $\kappa=2$, initial $\mathrm{u} = a\sin(7\pi x)$ +b

✓ **Scan** $\mathrm{u} = a\sin(7\pi x)$ +b

	Number of jumps N_{jump} formed at t=1
Increase a ∈[0.001, 0.01] with b=0.01	✓ The increase of a is beneficial for staircase ✓ Further increase of a just makes the jump and step characteristic more evident rather than increases N_{jump} too much
Increase b ∈ [0, 0.05] with a=0.04	✓ Finite b ≈ is **necessary** for the formation of staircase structure ✓ The increase of b will **increase N_{jump}** and can also cause the staircase structure become **not very uniform** ✓ **When b>a, staircase disappear**



Conclusions

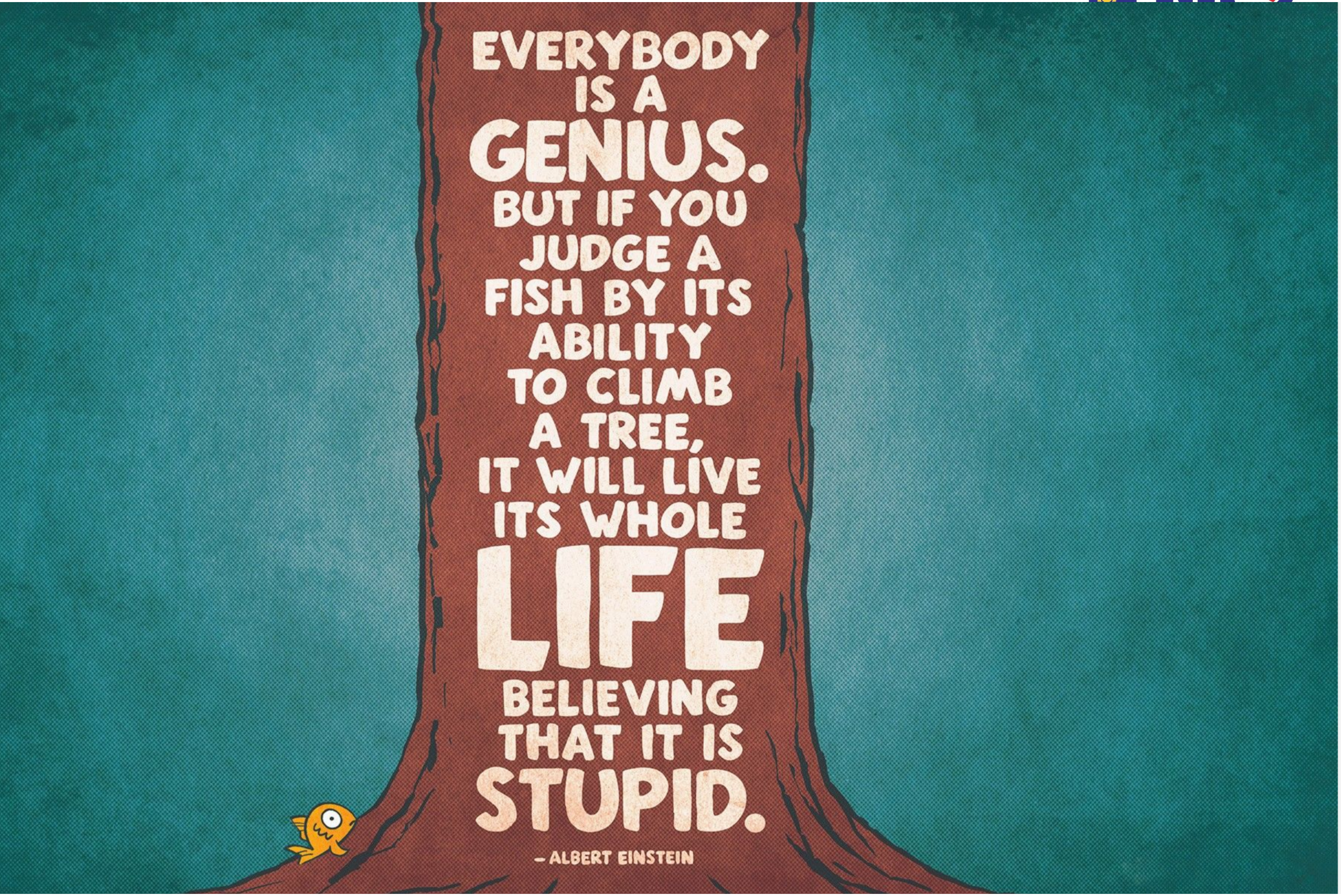
- **The the emergence and sustainment of staircase structure are sensitive to :**
  - ✓ **Feedback loop**
    - **Recovered and sustained the staircase for Rhines scale feedback**
    - **Sustained staircase is not observed for shearing feedback**
    - **Density gradient is essential to staircase formation:**

      $\nabla n \uparrow \longrightarrow D(\nabla n) \downarrow$
  - ✓ **Drive**
    - **Steepening ∇n and increasing the production rate protect staircase;**
    - **While, increasing viscosity, PE turbulence spreading β and mean vorticity weaken the staircase.**
  - ✓ **Initial condition**
    - **Increase initial mean vorticity (mean shear) weakens the staircase by suppressing drift wave;**
    - **Oscillating initial condition persist footprint on staircase pattern**
    - **Inhomogeneous staircase due to the interplay of zonal shear and mean shear**

Future works---Looking forward for another possible opportunity

- **Understand the feedback loops**
  →Staircase emergence via Rhines scale with ∇n only??
- **Why shearing feedback is not working?**
- **How to sustain the staircase?**
  →Flux driven studies
- **Understand these sensitivities more carefully and deeply!**
- **Why a wider zonal modulation pattern is not observed?**
- **Staircase vs ITB**
- **Finish the modification of the reduced model to study the external shear**

…….


EVERYBODY IS A GENIUS. BUT IF YOU JUDGE A FISH BY ITS ABILITY TO CLIMB A TREE, IT WILL LIVE ITS WHOLE LIFE BELIEVING THAT IT IS STUPID.
– ALBERT EINSTEIN

Thank you very much for your attention!

Thanks for all the sweet caring!