A closer look at turbulence spreading: how bistability admits intermittent, propagating turbulence pulses

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Introduction

- We introduce a new model for turbulence spreading in MFE plasma, an important phenomenon that delocalizes the relation between fluctuation intensity and temperature gradient
- Unlike conventional models, this model
  1. Accounts for observed hysteresis in the fluctuation intensity
  2. Predicts significantly stronger delocalization, via ballistic spreading into the stable zone
  3. Supports subcritical spreading of turbulence
- It also **(a)** serves as physical model for avalanching by supporting intermittently propagating turbulent excitations and **(b)** provides a quantitative estimate for the threshold for such pulses to propagate

What is turbulence spreading?

- Phenomenon in which turbulent fluctuations propagate radially [Garbet et al., 1994, Diamond and Hahm, 1995]
- Fluctuations can penetrate into linearly stable zone and excite turbulence there [Hahm et al., 2004, Naulin et al., 2005]
- Closely related to avalanching, transport barrier/staircase formation

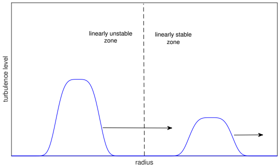

Figure Cartoon depicting a turbulence pulse propagating into the stable zone and exciting turbulence there.

Why is turbulence spreading important?

- Believed to be key actor in various nonlocality phenomena [Ida et al., 2015]
- Crucial: spreading results in the fluctuation intensity being influenced by dynamics outside of the turbulence correlation length
- Result: fluctuation level, heat flux have *nonlocal dependence* on driving gradient, e.g.

$$Q(r) = -\chi \nabla T(r) \longrightarrow Q(r) = -\chi \int dr' \, K(r, r') \nabla T(r')$$

- Spreading also believed to be involved in the observed breakdown of gyro-Bohm transport scaling [Lin and Hahm, 2004]

Conventional wisdom: Fisher fronts

- How to model spreading? Simplest, most common model is based on Fisher equation for normalized turbulence intensity I:

$$\partial_t I = \underbrace{\gamma_0 I}_{\text{local lin. growth/decay}} - \underbrace{\gamma_{nl} I^2}_{\text{local nonlin. coupling to dissipation}} + \underbrace{\partial_x (D(I) \partial_x I)}_{\text{nonlin. diffusion of turb. energy}}$$

- Typically take $D(I) = D_0 I$
- If lin. stable ($\gamma_0 < 0$): single stable root at $I = 0$ (low turbulence)
- If lin. unstable ($\gamma_0 > 0$): unstable root at $I = 0$, stable root at $I = \gamma_0 / \gamma_{nl}$ (high turbulence)—quadratic term saturates growth

Fisher fronts (cont'd)

- Upshot: if supercritical, turbulent fluctuations grow into traveling waves connecting the two roots
- Wavefronts propagate at constant speed

$$c = \sqrt{\frac{D_0 \gamma_0^2}{2\gamma_{nl}}}$$

- If subcritical, all fluctuations decay to $I = 0$ exponentially in time

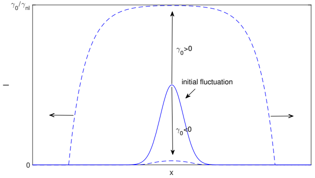

Figure Initial fluctuation in Fisher will grow into a wave and spread if $\gamma_0 > 0$ or decay to 0 if $\gamma_0 < 0$

How does Fisher do?

- Correctly predicts ballistic spreading, reasonable success predicting propagation speed
- However: penetration into stable zone is *weak*. Turbulence level decays exponentially to finite depth depth $\lambda \sim \sqrt{D_0/\gamma_{nl}}$, i.e. just a few correlation lengths at most [Gürcan et al., 2005]
- Suggests Fisher may be insufficient to explain nonlocality!
- Also, no possibility of spreading in subcritical zone: not a model of avalanching!

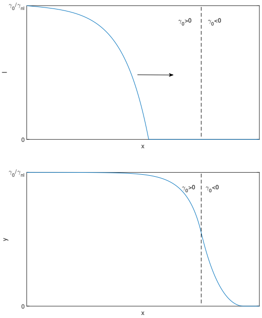

Figure A wave develops in the unstable zone and penetrates a short depth into the stable zone

Nail in the coffin: hysteresis in fluctuation intensity

- Experiments have clearly demonstrated hysteresis between flux/gradient *and* fluctuation intensity/gradient in the L-mode [Inagaki et al., 2013]
- Hysteresis strongly suggests *bistability* in the fluctuation intensity
- Fisher is unistable: cannot account for this!

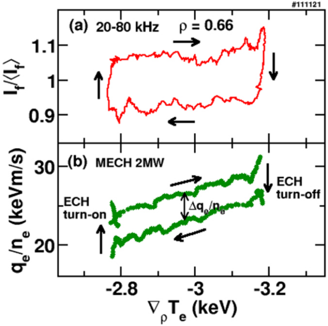

Figure Inagaki *et al.* 2013. Hysteresis!

A new model is born

- We thus propose a new phenomenological model equation for turbulence spreading

$$\partial_t I = \gamma_1 I + \gamma_2 I^2 - \gamma_3 I^3 + \partial_x (D(I) \partial_x I) \qquad (*)$$

  where again $D(I) = D_0 I$
- Motivation: simplest, generic 1D model incorporating bistability (thus accounting for hysteresis). Other forms possible, but qualitative features should be the same!
- In the spirit of [Barkley et al., 2015] model for onset of turbulence in pipe flow, also [Gil and Sornette, 1996] Landau-Ginzburg model for avalanching
- Roughly anticipate $\gamma_i \sim \omega_*$, $D_0 \sim \chi_{GB} \sim c_s \rho_i^2 / a$

Physical justification: whence bistability?

- [Guo and Diamond, 2017] showed that temperature profile corrugations can contribute an additional nonlinear drive, modifying Fisher equation to

$$\partial_t I = \gamma_0 I + \gamma_{corr} I^{3/2} - \gamma_{nl} I^2 + \partial_x (D(I) \partial_x I)$$

- Essentially same as cubic equation $(*)$
- Corrugations observed in GK simulation [Waltz et al., 2006]
- Physics of $I^{3/2}$ term: temperature gradient fluctuations can cause critical gradient to be locally exceeded, driving turbulence, but mean square gradient fluctuations themselves scale linearly with turbulence intensity

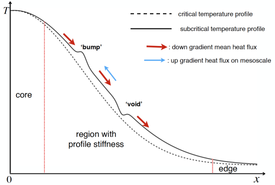

Figure Profile corrugations ('bumps' or 'voids') can cause the critical gradient to be exceeded locally

Summary of model regimes

regime	stable roots	unstable roots	waves	comments
$\gamma_1 > 0$	I_+	0	forward-propagating	unistable similar to Fisher
$\gamma_1 < 0$ $\lvert\gamma_1\rvert\gamma_3/\gamma_2^2 < 15/64$	0, I_+	I_-	foward-propagating	$\alpha < \alpha^*$ turbulent root abs. stable
$\gamma_1 < 0$ $15/64 < \lvert\gamma_1\rvert\gamma_3/\gamma_2^2 < 1/4$	0, I_+	I_-	receding	$\alpha > \alpha^*$ turbulent root metastable
$\gamma_1 < 0$ $\lvert\gamma_1\rvert\gamma_3/\gamma_2^2 > 1/4$	0	none	none	"strong damping"

Table Summary of features of the various parameter regimes in cubic model. Here $I_\pm = (\gamma_2 \pm \sqrt{\gamma^2 + \gamma_1\gamma_3})/2\gamma_3$.

Note: in the bistable case we can rewrite the equation in the simpler form

$$\partial_t I = f(I) + \partial_x(D(I)\partial_x I)$$

with $f(I) = \gamma I(I-\alpha)(1-I)$

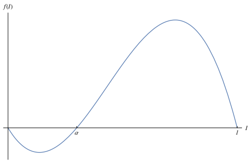

Figure Plot of the reaction function $f(I) = I(1 - I)(I - \alpha)$

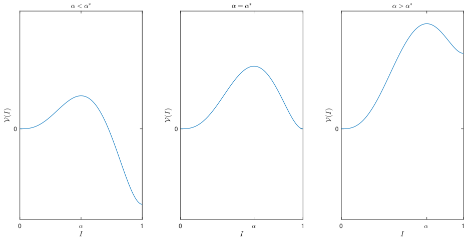

Figure Plot of potential part of free energy $\mathcal{V}(I) = -\int_0^I dI'\, D(I) f(I)$ for different α. Note the minima at $I = 0, 1$ and the barrier at $I = \alpha$

Spreading "free energy"

- Dynamics governed by dissipation of free energy: can rewrite in variational form

$$D(I)\partial_t I = -\frac{\delta \mathcal{F}}{\delta I}$$

with free energy functional

$$\mathcal{F} = \int dx \; \underbrace{[\frac{1}{2}(D(I)\partial_x I)^2}_{\text{kinetic/flux}} \underbrace{- \int_0^I dI' \, D(I') f(I')]}_{\text{potential}}$$

and $d\mathcal{F}/dt \leq 0$

Free energy and hysteresis

- Bifurcation: when $\alpha < \alpha^* = 3/5$, potential has metastable minimum at $I = 0$ and stable minimum at $I = 1$ — turbulence 'preferred.' Opposite for $\alpha > \alpha^*$. L-mode/H-mode transition?
- Potential barrier at $I = \alpha$ leads to threshold behavior and hysteresis
- Basic idea: thresholds for global heat flux increment (decrement) for forward (backward) transition. Thresholds unequal $\rightarrow$ hysteresis

$$\Delta Q_f = D_0 I_- \langle \nabla T \rangle, \ \Delta Q_b = D_0 (I_+ - I_-) \langle \nabla T \rangle$$

Traveling waves in bistable system

- Like Fisher, again have traveling waves [Sánchez-Garduño and Maini, 1994]. Unlike Fisher, supported even in damped system!
- Speed c of order $\sqrt{D\gamma}$, depends on α
- Can show that waves propagate forward for $\alpha < \alpha^*$, retreat when $\alpha > \alpha^*$—consistent with the bifurcation in the potential

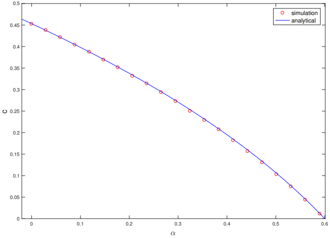

Figure Wave speed for $\alpha < \alpha^*$ in units of $\sqrt{D/\gamma}$. Analytical approx. due to [Pedersen, 2005] also shown

Threshold for spreading of a slug of turbulence

- For $\alpha < \alpha^*$, a localized perturbation from $I = 0$ (i.e. turbulent slug) in this model may either grow into a wave and spread or collapse exponentially
- Similar, but reverse situation for $\alpha > \alpha^*$ ('laminar slugs')
- Classic question in turbulence (spreading of a spot): how big, in amplitude and spatial extent, does the slug have to be in order to spread?

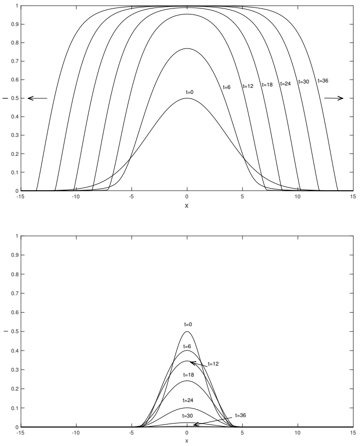

Figure A slug will either grow into a wave (above) or collapse (below)

Threshold for spreading of a slug of turbulence (cont'd)

- Threshold for amplitude is clear: intensity must exceed $I = \alpha$ somewhere
- Otherwise effective linear growth $\gamma_{eff} = (I - \alpha)(1 - I)$ is negative everywhere
- What about threshold in spatial extent? Question seems largely unexplored in literature!

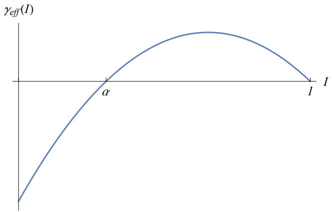

Figure Plot of effective local linear growth as function of turbulence intensity

Lengthscale threshold

- Can estimate by assuming initial growth of turbulent mass in "cap" (part $> \alpha$) of slug governs asymptotic spreading
- Threshold then determined by competition between outgoing diffusive flux from cap and local growth in cap
- This competition suggested by form of free energy functional
- Leads to power law $L_{min} \sim (I_0 - \alpha)^{-1/2}$. Result sees excellent agreement with simulation!

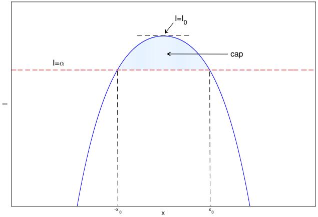

Figure Illustration of slug's "cap"

Lengthscale threshold: analytical vs. simulation

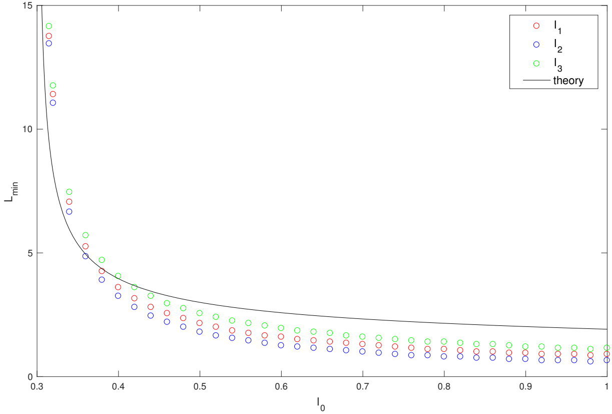

Figure Numerical result for threshold at $\alpha = 0.3$ for three types of initial condition (Gaussian (I_1), Lorentzian (I_2), parabola (I_3)), compared with analytical estimate

Threshold: what have we learned?

- An initially localized turbulent fluctuation with amplitude exceeding I_- and correlated over at least L_{min} will spread and excite the system to turbulence!
- Thus a bistable system naturally supports *intermittent* propagating turbulence pulses, especially near marginal. Captures basic features of avalanching!
- Near marginal linear stability, threshold is meager:

$$I_- \sim \frac{|\gamma_1|}{\gamma_2} \ll 1, \; L_{\min} \sim \left(\frac{\chi_{GB}}{\omega_*}\right)^{1/2} \sim \rho_i$$

- Suggests that near marginal, stability is not robust against noise, provided the fluctuation spectrum has a fat enough tail!

Penetration into bistable zone

- Let's revisit the problem of spreading from weakly supercritical into weakly subcritical ($\alpha < \alpha^*$), now with a bistabilizing effect (temperature corrugations, e.g.)
- Amplitude of wave in unstable region always exceeds amplitude threshold in stable region
- Thus, another wave forms in second region! Turbulence front propagates at constant speed (instead of finite depth), as long as weakly subcritical
- Conclude: delocalization effect much stronger than in Fisher!

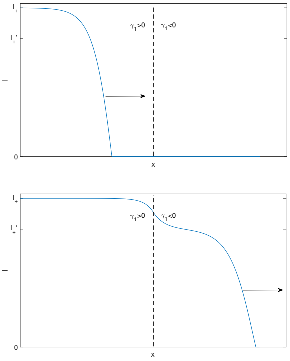

Figure A wave develops in the unstable zone, penetrates into the bistable zone, and forms a new traveling wave with reduced speed and turbulence level.

Conclusions

- Upgrading the unistable Fisher model to a bistable model simultaneously resolves several issues
  1. Can account for hysteresis in fluctuation intensity
  2. Penetration into stable zone much stronger
  3. Subcritical spreading is supported
- As a bonus, also predicts avalanche-like phenomena, along with a quantitative prediction for the threshold excitation require to trigger an avalanche

Future directions

- Full model needs to incorporate coupling to zonal flow and/or profiles
- Can we test for ballistic spreading into stable zone numerically? Possible inspiration: [Yi et al., 2014]
- Can we test for threshold numerically? Idea: initialize patches of turbulence in subcritical zone in GK
- Possible experiments: what does the fluctuation spectrum look like? How does its tail evolve as we move about the hysteresis loop? Spatial correlator?

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Avalanching 101

- Observed in MFE plasma [Politzer, 2000]
- Basic picture: a sufficiently large, localized increase in the turbulence level radially cascades into neighboring regions, ultimately causing a sudden burst of transport
- Closely related to turbulence spreading: avalanching and (subcritical) spreading essentially two ways of looking at same phenomenon
- Associated with self-organized criticality (occurs near marginal, $1/f$ spectra)
- Intermittent (long tails)

Bistable case: reduction to FitzHugh-Nagumo

- $(*)$ is bistable for weak damping $\gamma_1 < 0$ and $\gamma_2^2 > 4|\gamma_1|\gamma_3$
- Roots: $I = 0$, $I_\pm = (\gamma_2 \pm \sqrt{\gamma_2^2 - 4|\gamma_1|\gamma_3})/2\gamma_3$. $0, I_+$ stable (note: nonzero for marginal γ_1), I_- unstable
- If $\gamma_1 < 0$ and γ_2 sufficiently large, can be written

$$\partial_t I = f(I) + \partial_x(D(I)\partial_x I)$$

with $f(I) = \gamma I(I - \alpha)(1 - I)$ by defining

$$|\gamma_3|{I_+}^2 \to \gamma,\ \frac{I_-}{I_+} \to \alpha,\ I_+ D_0 \to D$$

- This is a version of the Nagumo equation, a simplification of the FitzHugh-Nagumo model for excitable media [FitzHugh, 1961, Nagumo et al., 1962]

Lengthscale threshold (details)

- Strategy: assume initial slug is even, has single max at I_0 and single lengthscale L
- Expand intensity curve about max to quadratic order, plug into dynamical equation, integrate over extent of cap
- Result: growth if

$$L > L_{\min} = \sqrt{\frac{\lambda D(\alpha) I_0}{f(I_0) - \frac{1}{3}(I_0 - \alpha) f'(I_0)}} = \sqrt{\frac{3\lambda D \alpha I_0}{\gamma (I_0 - \alpha)((1 - 2\alpha) I_0 + \alpha)}}$$