Dynamics of Zonal Shear Collapse in Hydrodynamic Electron Limit
↕
Transport Physics of the Density Limit

R. Hajjar, P. H. Diamond, M. Malkov

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Outline

- Introduction: Shear layer collapse at the density limit.

- Density limit $\bar{n}/\bar{n}_G \to 1$ as a transport phenomenon.

- Recent experimental studies related to the density limit.

- A model for the collapse of zonal flows as $\bar{n} \to \bar{n}_G$.
  A closer look at the:
  - Energy and Momentum fluxes in the adiabatic and hydrodynamic electron limit.
  - PV mixing in both electron limits.
  - Scaling of the transport fluxes and evolution of the shear flow layer.

- Implications and recommended experimental tests.

Introduction

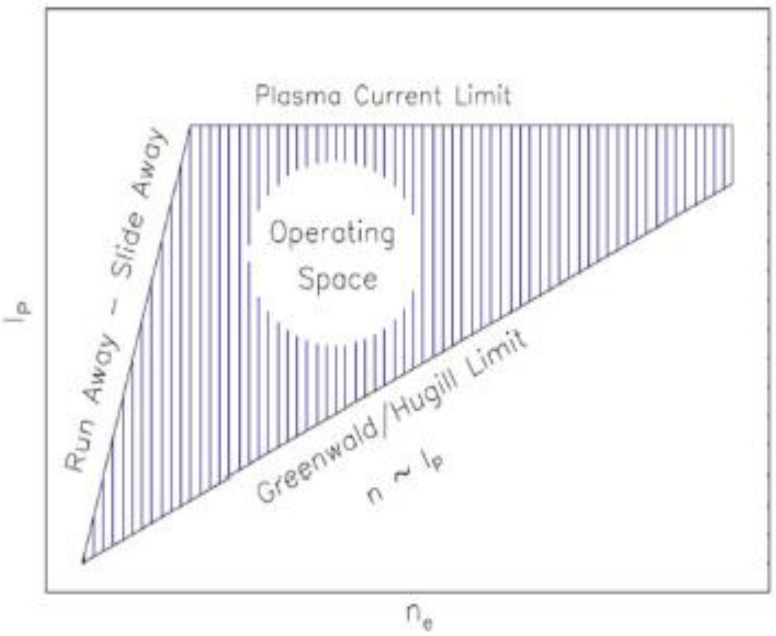
- An explanation of the density limit phenomenon using a simple mechanism of DW turbulence. Note that the density limit is manifested in tokamaks, stellerators and RFPs.

- Understand why ZFs collapse in the hydrodynamic electron limit

  Key parameter: Local adiabaticity parameter $\alpha = \frac{k_z^2 v_{th}^2}{|\omega| \nu_{ei}} \sim \frac{T_e^2}{n}$ (at fixed $k_\theta \rho_s$ and $\omega \sim \omega^*$)

- Previous work simply described the symptoms of the density limit but did not present a physical explanation of the enhancement of turbulence and particle transport as $\bar{n} \rightarrow \bar{n}_G$.

Density limit as a transport phenomenon-1



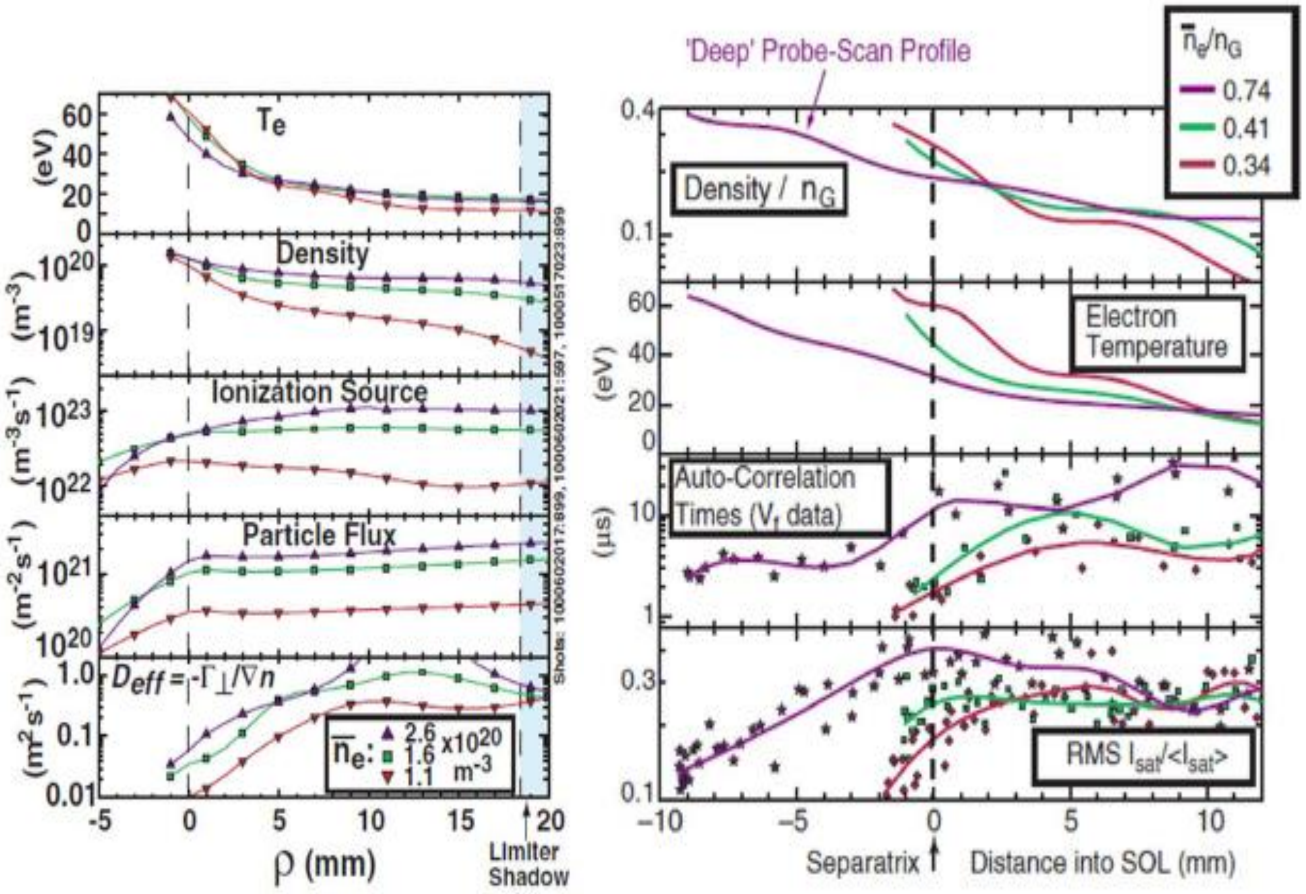
Tokamak Operating Space

- Greenwald density limit $\bar{n} = \bar{n}_g \sim \frac{I_p}{\pi a^2}$

associated with:

1. MARFE radiation = impurity flux (sometimes)
2. MHD disruptions.
3. Divertor detachment.
4. $H \rightarrow L$ back transition.

Density limit as a transport phenomenon-2

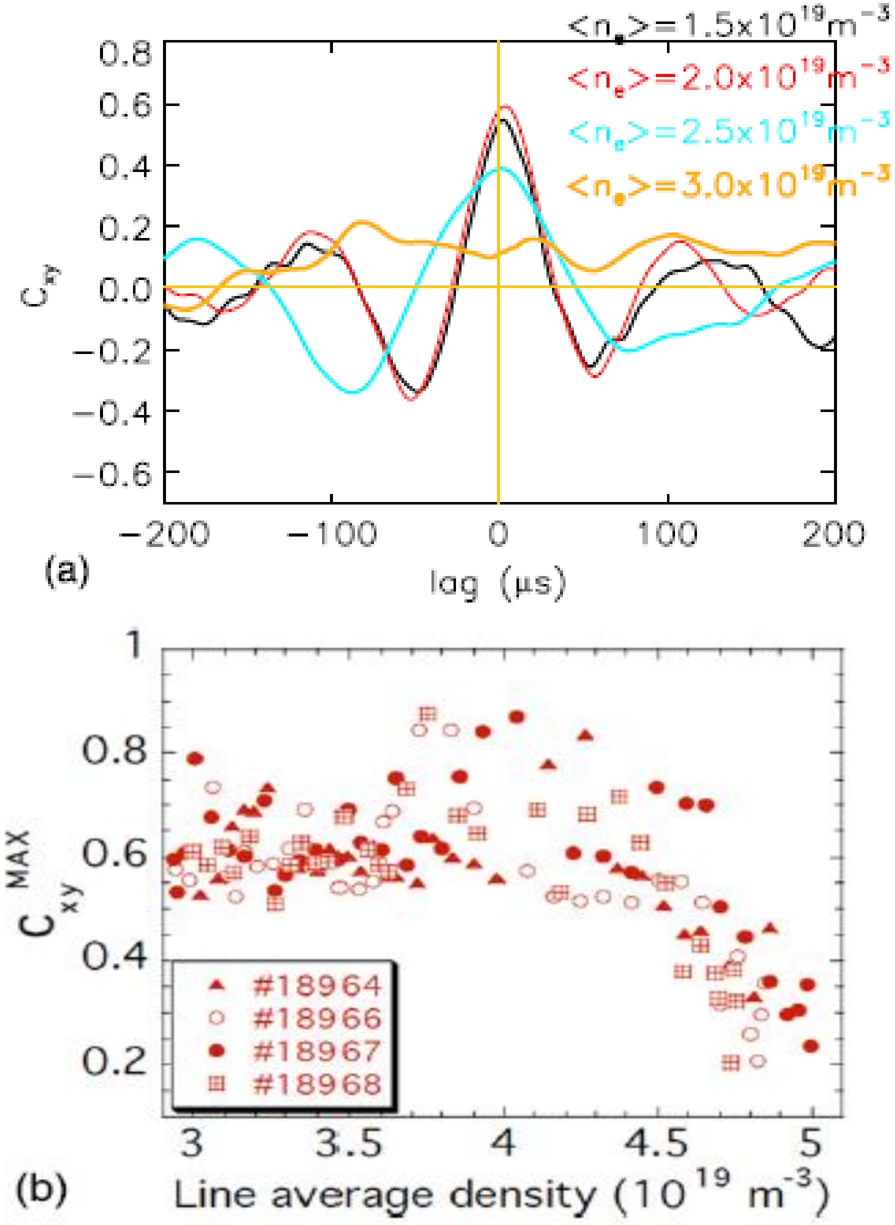


C-Mod profiles,
Greenwald et al, 2002, PoP

- Average plasma density increases as a result of edge fueling → edge transport is crucial to density limit.
- As *n* increases, high ⊥ transport region extends inward and fluctuation activity increases.
- Turbulence levels increase and perpendicular particle transport increases as $n/n_G \rightarrow 1$.

Recent Experiments - 1

(Y. Xu et al., NF, 2011)



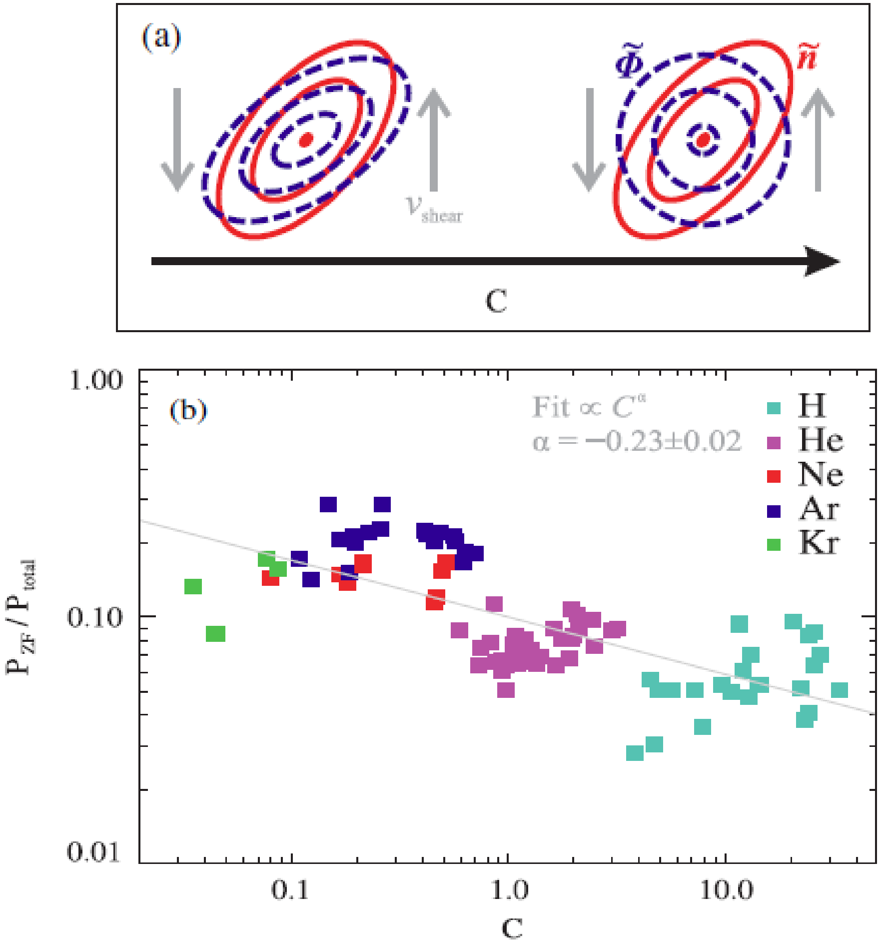
- Decrease in maximum correlation value of LRC (i.e. ZF strength) as line averaged density <n> increases at the edge (r/a=0.95) in both TEXTOR and TJ-II.

- At high density ($\langle n_e \rangle > 2 \times 10^{19}\ m^{-3}$), the LRC (also associated with GAMs) drops rapidly with increasing density.

- Interestingly, the reduction in LRC due to increasing density is also accompanied by a reduction in edge mean radial electric field (Relation to ZFs).

Recent Experiments - 2

(Schmid et al., PRL, 2017)

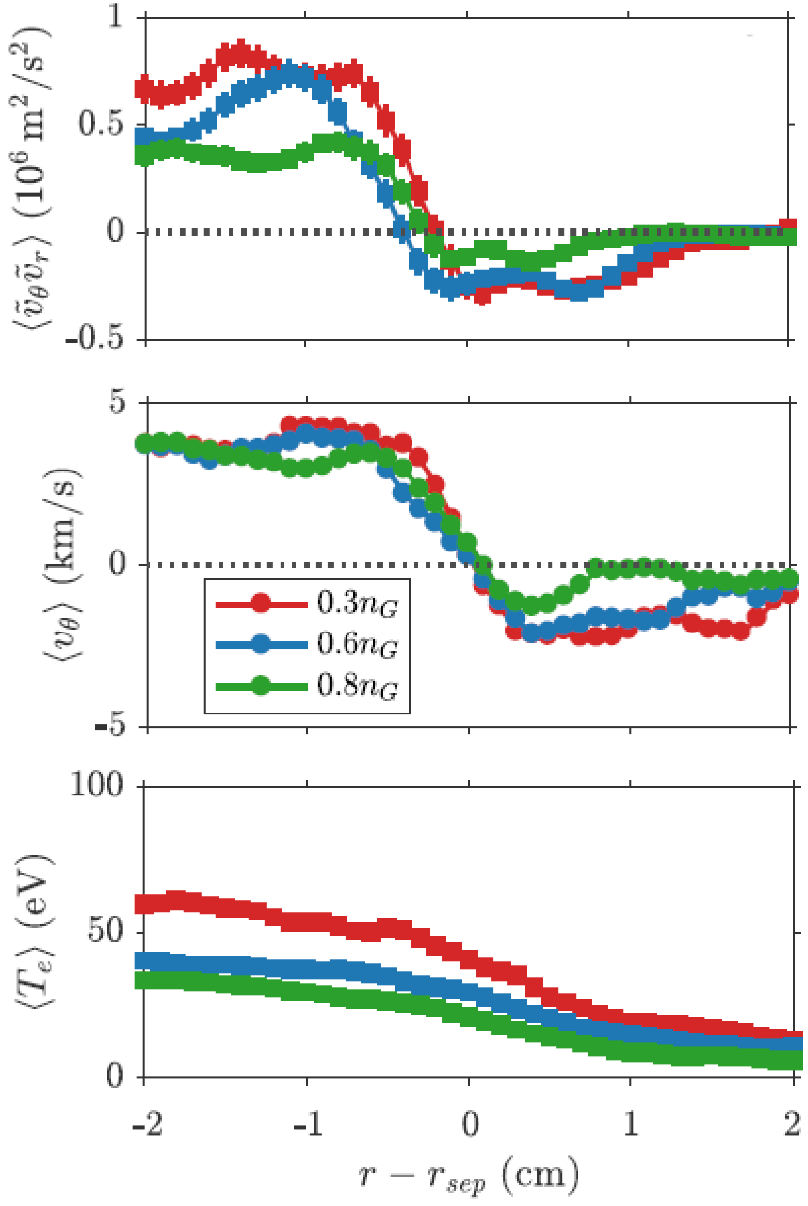


- First experimental verification of the importance of collisionality for large-scale structure formation in TJ-K.
- Analysis of the Reynolds stress and pseudo-Reynolds stress shows a decrease in the coupling between density and potential for increasing collisionality → hindering of zonal flow drive.
- Decrease of the zonal flow contribution to the complete turbulent spectrum with collisionality *C*.

a) Increase in decoupling between density (red) and potential (blue) coupling with collisionality C.
b) Increase in ZF contribution to the spectrum in the adiabatic limit (C→0)

Recent Experiments – 3

(Hong et al., NF, 2018)



- An Ohmic *L*-mode discharge experiment in HL-2A showed that, as n/n_G is raised:

  + Enhancement of edge turbulence.

  + Edge cooling.

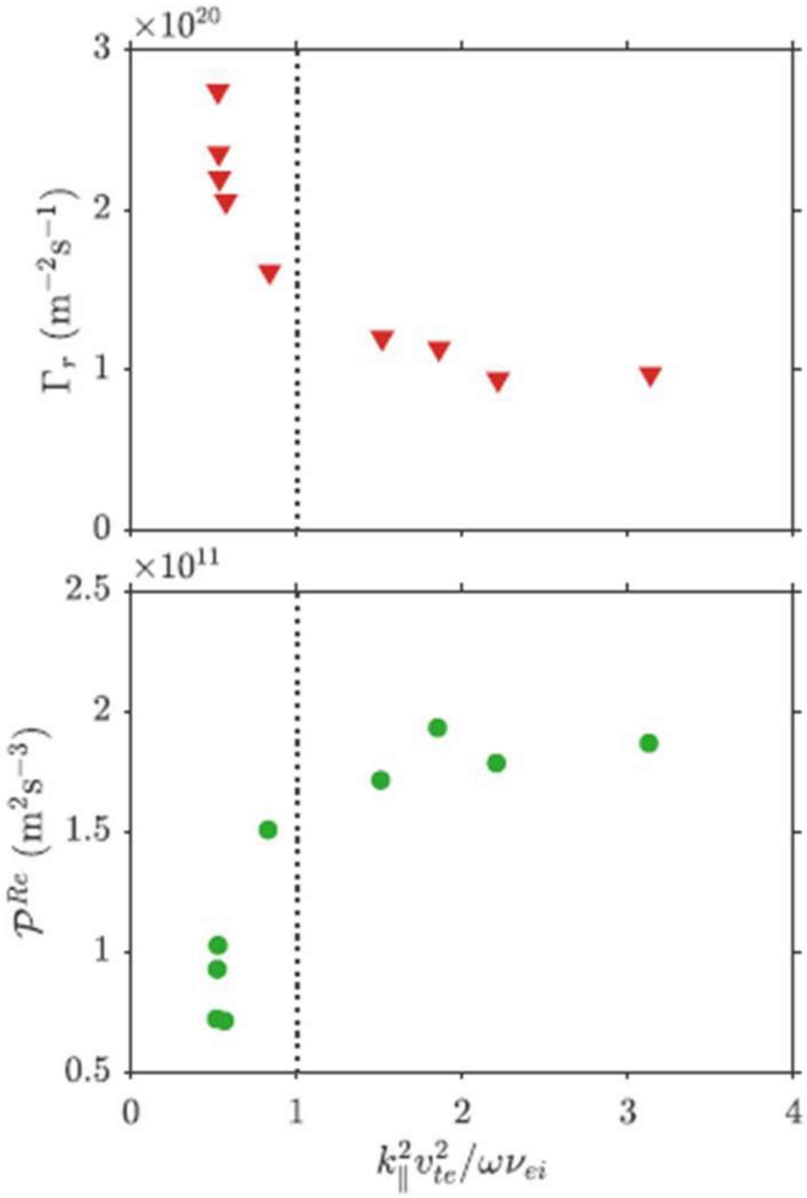
  + Drop in $\alpha = \frac{k_z^2 v_{th}^2}{|\omega| \nu_{ei}}$ from 3 to 0.5.

  + Drop in edge shear.

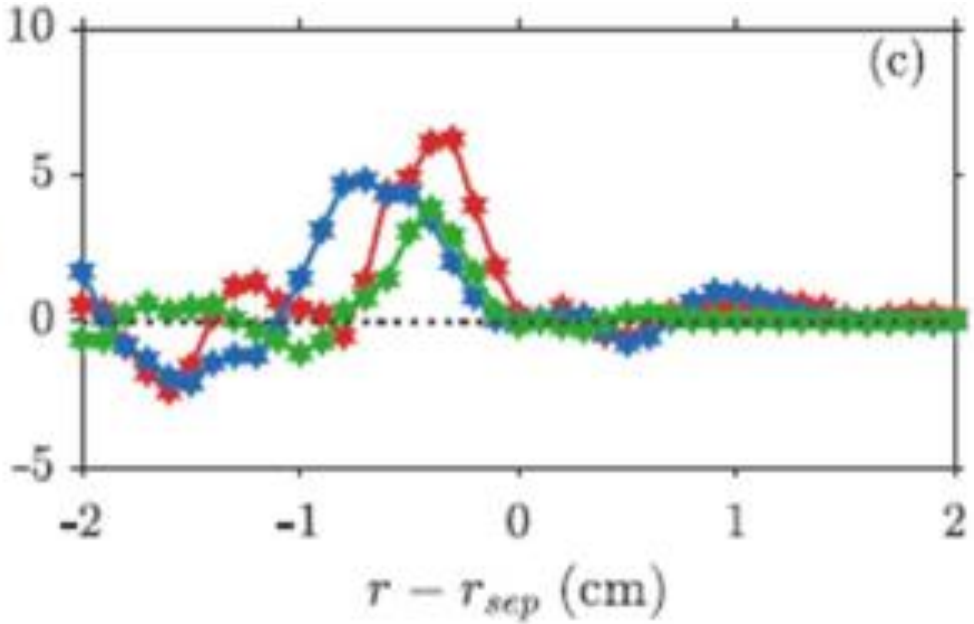
- Note the low values $0.01<\beta<0.02$ in this experiment

- Electron adiabaticity $\alpha = \frac{k_z^2 v_{th}^2}{|\omega| \nu_{ei}}$ emerges as an interesting local parameter.

- Particle flux ↑ and Reynolds power $P_{Re} = -\langle V_\theta \rangle \partial_r \langle \tilde{V}_r \tilde{V}_\theta \rangle$ ↓ as α drops below unity.



P_{Re}



Synthesis of the Experiments

- Shear layer collapse and turbulence and D (particle transport) rise as $\frac{\bar{n}}{\bar{n}_G} \to 1$.

- ZF collapse as $\alpha = \frac{k_z^2 v_{th}^2}{|\omega| \nu_{ei}}$ drops from $\alpha > 1$ to $\alpha < 1$.

- Degradation in particle confinement at density limit in L-mode is due to ZF collapse and rise in turbulence.

- Note that β in these experiments is too small for conventional Resistive Ballooning Modes (RBM) explanations.

A model for the collapse of the ZFs as $n \rightarrow n_G$

Hasegawa-Wakatani for Collisional DWT:

$$\frac{dn}{dt} = -\frac{v_{th}^2}{\nu_{ei}}\nabla_{\parallel}^2(\phi - n) + D_0\nabla^2 n$$

$$\frac{d\nabla^2\phi}{dt} = -\frac{v_{th}^2}{\nu_{ei}}\nabla_{\parallel}^2(\phi - n) + \mu_0\nabla^2(\nabla^2\phi)$$

$$\alpha = \frac{k_z^2 v_{th}^2}{|\omega|\nu_{ei}}$$

Fluctuations

Mean Fields

$$\partial_t\tilde{n} + \tilde{v}_x.\nabla\bar{n} = -\frac{v_{th}^2}{\nu_{ei}}\nabla_{\parallel}^2(\tilde{\phi} - \tilde{n}) - \{\tilde{\phi}, \tilde{n}\} + D_0\nabla^2\tilde{n}$$

$$\partial_t\nabla^2\tilde{\phi} + \tilde{v}_x.\nabla\overline{\nabla^2\phi} = -\frac{v_{th}^2}{\nu_{ei}}\nabla_{\parallel}^2(\tilde{\phi} - \tilde{n}) - \{\tilde{\phi}, \nabla^2\tilde{\phi}\} + \mu_0\nabla^2(\nabla^2\tilde{\phi})$$

$$\partial_t\bar{n} = -\partial_x\langle\tilde{V}_x\tilde{n}\rangle + D_0\bar{\nabla}_x^2\bar{n}$$

$$\partial_t\overline{\nabla_x^2\phi} = -\partial_x\langle\tilde{V}_x\nabla^2\tilde{\phi}\rangle + \mu_0\nabla_x^2\overline{\nabla_x^2\phi}$$

Dispersion Relation for $\alpha < 1$ *and* $\alpha > 1$

Dispersion relation:

$$\omega = \frac{1}{2}\left(-i\frac{\hat{\alpha}(1+k_\perp^2\rho_s^2)}{k_\perp^2\rho_s^2} + \sqrt{\frac{4i\omega^\star\hat{\alpha}}{k_\perp^2\rho_s^2} - \left(\frac{\hat{\alpha}(1+k_\perp^2\rho_s^2)}{k_\perp^2\rho_s^2}\right)^2}\right)$$

$$\hat{\alpha} = -\frac{v_{th}^2}{\nu_{ei}}\nabla_\parallel^2$$

$$\alpha = \frac{k_z^2 v_{th}^2}{\nu_{ei}|\omega|}$$

Adiabatic Limit:

($\alpha \gg 1$ and $\hat{\alpha} \gg |\omega|$)

$$\omega_{adiabatic} = \frac{\omega^\star}{1+k_\perp^2\rho_s^2} + i\frac{\omega^{\star 2}k_\perp^2\rho_s^2}{\hat{\alpha}}$$

Wave + inverse dispersion

Hydro Limit:

($\alpha \ll 1$ and $\hat{\alpha} \ll |\omega|$)

$$\omega_{hydrodynamic} \simeq \sqrt{\frac{\omega^\star\hat{\alpha}}{2k_\perp^2\rho_s^2}}(1+i)$$

Convective Cell

An Idiot proof argument for ZF collapse for Hydrodynamic Electrons: Wave propagation

Adiabatic regime ($k_z^2 v_{th}^2/|\omega|\nu_{ei} \gg 1$):

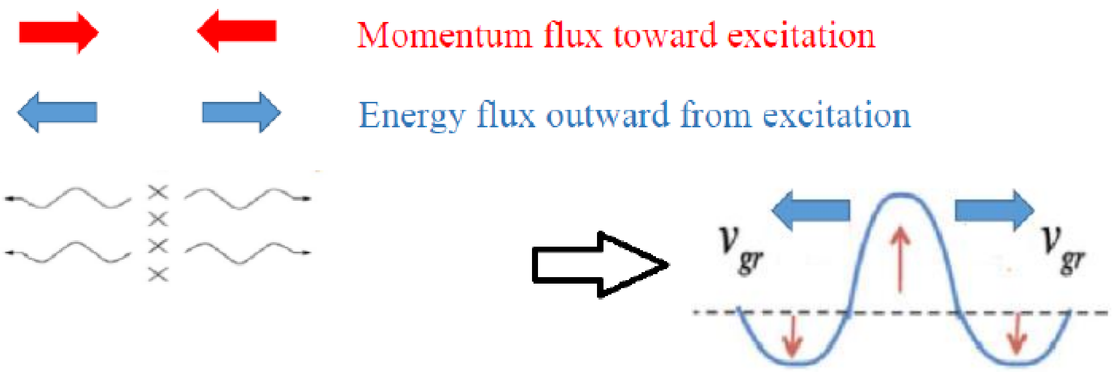
$$\langle \tilde{v}_x \tilde{v}_y \rangle = -\sum_k k_r k_m |\tilde{\varphi}_k|^2 \qquad \langle v_{gr}\varepsilon \rangle = -\sum_k \frac{k_r k_m}{1+k_\perp^2 \rho_s^2} v_{De}$$

- $v_{De} \propto \frac{dn}{dx} < 0$ and $v_{gr} > 0$ → $k_r k_m > 0$
- Momentum flux <0 and energy flux>0
- Causality implies a counter flow spin-up → eddy shearing and ZF formation

Hydrodynamic regime ($k_z^2 v_{th}^2/|\omega|\nu_{ei} \ll 1$):

$$\langle \tilde{v}_x \tilde{v}_y \rangle = -\sum_k k_r k_m |\tilde{\varphi}_k|^2 \qquad v_{gr} = \frac{\partial \omega_{hydro}^r}{\partial k_r} = -\frac{k_r}{k_\perp^2}\omega_{hydro}^r$$

- v_{gr} is not proportional to k_m
- Condition of outgoing wave energy flux does not constrain the momentum flux, as v_{gr} is not proportional to k_m → no implication for Reynolds stress



BOTTOM LINE:

The Tilt and Stretch mechanism fails in the Hydrodynamic limit, as causality does not constrain the Reynolds stress.

Reduced Model (Hajjar et al., PoP, 2017 and Hajjar et al., PoP, 2018)

- $\partial_t n = -\partial_x \Gamma_n + D_0 \nabla_x^2 n$

$$l_{mix} = \frac{l_0}{\left(1+\frac{(l_0 \nabla u)^2}{\varepsilon}\right)^{\delta}} \rightarrow l_0$$

- $\partial_t u = -\partial_x \Pi + \mu_0 \nabla_x^2 u$

- $\partial_t \varepsilon + \partial_x \Gamma_\varepsilon = -(\Gamma_n - \Pi)(\partial_x n - \partial_x u) - \varepsilon^{\frac{3}{2}} + P$

<u>Fluxes:</u>

- Particle flux = $\Gamma_n = \langle \tilde{n}\, \tilde{v}_x \rangle$
- Vorticity flux = $\Pi = \langle \nabla^2 \tilde{\phi}\, \tilde{v}_x \rangle = -\chi_y \partial_x u + \Pi^{res} = -\partial_x \langle \tilde{v}_x \tilde{v}_y \rangle$ (Taylor ID)
- Potential enstrophy density flux = Γ_ε = turbulence spreading due to triad coupling

Expression of Transport Fluxes as calculated by QLT:

$\rightarrow \Gamma_n = -D\,\partial_x n = -\frac{(\hat{\alpha}+|\gamma_m|)}{|\omega+i\hat{\alpha}|^2}\frac{d\ln n}{dx}\langle\delta v_x^2\rangle$ → Diffusive Flux

$\rightarrow \Pi = -\chi_y\,\partial_x u + \Pi^{res}$

Shear relaxation by turbulent viscosity

Shear production and acceleration of flow by ∇n

$$\chi_y = \frac{|\gamma_m|\langle\delta v_x^2\rangle}{|\omega|^2}$$

$$\Pi^{res} = \frac{k_\theta\rho_s c_s\omega_{ci}\hat{\alpha}\left[(\omega^r)^2(\omega^\star-\omega^r)-|\gamma_m|^2(\omega^r+\omega^\star)-\omega^\star\hat{\alpha}|\gamma_m|\right]}{|\omega|^2\times|\omega+i\hat{\alpha}|^2}\langle\tilde{\phi}^2\rangle$$

$\rightarrow \Gamma_\varepsilon = -l_{mix}^2\sqrt{\varepsilon}\,\partial_x\varepsilon$ → Turbulence Spreading

Clear dependence of D, χ_y, Π^{res} on $|\omega|$ and $\hat{\alpha}$

Transport Fluxes

Adiabatic limit

$$n_0\Gamma_n = -\frac{\langle \delta v_x^2\rangle}{\hat{\alpha}}\frac{d\bar{n}}{dx} \simeq -\frac{\varepsilon l_{mix}^2}{\hat{\alpha}}\frac{d\bar{n}}{dx}$$

$$\Pi = -\frac{|\gamma_m|\langle \delta v_x^2\rangle}{|\omega|^2}\frac{d^2\bar{v}_y}{dx^2} - \frac{\omega_{ci}\langle \delta v_x^2\rangle}{\hat{\alpha}}\frac{d\bar{n}}{dx}\left(\frac{k_\perp^2\rho_s^2}{1+k_\perp^2\rho_s^2}\right)$$

$$\simeq -\frac{\varepsilon l_{mix}^2}{\hat{\alpha}}\frac{d^2\bar{v}_y}{dx^2} - \frac{\omega_{ci}\varepsilon l_{mix}^2}{\hat{\alpha}}\frac{d\bar{n}}{dx}$$

$$\Gamma_n \simeq -(\varepsilon l_{mix}^2/\hat{\alpha})\nabla\bar{n}$$

$$\chi_y \simeq \varepsilon l_{mix}^2/\hat{\alpha}$$

$$\Pi^{res} \simeq -(\omega_{ci}\varepsilon l_{mix}^2/\hat{\alpha})\nabla\bar{n}$$

Hydrodynamic limit

$$n_0\Gamma_n \simeq -\sqrt{\frac{k_\perp^2\rho_s^2}{2k_\theta\rho_s c_s}}\sqrt{\frac{|d\bar{n}/dx|}{\hat{\alpha}}}\langle \delta v_x^2\rangle \simeq -\frac{\varepsilon l_{mix}^2}{\sqrt{\hat{\alpha}|\omega^\star|}}\frac{d\bar{n}}{dx}$$

$$\Pi = -\frac{|\gamma_m|\langle \delta v_x^2\rangle}{|\omega|^2}\frac{d^2\bar{v}_y}{dx^2} - \frac{\omega_{ci}\langle \delta v_x^2\rangle}{k_\theta\rho_s c_s}.\sqrt{\frac{k_\perp^2\rho_s^2}{2}}\sqrt{\frac{\hat{\alpha}}{|\omega^\star|}}$$

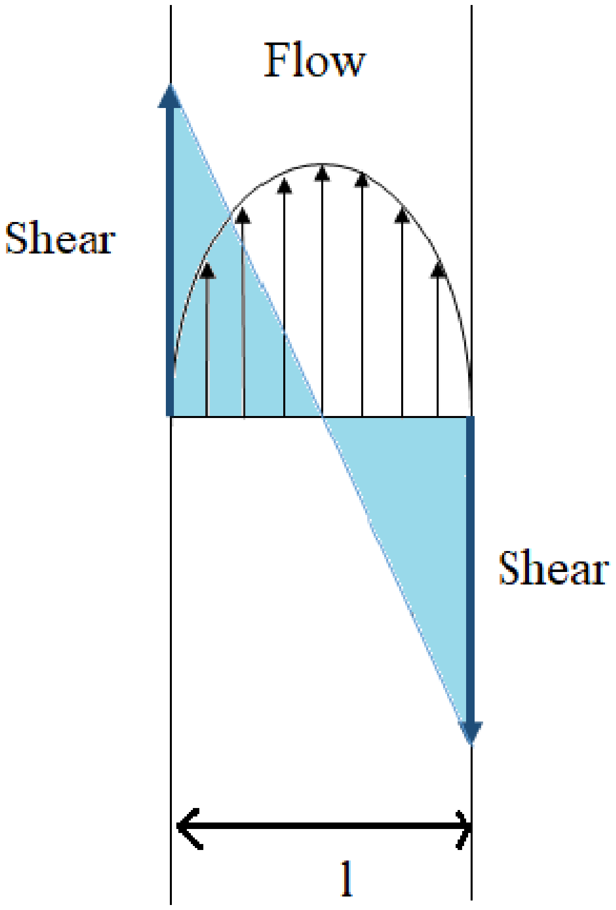
$$\simeq -\frac{\varepsilon l_{mix}^2}{\sqrt{\hat{\alpha}|\omega^\star|}}\frac{d^2\bar{v}_y}{dx^2} - \frac{\omega_{ci}\varepsilon\sqrt{\hat{\alpha}}l_{mix}^2}{|\omega^\star|^{3/2}}\frac{d\bar{n}}{dx}$$

$$\Gamma_n \simeq -(\varepsilon l_{mix}^2/\sqrt{\hat{\alpha}|\omega^\star|})\nabla\bar{n}$$

$$\chi_y \simeq \varepsilon l_{mix}^2/\sqrt{\hat{\alpha}|\nabla\bar{n}|}$$

$$\Pi^{res} \simeq -(\omega_{ci}\varepsilon\sqrt{\hat{\alpha}}l_{mix}^2/|\omega^\star|^{3/2})\nabla\bar{n}$$

Evolution of the Stationary vorticity flux



- Vorticity gradient emerges as natural measure of production vs. turbulent mixing.
- $\Pi = 0 \rightarrow \nabla u = \Pi^{res}/\chi_y$
- The vorticity gradient is characteristic of the flow shear layer strength.

A jump in the flow shear over a scale length *l* is equivalent to a vorticity gradient over that scale length

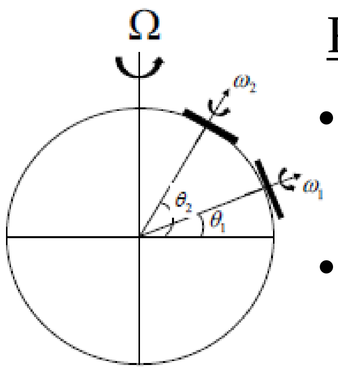
Scaling of transport fluxes with α

Plasma Response	Adiabatic (α >>1)	Hydrodynamic (α <<1)
Particle Flux Γ	$\Gamma_{\text{adia}} \sim \frac{1}{\alpha}$	$\Gamma_{hydro} \sim \frac{1}{\sqrt{\alpha}}$
Turbulent Viscosity χ	$\chi_{adia} \sim \frac{1}{\alpha}$	$\chi_{hydro} \sim \frac{1}{\sqrt{\alpha}}$
Residual stress Π^{res}	$\Pi^{res}{}_{\text{adia}} \sim -\frac{1}{\alpha}$	$\Pi^{res}{}_{\text{hydro}} \sim -\sqrt{\alpha}$
$\frac{\Pi^{\text{res}}}{\chi} = (\omega_{\text{ci}} \nabla n) \times$	$\left(\frac{\alpha}{\lvert\omega\star\rvert}\right)^0$	$\left(\frac{\alpha}{\lvert\omega\star\rvert}\right)^1$

$\Gamma_n, \chi_y \uparrow$ and $\Pi^{\text{res}} \downarrow$ as the electron response passes from adiabatic (α >1) to hydrodynamic (α <1)

- Mean vorticity gradient ∇u (i.e. ZF production) becomes proportional to $\alpha \ll 1$ in the hydrodynamic limit.
- Weak ZF formation for $\alpha \ll 1$ → weak regulation of turbulence and enhancement of particle transport and turbulence.

How does ZF collapse square with PV Mixing?

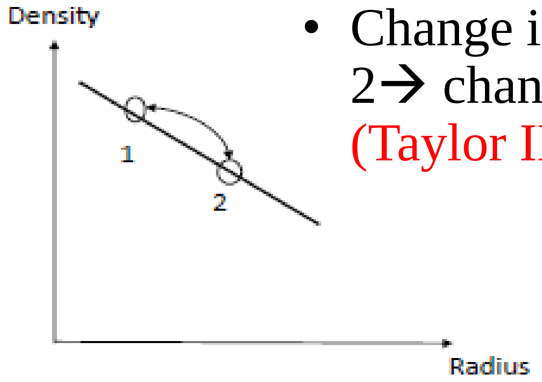


Rossby waves:

- $PV = \nabla^2\phi + \beta y$ is conserved between θ_1 and θ_2.
- Total vorticity $2\vec{\Omega} + \vec{\omega}$ is frozen in→ Change in mean vorticity Ω leads to a change in local vorticity ω → Flow generation (Taylor's ID)

Drift waves:

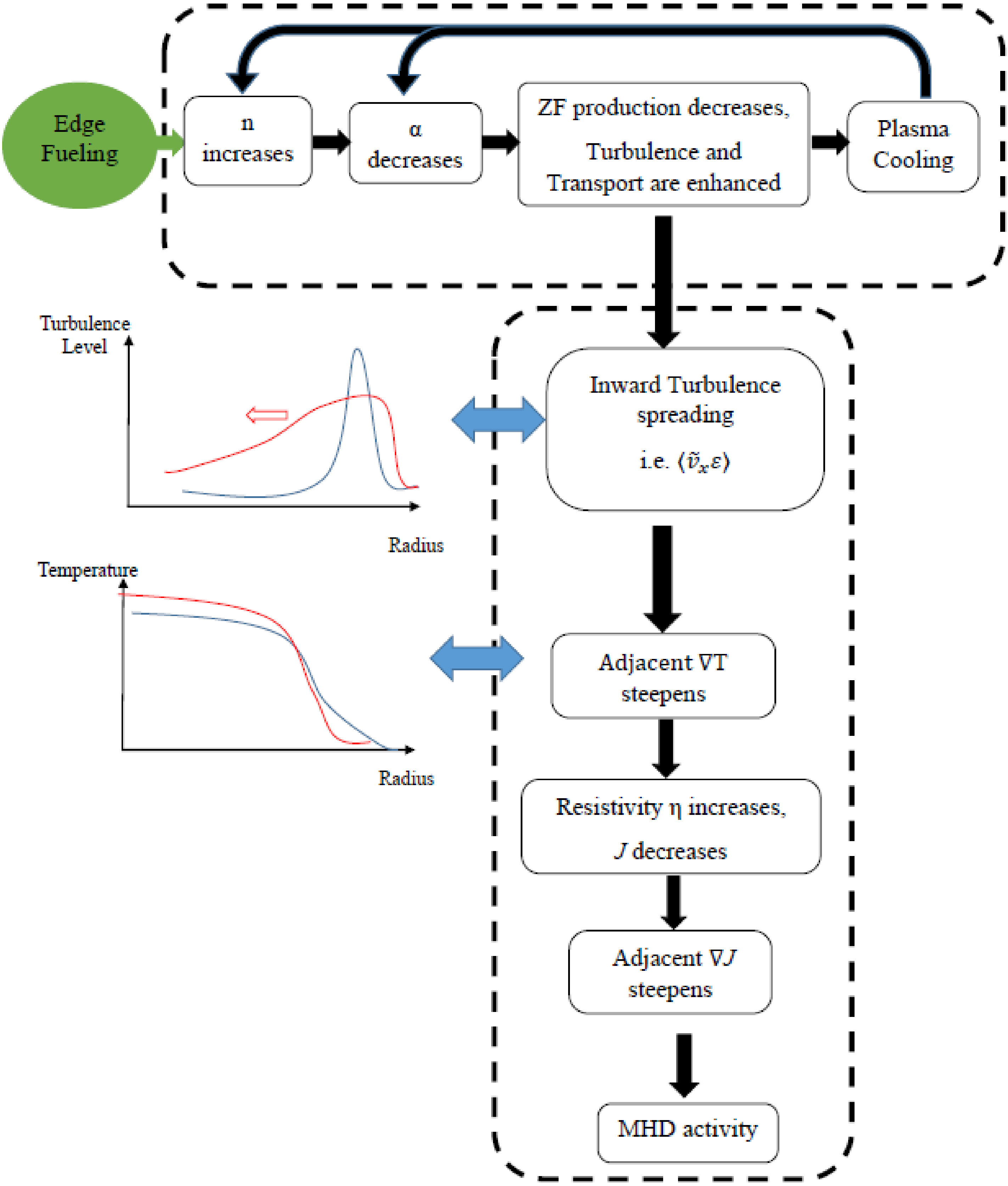
- In HW, $q = \ln n - \nabla^2\phi = \ln n_0 + h + \tilde{\phi} - \nabla^2\phi$ is conserved along the line of density gradient.
- Change in density from position 1 to position 2→ change in vorticity → Flow generation (Taylor ID)



Quantitatively

- The PV flux $\Gamma_q = \langle \tilde{v}_x h\rangle - \rho_s^2\langle \tilde{v}_x \nabla^2\phi\rangle$
- Adiabatic limit $\alpha \gg 1$:
  +Particle flux and vorticity flux are tightly coupled (both are prop. to $1/\alpha$)
- Hydrodynamic limit $\alpha \ll 1$:
  +Particle flux is proportional to $1/\sqrt{\alpha}$.
  +Residual vorticity flux is proportional to $\sqrt{\alpha}$.
- PV mixing is still possible without ZF formation → Particles carry PV flux

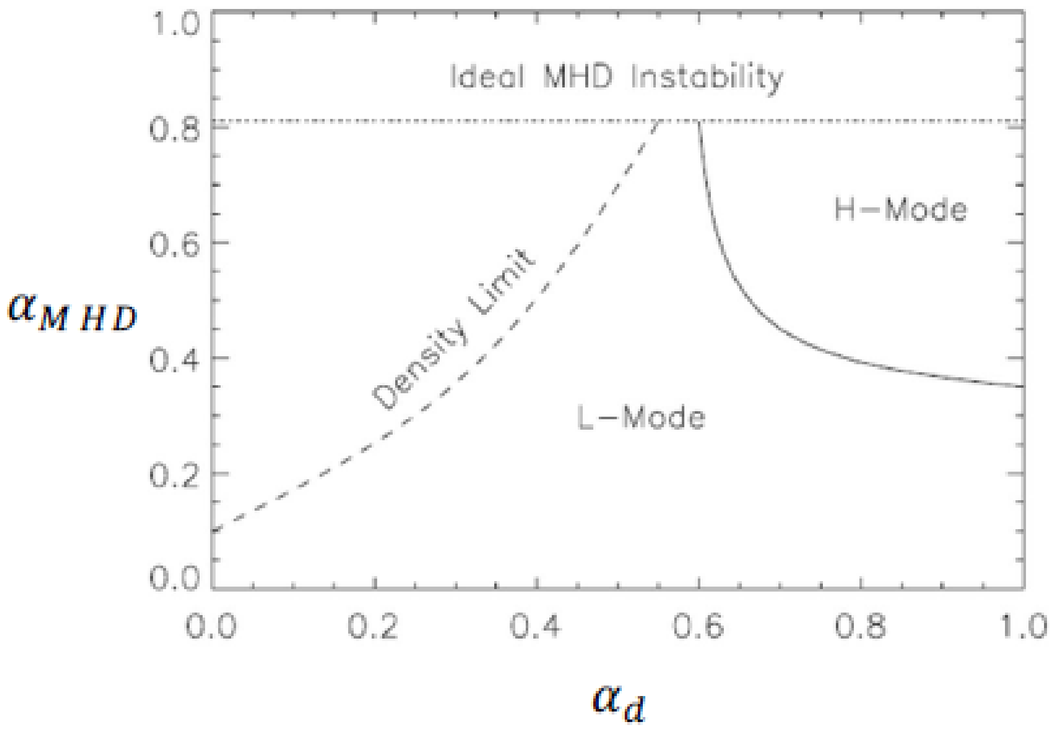
Feedback loop for plasma cooling: transport can lead to MHD activity



The Old Story / A Better Story

Modes, Glorious Modes / Self-Regulation and its Breakdown

(Drake and Rogers, PRL, 1998)



- $\alpha_{MHD} = -\frac{Rq^2 d\beta}{dr} \rightarrow$ ∇P and ballooning drive to explain the phenomenon of density limit.
- Invokes yet another linear instability of RBM.
- What about density limit phenomenon in plasmas characterized by a low β?

(Hajjar et al., PoP, 2018)

State	Electrons	Turbulence Regulation
Base State - *L*-mode	Adiabatic or Collisionless $\alpha > 1$	Secondary modes (ZFs and GAMs)
H-mode	Irrelevant	Mean $E \times B$ shear (∇p_i)
Degraded particle confinement (Density Limit)	Hydrodynamic $\alpha < 1$	None - ZF collapse due weak production for $\alpha < 1$

Secondary modes and states of particle confinement

L-mode: Turbulence is *regulated* by shear flows but not suppressed.
H-mode: *Mean ExB* shear ↔∇p_i suppresses turbulence and transport.
Approaching Density Limit: High levels of turbulence and particle transport, as shear flows collapse.

Conclusions - 1

L-mode density limit experiments:

- Density limit is consequence of particle transport processes.

- Edge, turbulence-driven shear layer collapses as $n \rightarrow n_G$

    - Relation to the local parameter α

- ZF production drops as α decreases below unity, while edge particle transport and turbulence increase.

- Cooling front:

    - Extent penetration of turbulence spreading?

    - Strength $\rightarrow$ operation regime

Conclusions - 2

<u>H-mode density limit experiments:</u>

- Density limit a 'back-transition' phenomenon i.e. drift-ZF state → convective cell, strong fluctuation turbulence

  → scaling of collapse? (spatio-temporal)

  → bifurcation? Trigger?, hysteresis?!

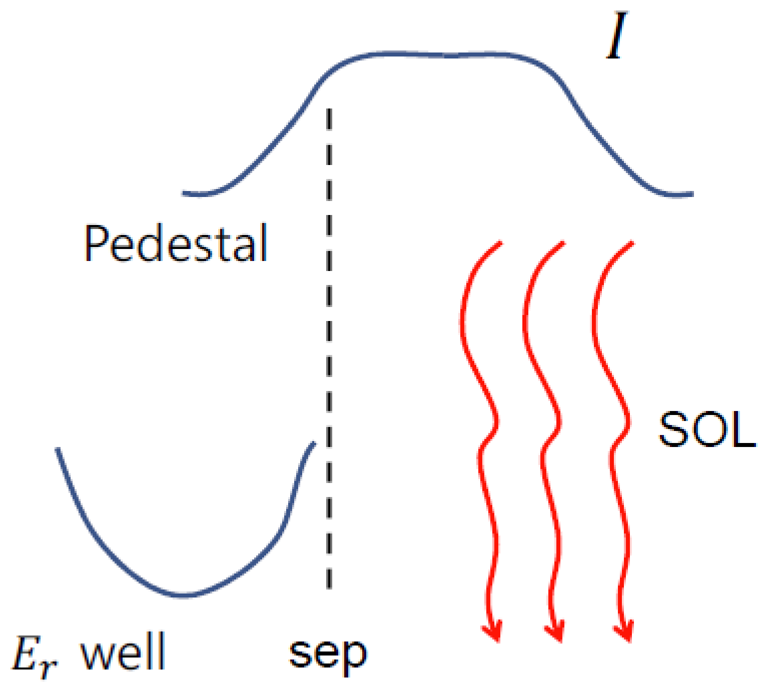
  → control parameter ↔ α

- Pedestal quiescent while SOL turbulence set by:

  → Q

  → Fueling

  → Divertor conditions



Future work

- Numerical investigation of the evolution of a plasma transition from one limit to the other.

- Experimental investigation of which happens first: a drop in α or a decrease in the ZF production:
  1. Experimental verification of the drop in the total Reynolds work as $n/n_G \rightarrow 1$.

  2. Increase n and decrease T_e so to keep $\alpha \sim T_e^2/n$ constant.
     In theory, no collapse of ZFs should be observed, as α constant.

  3. Investigation of the role of high edge ∇p and high β values in H-modes on the enhancement of turbulence and prole evolution in density limit experiments.

- Verify the decrease in bi-spectra of <ZF|DW,DW> as $n/n_G \rightarrow 1$.