Ecology of Flows and Drift Wave Turbulence: Reduced Models and Applications

PhD Dissertation Defense by Rima Hajjar

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Publications + Plan of Dissertation

- Background.
- Chapter 2: The Ecology of Flows and Drift Wave Turbulence in CSDX: a Model. Physics of Plasmas, 2018.
- Chapter 3: Modeling the Enhancement in Drift Wave Turbulence. Physics of Plasmas, 2017.
- Chapter 4: Zonal Shear Layer Collapse in the Hydrodynamic Electron Limit. Physics of Plasmas, 2018 (in preparation)
- Conclusions and Future Work

On the side:

- Modeling of Aluminum Impurity Entrainment in the PISCES-A He^+ Plasma. Journal of Nuclear Material, 2015.

Fusion 101

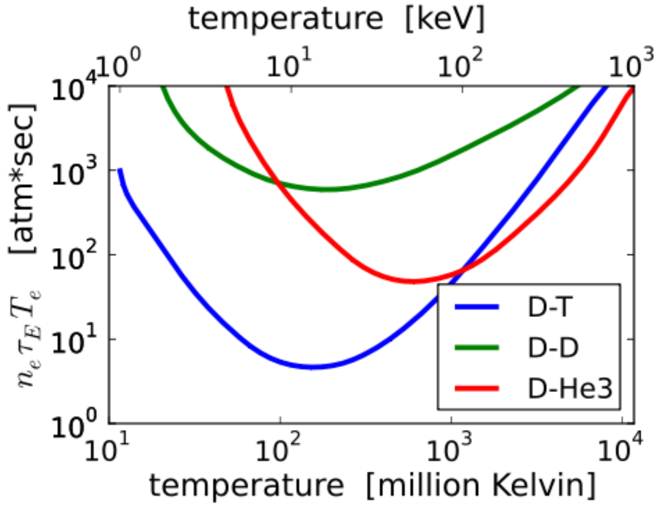
- Increasing need for sustainable and clean energy.
- Nuclear fusion releases high outputs of energy that can be converted into electric power. The fusion reaction with the highest cross-section is:

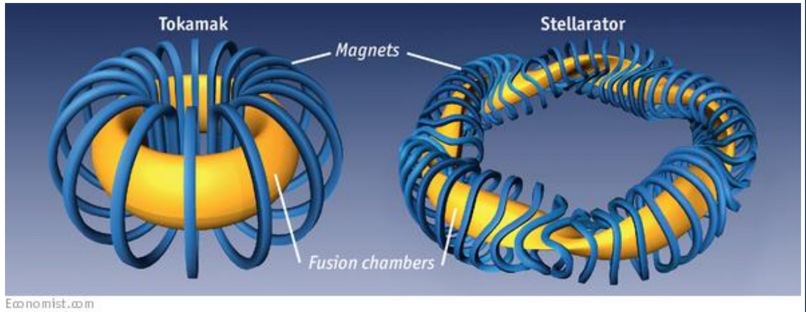
$$ {}_1^2H + {}_1^3H \longrightarrow {}_2^4He + {}_0^1n + 18MeV $$

- Challenge: → Ignition ($E^{out} > E^{in}$)
  → Confinement and Lawson criterion:

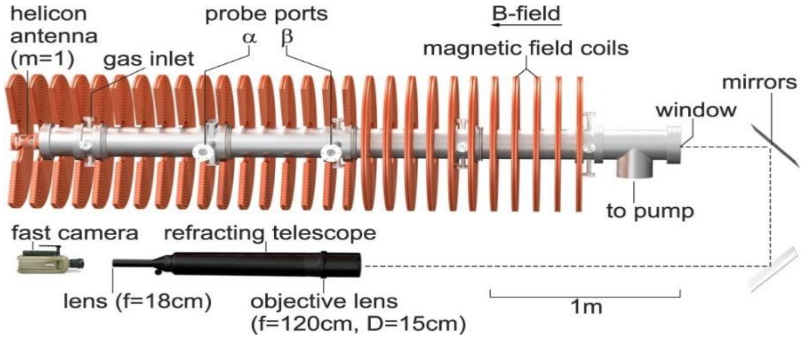
$$ n\tau_E T > 3 \times 10^{21} keV.s.m^{-3} \qquad \tau_E = \frac{W_{plasma}}{P_{loss}} = \frac{3\overline{nT}V}{P_{in}} $$

- Use externally imposed magnetic field lines to confine the plasma in toroidal or linear devices.
- Turbulent transport of particles and energy (mainly due to instabilities) destroys confinement.





Not to scale



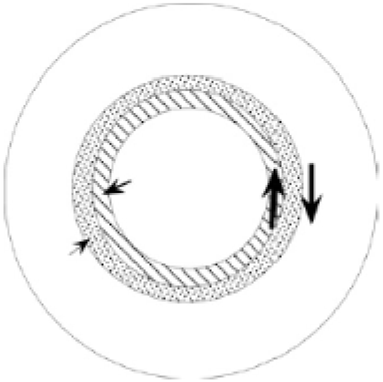
Drift Waves and Zonal Flows

- DWs: → plasma fluctuations caused by radial density gradients.
  → propagate in the electron direction at v_{De}

$$\omega = \frac{\omega^*}{1+k_\perp^2\rho_s^2} = \frac{k_m v_{De}}{1+k_\perp^2\rho_s^2}$$

- Parallel resistivity is one mechanism that can destabilize DWs by introducing a phase shift between $\tilde{n}$ and $\tilde{\phi}$, thus creating a DW instability.

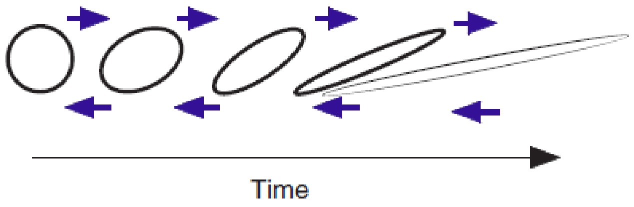
$$v_{De} = -\frac{T_e}{eB}\frac{d\ln n(x)}{dx}\hat{y}$$



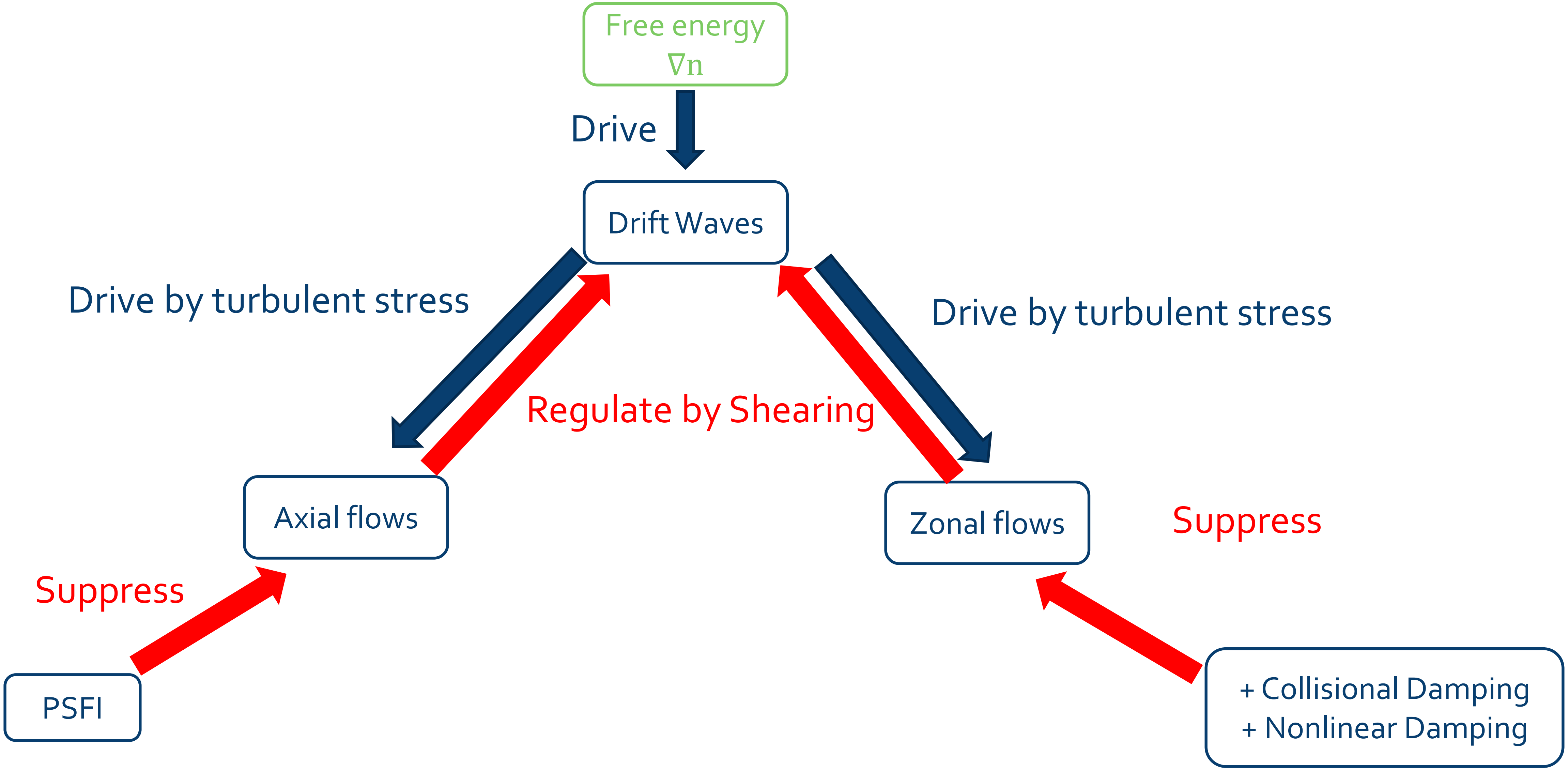
Diamond *et al*, 2005, PoP

- Fortunately, one mechanism that regulates these fluctuations is the self generation and amplification of Zonal Flows by turbulent stresses.

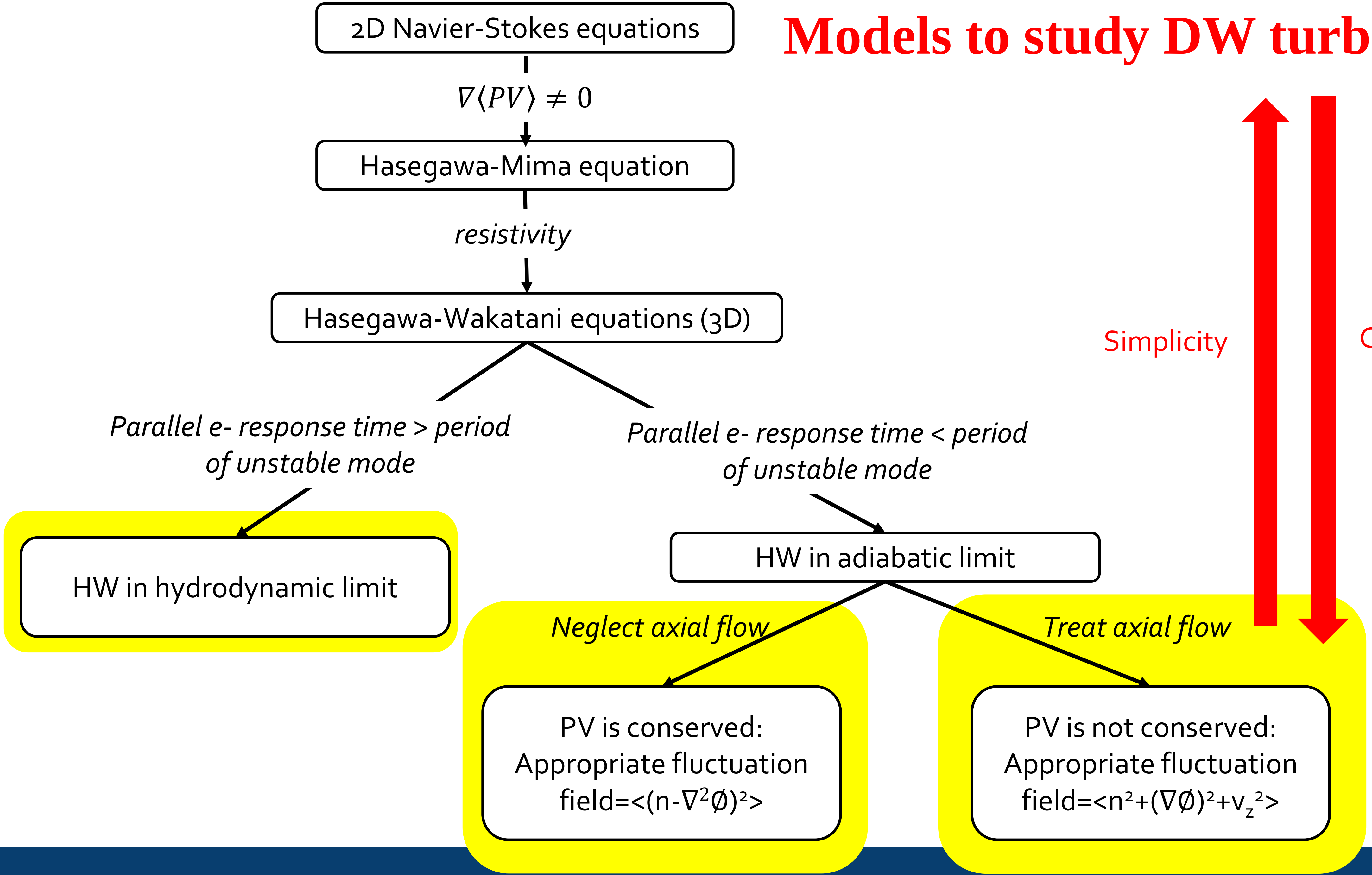
- Zonal Flows: → Large scale sheared $E \times B$ layers
  → Decorrelate the turbulent eddies by shearing.
  → Reduce turbulence and transport.



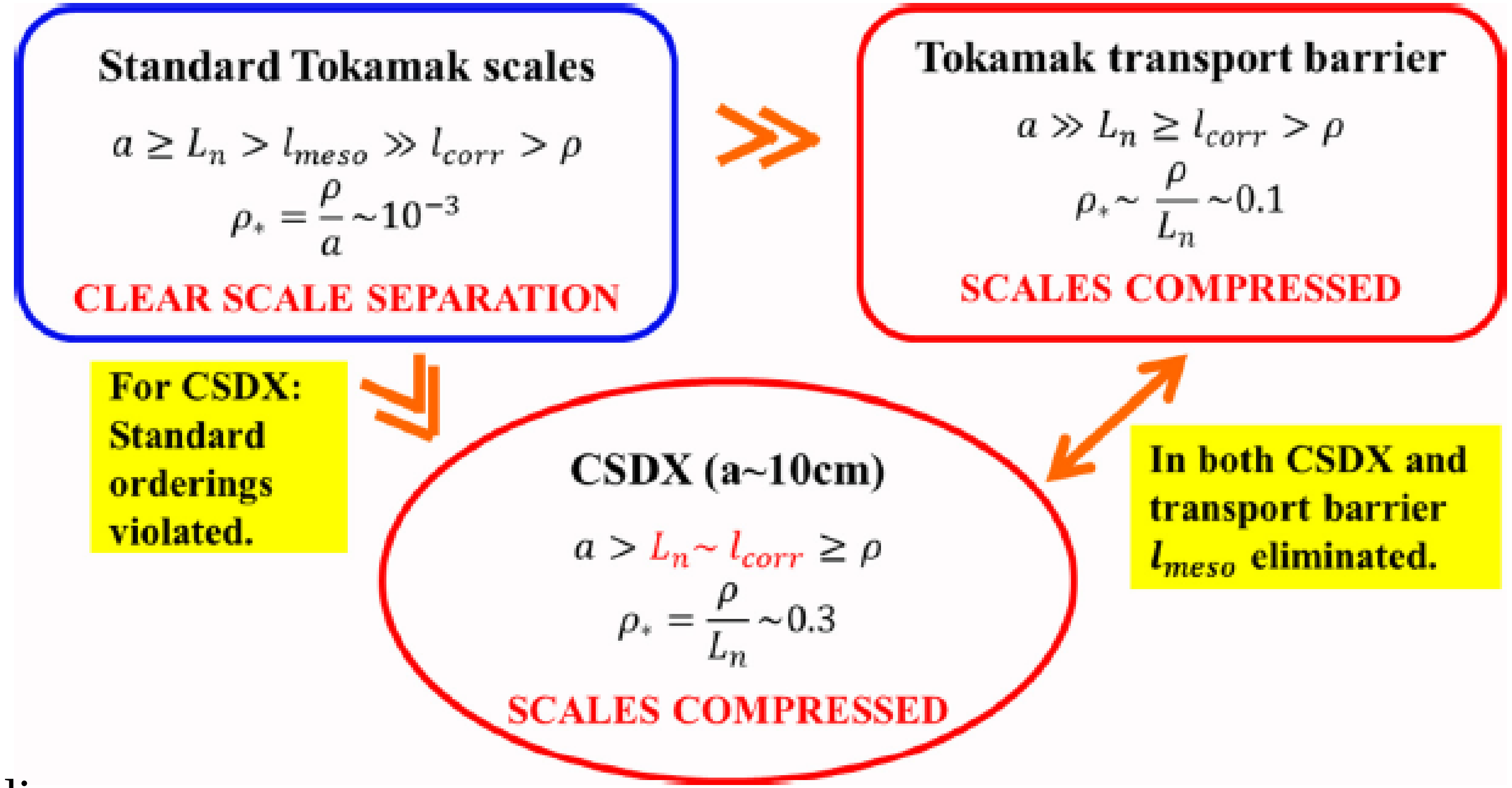
Drift Waves/Flows = Predator/Prey



Models to study DW turbulence



CSDX: a promising testbed for exploring DW turbulence models over compressed ranges of scales.



Cui *et al*, 2016, PoP

a = plasma radius
L_n = density scale length
ρ = modified ion Larmor Radius
l_{corr} = turbulence correlation length

Models and Results obtained from CSDX can be extrapolated to larger scale devices

What am I doing?

- Explore the status of flows and fluctuations ecology.

- Investigate the relationship between microscopic DW turbulence and macroscopic flows in magnetically confined plasmas.
- In particular, study the coupling relation between parallel and perpendicular flow dynamics in the plasma of CSDX .

- Model the evolution of plasma mean profiles and fluctuations in CSDX, as the magnitude of the magnetic field **B** increases.
- Analytically confirm the transport bifurcation phenomenon reported in CSDX as **B** is raised.

- Examine the Drift Wave/Zonal Flow relation in the hydrodynamic electron limit → Relevance to density limit experiment.

Why do I care?

- Mean flow structures, including both Zonal and Axial Flows, play an important role in regulating turbulence (*L-H* transitions, ITB formation) → understanding the mechanism of formation of these flows is crucial in achieving better confinement in ITER.

- Explain and understand the physics behind the collapse of ZFs and the enhancement of turbulence in the hydrodynamic electron limit which is an important and under-explored problem → interpret the density limit experiments using a simple robust mechanism of DW turbulence.

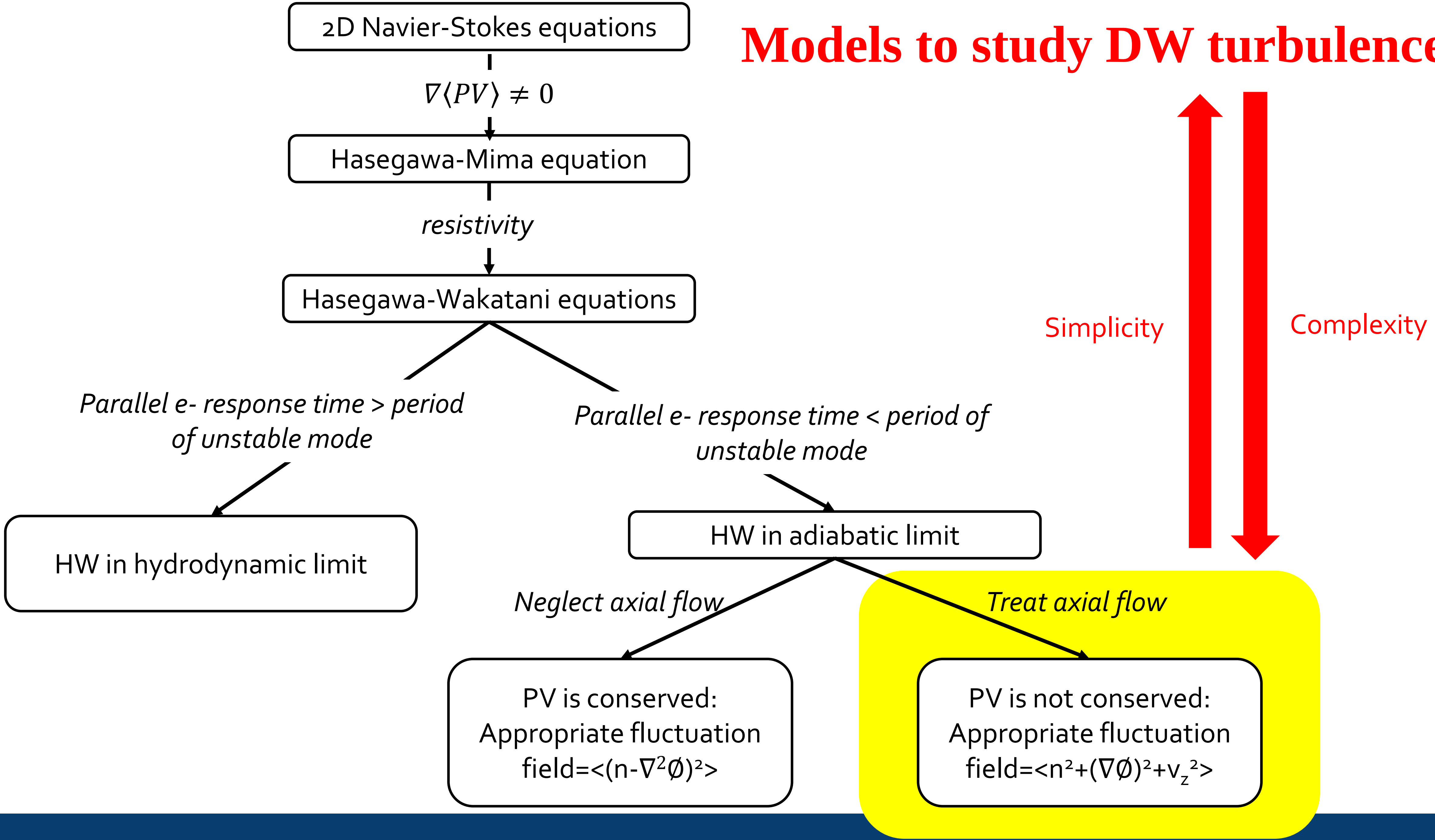
How to do it?

- Formulate reduced models that self-consistently relate variations in mean plasma fields to fluctuation intensity (*total energy/potential enstrophy*).

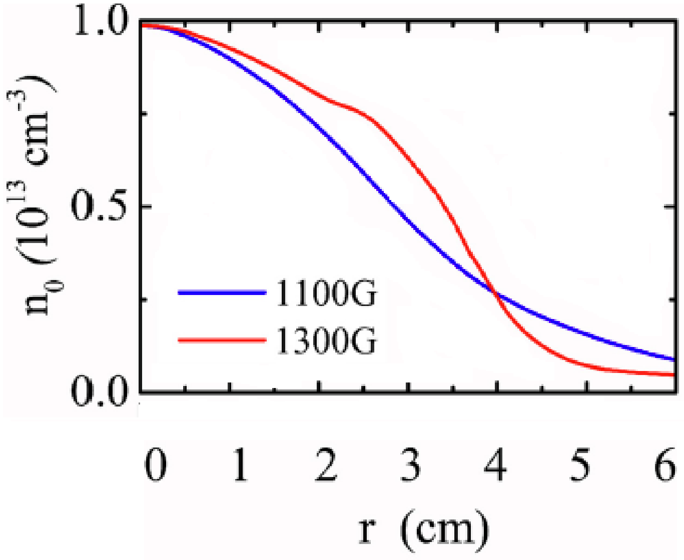
- Reduced models are the excellent candidate:
    1. Low computational cost if compared to DNS or LES
    2. Good candidate to describe the physics of a multiscale plasma such as CSDX plasma.
    3. Essential to understand the feedback loops between mean profiles (macro) and fluctuations (micro).
    4. Easily coupled to other PMI codes.
    5. Failure in model reduction suggests a gap in understanding → Need to update the codes

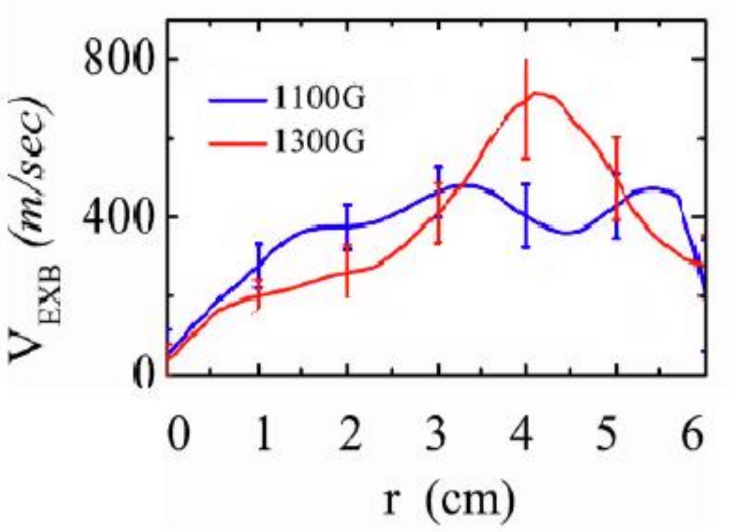
The Ecology of Flows and Drift Wave Turbulence: a Model for CSDX

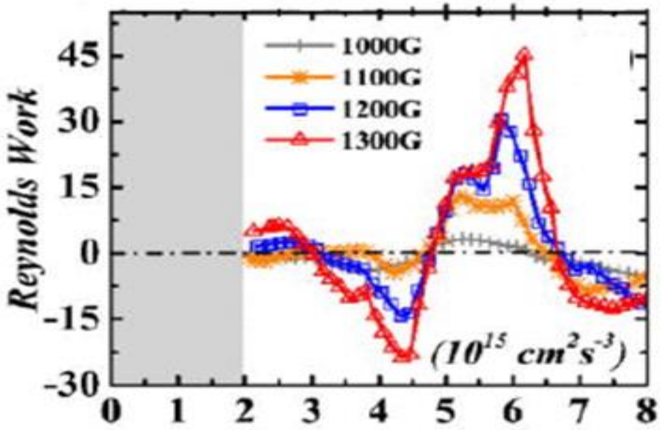
Models to study DW turbulence



Experimental results in CSDX - 1







As magnitude of **B** increases:

1. Development of radial velocity shear

   +

2. Decrease in turbulence level

   +

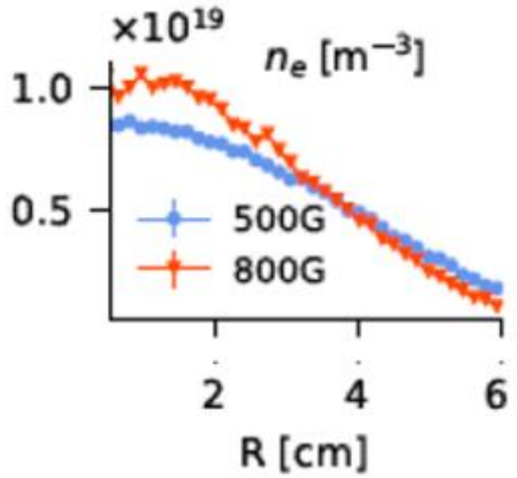
3. Steepening of density profile

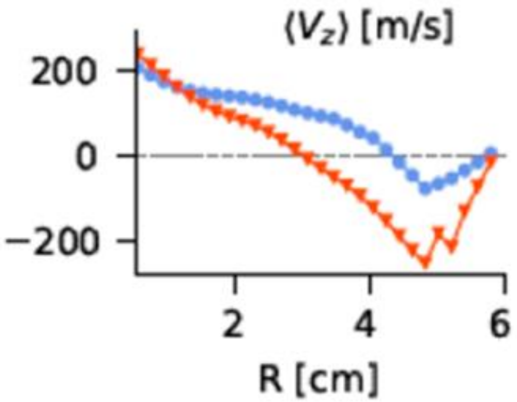
=

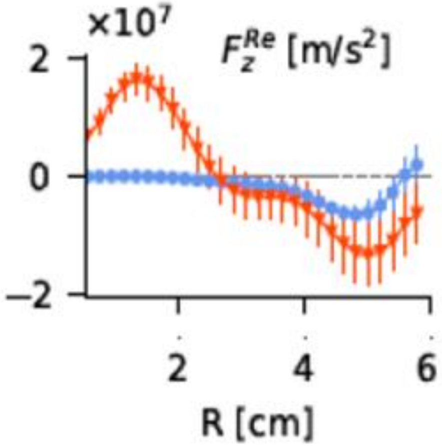
Transition to a state of enhanced energy in the perpendicular plane (Analogy to larger MFE devices)

Cui *et al*, 2015 and 2016, PoP

Experimental results in CSDX - 2







Reynolds Work= Reynolds force x velocity

As magnitude of **B** increases:

1. Development of axial velocity shear

 +

2. Increase in parallel Reynolds force

 +

3. Steepening of density profile

= Transition to a state of enhanced energy in the parallel direction

Hong, Hajjar *et al*, 2018, PoP (submitted)

Formulation of the Model

Hasegawa
Wakatani
+
Parallel
Compression

$$\begin{cases} \dfrac{d\tilde{n}}{dt} + v_E.\nabla\langle n\rangle + \boxed{n_0\nabla_z\tilde{v}_z} = -\dfrac{v_{th}^2}{\nu_{ei}}\nabla_z^2(\tilde{\varphi}-\tilde{n}) + D_0\nabla^2\tilde{n} + \{\tilde{n},\tilde{\varphi}\} \\ \dfrac{d\nabla^2\tilde{\varphi}}{dt} + v_E.\nabla\langle\nabla^2\varphi\rangle = -\dfrac{v_{th}^2}{\nu_{ei}}\nabla_z^2(\tilde{\varphi}-\tilde{n}) + \mu_0\nabla^4\tilde{\varphi} - \nu_{in}(\langle v_y\rangle - \langle v_n\rangle) + \{\nabla^2\tilde{\varphi},\tilde{\varphi}\} \\ \dfrac{d\tilde{v}_z}{dt} + \boxed{v_E.\nabla\langle v_z\rangle} = -c_s^2\nabla_z\tilde{n} + \nu_0\nabla^2\tilde{v}_z + \{\tilde{v}_z,\tilde{\varphi}\} \end{cases}$$

Parallel Compression breaks parallel symmetry → Breaking of PV conservation → Define a new conserved energy:

$$\varepsilon = \frac{\langle \tilde{n}^2 + \tilde{v}_z^2 + (\nabla\tilde{\varphi})^2 \rangle}{2} = \int_0^{L_z} dz \int_0^{L_y} dy\, \varepsilon(x)$$

The model (mean fields + turb. Fluctuation)

$$\frac{\partial n}{\partial t} = -\frac{\partial\langle\tilde{n}\tilde{v}_x\rangle}{\partial x} + D_c\frac{\partial^2 n}{\partial x^2} + S_n$$

$$\frac{\partial v_z}{\partial t} = -\frac{\partial\langle\tilde{v}_x\tilde{v}_z\rangle}{\partial x} + \nu_{c//}\frac{\partial^2 v_z}{\partial x^2} - \nu_{in}v_z + S_{v_z}$$

$$\frac{\partial v_y}{\partial t} = -\frac{\partial\langle\tilde{v}_x\tilde{v}_y\rangle}{\partial x} + \nu_{c\perp}\frac{\partial^2 v_y}{\partial x^2} - \nu_{in}v_y + S_{v_y}$$

$$\frac{\partial \varepsilon}{\partial t} - \partial_x(\varepsilon^{1/2}l_{mix}\partial_x\varepsilon) = -\langle\tilde{n}\tilde{v}_x\rangle\frac{dn}{dx} - \langle\tilde{v}_x\tilde{v}_y\rangle\frac{dv_y}{dx} - \langle\tilde{v}_x\tilde{v}_z\rangle\frac{dv_z}{dx} - \frac{\varepsilon^{3/2}}{l_{mix}} + P$$

Diffusion
Sources
Dissipation
Mean/Fluctuation coupling terms
Sink

Hajjar *et al*, 2018, PoP

Using QL theory and turbulent mixing concepts

1) Particle Flux:

$$\langle \tilde{n}\tilde{v}_x \rangle = -D\frac{dn}{dx} = -\frac{f\varepsilon}{\hat{\alpha}}\frac{dn}{dx}$$

- The electron parallel diffusion rate: $\hat{\alpha} = \frac{k_z^2 v_{th}^2}{\nu_{ei}} \gg |\omega|$. (Near adiabatic electrons)
- The factor f represents the fraction of total energy allocated for kinetic energy in the radial direction:

$$f = \frac{k_\perp^2\rho_s^2}{(1+\Delta^2) + k_\perp^2\rho_s^2 + {|k_m\rho_s\nabla\bar{v}_z - k_z c_s(1-i\Delta)|^2}\big/{\omega^2 + \varepsilon l_{mix}^{-2}}}$$

$\langle k_m k_z \rangle$

Adiabatic electron without axial flow shear

$$f = \frac{k_\perp^2\rho_s^2}{1 + k_\perp^2\rho_s^2 + {k_z^2 c_s^2}\big/{\omega^2 + \varepsilon l_{mix}^{-2}}}$$

Pure DWs

$$f = \frac{k_\perp^2\rho_s^2}{1 + k_\perp^2\rho_s^2}$$

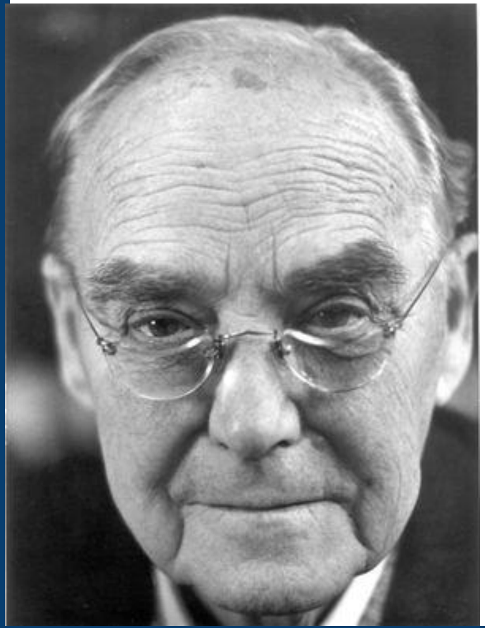
2) Perpendicular Reynolds Stress

$$\langle \tilde{v}_x \nabla_\perp^2 \varphi \rangle = -\partial_x \langle \tilde{v}_x \tilde{v}_y \rangle = -l_{mix}\sqrt{\varepsilon}\,\frac{d^2 v_y}{dx^2} + \frac{l_{mix}\sqrt{\varepsilon}\,\omega_{ci}}{n}.\nabla n$$

Taylor's ID

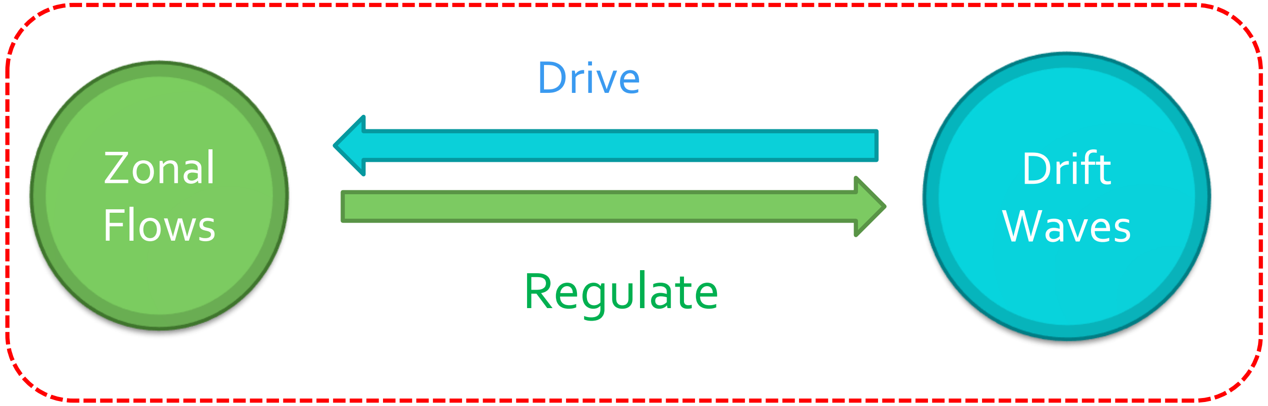
Diffusive Stress relaxes the flow

Residual stress drives the flow via density gradient



3) Reynolds Power rate

$$\langle \tilde{v}_x \tilde{v}_y \rangle \frac{dv_y}{dx} = v_y[-l_{mix}\sqrt{\varepsilon}\,\frac{d^2 v_y}{dx^2} + \frac{l_{mix}\sqrt{\varepsilon}\,\omega_{ci}}{n}.\nabla n]$$



Predator-Prey Relation

4) Parallel Reynolds Stress

$$\langle \tilde{v}_x \tilde{v}_z \rangle = -l_{mix}\sqrt{\varepsilon}\frac{dv_z}{dx} + \langle k_m k_z \rangle \rho_s c_s^3 [\frac{l_{mix}}{\sqrt{\varepsilon}} + \frac{\rho_s^2 k_\perp^2}{\alpha}]$$

Difficult to measure experimentally

Empirically, in analogy with turbulence in pipe flows (à la Prandtl):

$$\tilde{v}_z = -l_{mix}\frac{dv_z}{dx} + \frac{\sigma_{VT} c_s^2 \tau_c}{L_{//}}(-l_{mix}\frac{dn}{dx})$$

Measures parallel to perpendicular coupling

Turbulent diffusivity from Prandtl theory

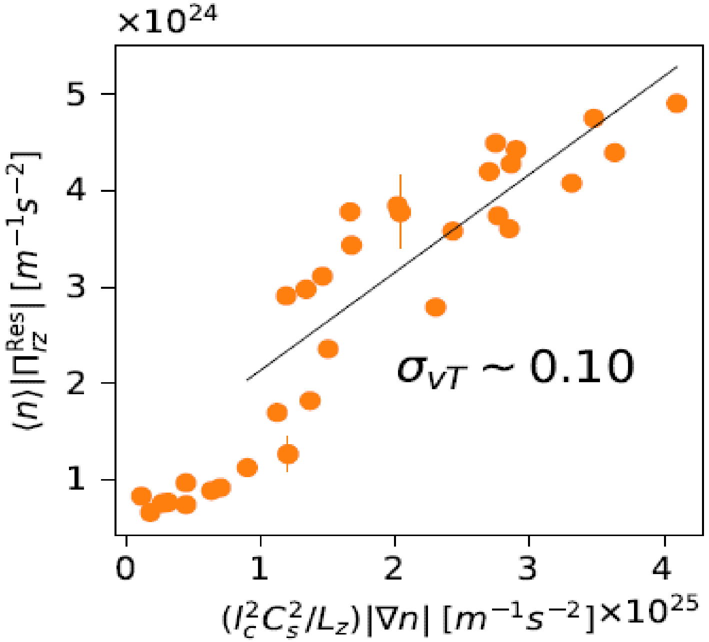
Energy source proportional to density gradient that accelerates the parallel flow

$$\langle \tilde{v}_x \tilde{v}_z \rangle = -\chi_z \frac{dv_z}{dx} - \frac{\sigma_{VT} c_s^2 \langle l_{mix}^2 \rangle}{L_z} \cdot \frac{\nabla n}{n}$$

Hajjar *et al*, 2018, PoP

Measurements in CSDX

$$n\langle\tilde{v}_x\tilde{v}_z\rangle^{Res} = -\sigma_{VT}\frac{c_s^2\langle l_{mix}^2\rangle\nabla n}{L_z}$$



Hong, Hajjar *et al*, 2018, PoP (submitted)

- σ_{VT} is the counterpart of the correlator $\langle k_m k_z\rangle$.

- σ_{VT} represents the degree of symmetry breaking in $\langle k_m k_z\rangle$, and quantifies the efficiency of ∇n in driving an axial flow:

$$\nabla v_z = -\frac{\sigma_{VT}c_s^2\tau_c}{L_z n}\nabla n$$

- σ_{VT} couples parallel to perpendicular flow dynamics via:

$$\frac{d}{dx}(\nabla v_y) = -\frac{\omega_{ci}L_z}{\sigma_{VT}c_s^2\tau_c}\nabla \mathrm{v_z} \propto \frac{\Pi_{xy}^{res}}{\Pi_{xz}^{res}}$$

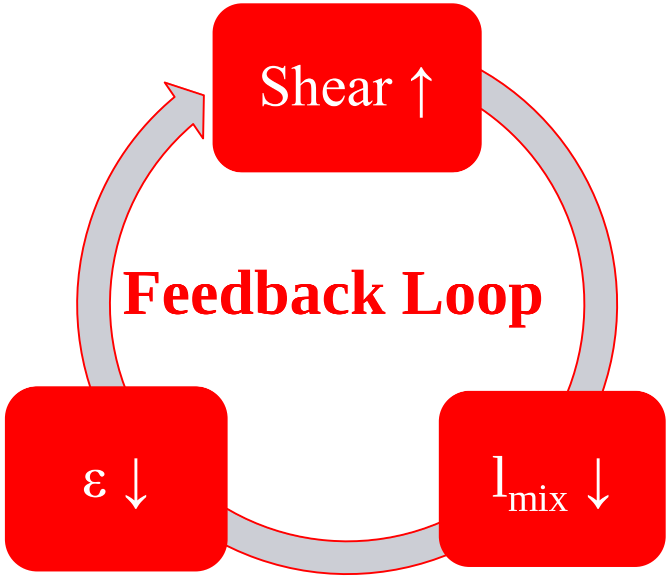
5) The mixing length l_{mix}:

- The mixing length exhibitis turbulence suppression via axial and azimuthal flow shear:

$$l_{mix}^2 = \frac{l_0^2}{1+\frac{l_0^2}{f\varepsilon}\left(\frac{\nabla\bar{v}_y}{l_0}+\frac{\nabla\bar{v}_z}{L_{//}}\right)^2}$$

No Axial Shear

$$l_{mix}^2 = \frac{l_0^2}{1+\frac{(\nabla\bar{v}_y)^2}{f\varepsilon}}$$



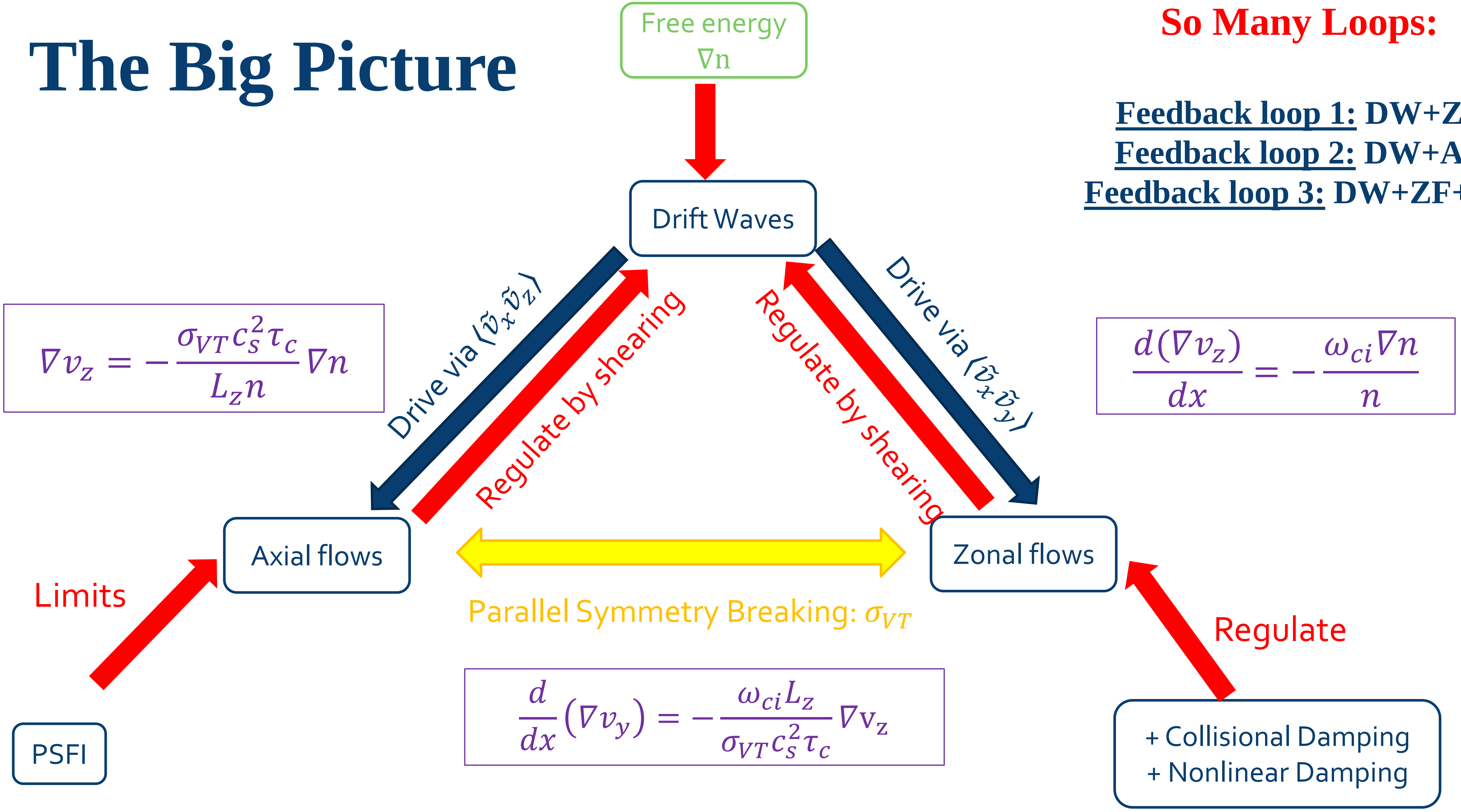
- In CSDX, the mixing scale for turbulence l_0 in the absence of shear ($\rho_*=\rho/L_n$):

$$l_0 \cong 2.3\rho_s^{0.6}L_n^{0.3}$$ — In between Bohm and gyro-Bohm Diffusion → $$D_{CSDX} \cong D_B\rho_*^{0.6}$$

The Big Picture

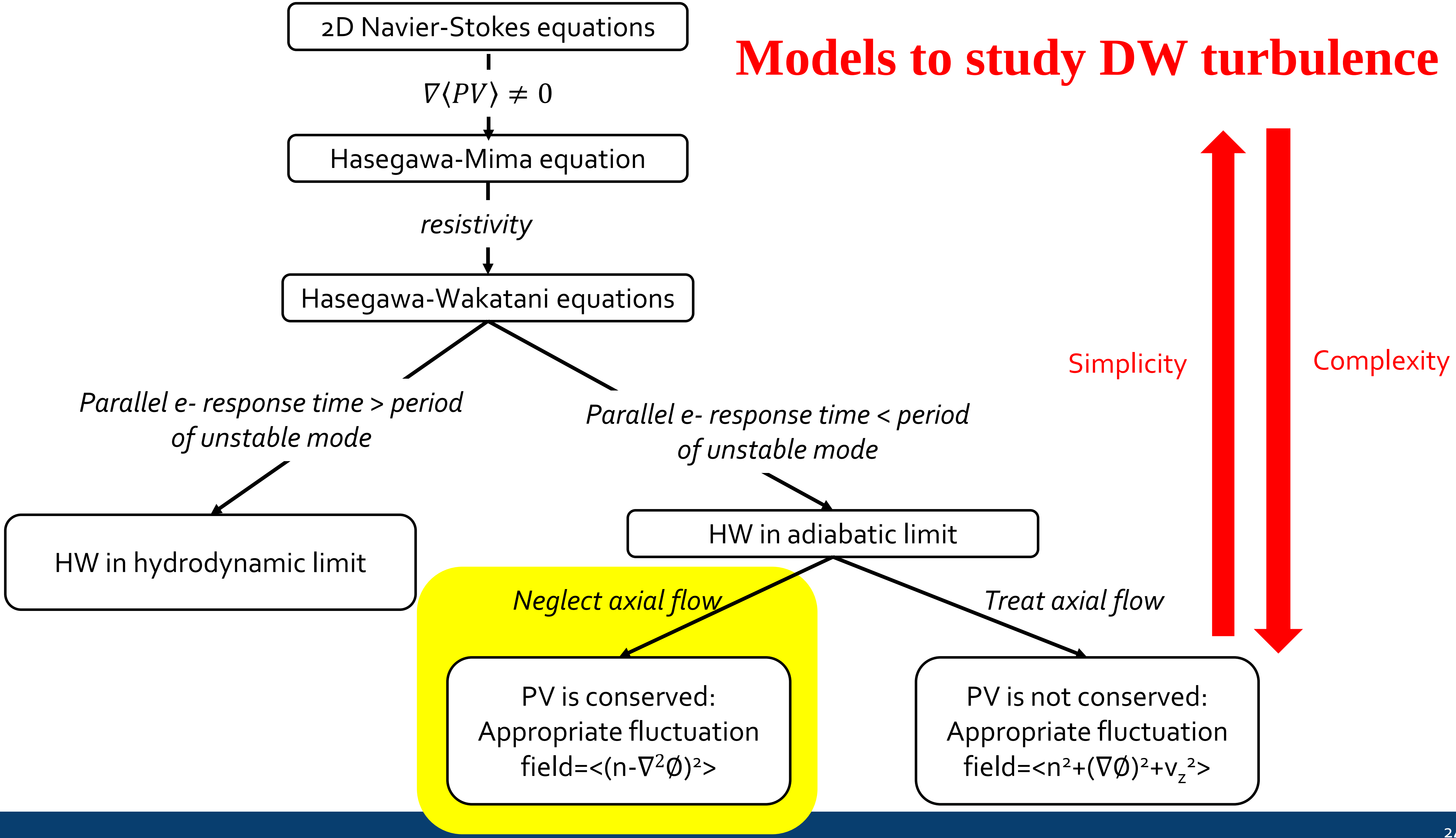
So Many Loops:

Feedback loop 1: DW+ZF
Feedback loop 2: DW+AF
Feedback loop 3: DW+ZF+AF



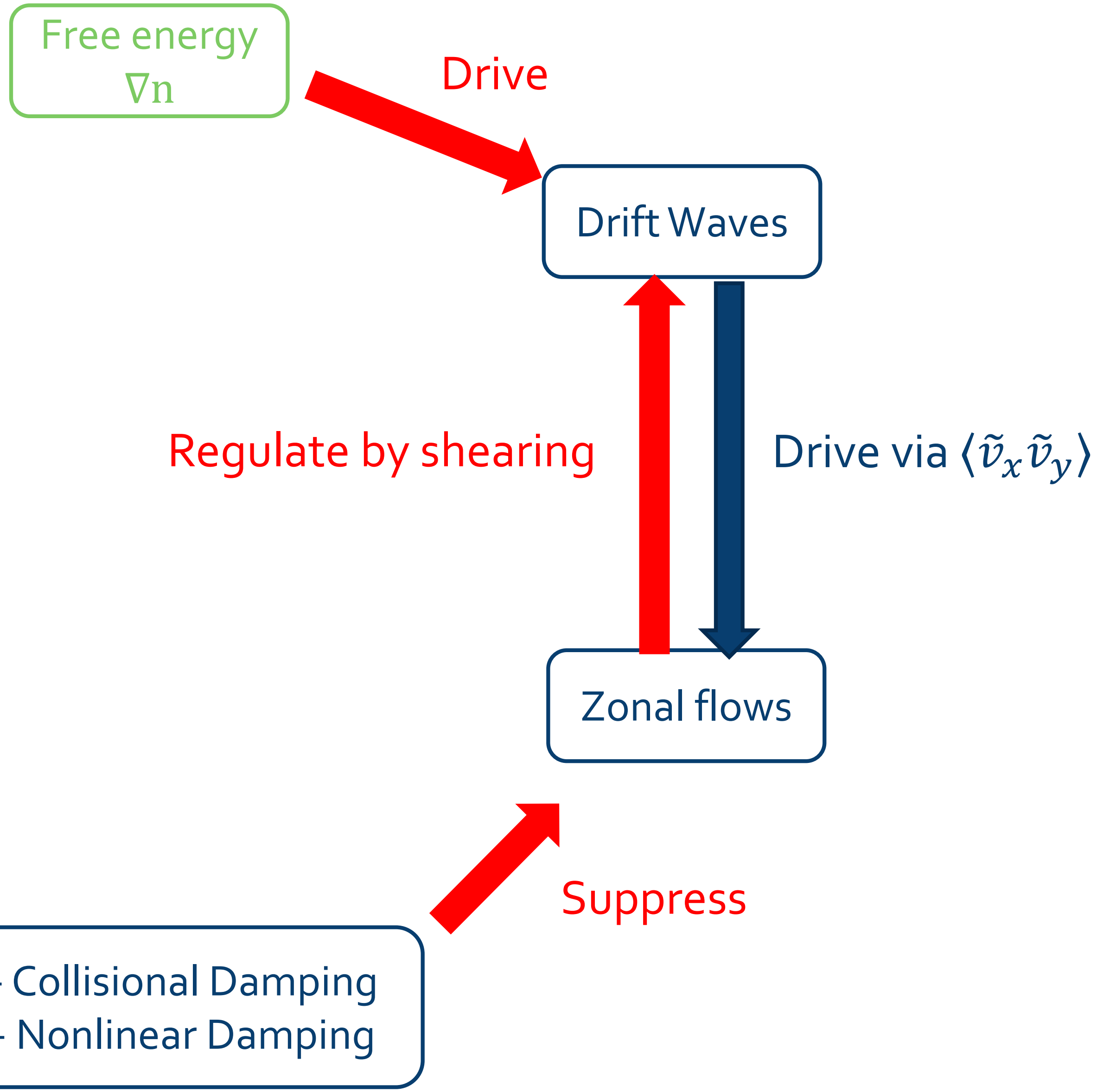
Modeling Enhanced Confinement in Drift Wave Turbulence

Models to study DW turbulence



- When parallel Reynolds power is negligible, and when energy exchange occurs mainly between DWs and ZFs, axial flow is treated as parasitic.

- Back to the predator/prey relation between DWs and ZFs



Formulation of the Model

Basic form of Hasegawa Wakatani

$$\begin{cases} \dfrac{d\tilde{n}}{dt} + v_E.\nabla\langle n\rangle + n_0\nabla_z\tilde{v}_z = -\dfrac{v_{th}^2}{\nu_{ei}}\nabla_z^2(\tilde{\varphi}-\tilde{n}) + D_0\nabla^2\tilde{n} + \{\tilde{n},\tilde{\varphi}\} \\ \dfrac{d\nabla^2\tilde{\varphi}}{dt} + v_E.\nabla\langle\nabla^2\varphi\rangle = -\dfrac{v_{th}^2}{\nu_{ei}}\nabla_z^2(\tilde{\varphi}-\tilde{n}) + \mu_0\nabla^4\tilde{\varphi} - \nu_{in}(\langle v_y\rangle - \langle v_n\rangle) + \{\nabla^2\tilde{\varphi},\tilde{\varphi}\} \\ \dfrac{d\tilde{v}_z}{dt} + v_E.\nabla\langle v_z\rangle = -c_s^2\nabla_z\tilde{n} + \nu_0\nabla^2\tilde{v}_z + \{\tilde{v}_z,\tilde{\varphi}\} \end{cases}$$

HW equations locally conserve the total Potential Vorticity $\tilde{q} = \tilde{n} - \nabla^2\tilde{\phi}$ → potential enstrophy ε is also conserved:

$$\varepsilon = \frac{\langle(\tilde{n}-\nabla^2\tilde{\varphi})^2\rangle}{2} = \int_0^{L_z} dz \int_0^{L_y} dy\,\varepsilon(x)$$

The Model

Diffusion
Sources
Dissipation
Mean/Fluctuation coupling terms

$$\frac{\partial n}{\partial t} = -\frac{\partial \langle \tilde{n} \tilde{v}_x \rangle}{\partial x} + D_c \frac{\partial^2 n}{\partial x^2} + S_n$$

$$\frac{\partial u}{\partial t} = -\frac{\partial \langle \tilde{v}_x \tilde{u} \rangle}{\partial x} + \mu_c \frac{\partial^2 u}{\partial x^2} + S_u$$

$$\frac{\partial \varepsilon}{\partial t} - \partial_x (l_{mix}^2 \varepsilon \partial_x \varepsilon / \alpha) = -(\langle \tilde{v}_x \tilde{n} \rangle - \langle \tilde{v}_x \tilde{u} \rangle)(\partial_x n - \partial_x u) + P - \varepsilon^{3/2}$$

$$P \rightarrow \sqrt{\varepsilon}(u_0^2 - \varepsilon)$$

Hajjar *et al*, 2017, PoP

Closure using QL theory and mixing length concepts

1) Particle Flux:

$$\langle \tilde{n}\tilde{v}_x \rangle = -D\frac{dn}{dx} = -\frac{f\varepsilon l_{mix}^2}{\hat{\alpha}}\frac{dn}{dx}$$

where

$$f = \frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2}$$

2) Vorticity Flux: (Taylor ID)

$$-\partial_x \langle \tilde{v}_x \tilde{v}_y \rangle = \langle \tilde{v}_x \tilde{u} \rangle = -\frac{f\varepsilon l_{mix}^2}{\sqrt{\hat{\alpha}^2 + c_u u^2}}\left[\frac{du}{dx} + \frac{dn}{dx}\right] - \frac{f\varepsilon l_{mix}^2}{\hat{\alpha}}\frac{dn}{dx}$$

Diffusive Stress

Residual Stress

The coefficient c_u reflects the shearing feedback on the mean profiles

3) The mixing length:

- 2D turbulence, the Rhines' scale emerges as a convenient mixing length for turbulence.

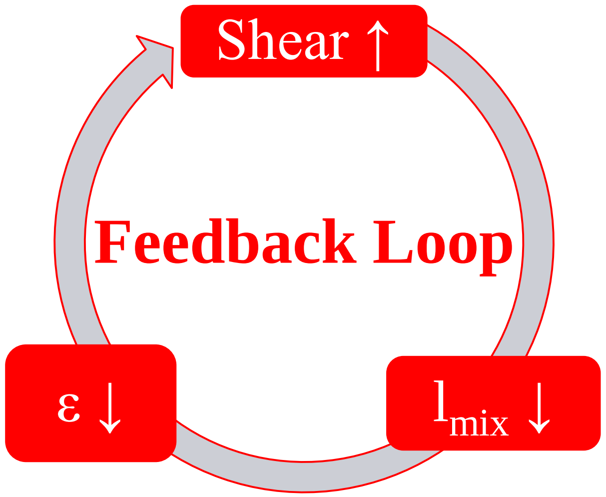
$$l_{Rh} = \sqrt{\varepsilon} / \nabla(n-u)$$

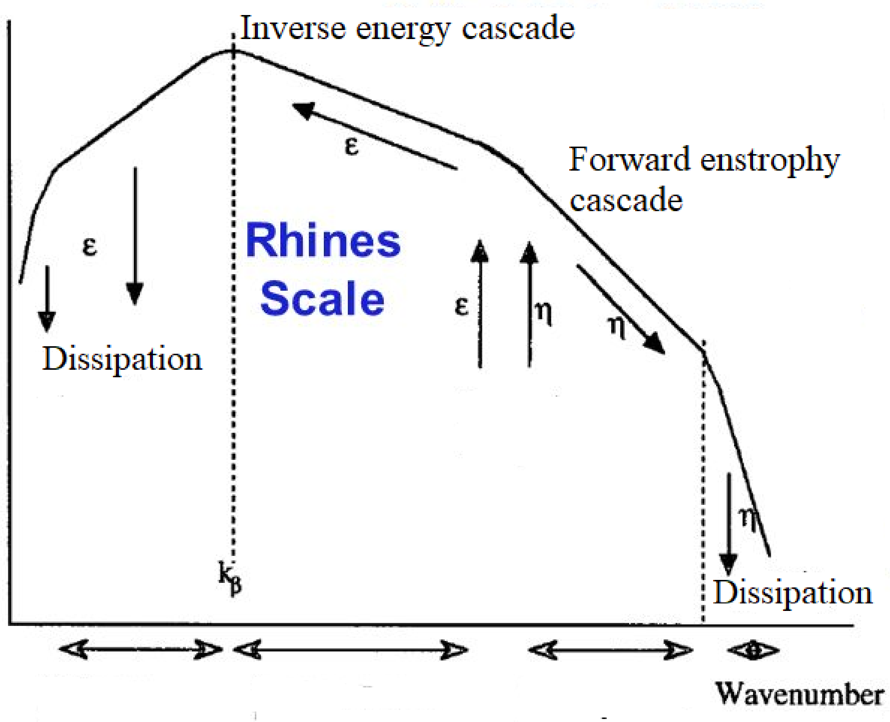
- Choose a hybrid mixing length:

$$l_{mix}^2 = \frac{l_0^2}{1+(l_0/l_{Rh})^2} = \frac{l_0^2}{1+l_0^2(\nabla(n-u))^2/\varepsilon}$$

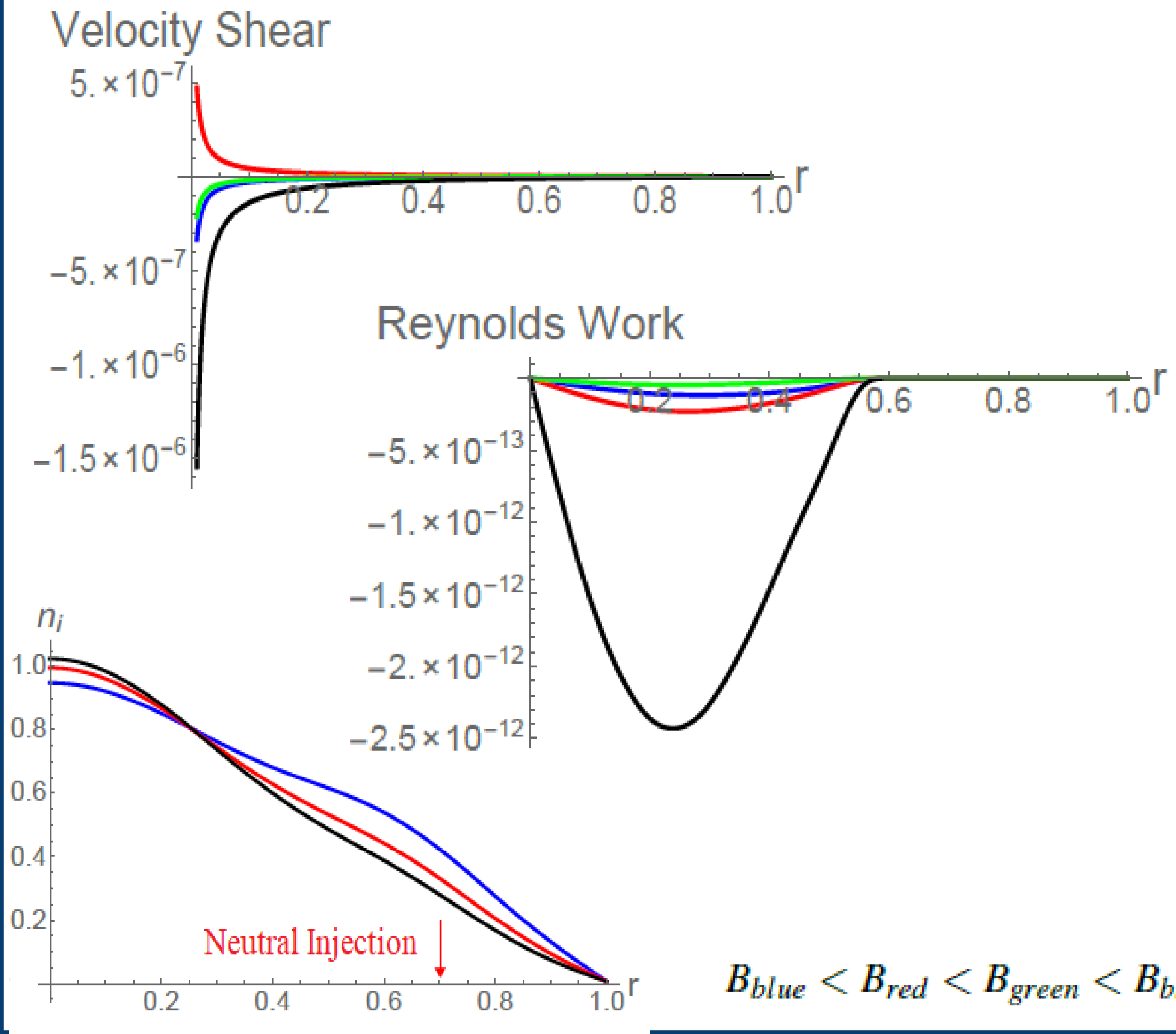
+ Weak PV mixing → $l_{mix} \sim l_0$

+ Strong PV mixing → $l_{mix} \sim l_{Rh}$





Recovery of experimental trends in CSDX



$B_{blue} < B_{red} < B_{green} < B_{black}$.

$$\frac{\Delta(1/L_n)}{L_{n_i}} = \frac{1/L_{n_f} - 1/L_{n_f}}{1/L_{n_i}} = \begin{cases} 0.70 & \text{numerically} \\ 0.55 & \text{experimentally} \end{cases}$$

$$\frac{\Delta(1/L_v)}{L_{v_i}} = \frac{1/L_{v_f} - 1/L_{v_f}}{1/L_{v_i}} = \begin{cases} 0.73 & \text{numerically} \\ 0.57 & \text{experimentally} \end{cases}$$

Hajjar *et al*, 2017, PoP

Summary on numerical results

- As ***B*** increases:

  + Steepening of the density profile with B.

  + Development of azimuthal velocity shear with B.

  + Increase in the magnitude of the Reynolds work, i.e., turbulence regulation with B.

- These trends are **qualitatively** insensitive to:

  + Magnitude of the shearing coefficient c_u

  + Outer edge Boundary Condition on vorticity.

  + Magnitude of l_0

  + The presence of a residual stress

R_T: a criterion for turbulence suppression?

- Need to quantitatively predict when transport barriers are formed.

$$R_T = \frac{\langle \tilde{v}_x \tilde{v}_y \rangle' v_{E\times B}}{|\gamma_{eff}| \langle \tilde{v}_\perp^2 \rangle} = \frac{local\ Reynolds\ power\ density}{effective\ increase\ in\ turb.kinetic\ energy}$$

- When $R_T > 1$ → energy transfer to the shear flow exceeds the effective increase in turbulent energy → reduction of transport and formation of a barrier.

- BUT, $|\gamma_{eff}|$= ? What does it really depend on?
- What about non-kinetic turbulent energy (such as internal turbulent energy):

$$E = \langle \tilde{n}^2 + (\nabla_\perp \tilde{\phi})^2 \rangle?$$

Manz *et al*, 2011, NF

R_{DT}: a better criterion for turbulence suppression

$$R_{DT} = \frac{\tau_{prod}}{\tau_{transfer}} = \frac{\int \partial_x \langle \tilde{v}_x \tilde{u} \rangle u}{-\int \Gamma_n \nabla n}$$

- Here $1/\tau_{prod} = -\int \Gamma_n \nabla n$ is the rate of turbulent enstrophy production due to relaxation of mean density profile (relation with γ_{eff} in R_T).

- And $1/\tau_{transfer} = \int \partial_x \langle \tilde{u}\, \tilde{v}_x \rangle\, u$ is the rate of turbulent enstrophy destruction via coupling with the mean flow (relation with Reynolds power density in R_T via Taylor ID).

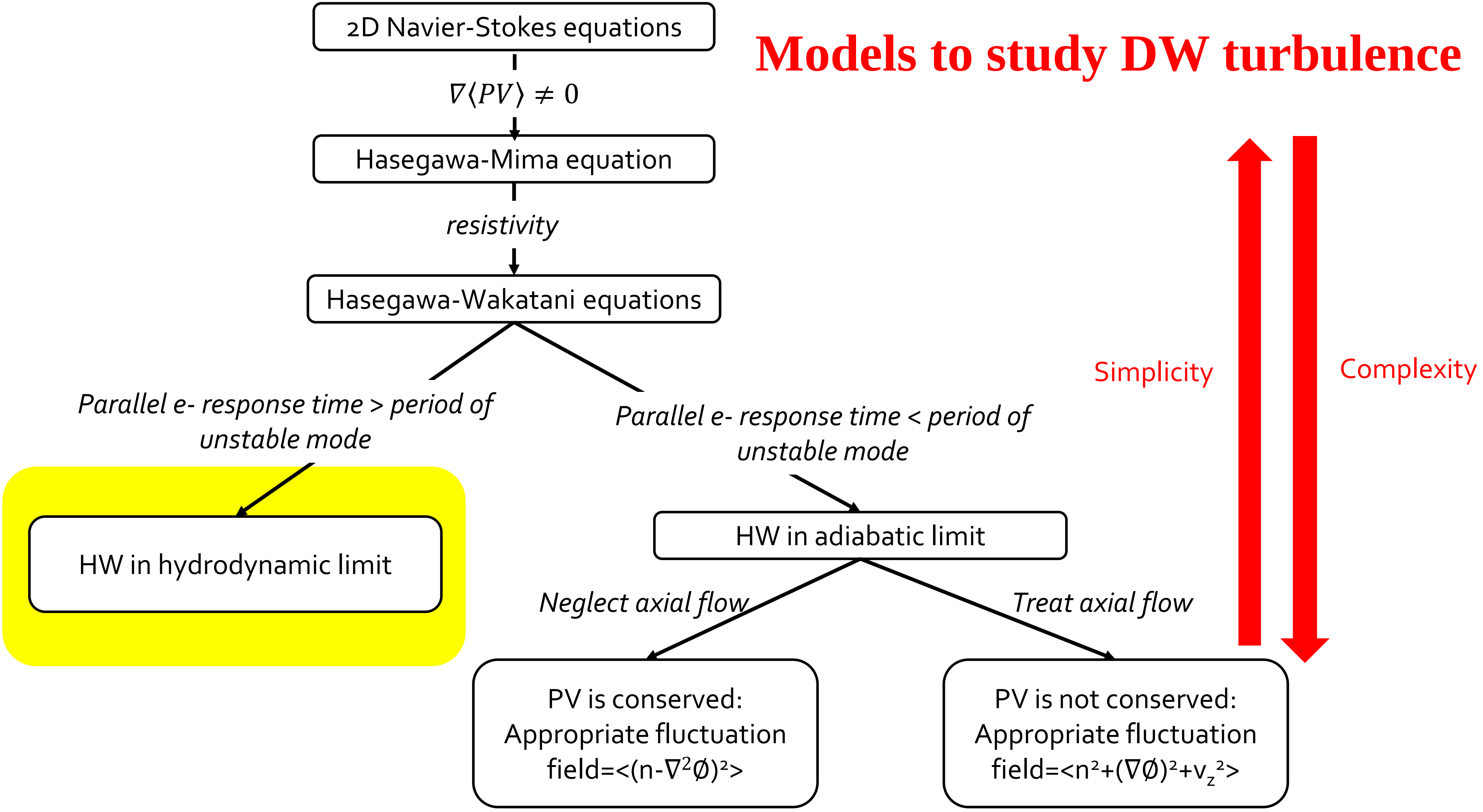
- R_{DT} emerges naturally in this model from the turbulent enstrophy equation:

$$\frac{d\varepsilon}{dt} = -\left(\langle \tilde{v}_x \tilde{n} \rangle - \langle \tilde{v}_x \tilde{u} \rangle\right)(\partial_x n - \partial_x u) + P - \varepsilon^{3/2}$$

- R_{DT} can be easily generalized to complex models by expanding the comparison of sources and sinks for potential enstrophy.

Zonal Flow Shear Layer Collapse in the Hydrodynamic Electron Limit

Models to study DW turbulence



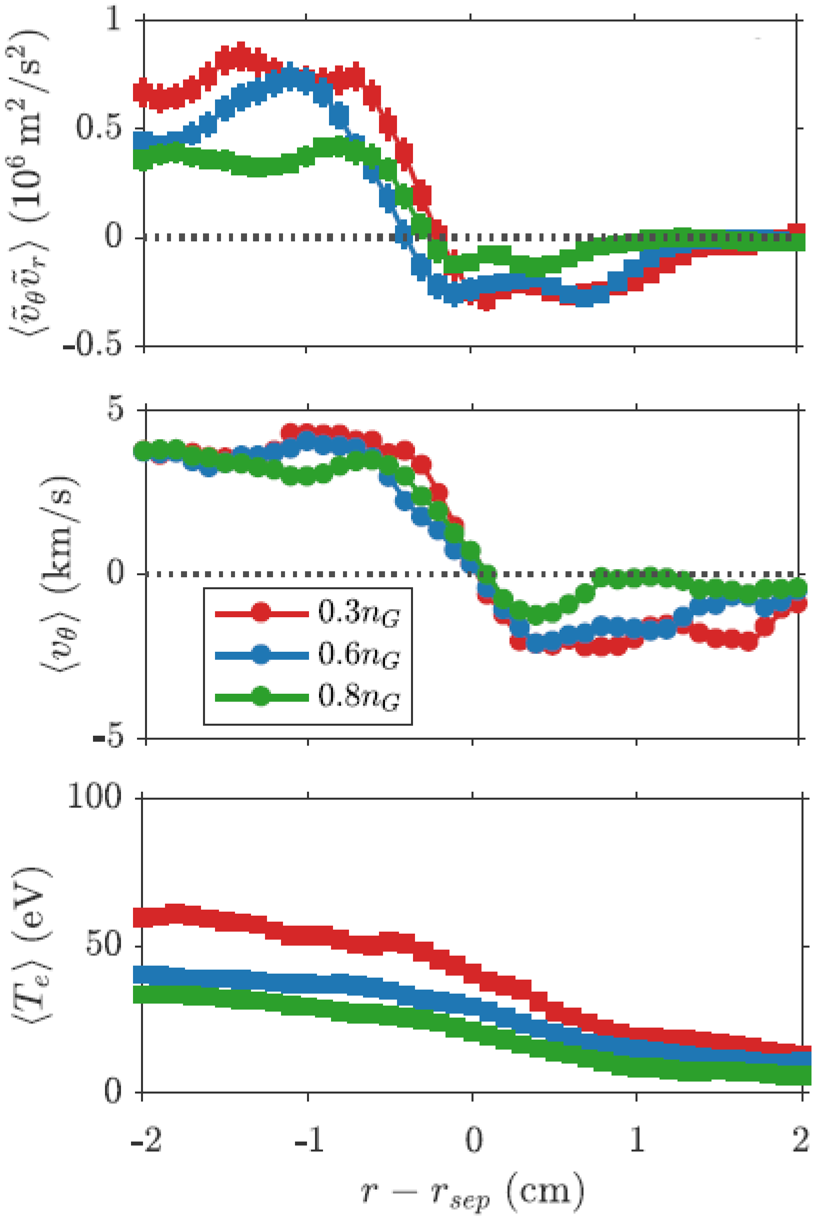
Background: Density Limit Experiments

- Experiments show that as n approaches $n_G = I/\pi a^2$, MHD activity is triggered along with strong disruptions, edge cooling, MARFE…

- Recently, an Ohmic L-mode discharge experiment in HL-2A showed that, as n/n_G is raised:

  + Enhancement of edge turbulence.
  + Edge cooling.
  + Drop in $\alpha = k_z^2 v_{th}^2/(\nu_{ei}|\omega|)$ from 3 to 0.5.
  + Drop in edge shear.

- Note the low values $0.01 < \beta < 0.02$ in this experiment.

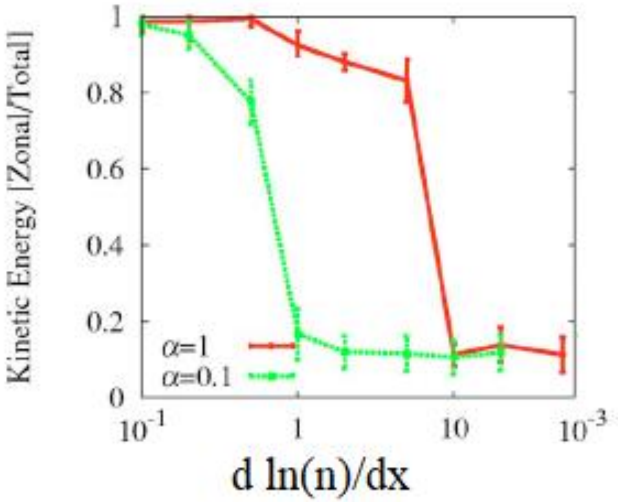


Hong *et al*, NF, 2018)

Hydrodynamic Plasma Limit

$$\alpha = \frac{k_z^2 v_{th}^2}{\nu_{ei}|\omega|} = \frac{Parallel\ Diffusion\ rate}{DW\ frequency}$$

- $\alpha \gg 1$ → adiabatic plasma limit → $\tilde{n}$ and $\nabla^2\tilde{\phi}$ are strongly coupled
- $\alpha \ll 1$ → hydrodynamic plasma limit → $\tilde{n}$ and $\nabla^2\tilde{\phi}$ tend to decouple
- Simulations results show enhancement of turbulence and weakening of edge shear layer as the plasma response passes from the adiabatic to the hydrodynamic limit .



Numata *et al*, 2007

However, these results do not explain WHY turbulence is enhanced in the hydrodynamic limit

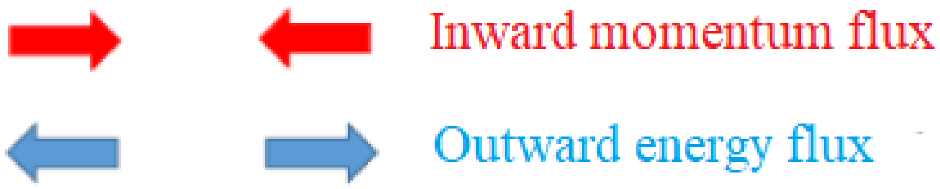
Hypothesis:

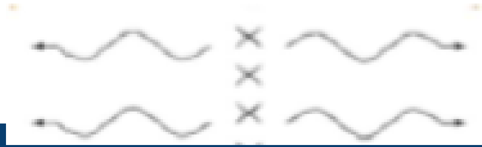
Flow Production drops in Hydrodynamic Limit

Energy and Momentum Fluxes

<u>Adiabatic regime</u> $(k_z^2 v_{th}^2/|\omega|\nu_{ei} \gg 1)$:

- $\langle \tilde{v}_x \tilde{v}_y \rangle = -\sum_k k_r k_m |\tilde{\varphi}_k|^2$
- $\langle v_{gr} \varepsilon \rangle = -\sum_k \frac{k_r k_m}{1+k_\perp^2 \rho_s^2} v_{De}$ $\qquad \omega_k = \frac{\omega^*}{1+k_\perp^2 \rho_s^2} + i \frac{\omega^{*2} k_\perp^2 \rho_s^2}{\alpha}$
- $v_{De} \propto \frac{dn}{dx} < 0$ and $v_{gr} > 0$ → $\mathbf{k_r k_m > 0}$
- Momentum flux <0 and energy flux>0
- Causality implies a counter flow spin-up → eddy shearing and ZF formation





<u>Hydrodynamic regime</u> $(k_z^2 v_{th}^2/|\omega|\nu_{ei} \ll 1)$:

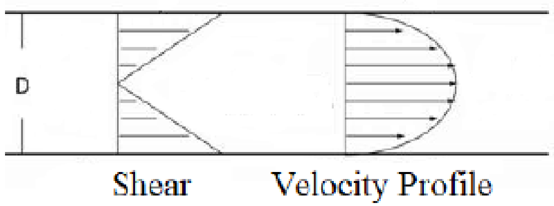
- $\langle \tilde{v}_x \tilde{v}_y \rangle = -\sum_k k_r k_m |\tilde{\varphi}_k|^2$
- $v_{gr} = \frac{\partial \omega_{hydro}^r}{\partial k_r} = -\frac{k_r}{k_\perp^2} \omega_{hydro}^r$
- $\omega_k = \sqrt{\frac{\omega^* \alpha}{2 k_\perp^2 \rho_s^2}} (1+i)$
- v_{gr} is independent of k_m
- Condition of outgoing wave energy flux does not constrain the momentum flux, as v_{gr} is independent of k_m → no implication for Reynolds stress

PV conservation can also be used to square PV mixing with ZF formation

Scaling of transport fluxes with α

Plasma Response	Adiabatic (α >>1)	Hydrodynamic (α <<1)
Particle Flux Γ	$\Gamma_{\mathrm{adia}} \sim \frac{1}{\alpha}$	$\Gamma_{hydro} \sim \frac{1}{\sqrt{\alpha}}$
Turbulent Viscosity χ	$\chi_{adia} \sim \frac{1}{\alpha}$	$\chi_{hydro} \sim \frac{1}{\sqrt{\alpha}}$
Residual stress Π^{res}	$\Pi^{res}{}_{\mathrm{adia}} \sim -\frac{1}{\alpha}$	$\Pi^{res}{}_{\mathrm{hydro}} \sim -\sqrt{\alpha}$
$\frac{\Pi^{\mathrm{res}}}{\chi} = (\omega_{\mathrm{ci}} \nabla n) \times$	$\left(\frac{\alpha}{\lvert\omega\star\rvert}\right)^0$	$\left(\frac{\alpha}{\lvert\omega\star\rvert}\right)^1$

- Mean vorticity gradient $\frac{d(\nabla v_y)}{dx} = \frac{\Pi^{res}}{\chi}$ - which represents the production of ZF - decreases and becomes proportional to $\alpha \ll 1$ in the hydrodynamic limit.
- Weak ZF formation for $\alpha \ll 1$ → weak regulation of turbulence and enhancement of transport.



One step backward: Relevance to the Density Limit Experiments

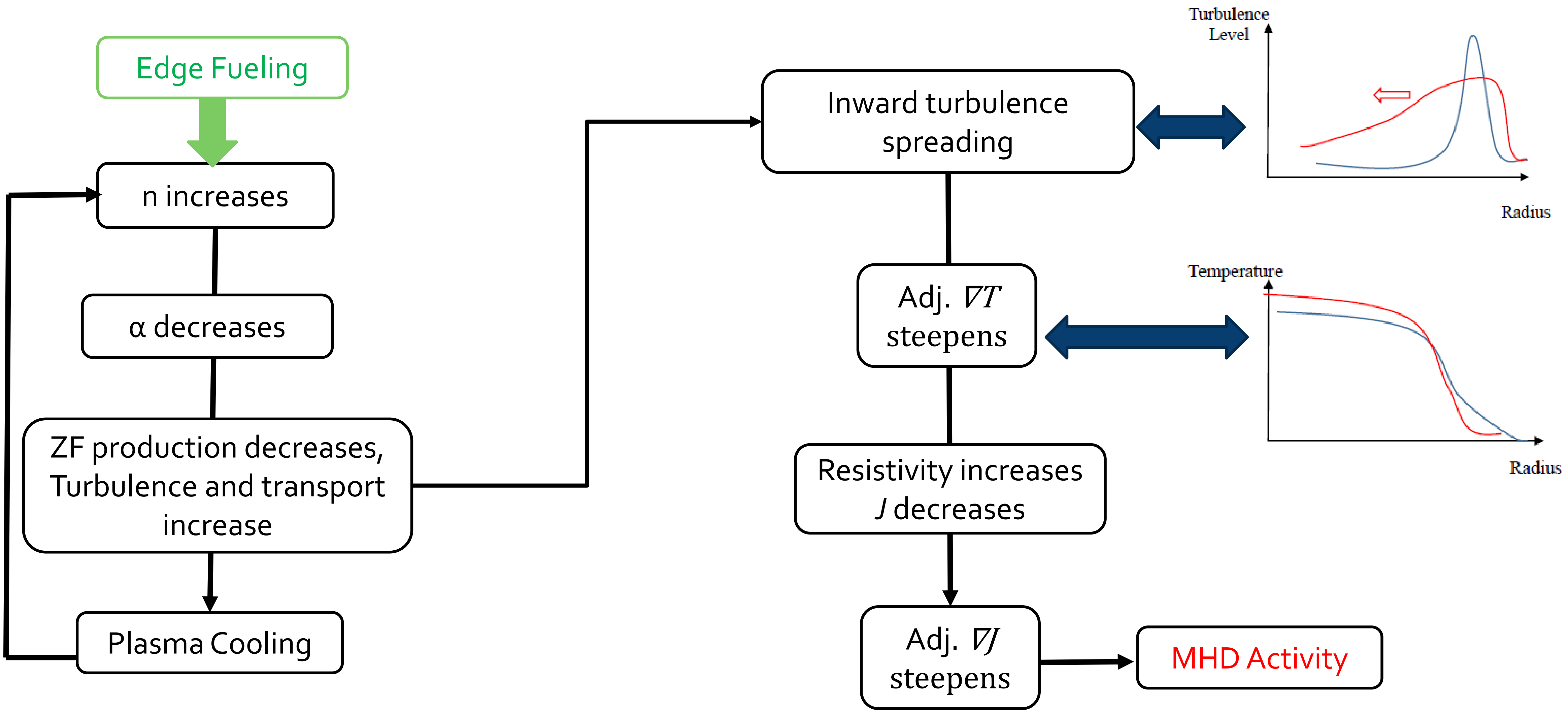
- $\alpha \sim \nu_{ei}^{-1} \sim n^{-1}$ → when n increases, α decreases, the ZF production weakens and turbulence is enhanced.

No appeal to:

1) ZF damping effects associated with plasma collisionality, charge exchange – (murky, case sensitive).

2) The development of other instabilities, such as resistive ballooning modes which are not relevant in this experiment because of the low β values.

All Roads Lead to MHD instabilities



What did I learn while pursuing a PhD?

- Reduced models are a powerful tool to describe complex turbulent systems.
- They describe feedback loops and allow the study of plasma profiles across timescales ranging from a few turbulent correlation times up to system equilibrium time scales.
- Reduced models distill what is learned from simulations, basic theory and experiments.
- Capacity of drift wave turbulence to accelerate both zonal and axial flows via the Reynolds stresses in both parallel and perpendicular directions.
- Importance of parallel symmetry breaking in determining the energy branching in the system as well as the strength of the parallel to perpendicular flow coupling.
- Relation between wave energy flux, Reynolds stress and PV mixing is essential in regulating turbulence in both adiabatic and hydrodynamic plasma limits, where predators feed on the prey in the former case, or are simply not produced in the latter.
- Mechanism for onset of turbulence when $\frac{k_z^2 v_{th}^2}{|\omega| \nu_{ei}} \ll 1$ is the collapse of the ZF regulation

Recommendations for future work

- Numerical simulations of a slow evolution plasma transition form the adiabatic to the hydrodynamic plasma limit.

- Adding charge-exchange effects, and ion-neutral collisions to the model, so to numerically study the role of collisional ZF damping

- Generalize the model to include an investigation of both flows and fluctuations in *H*-mode hydrodynamic plasma limit (Need to add temperature equations for both ions and electrons, EM effects).

You made a difference.
THANK YOU

- Pat, George for your patience and immense knowledge.
- My family.
- Lunch group people.
- Awesome San Diegans.
- Avram Dalton.

Backup

Scaling of l_0 from experimental results

$B(G)$	800	900	1000	1200	1300
$\rho_s(cm)$	1.40	1.24	1.12	0.93	0.86
$L_n^{-1}(cm^{-1})$	0.53	0.55	0.6	0.62	0.5
$\bar{k}_r(cm^{-1})$	0.33	0.33	0.37	0.32	0.34
$1/[2.3\rho_s^{0.6}L_n^{0.3}]$	0.29	0.32	0.34	0.39	0.37

$$l_0 = \bar{k}_r^{-1} = 2.3\rho_s^{0.6}L_n^{0.3} \sim \rho_s$$

- k_r is calculated form density fluctuations.

Additional Numerical Results

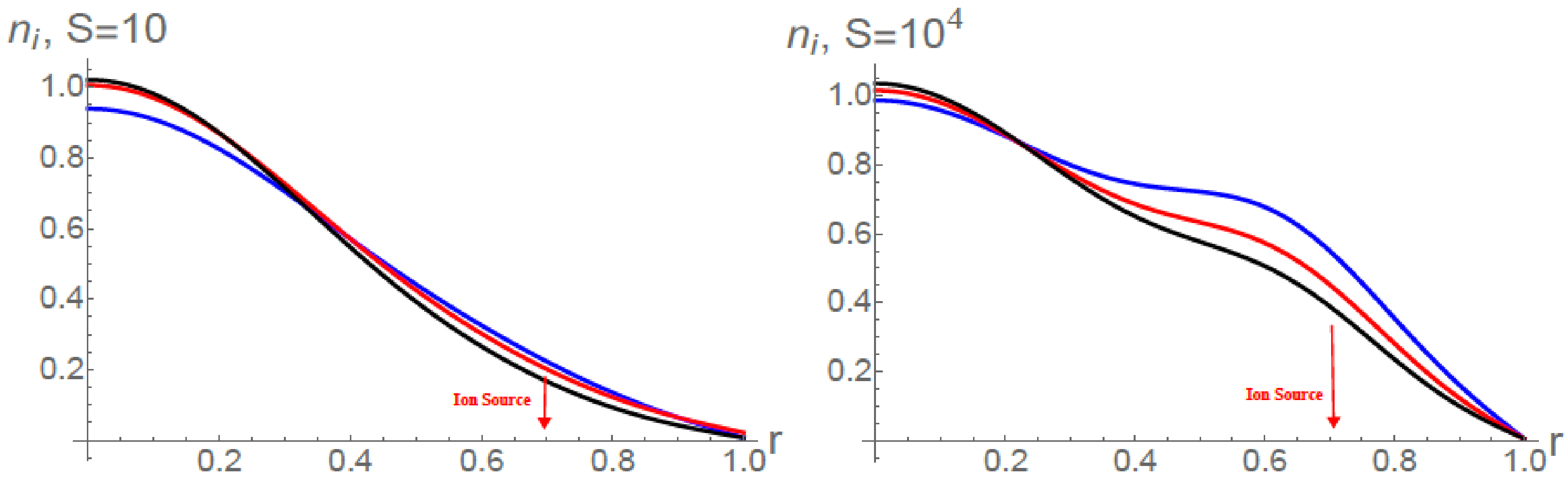
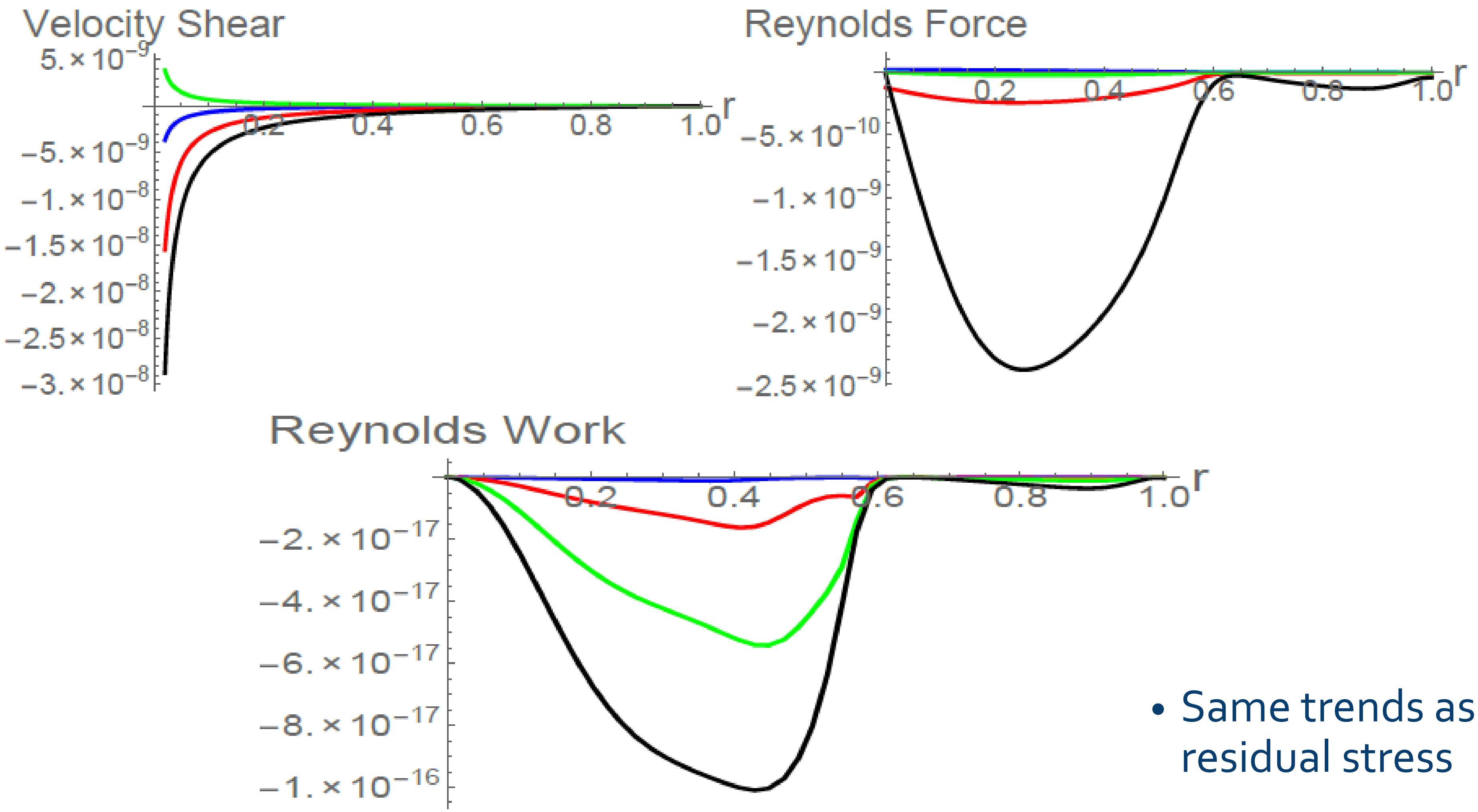


FIG. 2: Density profiles for $S = 10$ and $S = 10^4$ for increasing B.

- Steepening of density profile for different amplitudes of the density source S_n

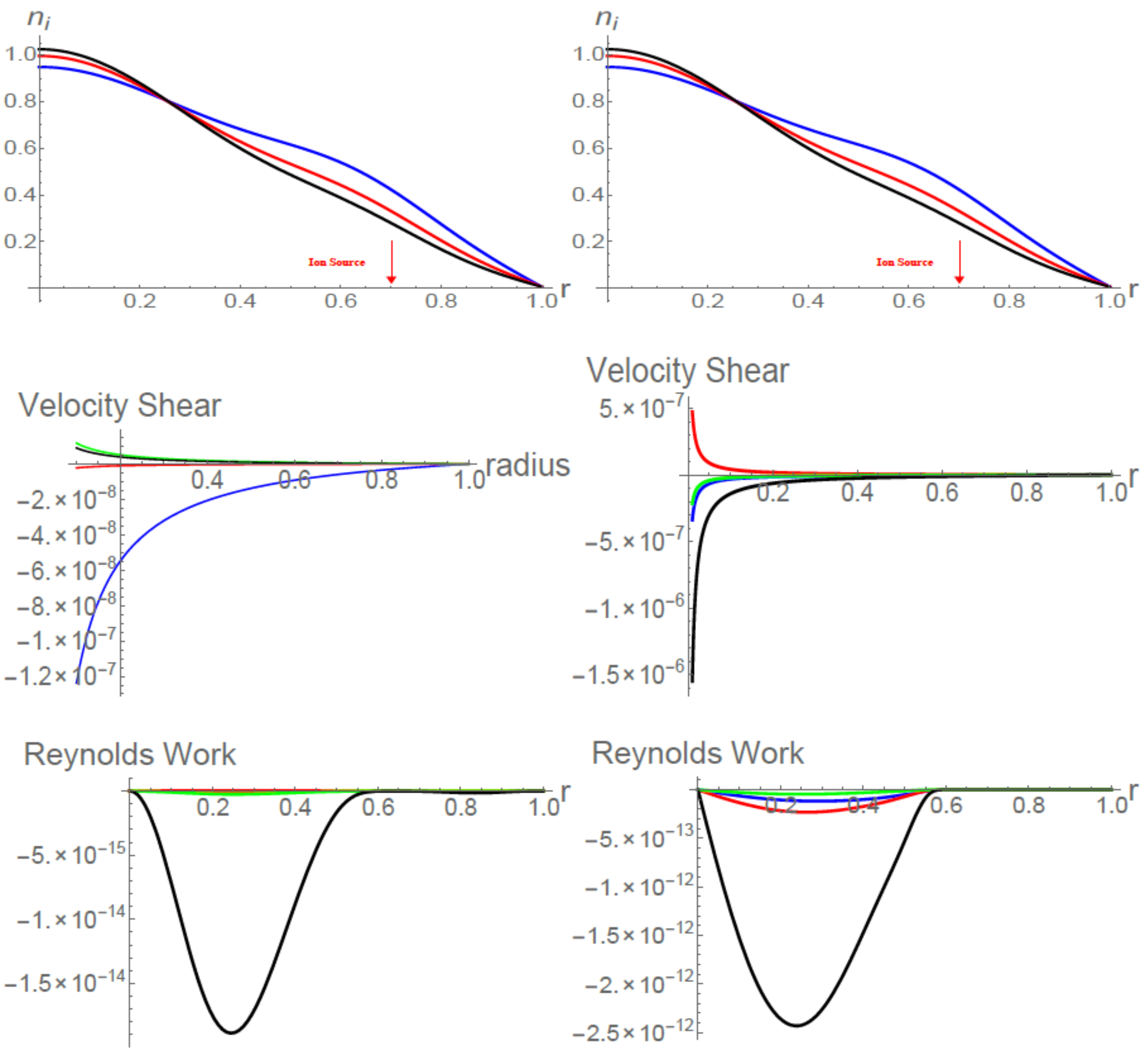
Numerical Results without residual vorticity flux



- Same trends as with a residual stress

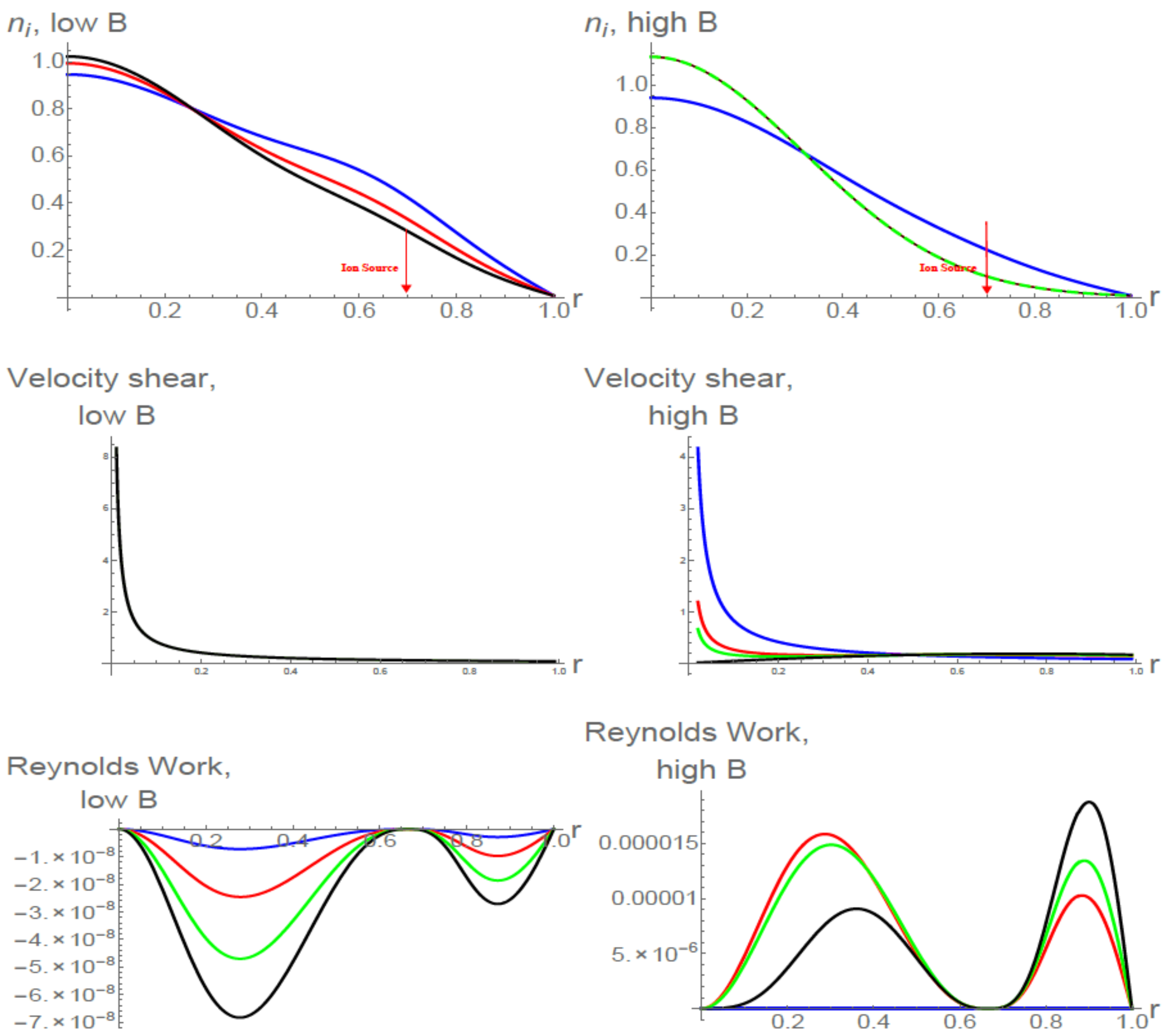
Variations of the shearing factor c_u

$c_u=6$



$c_u=600$

Results for Neumann Boundary conditions for vorticity



<u>For Low B</u>

- Steepening of density.
- Increase in Reynolds work (magnitude)
- Development of velocity shear.

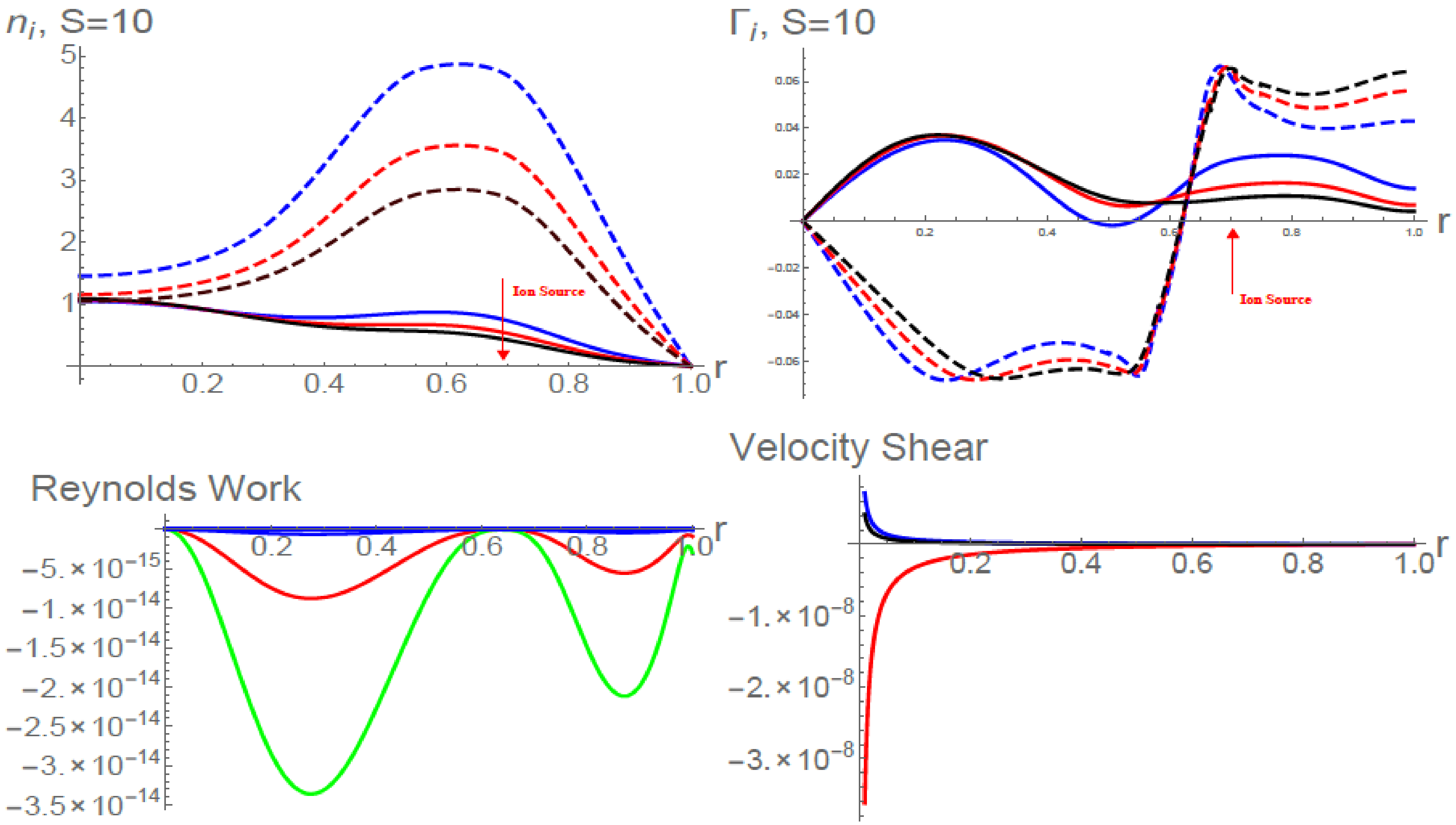
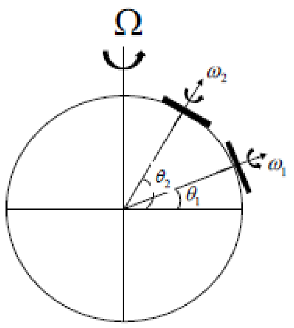


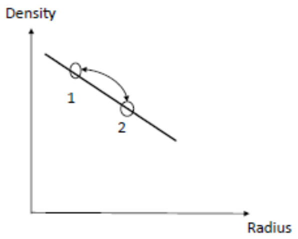
FIG. 11: Profiles for $Pr = 65000$ and increasing B. Solid and dashed plots correspond to data at t_1 and t_2 respectively.

How does ZF collapse square with PV Mixing



Rossby waves:

- $PV = \nabla^2\phi + \beta y$ is conserved between θ_1 and θ_2.
- Total vorticity $2\vec{\Omega} + \vec{\omega}$ is frozen in→ Change in mean vorticity Ω leads to a change in local vorticity ω → Flow generation, via Taylor's ID.



Drift waves:

- In HW, the $q = \ln n - \nabla^2\phi = \ln n_0 + h + \tilde{\phi} - \nabla^2\phi$ is conserved along the line of density gradient.
- Change in density from position 1 to position 2→ change in vorticity → Flow generation via Taylor ID

Quantitatively

- The PV flux $\Gamma_q = \langle \tilde{v}_x h \rangle - \rho_s^2 \langle \tilde{v}_x \nabla^2\phi \rangle$
- Adiabatic limit $\alpha \gg 1$:
  +Particle flux and vorticity flux are tightly coupled (both are prop. to $1/\alpha$)
- Hydrodynamic limit $\alpha \ll 1$:
  +Particle flux is proportional to $1/\sqrt{\alpha}$.
  +Residual vorticity flux is proportional to $\sqrt{\alpha}$.
- PV mixing is still possible without ZF formation → Particles carry PV flux