Turbulence and Transport in Stochastic Magnetic Fields

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Disclaimer

- NOT a “My latest homework set” presentation,

  NOR a review

- Aims to:
  - Scope out some fundamental issues and problems
  - Discuss some ongoing work

Outline

- Why this topic is relevant and important
- Aspects of the problem: Scales and Dimensionless Ratios
- Emphasis: $Ku > 1$ regime
- Drift-Alfven Turbulence in a Random Magnetic Field

  ➔ Implications for Reynolds stress cross-phase
- Discussion

Why?

- Several important phenomena in magnetic confinement involve turbulence, turbulent transport in a stochastic field
- Examples
  - RMP
    - Edge plasma (i.e. pedestal and beyond) stochasticized by externally induced $\widetilde{B_r}$

    ⇔ Screening of external field important ⇔ plasma response

Why?

– RMP, cont'd

  • ELM mitigation/control incurs "cost" of increased P_{LH} → $\left\{ \begin{array}{l} \langle E_r \rangle \\ \text{Flows} \end{array} \right.$

– Stellarator 'Family'

  • Stochastic fields due coils, etc

  • De-facto ergodic divertor

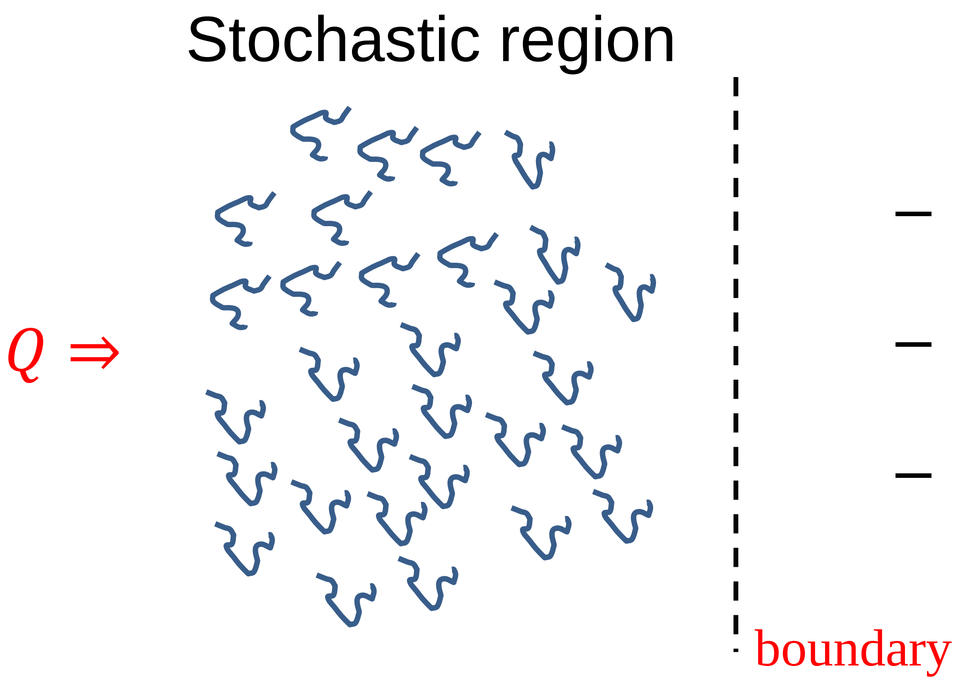
  • LH transition in stellarator?

– Disruptions, MHD activities

Note Distinction

- <u>Externally</u> induced stochasticity:
  - Plasma screening, response etc. critical
  - Need understand plasma dynamics, including turbulent transport in stochastic base state.
- Self-induced stochasticity (usual topic)
  - EM instabilities, turbulence
  - Zonal flows, fields
  - Reynolds, Maxwell, Kinetic stresses

Minimal Paradigm



– pre-existing stochastic field

– finite heat flux across system

– ES micro-instability (i.e. low-β)

– profiles (T_e, T_i, n) ?

– $\langle E_r \rangle$ and shear ?

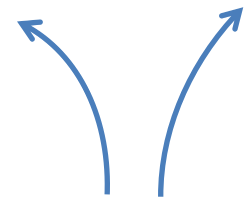
– fluctuation structure ?

Scales (see classics)

- l_{ac} : stochastic magnetic field auto-correlation

  → scatterer coherence

  $l_{ac} \sim \left|\Delta(k_{\|})\right|^{-1}$ → inverse bandwidth



- l_c : field de-correlation → field line scattering

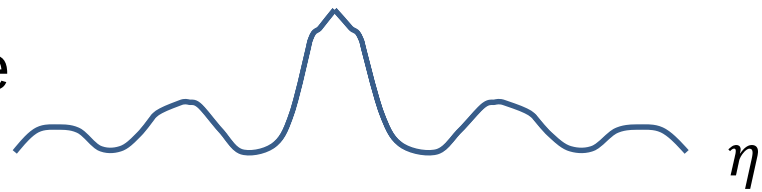
  $l_c \sim (k_\perp^2 \, D_M \, / \, L_s^2)^{-1/3}$ $\qquad D_M \equiv$ field line diffusivity

- l_{mfp} : mean-free path

Scales (see classics)

- l_T : turbulence correlation length, along field

mode structure



Relevant, Interesting Orderings:

- $l_{ac} < l_c,\ l_T < l_{mfp}$ → collisionless regime

  $l_c,\ l_T$ competition TBD (l_T coarse graining)

  Is $l_c < l_T$ viable?

- $l_{ac} < l_{mfp} < l_c, l_T$ → collisional regime

Dimensionless Parameters

- Kubo Number

  $K = \delta \,/\, \Delta_{\perp} \sim \; l_{ac}\, \delta B / B_r\, \Delta_{\perp b}$

- Scattering length vs scatter coherence

- $K < 1$ → diffusion

  $K > 1$ → unclear, for random scatter field. Quenched

  disorder → percolation (Kadomtsev, Zeldovich;

  Isichenko)

- Which Kubo?

  $K_c = \delta / \Delta_{\perp} \sim \;\; \tau_{ac} \tilde{V} / \Delta_{\perp c}$

Dimensionless Parameters, cont'd

- Begs "Prandtl #" type ratio:

$$K_c/K_B \sim \tilde{V}\tau_{ac}/l_{ac}\left(\frac{\delta B}{B_0}\right)$$

$\sim l_c/l_T$ equivalent

- "The conventional wisdom..."

$Ku < 1$ → diffusion; well understood

$Ku > 1$ → poorly understood

- Reality: $Ku \sim 1$ ($Ku \sim 1 \Leftrightarrow$ mixing length level)

Dimensionless Parameters, cont'd

- "..... is little more than convention."

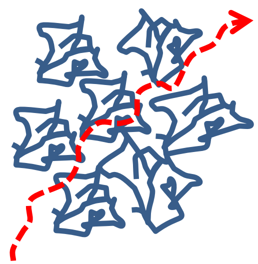
  – J.K. Galbraith

  i.e. Push $K < 1$ to $K \approx 1$ → "Renormalized" quasi-linear theory

- A bit better:
  - Explorer $\left\{ \begin{matrix} Ku > 1 \\ Ku < 1 \end{matrix} \right\}$ and examine trends as $Ku \rightarrow 1$
  - Physics for $Ku > 1$ is very different from usual diffusion crank

A Peek at $Ku > 1$

- (See physics 235 notes and materials, UCSD spring 2019 https://physics.ucsd.edu)
- Contrast:
- $Ku \ll 1$ – diffusion

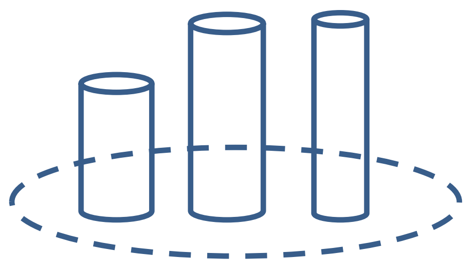


$$Ku \sim \frac{l_{ac}}{\Delta_\perp} \tilde{b}$$

$$l_{ac} \to 0$$

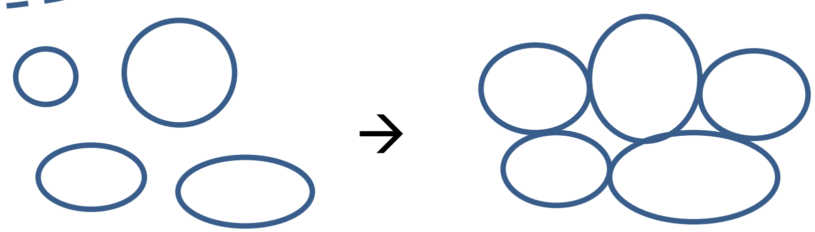
- $Ku \gg 1$ – "percolation"

  Static, disordered
  ~ 2D cells



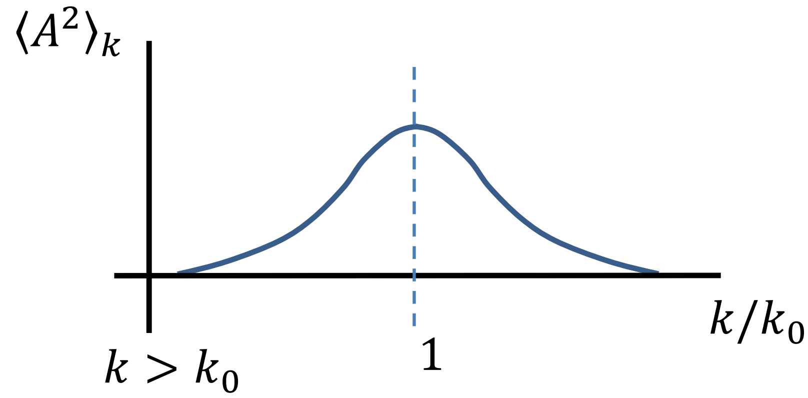
$$l_{ac} \to \infty$$

base →



Does macroscopic connection form?

Connections



$\langle A^2 \rangle_k$ - random phases

$b = \nabla_\perp A \times \hat{z}$ - $\langle A \rangle = 0$

- $\langle A^2 \rangle = 1$

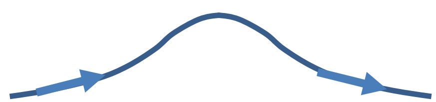
i.e. $\langle A^2 \rangle_k \approx \exp[-k^2/k_0^2]$ $k > k_0$ (small scale)

$\langle A^2 \rangle_k \approx (k/k_0)^{2n}$ $n > 0$ and $k < k_0$ (large scale)

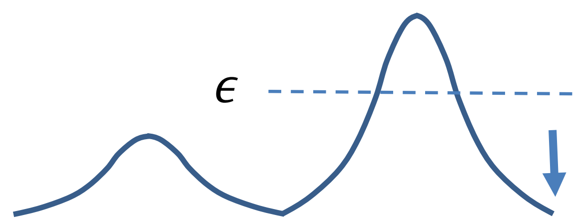
$A > \epsilon$

magnetic vortex,
island

$A < \epsilon$



percolating lines
$l(\epsilon) \rightarrow$ length of isoline



Topographic map

- Point: l ↑ as ϵ ↓ $l \rightarrow L \rightarrow \infty$ percolation threshold

Connections, cont'd

- Absent kicks off field line:

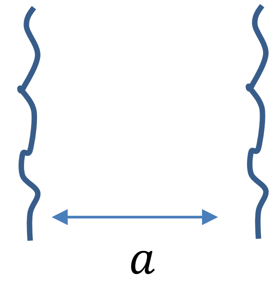
  $D_{eff} \to 0$; $l(\epsilon) \ll L$ → trapping

←→ or:

  burst along macroscopic connection link ; $l(\epsilon) \to L$

- Key question: Is lost energy focused in few strike points or distributed, in space
- Begs two questions:
  - Conditions for percolation?
  - What if 'near miss'?

Conditions (Zeldovich; '83)



- $A_k \sim k^m$
- Seek $\langle B \rangle = \left(\langle b^2 \rangle_{k<\frac{1}{a}} \right)^{1/2}$

  long wavelength component of mean field

- So

$$\langle B \rangle = \left(\int_0^{\frac{1}{a}} dk\, k\, k^{2m}\, k^2 \right)^{1/2} \approx (1/a)^{-m-2}$$

for $\langle B \rangle \neq 0$ as $a \rightarrow 0$, need $m \geq -2$

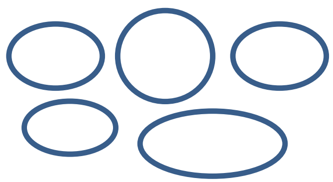
but $m = -2$ → $j_{zk} \approx k^2 A_k \sim k^0 \sim const$

Random currents ←→ white noise

→ percolation of large scale field

General Result: Current correlation function

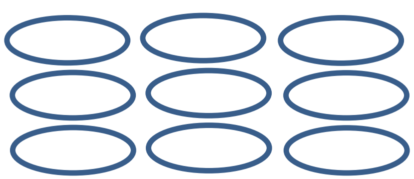
- $\langle j(x)j(x+r)\rangle \geq 0$ for percolating field
- i.e. no percolation for anti-correlated currents
- What of 'near miss' D_0?

What of ? 

- First transit thru cell, but reversible
- Slow diffusion thru interstices (kick cell-to-cell)

Classic of G.I. Taylor, Rosenbluth ...

For ordered array of convective cells:



$D_{eff} = \langle D\rangle \approx (D_T D_o)^{1/2}$ → Geometric mean!

$D_T \sim V_0 l_0$ → $D_T \gg D_0$

'Near Miss', cont'd

- Comments
  - Geometric mean is consequence of simple geometry
  - In general, $\langle D \rangle \approx \left(D_T^{\alpha} \, D_0^{\beta} \right)^{1/\alpha+\beta}$

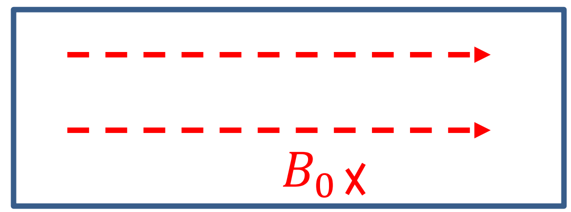
    hybrid $\quad \alpha, \beta > 0 \quad (D_T \gg D_0)$
  - Relevant to stiff profile regimes?!

    → Cells, turbulence hover near threshold of overlap

    → Neoclassical kicks across the gap
  - Lesson: Turbulence and neoclassical NOT additive

Drift-Alfven Turbulence in a Random Magnetic Field (C. Chen, P.D., S. Tobias '19 Ap.J.)

- Abbreviated description:
  - Drift-Alfven = { Rossby-Alfven, on β –plane; 2D MHD + β –effect }
  - Weak mean magnetic field



➔

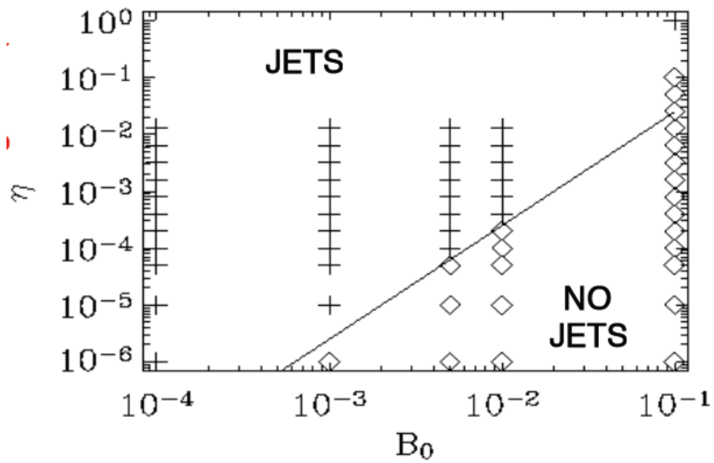
- Classical inviscid dispersion relation

$$\omega^2 - \omega_R \omega - k_{\parallel}^2 V_A^2 = 0 \quad \text{(R. Hide, '60's)}$$

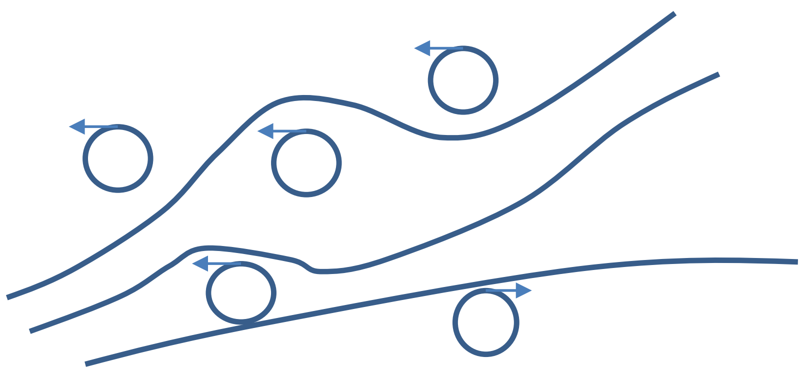
- Key issue: $\langle \tilde{B}^2 \rangle \gg \langle B \rangle^2$

RMS field large → Quasi-linear theory crap

- Challenge: Tobias, P.D., Hughes; Ap.J. '07



- Field configuration



- Cells + 'Sinews' (Zeldovich)

What is going on?

- η dissipates field, weakening magnetic stress
- B_0 increases Alfvenic coupling
- $B_{rms} \gg B_0$ → large scale field distorted by small scales
- Configuration → ensemble of magnetic cells + extended 'sinews'

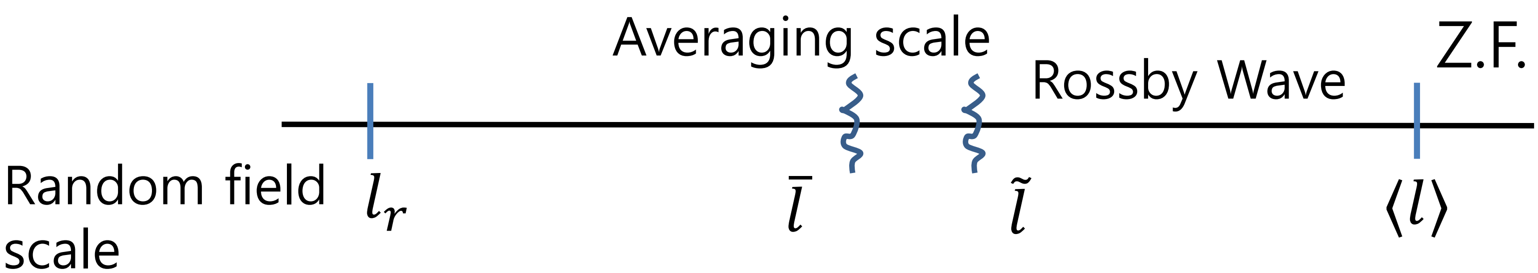
Short summary of plot:

- $\langle \tilde{B}^2 \rangle > B_0^2$ → quasi-linear, w.t.t. etc. all <u>fail</u>
- Computation indicates Reynolds stress decays <u>prior</u> to point when Maxwell stress equalizes
  - Forced (can't appeal to linear theory)
  - B_0 feeble
- $\langle \tilde{B}^2 \rangle$ entering phase relation in $\langle \tilde{V}_r \tilde{V}_\theta \rangle$?! How?
- Key question: RMP ↑ P_{LH} ⇔ effect stochastic field on trigger ?!
- Poses general problem: PV mixing in tangled magnetic field

PV mixing in Tangled Magnetic Field ⇔ relevant to lab and astrophysics

→ approach ala' Rechester and Rosenbluth '78

- "A more realistic approach might be to asses the effects of the thermal conduction we have estimated and related transport coefficients such as electron viscosity, on the linear theory of these modes – estimating saturation of stochasticity to occur at the marginal stability point."
- Scales:



$\bar{l} \ll \langle l \rangle$

Key: Seek mean field equations, averaged over random field

$$\bar{F} = \int dR^2 \int dB_r \, P(B_{rx}, B_{ry}) F$$

defines random field average

averaged

Pdf of random field

- Dispersion relation:

$$\left(\omega - \omega_R + \frac{i\bar{B}_{r,j}^2 k_j^2}{\mu_0 \rho \, \eta k^2} + i\nu k^2 \right) (\omega + i\eta k^2) = \frac{B_{\rho,x}^2 k_x^2}{\mu_0 \rho}$$

Dissipative response to MSR field

usual AW

~ emerges from mean field treatment of $J \times B$ force

For Zonal Flow: ("double average theory")

- $u \equiv$ vorticity

Drag, emergent from random field

- $\partial_t \langle \bar{u} \rangle = -\partial_y \langle \bar{\Gamma} \rangle + \frac{1}{\eta \mu_0 \rho} \partial_y \left(\langle \bar{B}_{r,y}^2 \rangle \partial_y \phi \right) + \nu \nabla^2$

- $\Gamma = -D \left(\frac{\partial}{\partial y} \langle \bar{u} \rangle + \beta \right)$

PV flux

- $D = \sum_k \left| \tilde{V}_{k,y} \right|^2 \frac{\nu k^2 + \frac{\eta k^2}{\omega^2 + \eta^2 k^2} + \frac{\overline{B_r^2} k_j^2}{\mu_0 \rho \eta k^2}}{\left(\omega - \omega_A^2 \frac{\omega}{\omega^2 + (\eta k^2)^2} \right)^2 + \left(\nu k^2 + \frac{\eta k^2}{\omega^2 + (\eta k^2)^2} + \frac{\overline{B_r^2} k_j^2}{\mu_0 \rho \eta k^2} \right)}$

Simplifying

$$D = \sum_k \left|\tilde{V}_k\right|^2 \frac{\frac{\overline{B_r^2} k_j^2}{\mu_0 \rho \eta k^2}}{\omega^2 + \left(\nu k^2 + \frac{\overline{B_r^2} k_j^2}{\mu_0 \rho \eta k^2}\right)^2}$$

Random field

- Cross phase drops for stronger random field → P_{LH} increase ?!
- Resisto-elastic MHD medium
- Alfven waves in random field over-damped in regime of interest
- Both drag and cross-phase effects induced by random field

Discussion

- Problem of turbulence and transport in stochastic field is interesting, important and terra nova.
- Important to face $Ku > 1$ regime, as a pathway to understanding of $Ku \sim 1$
- $Ku > 1$ requires deeper insight than $Ku < 1$ → non-perturbative
- PV transport and flows in tangled field is central to LH transition with RMP
- ‘Double average’ formalism is promising