Elastic Turbulence in Flatland: ‘Blobs and Barriers’ Encode Memory, and so Determine Transport

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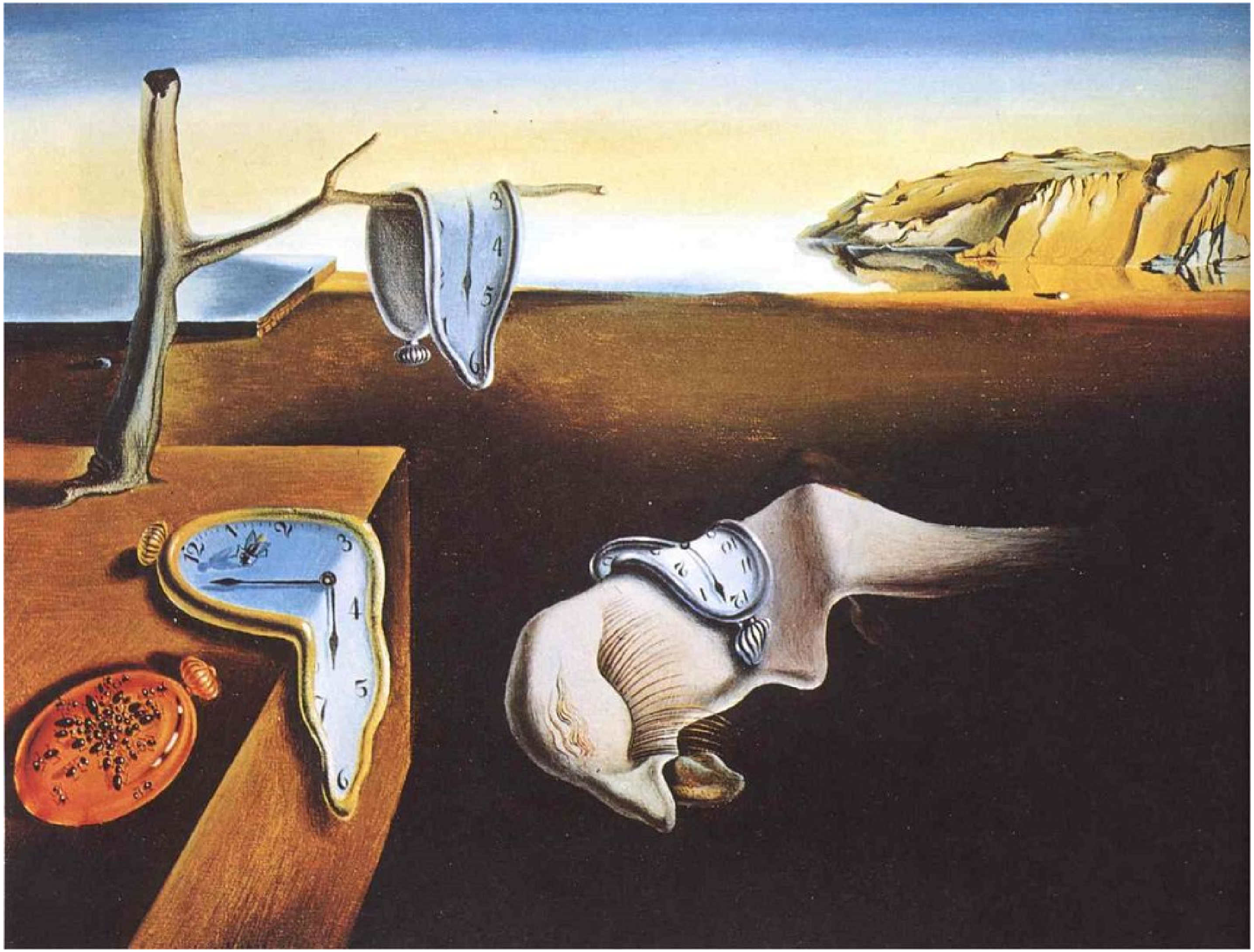


Persistence of Memory
- Salvador Dali

Special Acknowledgement:

- School of Uriel Frisch, especially: Annick Pouquet
- Two seminal papers:
  - Pouquet, Frisch, Leorat ‘76
  - Pouquet, ‘78

Outline (Tutorial)

- What and Why are Elastic Fluid?
- Active Scalar Transport in 2D MHD: Background and Conventional Wisdom
- New Development: “Blobs and Barriers”
  - Intermittent Field
  - Transport Barriers Form
- Revisting Quenching: Role of Barriers and Blobs

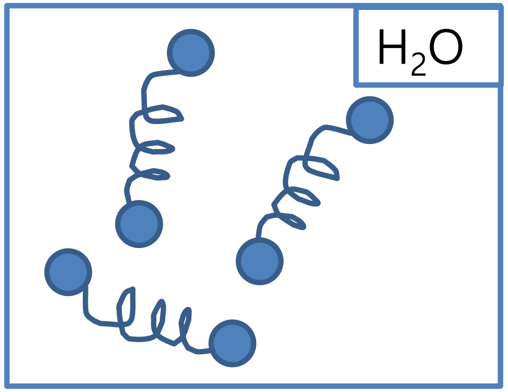
Outline, cont'd

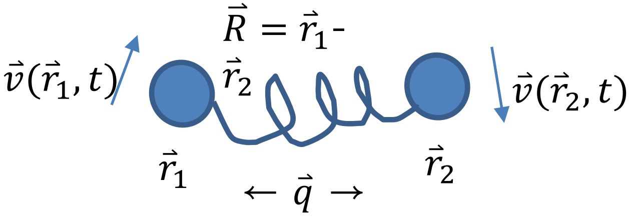
- Barrier Formation: Negative Diffusion and Bifurcation
- Hints of Staircases
- Open Questions

- Back-up Material and CHNS

What is an Elastic Fluid?

Elastic Fluid -> Oldroyd-B Family Models





→ Solution
of Dumbells

Internal DoF
i.e. polymers

- $\gamma\left(\frac{d\vec{r}_{1,2}}{dt} - \vec{v}(\vec{r}_{1,2},t)\right) = -\frac{\partial U}{\partial \vec{r}_{1,2}} + \vec{\xi}$, where $U = \frac{k}{2}(\vec{r}_1 - \vec{r}_2)^2 + \cdots$

stokes
drag

entropic
spring

noise

- so $\frac{d\vec{R}}{dt} = \vec{v}(\vec{R},t) + \vec{\xi}/\gamma$, and $\frac{d\vec{q}}{dt} = \vec{q} \cdot \nabla\, \vec{v}(\vec{R},t) - \frac{2}{\gamma}\frac{\partial U}{\partial \vec{q}}$ + noise

Seek $f(\vec{q},\vec{R},t|\vec{v},\dots)$ → distribution

- $$\partial_t f + \partial_{\vec{R}}\cdot\left[\vec{v}(\vec{R},t)f\right] + \partial_{\vec{q}}\cdot\left[\vec{q}\cdot\nabla\vec{v}(\vec{R},t)f - \frac{2}{\gamma}\frac{\partial U}{\partial\vec{q}}f\right]$$
  $$= \partial_{\vec{R}}\cdot\mathbf{D}_0\cdot\frac{\partial f}{\partial\vec{R}} + \partial_{\vec{q}}\cdot\mathbf{D}_\mathrm{q}\cdot\frac{\partial f}{\partial\vec{q}}$$
  N.B.: Is F.P. valid?
- and moments:
  $Q_{ij}(\vec{R},t) = \int d^3q\, q_i q_j f(\vec{q},\vec{R},t)$ → elastic energy field (tensor)
- so:
  strain
  $$\partial_t Q_{ij} + \vec{v}\cdot\nabla Q_{ij} = Q_{i\gamma}\partial_\gamma v_j + Q_{j\gamma}\partial_\gamma v_i$$
  relaxation
  $$-\omega_z Q_{ij} + D_0\nabla^2 Q_{ij} + 4\frac{k_B T}{\gamma}\delta_{ij}$$
  and concentration equation
- Defines Deborah number: $\nabla\vec{v}/\omega_z$

- D ~ Deborah Number ~ $|\nabla V|/\omega_Z$ ~ τ_{relax}/τ_{dyn}
- Limit for elasticity: $D \gg 1$
- Why “Deborah”? →

Hebrew Prophetess Deborah:

“The moutains flowed before the Lord.” (Judges)

∴

- Revisit Heraclitus (1500 years later):

“All things flow” – if you can wait long enough

Reaction on Dynamics

- $\rho[\partial_t v_i + \vec{v} \cdot \nabla v_i] = -\nabla_i P + \nabla_i \cdot [c_p k Q_{ij}] + \eta \nabla^2 v_i + f_i$

- Classic systems; Oldroyd-B (1950).
- Extend to nonlinear springs (FENE), rods, rods + springs, networks, director fields, etc…
- Supports elastic waves and fluid dynamics, depending on Deborah number.
- Oldroyd-B ↔ *active tensor* field → elastic stress

See: Ogilvie, Proctor; Bird et. al.

Constitutive Relations

- J. C. Maxwell:

$$\text{(stress)} + \overset{\substack{\text{relaxation}\\ \downarrow}}{\tau_R} \frac{d(\text{stress})}{dt} = \overset{\substack{\text{viscosity}\\ \downarrow}}{\eta} \frac{d}{dt}\,\text{(strain)}$$

- If $\tau_R/T = D \ll 1$, stress = $\eta \frac{d}{dt}$ (strain)

$T \equiv$ dynamic time scale

$$\Pi = -\,\eta \nabla \vec{v}$$ viscous

- If $\tau_R/T = D \gg 1$, stress $\cong \frac{\eta}{\tau_R}$ (strain)

~ E (strain) elastic

- Limit of “freezing-in”: D≫1 is criterion.

Relation to MHD?!

- Re-writing Oldroyd-B:

$\mathbf{T} \equiv$ stress

$$\frac{\partial}{\partial_t}\mathbf{T} + \vec{v}\cdot\nabla\mathbf{T} - \mathbf{T}\cdot\nabla\vec{v} - (\nabla\vec{v})^T\cdot\mathbf{T} = \frac{1}{\tau}(\mathbf{T} - \frac{\mu}{\tau}\mathbf{I})$$

- MHD: $\mathbf{T}_m = \frac{\vec{B}\vec{B}}{4\pi}$ → Maxwell Stress Tensor

$$\partial_t\vec{B} + \vec{v}\cdot\nabla\vec{B} = \vec{B}\cdot\nabla\vec{v} + \eta\nabla^2\vec{B}$$

- So

$$\frac{\partial}{\partial_t}\mathbf{T}_m + \vec{v}\cdot\nabla\mathbf{T}_m - \mathbf{T}_m\cdot\nabla\vec{v} - (\nabla\vec{v})^T\cdot\mathbf{T}_m = \eta[\vec{B}\nabla^2\vec{B} + (\nabla^2\vec{B})\vec{B}]$$

- $\boxed{\lim_{D\to\infty}\text{(Oldroyd-B)} \Leftrightarrow \lim_{R_m\to\infty}\text{(MHD)}}$

High R_m MHD is a good example of an Elastic Fluid!

- High Rm ←→ High D

  → <u>Elasticity</u>

- High Rm ~ Freezing in

  ~ <u>Memory</u>

- Elastic fluids have memory

➔ Implications of memory for Transport, Mixing

Active Scalar Transport in 2D MHD: Background and Conventional Wisdom

2D MHD

ϕ: Potential

A: Magnetic Potential

$$\partial_t \nabla^2 \phi + \nabla \phi \times \hat{z} \cdot \nabla \nabla^2 \phi = \nabla A \times \hat{z} \cdot \nabla \nabla^2 A + \nu \nabla^2 \nabla^2 \phi + \tilde{f}$$

$$\partial_t A + \vec{V} \cdot \nabla A = \eta \nabla^2 A$$

Conserved Quantities (Quadratic)

1. Energy

$$E = E_K + E_B = \int (\frac{v^2}{2} + \frac{B^2}{2\mu_0}) d^2x$$

2. Mean Square Magnetic Potential

$$H_A = \int A^2 \, d^2x$$

→ critical constraint

→ induces dual cascade

3. Cross Helicity

$$H_C = \int \vec{v} \cdot \vec{B} d^2x$$

\- zeroed ab-initio

➔ N.B.: What ‘cascade’ is fundamental in 2D MHD?

(A. Pouquet)

- Conventional Wisdom: Energy
- Is this merely the convention from fluids?
- $\langle A^2 \rangle$ conservation reflects freezing-in, etc.
- Is inverse cascade $\langle A^2 \rangle$ fundamental?
- $\langle A^2 \rangle \leftrightarrow$ 2D
- $\langle \vec{A} \cdot \vec{B} \rangle \leftrightarrow$ 3D

Background

- Motivation: Why study 2D MHD and Anomalous transport/Resistivity?
- All MFE models = Reduced MHD + Assorted Scalar Advection Equations
- Reduced MHD = 2D MHD + Shear Alfven Wave
- Key Issues in Fast Relaxation (i.e. ELMs), Reconnection:

  → Hyper-resistivity, anomalous dissipation
- Related to Nonlinear Dynamos and α –quenching

Ideology of Turbulent Mixing

- L. Prandtl: Analogy with kinetic theory

$$\left.\begin{array}{l} V_{Th} \to \tilde{V} \\ \\ l_{mfp} \to l \end{array}\right\} \text{define } D_T \sim \tilde{V}\, l$$

- $l \to$ mixing length. What l is sets the result
- $\eta_k \approx \tilde{V}\, l$ → kinematic turbulent resistivity

also obtained via Mean Field Electrodynamics

Physics: Active Scalar Transport

- Magnetic diffusion, ψ transport are cases of active scalar transport
- (Focus: 2D MHD) (Cattaneo, Vainshtein '92, Gruzinov, P. D. '94, '95)

scalar mixing – the usual

$$\partial_t A + \nabla\phi \times \hat{z} \cdot \nabla A = \eta \nabla^2 A$$

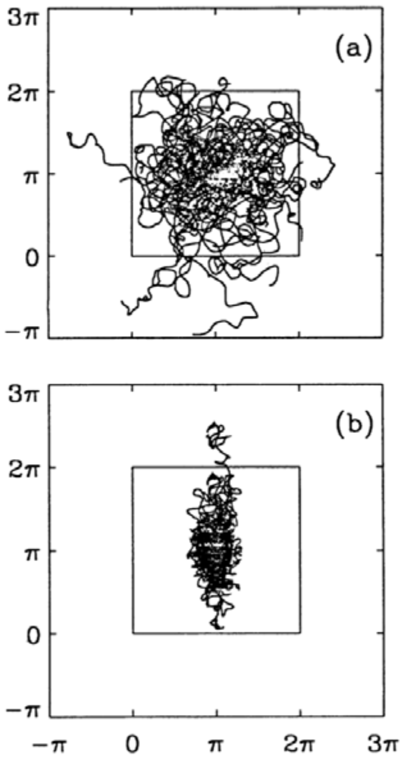
$$\partial_t \nabla^2\phi + \nabla\phi \times \hat{z} \cdot \nabla\nabla^2\phi = \nabla A \times \hat{z} \cdot \nabla\nabla^2 A + \nu\nabla^2\nabla^2\phi$$

back-reaction

turbulent resistivity

- Seek $\langle v_x A\rangle = -D_T \dfrac{\partial\langle A\rangle}{\partial x} - \eta \dfrac{\partial\langle A\rangle}{\partial x}$
- Point: $D_T \neq \sum_{\vec{k}} |v_{\vec{k}}|^2 \, \tau_{\vec{k}}^{K}$, often substantially less
- Why: <u>Memory</u>! ⟷ Freezing-in
- Cross Phase

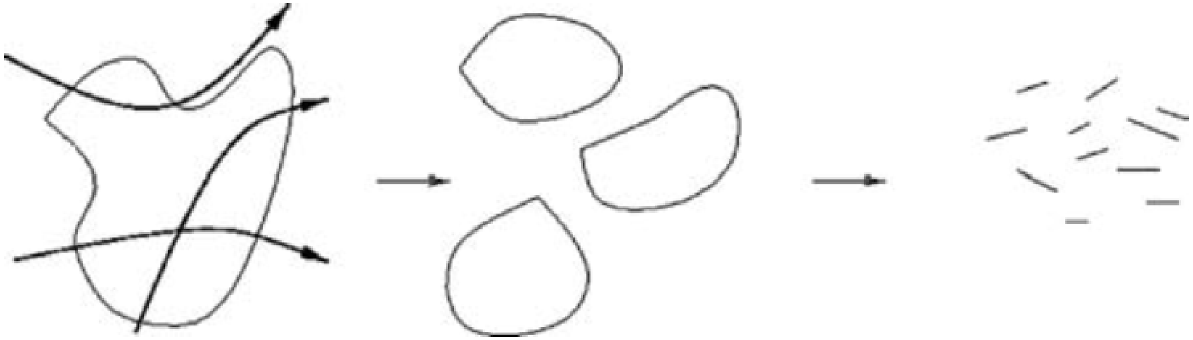
Conventional Wisdom



- [Cattaneo and Vainshtein 1991]: turbulent transport is suppressed even for a <u>weak</u> large scale magnetic field is present.
- Starting point: $\partial_t \langle A^2 \rangle = -2\eta \langle B^2 \rangle$
- Assumptions:
  - Energy equipartition: $\frac{1}{\mu_0 \rho} \langle B^2 \rangle \sim \langle v^2 \rangle$
  - Average B can be estimated by: $|\langle \mathbf{B} \rangle| \sim \sqrt{\langle A^2 \rangle}/L_0$
- Define Mach number as: $M^2 = \langle v_A \rangle^2 / \langle \tilde{v}^2 \rangle = \langle v^2 \rangle / v_A^2 = \langle v^2 \rangle / \frac{1}{\mu_0 \rho} \langle B^2 \rangle$
- Result for suppression stage: $\eta_T \sim \eta M^2$
- Fit together with kinematic stage result: $\boxed{\eta_T \sim \frac{ul}{1 + \mathrm{Rm}/M^2}}$
- Lack physics interpretation of η_T !

Origin of Memory?

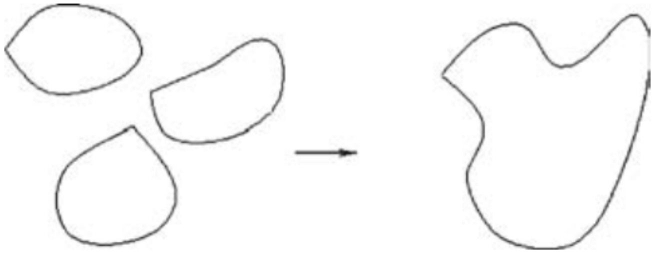
- (a) flux advection vs flux coalescence
  - intrinsic to 2D MHD (and CHNS)
  - rooted in <u>inverse cascade</u> of $\langle A^2 \rangle$ - dual cascades
- (b) tendency of (even weak) *<u>mean</u>* magnetic field to “Alfvenize” turbulence [cf: vortex disruption feedback threshold!]
- Re (a): Basic physics of 2D MHD



Forward transfer: fluid eddies chop up scalar A.

Memory Cont'd

- v.s.



Inverse transfer: current filaments and *A*-blobs attract and coagulate.

- Obvious analogy: straining vs coalescence; CHNS
- Upshot: closure calculation yields:

$$\Gamma_A = -\sum_{\vec{k}'}[\tau_c^{\phi}\langle v^2\rangle_{\vec{k}'} - \tau_c^{A}\langle B^2\rangle_{\vec{k}'}]\frac{\partial\langle A\rangle}{\partial x} + \cdots$$

flux of potential

competition

scalar advection vs. coalescence ("negative resistivity")

(+) (-)

N.B.:
Coalescence
→ Negative diffusion
→ Bifurcation

Conventional Wisdom, Cont'd

- Then calculate $\langle B^2 \rangle$ in terms of $\langle v^2 \rangle$. From:

$$\partial_t A + \mathbf{v} \cdot \nabla A = -v_x \frac{\partial \langle A \rangle}{\partial x} + \eta \nabla^2 A$$

- Multiplying by A and sum over all modes:

$$\frac{1}{2}[\cancel{\partial_t \langle A^2 \rangle} + \cancel{\langle \nabla \cdot (\mathbf{v} A^2) \rangle}] = -\Gamma_A \frac{\partial \langle A \rangle}{\partial x} - \eta \langle B^2 \rangle$$

Dropped stationary case

Dropped periodic boundary → introduce nonlocality?!

- Therefore: $$\langle B^2 \rangle = -\frac{\Gamma_A}{\eta} \frac{\partial \langle A \rangle}{\partial x} = \frac{\eta_T}{\eta} B_0^2$$
- Define Mach number as: $$M^2 \equiv \langle v^2 \rangle / v_{A0}^2 = \langle v^2 \rangle / (\frac{1}{\mu_0 \rho} B_0^2)$$
- Result: $$\boxed{\eta_T = \frac{\sum_{\mathbf{k}} \tau_c \langle v^2 \rangle_{\mathbf{k}}}{1 + \mathrm{Rm}/M^2} = \frac{ul}{1 + \mathrm{Rm}/M^2}}$$
- This theory is not able to describe $B_0 \to 0$

Is this story “the truth, the whole truth and nothing but the truth’?

➔ A Closer Look

Simulation Setup

- PIXIE2D: a DNS code solving 2D MHD equations in real space:

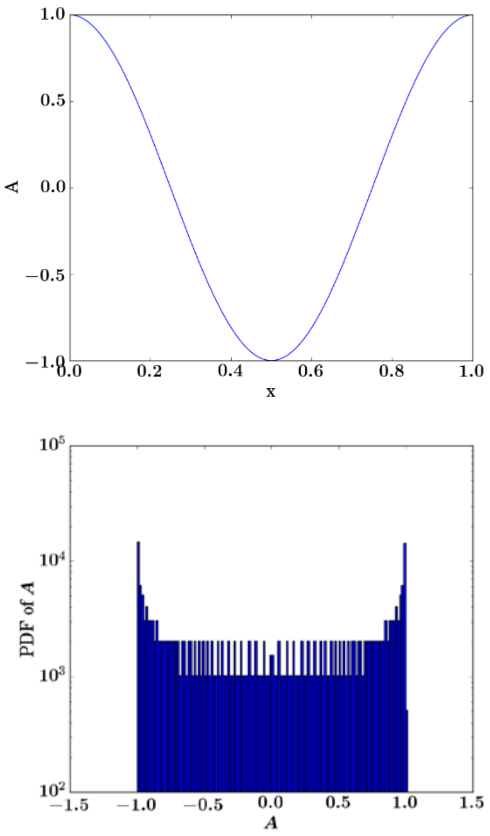
$$\partial_t A + \mathbf{v}\cdot\nabla A = \eta\nabla^2 A$$

$$\partial_t\omega + \mathbf{v}\cdot\nabla\omega = \frac{1}{\mu_0\rho}\mathbf{B}\cdot\nabla\nabla^2 A + \nu\nabla^2\omega + f$$

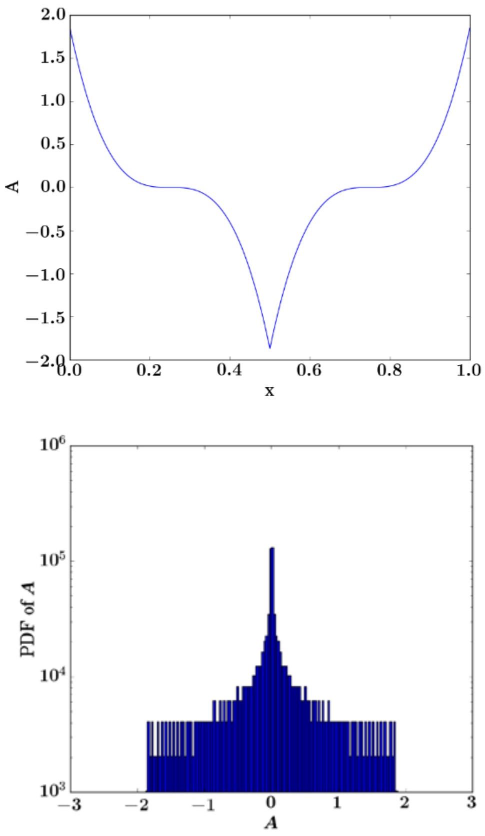
- 1024^2 resolution.
- External forcing f is isotropic homogeneous.
- Periodic boundary conditions (both).
- Initial conditions:
  - (1) bimodal: $A_I(x,y) = A_0\cos 2\pi x$
  - (2) unimodal: $A_I(x,y) = A_0 * \begin{cases} -(x-0.25)^3 & 0 <= x < 1/2 \\ (x-0.75)^3 & 1/2 <= x < 1 \end{cases}$

Initial Conditions

Bimodal

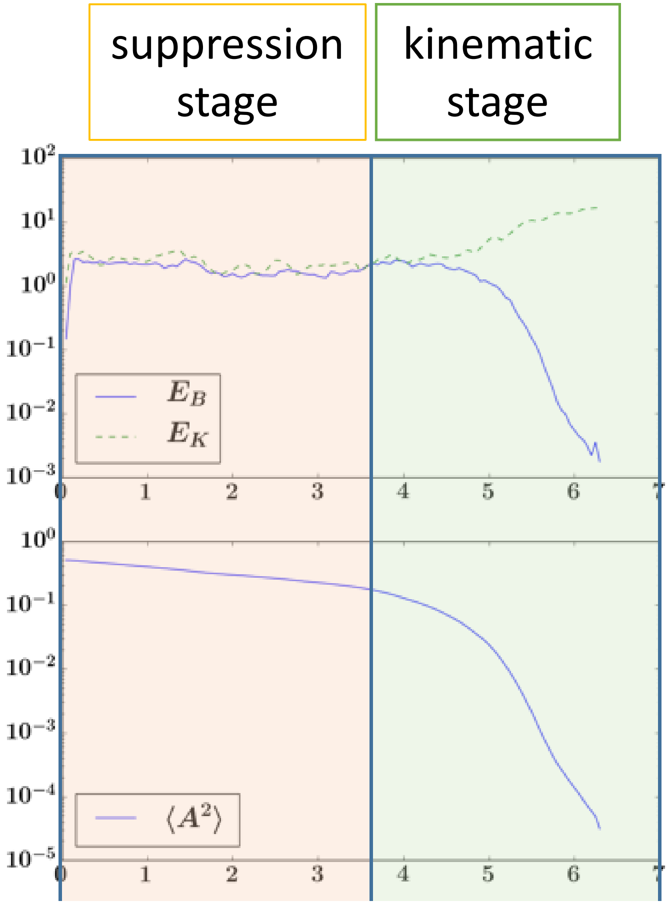


Unimodal



Two Stage Evolution:

- 1. The <u>suppression</u> <u>stage</u>: the (large scale) magnetic field is sufficiently strong so that the diffusion is suppressed.
- 2. The <u>kinematic decay stage</u>: the magnetic field is dissipated so the diffusion rate returns to the kinematic rate.
- Suppression is due to the memory induced by the magnetic field.

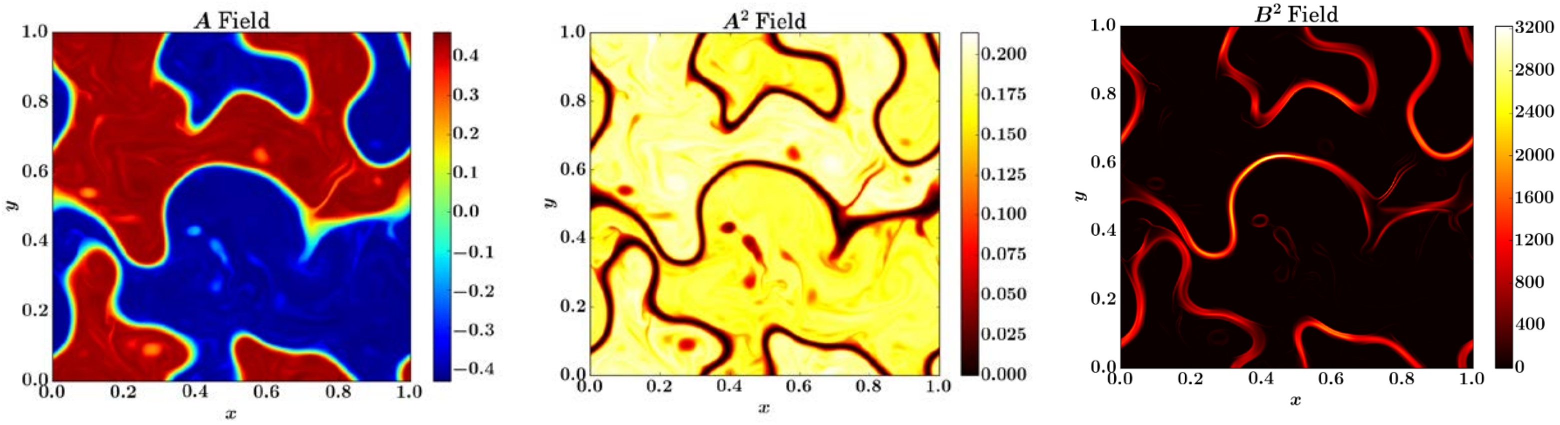


New Wrinkles

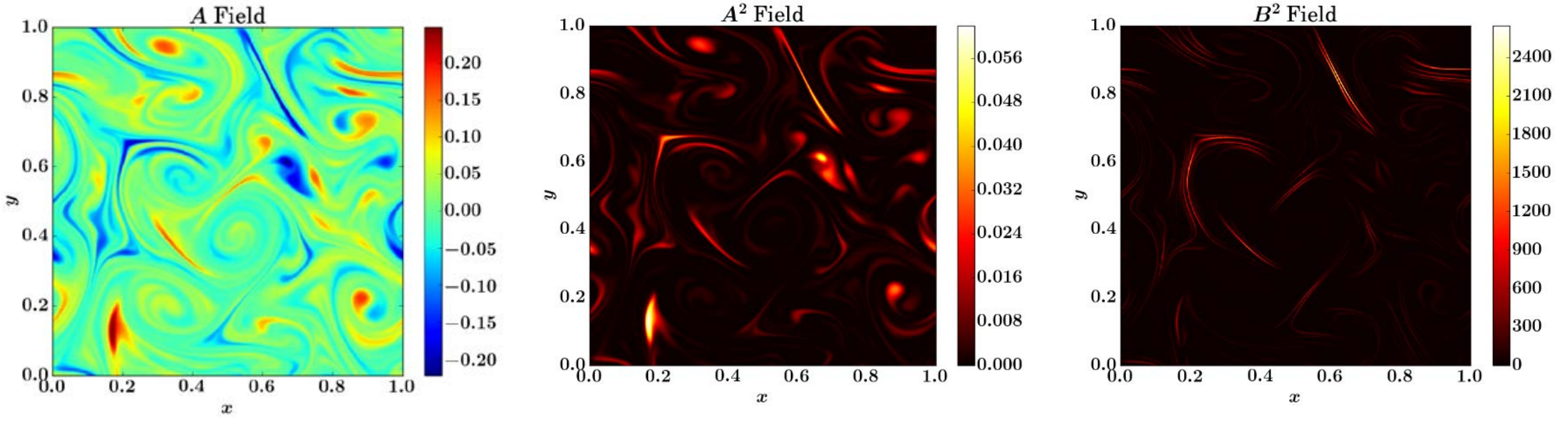
New Observations

- With no imposed B_0, in suppression stage:

Field
Concentrated!



- v.s. same run, in kinematic stage (trivial):



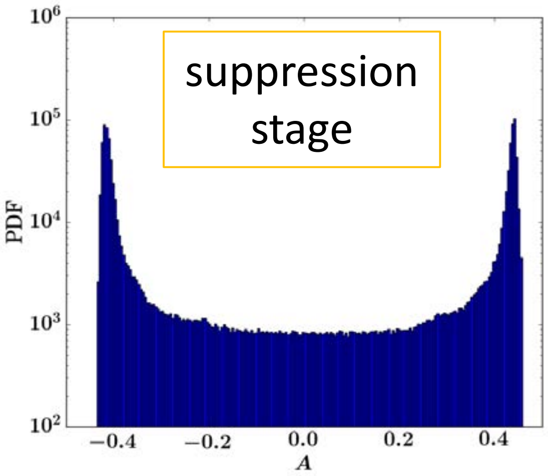
New Observations Cont'd

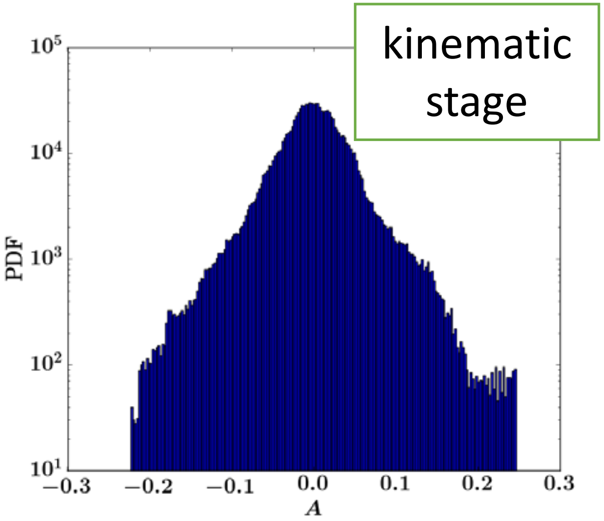
- Nontrivial structure formed in real space during the suppression stage.
- A field is evidently composed of "blobs".
- The low A^2 regions are 1-dimensional.
- The high B^2 regions are strongly correlated with low A^2 regions, and also are 1-dimensional.
- We call these 1-dimensional high B^2 regions ``barriers'', because these are the regions where mixing is reduced, relative to η_K.

➔ Story one of 'blobs and barriers'

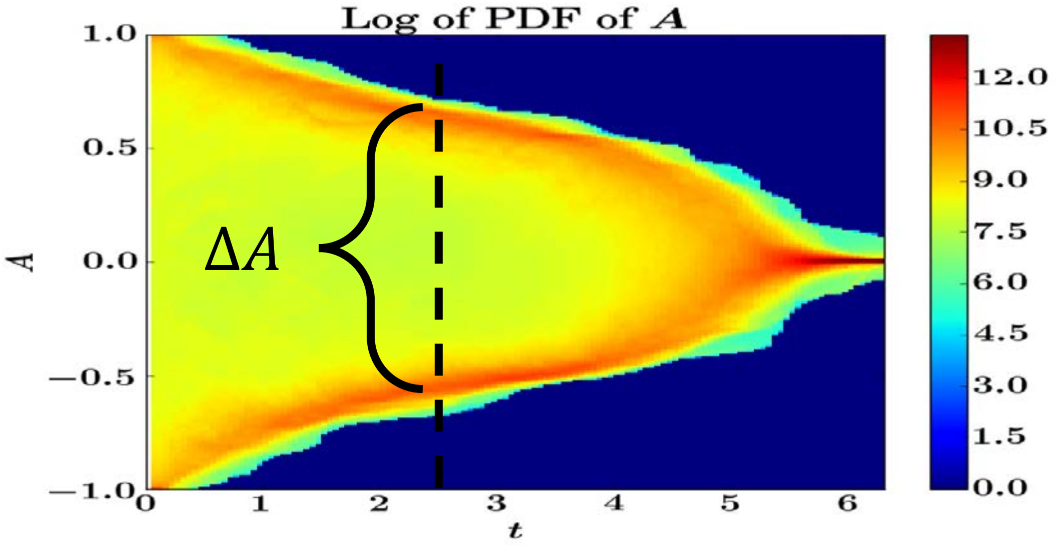
Evolution of PDF of A

- Probability Density Function (PDF) in two stage:


suppression stage


kinematic stage

- Time evolution: horizontal “Y”.

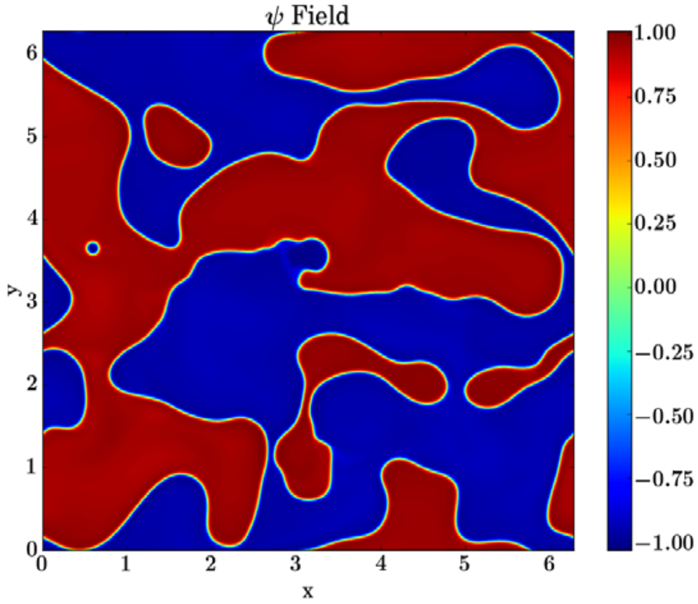

Log of PDF of A

- The PDF changes from double peak to single peak as the system evolves from the suppression stage to the kinematic stage.

2D CHNS and 2D MHD

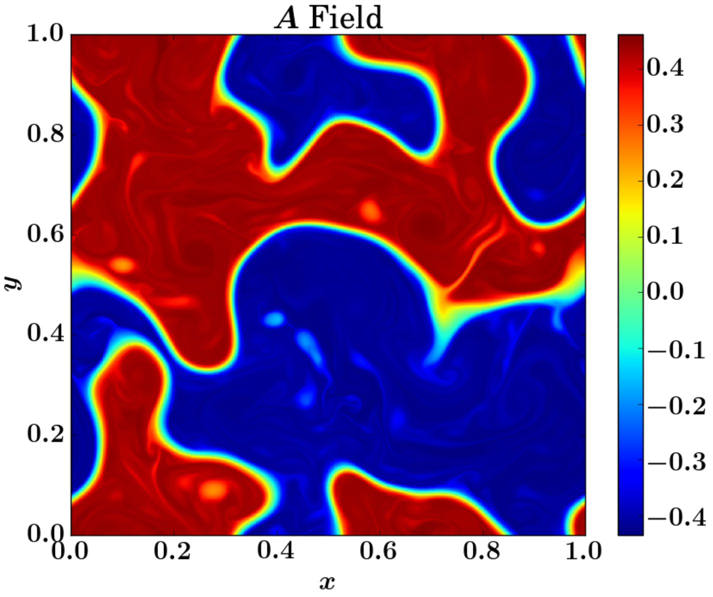
- The A field in 2D MHD in suppression stage is strikingly similar to the ψ field in 2D CHNS (Cahn-Hilliard Navier-Stokes) system:

ψ field in 2D CHNS



v.s.

A field in 2D MHD



2D CHNS and 2D MHD

	2D MHD	2D CHNS
Magnetic Potential	A	ψ
Magnetic Field	$\mathbf{B}$	$\mathbf{B}_\psi$
Current	j	j_ψ
Diffusivity	η	D
Interaction strength	$\frac{1}{\mu_0}$	ξ^2

- 2D CHNS Equations:

$$\partial_t \psi + \vec{v} \cdot \nabla \psi = D \nabla^2 (-\psi + \psi^3 - \xi^2 \nabla^2 \psi)$$

$$\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{\xi^2}{\rho} \vec{B}_\psi \cdot \nabla \nabla^2 \psi + \nu \nabla^2 \omega$$

$-\psi$: Negative diffusion term

ψ^3: Self nonlinear term

$-\xi^2 \nabla^2 \psi$: Hyper-diffusion term

With $\vec{v}=\hat{z} \times \nabla \phi$, $\omega = \nabla^2 \phi$, $\vec{B}_\psi = \hat{z} \times \nabla \psi$, $j_\psi = \xi^2 \nabla^2 \psi$. $\psi \in [-1,1]$.

- 2D MHD Equations:

$$\partial_t A + \vec{v} \cdot \nabla A = \eta \nabla^2 A$$

$$\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{1}{\mu_0 \rho} \vec{B} \cdot \nabla \nabla^2 A + \nu \nabla^2 \omega$$

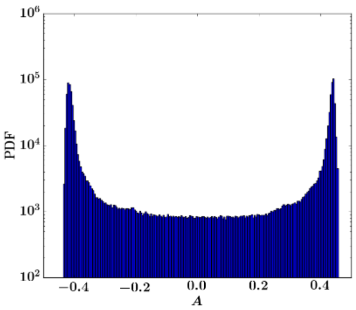
A: Simple diffusion term

See [Fan et.al. 2016] for more about CHNS.

With $\vec{v}=\hat{z} \times \nabla \phi$, $\omega = \nabla^2 \phi$, $\vec{B} = \hat{z} \times \nabla A$, $j = \frac{1}{\mu_0} \nabla^2 A$

Unimodal Initial Condition

- One may question whether the bimodal PDF feature is purely due to the initial condition. The answer is No.
- Two non-zero peaks in PDF of A still arise, even if the initial condition is unimodal.



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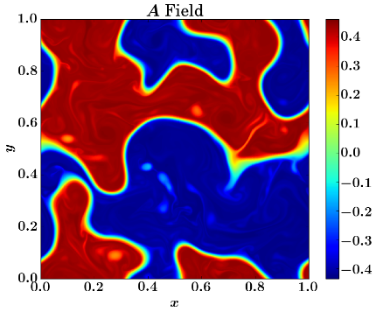

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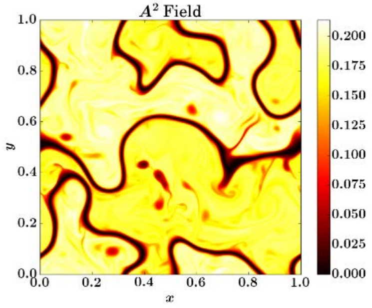

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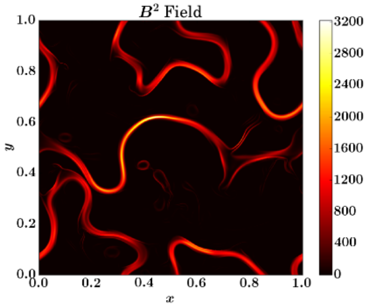

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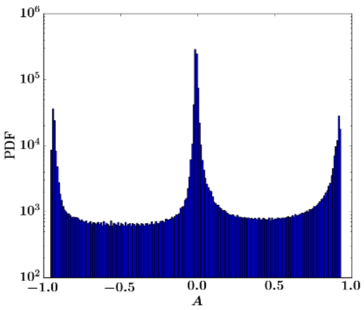
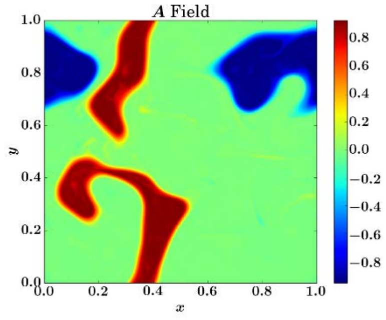
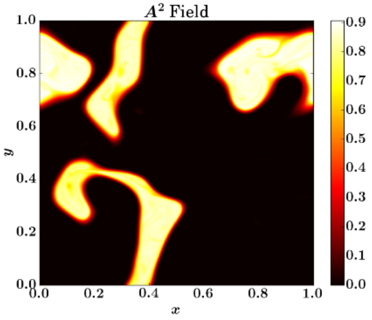


(b1)



(b2)



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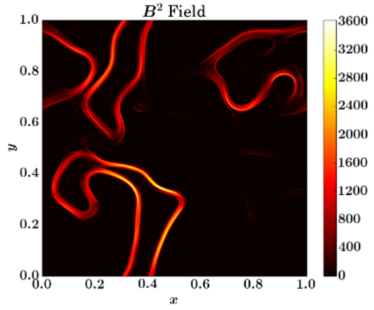
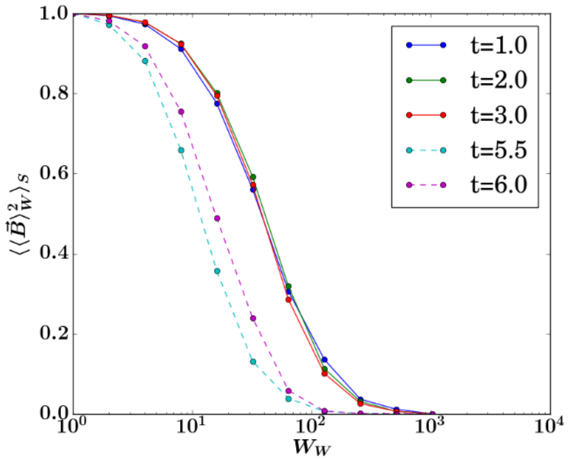
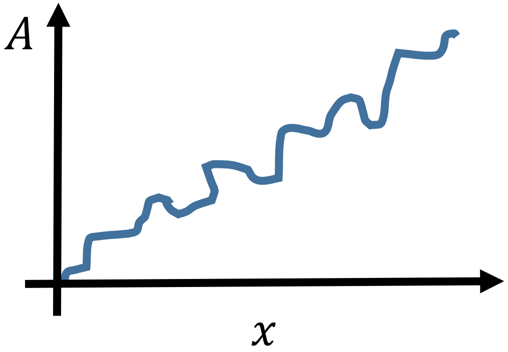


(b4)

The problem of the mean field $\langle \boldsymbol{B} \rangle$ → What does mean mean?

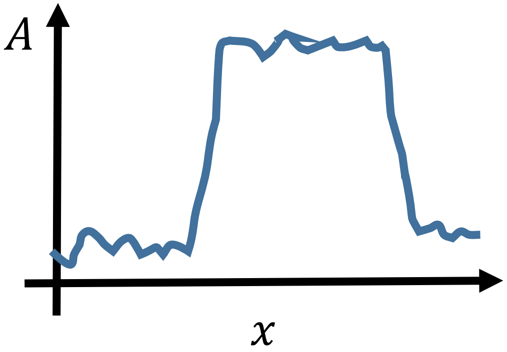
- $\langle B \rangle$ depends on the averaging window.
- With no imposed external field, B is highly intermittent, therefore the $\langle B \rangle$ is not well defined.





$|\langle B \rangle| \sim \sqrt{\langle A^2 \rangle}/L_0$ ✓

v.s.



$\langle B \rangle$ not well defined

Reality

Revisiting Quenching

New Understanding

- Summary of important length scales: $l < L_{stir} < L_{env} < L_0$
  - System size L_0
  - Envelope size L_{env} → emergent (blob)
  - Stirring length scale L_{stir}
  - Turbulence length scale l, here we use Taylor microscale λ
  - Barrier width W → emergent
- Quench is not uniform. Transport coefficients differ in different regions.
- In the regions where magnetic fields are strong, Rm/M^2 is dominant. They are regions of ***<u>barriers</u>***.
- In other regions, i.e. Inside blobs, Rm/M'^2 is what remains. $M'^2 \equiv \langle V^2\rangle / \left(\frac{1}{\rho}\langle A^2\rangle / L_{env}^2\right)$

New Understanding, cont'd

- From $\partial_t \langle A^2 \rangle = -\langle \mathbf{v} A \rangle \cdot \nabla \langle A \rangle - \nabla \cdot \langle \mathbf{v} A^2 \rangle - \eta \langle B^2 \rangle$
- Retain 2nd term on RHS. Average taken over an envelope/blob scale.
- Define diffusion (closure):

$$\langle \mathbf{v} A \rangle = -\eta_{T1} \nabla \langle A \rangle$$
$$\langle \mathbf{v} A^2 \rangle = -\eta_{T2} \nabla \langle A^2 \rangle$$

- Plugging in: $\partial_t \langle A^2 \rangle = \eta_{T1} (\nabla \langle A \rangle)^2 + \nabla \eta_{T2} \cdot \nabla \langle A^2 \rangle - \eta \langle B^2 \rangle$
- For simplicity: $\langle B^2 \rangle \sim \frac{\eta_T}{\eta} (\langle B \rangle^2 + \langle A^2 \rangle / L_{env}^2)$

  where L_{env} is the envelope size. Scale of $\nabla^2 \langle A^2 \rangle$.
- Define new strength parameter: $M'^2 \equiv \langle v^2 \rangle / (\frac{1}{\mu_0 \rho} \langle A^2 \rangle / L_{env}^2)$
- Result: $\boxed{\eta_T = \frac{ul}{1 + \mathrm{Rm}/M^2 + \mathrm{Rm}/M'^2} = \frac{ul}{1 + \mathrm{Rm} \frac{1}{\mu_0 \rho} \langle \mathbf{B} \rangle^2 / \langle v^2 \rangle + \mathrm{Rm} \frac{1}{\mu_0 \rho} \langle A^2 \rangle / L_{env}^2 \langle v^2 \rangle}}$

$$\eta_T = V\,l \,/\left[1+\frac{R_m}{M^2}+\frac{R_m}{M'^2}\right]$$

- Barriers:

Strong field

$$\eta_T \approx V\,l \,/\,\left[1+R_m\,\frac{\langle B\rangle^2}{\rho\langle\tilde{V}^2\rangle}\right]$$

- Blobs:

Weak effective field

$$\eta_T \approx V\,l \,/\left[1+R_m\,\frac{\langle A^2\rangle}{\rho L_{env}^2\,\langle\tilde{V}^2\rangle}\right]$$

- Quench stronger in barriers, ,non-uniform

Barrier Formation

Formation of Barriers

- How do the barriers form?

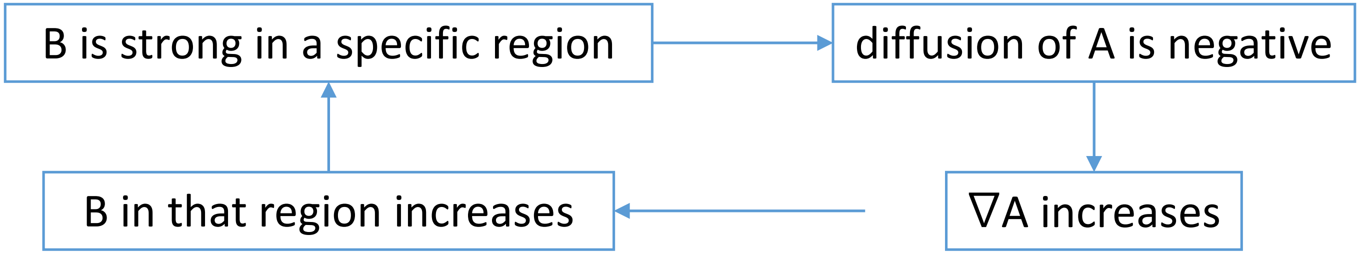
flux coalescence

$$\eta_T = \sum_{\mathbf{k}} \tau_c [\langle v^2 \rangle_{\mathbf{k}} - \frac{1}{\mu_0 \rho} \langle B^2 \rangle_{\mathbf{k}}]$$

- From above, strong B regions can support negative incremental $\eta_T \;\; \delta\Gamma_A / \delta(-\nabla A) < 0$, suggesting clustering

- $\langle \eta_T \rangle \; > \; 0$

- Positive feedback: a twist on a familiar theme



B is strong in a specific region

diffusion of A is negative

∇A increases

B in that region increases

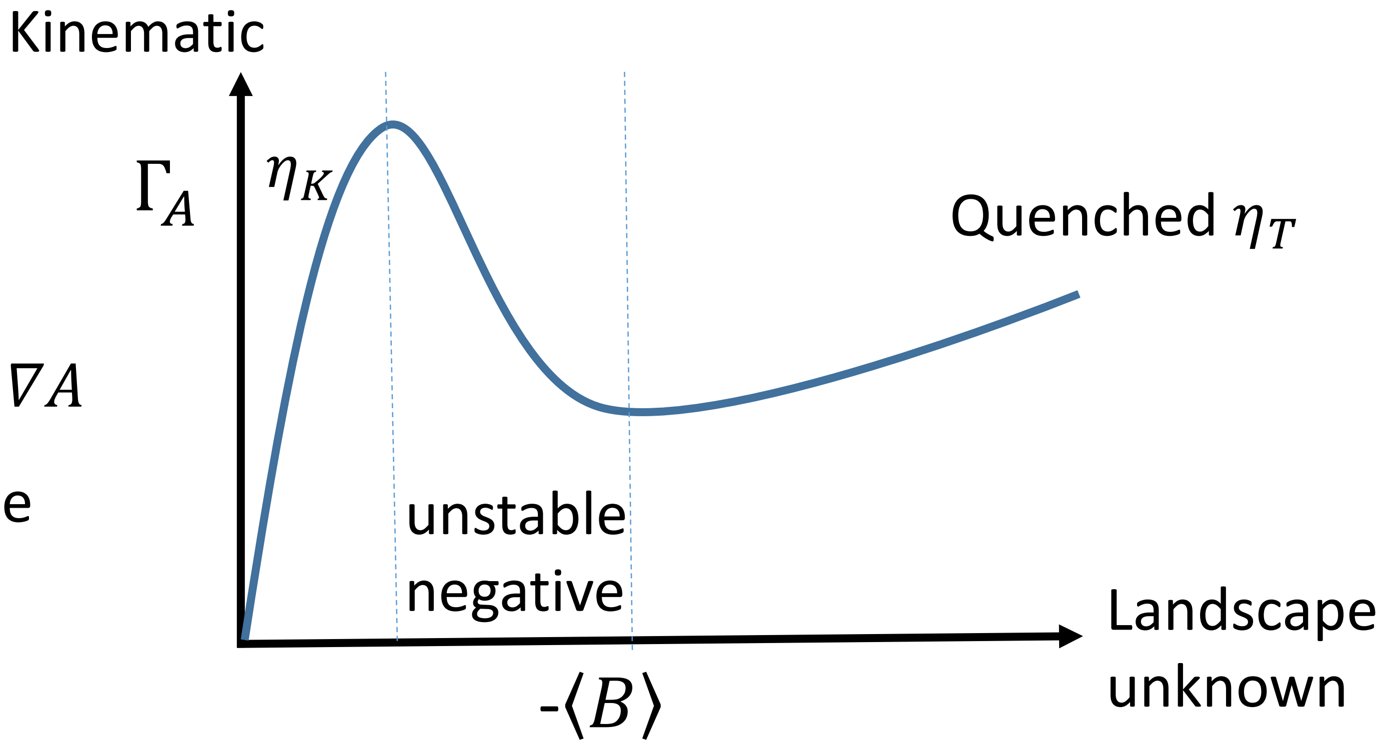
Formation of Barriers, Cont'd

- Negative resistivity leads to barrier formation.
- The S-curve reflects the dependence of Γ_A on B.
- When slope negative →negative (incremental) resistivity.

Barriers ←→

Bistability of Γ_A vs ∇A

→ a familiar theme



Describing the Barriers

- How to measure the barrier width W.
- Starting point: $W \sim \Delta A / B_b$
- Use $\sqrt{\langle A^2 \rangle}$ to calculate ΔA
- Define the barrier regions as: $B(x,y) > \sqrt{\langle B^2 \rangle} * 2$

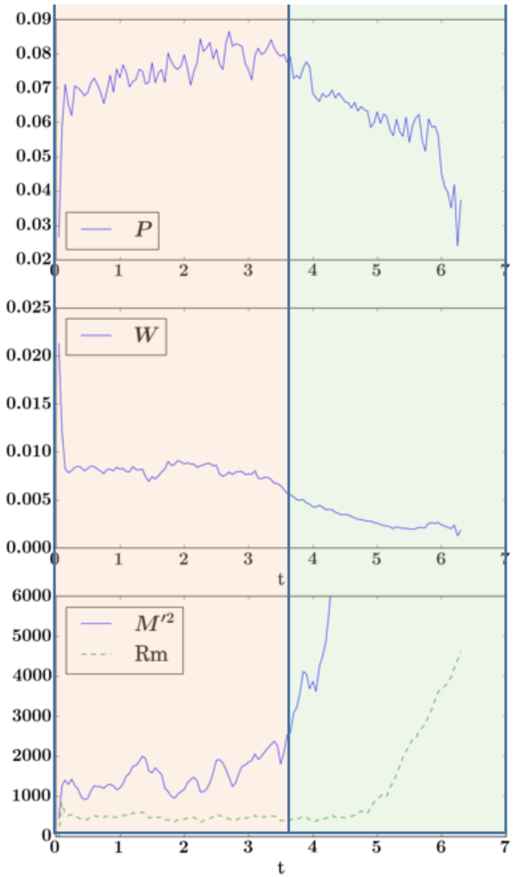
arbitrary threshold

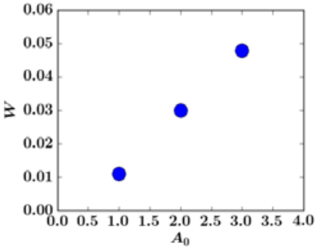
- Define barrier packing fracti $P \equiv \frac{\#\text{ of grid points for barrier regions}}{\#\text{ of total grid points}}$
- Use use the magnetic fields in the barrier regions to calculate the magnetic energy: $\boxed{\sum_{\text{barriers}} B_b^2 \sim \sum_{\text{system}} B^2}$
- Thus $\langle B_b^2 \rangle \sim \langle B^2 \rangle / P$
- So barrier width can be estimated by: $\boxed{W^2 \equiv \langle A^2 \rangle / (\langle B^2 \rangle / P)}$

N.B. All magnetic energy in the barriers

Describing the Barriers

- Time evolution of P and W:
  - P, W collapse in decay
  - M' rises
- Sensitivity of W:
  - A_0 or $1/\mu_0\rho$ greater → W greater;
  - f_0 greater, W smaller; (ala' Hinze)
  - W not sensitive to η or ν.





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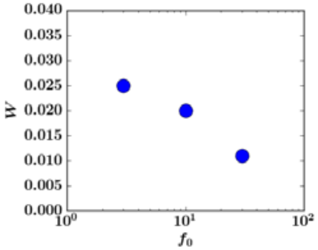

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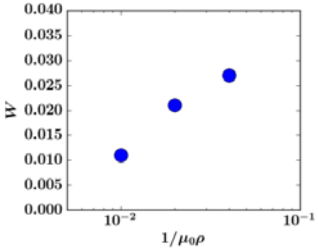
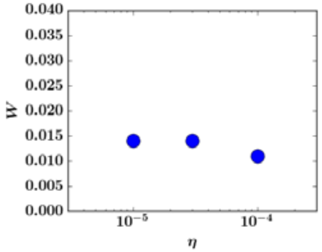


(c)



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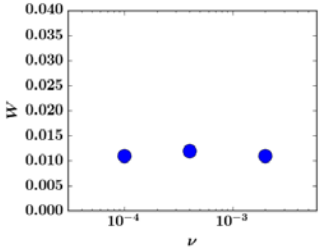
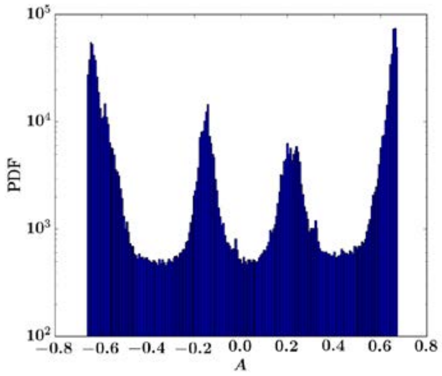


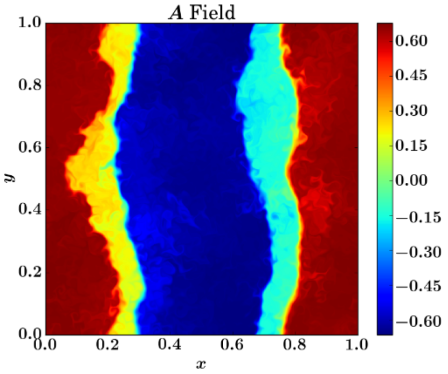
(e)

Staircase (inhomogeneous Mixing, Bistability)

- Staircases emerge spontaneously! - <u>Barriers</u>
- Initial condition is the usual cos function (bimodal)
- The only major sensitive parameter (from runs above) is the forcing scale is k=32 (for all runs above k=5).
- Resembles the staircase in MFE.



(1)



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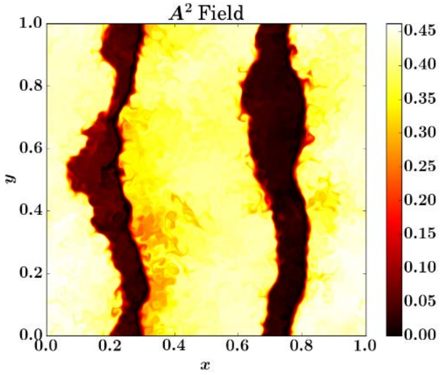

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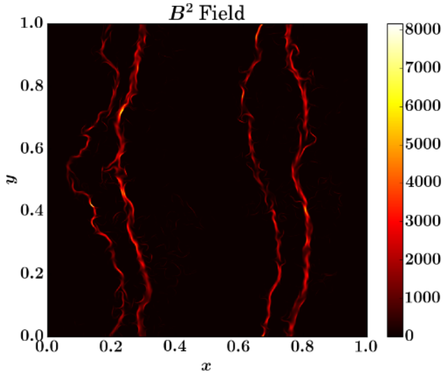


(4)

Conclusions / Summary

- Magnetic fields suppress turbulent diffusion in 2D MHD by: formation of intermittent ***transport barriers***.
- Magnetic structures:
  - Barriers – thin, 1D strong field regions
  - Blobs – 2D, weak field regions
- Quench not uniform:

$$\eta_T = \frac{ul}{1 + \mathrm{Rm}\frac{1}{\mu_0\rho}\langle \mathbf{B}\rangle^2/\langle v^2\rangle + \mathrm{Rm}\frac{1}{\mu_0\rho}\langle A^2\rangle/L_{env}^2\langle v^2\rangle}$$

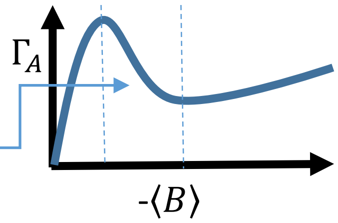
barriers, strong B

blobs, weak B, $\nabla^2\langle A^2\rangle$ remains

- Barriers form due to negative resistivity:

$$\eta_T = \sum_{\mathbf{k}} \tau_c[\langle v^2\rangle_{\mathbf{k}} - \frac{1}{\mu_0\rho}\langle B^2\rangle_{\mathbf{k}}]$$

flux coalescence



- Formation of "magnetic staircases" observed for some stirring scale

General Conclusions (MHD and CHNS)

- Dual (or multiple) cascades can interact with each other, and one can modify another.
- We also show how a length scale, e.g. the Hinze scale in 2D CHNS, emerges from the balance of kinetic energy and elastic energy in blobby turbulence. → blob scale in MHD?!
- We see that negative incremental diffusion (flux/blob coalescence) can lead to novel real space structure in a simple system.
- Negative incremental resistivity can exist in a simple system such as 2D MHD. This results in the formation of nontrivial real space structure.

Future Works

- Extension of the transport study in MHD:
    - Numerical tests of the new η_T expression ?
    - What determines the barrier width and packing fraction ?
    - Why does layering appear when the forcing scale is small ?
    - What determines the step width, in the case of layering
    - The transport study may also be extended to 3D MHD ($\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle$ important instead of $\langle A^2 \rangle$)
- Other similar systems can also be studied in this spirit. e.g. Oldroyd-B model for polymer solutions. (drag reduction)
- Reduced Model of Magnetic Staircase

Reading

Fan, P.D., Chacon:

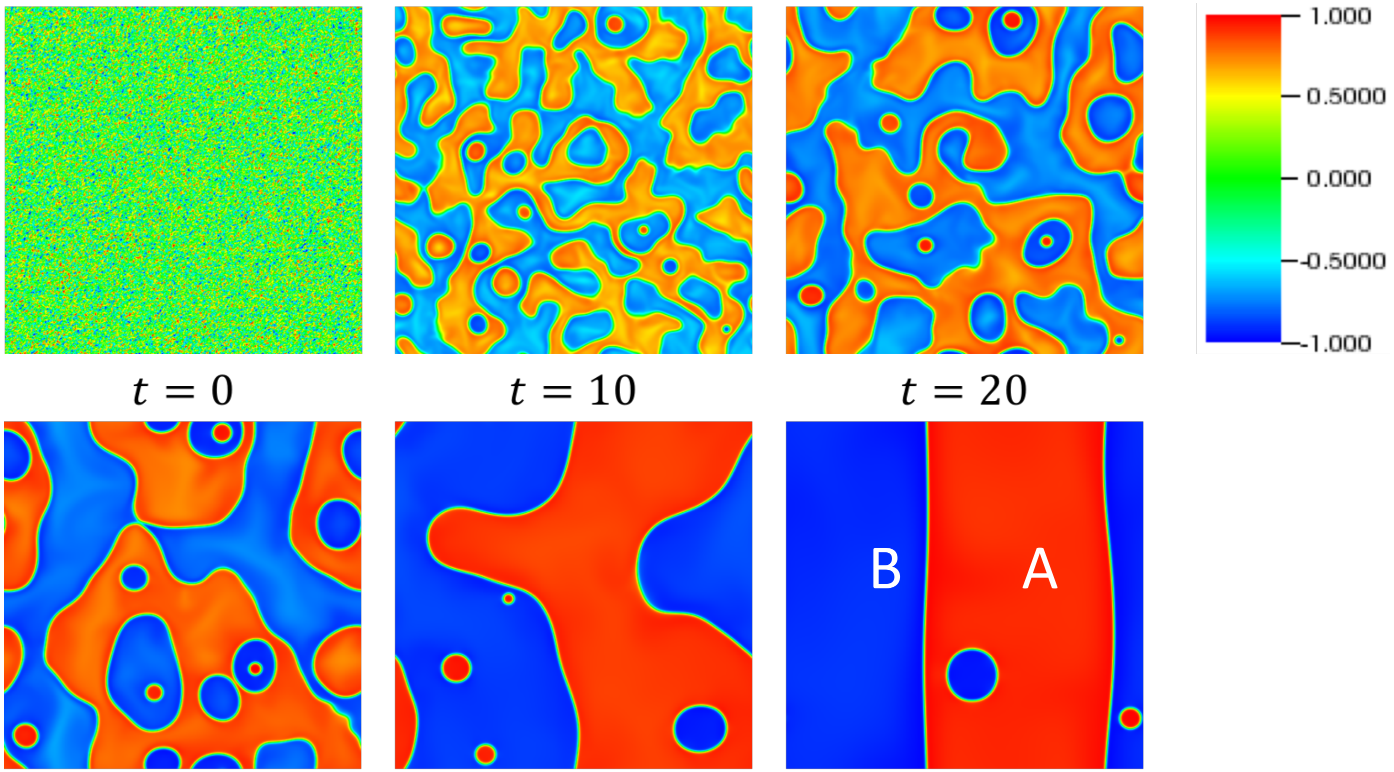
- PRE Rap Comm 99, 041201 (2019)
- PoP 25, 055702 (2018)
- PRE Rap Comm 96, 041101 (2017)
- Phys Rev Fluids 1, 054403 (2016)

Thank you!

Back-Up

2D CHNS (Cahn-Hilliard Navier-Stokes)

- The Cahn-Hilliard Navier-Stokes (CHNS) system describes ***separation of components*** for binary fluid (i.e. ***Spinodal Decomposition***)
- Miscible phase -> Immiscible phase



$t = 0$ $t = 10$ $t = 20$

$t = 40$ $t = 160$ $t = 350$

2D CHNS

- How to describe the system: the concentration field
- $\psi(\vec{r},t) \overset{\text{def}}{=} [\rho_A(\vec{r},t) - \rho_B(\vec{r},t)]/\rho$: scalar field
- $\psi \in [-1,1]$
- CHNS equations:

$$\partial_t \psi + \vec{v} \cdot \nabla \psi = D \nabla^2 (-\psi + \psi^3 - \xi^2 \nabla^2 \psi)$$

$$\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{\xi^2}{\rho} \vec{B}_\psi \cdot \nabla \nabla^2 \psi + \nu \nabla^2 \omega$$

- 2D MHD and 2D CHNS: analogous. Elasticity; elastic wave; conserved quantities; cascades; etc.

Challenges – Dual Cascade

- Some key issues to understanding active scalar turbulence:
    1. the physics of dual (or multiple) cascades;
    2. the nature of “blobby” turbulence;
    3. the effects of negative diffusion/resistivity;
    4. the understanding of turbulent transport.

1. Dual Cascade
    - Physics of dual cascades and constrained relaxation → relative importance, selective decay…
    - Physics of wave-eddy interaction effects on nonlinear transfer (i.e. Alfven effect ←→ Kraichnan)
    - How do dual cascades interact?

Challenges – Blobby Turbulence

2. "Blobby Turbulence"
   - Blobs observed in SOL in Tokamaks.
   - CHNS is a naturally blobby system of turbulence.
   - What makes a blob a blob?
   - What is the role of structure in interaction?
   - How to understand blob coalescence and relation to cascades?

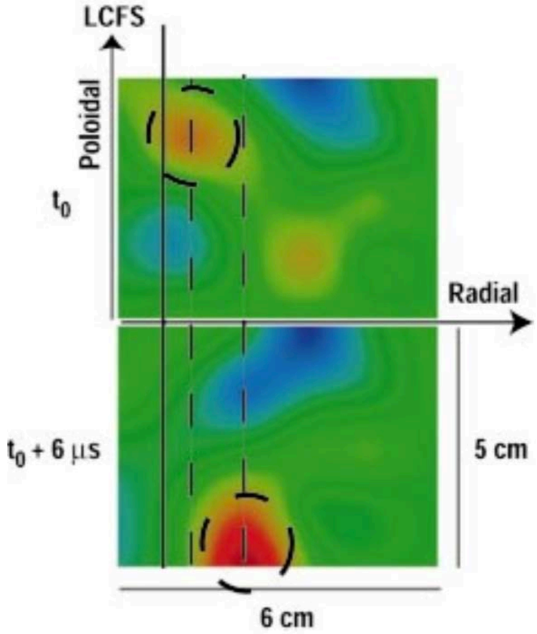


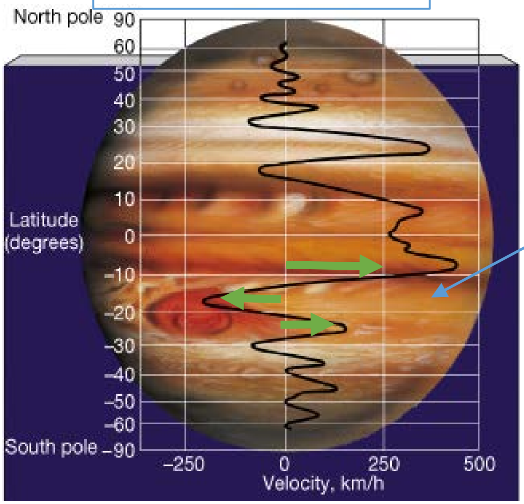
FIG. 4. (Color) Two frames from BES showing 2-D density plots. There is a time difference of 6 μs between frames. Red indicates high density and blue low density. A structure, marked with a dashed circle and shown in both frames, features poloidal and radial motion.

[J. A. Boedo et.al. 2003]

Challenges – Negative Diffusion

3. Zonal flow formation → negative viscosity phenomena
   - ZF can be viewed as a "spinodal decomposition" of momentum.
   - What determines scale?

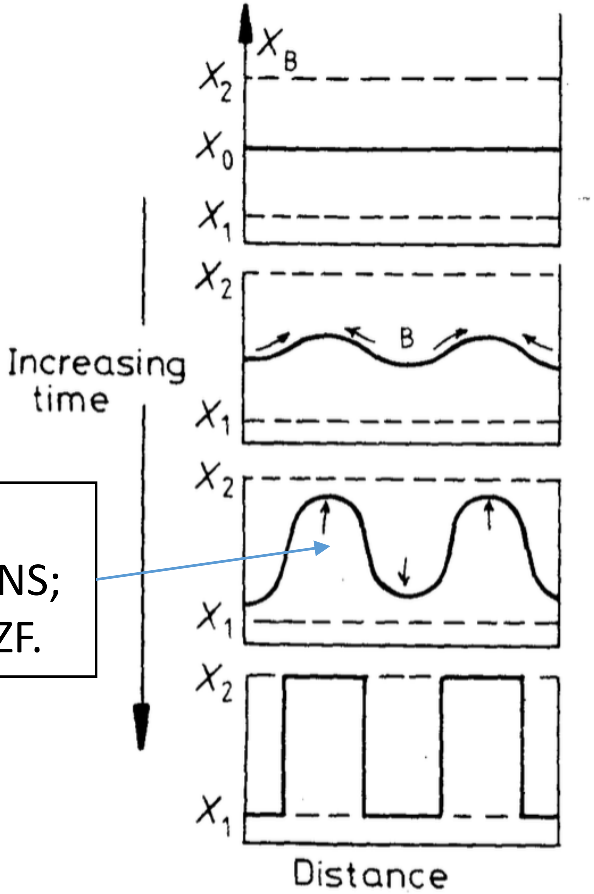
Zonal Flow



http://astronomy.nju.edu.cn/~lixd/GA/AT4/AT411/HTML/AT41102.htm

Arrows:
ψ for CHNS;
flow for ZF.

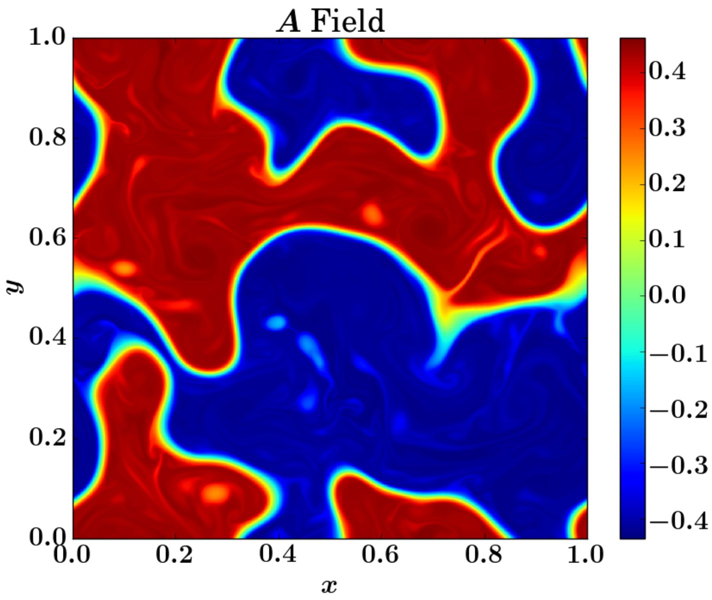
Spinodal Decomposition



[Porter 1981]

Challenges – Turbulent Transport

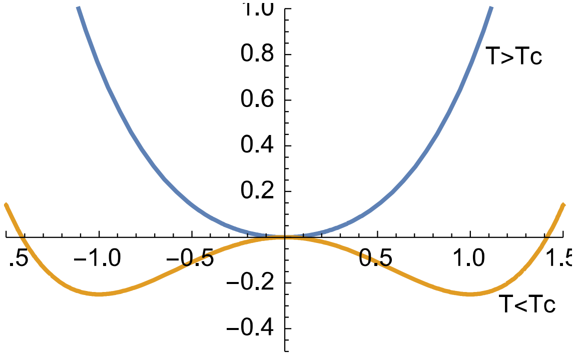
- 4. Turbulent transport
  - Suppressed in 2D MHD by magnetic field.
  - Previous understandings: mean field theory
  - New observation: blob-and-barrier structure
  - Need new understanding



A Brief Derivation of the CHNS Model

- Second order phase transition → Landau Theory.
- Order parameter: $\psi(\vec{r},t) \overset{\text{def}}{=} [\rho_A(\vec{r},t) - \rho_B(\vec{r},t)]/\rho$
- Free energy:

$$\mathrm{F}(\psi) = \int d\vec{r}(\underbrace{\frac{1}{2}C_1\psi^2 + \frac{1}{4}C_2\psi^4}_{\text{Phase Transition}} + \underbrace{\frac{\xi^2}{2}|\nabla\psi|^2}_{\text{Gradient Penalty}})$$



- $C_1(T), C_2(T)$.
- Isothermal $T < T_C$. Set $C_2 = -C_1 = 1$:

$$\mathrm{F}(\psi) = \int d\vec{r}(-\frac{1}{2}\psi^2 + \frac{1}{4}\psi^4 + \frac{\xi^2}{2}|\nabla\psi|^2)$$

A Brief Derivation of the CHNS Model

- Continuity equation: $\frac{d\psi}{dt} + \nabla \cdot \vec{J} = 0$.
- Fick's Law: $\vec{J} = -D\nabla\mu$.
- Chemical potential: $\mu = \frac{\delta F(\psi)}{\delta\psi} = -\psi + \psi^3 - \xi^2\nabla^2\psi$.
- Combining → Cahn Hilliard equation:

$$\boxed{\frac{d\psi}{dt} = D\nabla^2\mu = D\nabla^2(-\psi + \psi^3 - \xi^2\nabla^2\psi)}$$

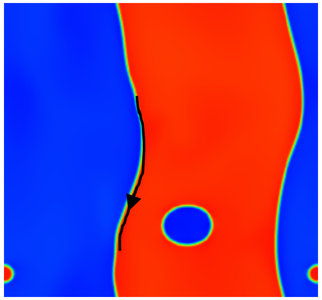
- $d_t = \partial_t + \vec{v} \cdot \nabla$.
- Surface tension: force in Navier-Stokes equation:

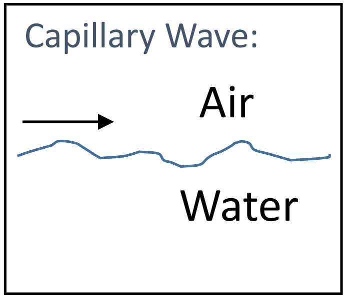
$$\boxed{\partial_t\vec{v} + \vec{v} \cdot \nabla\vec{v} = -\frac{\nabla p}{\rho} - \psi\nabla\mu + \nu\nabla^2\vec{v}}$$

- For incompressible fluid, $\nabla \cdot \vec{v} = 0$.

Linear Wave

- CHNS supports linear “elastic” wave:





- Akin to capillary wave at phase interface.
- Propagates ***only*** along the interface of the two fluids, where $|\vec{B}_\psi| = |\nabla\psi| \neq 0.$
- Analogue of Alfven wave in MHD (propagates along B lines).
- Important differences:
  - $\vec{B}_\psi$ in CHNS is large only in the interfacial regions.
  - Elastic wave activity does not fill space.

Ideal Quadratic Conserved Quantities

- **2D MHD**

1. Energy

$$E = E^K + E^B = \int (\frac{v^2}{2} + \frac{B^2}{2\mu_0}) d^2x$$

2. Mean Square Magnetic Potential

$$H^A = \int A^2 \, d^2x$$

3. Cross Helicity

$$H^C = \int \vec{v} \cdot \vec{B} d^2x$$

- **2D CHNS**

1. Energy

$$E = E^K + E^B = \int (\frac{v^2}{2} + \frac{\xi^2 B_\psi^2}{2}) d^2x$$

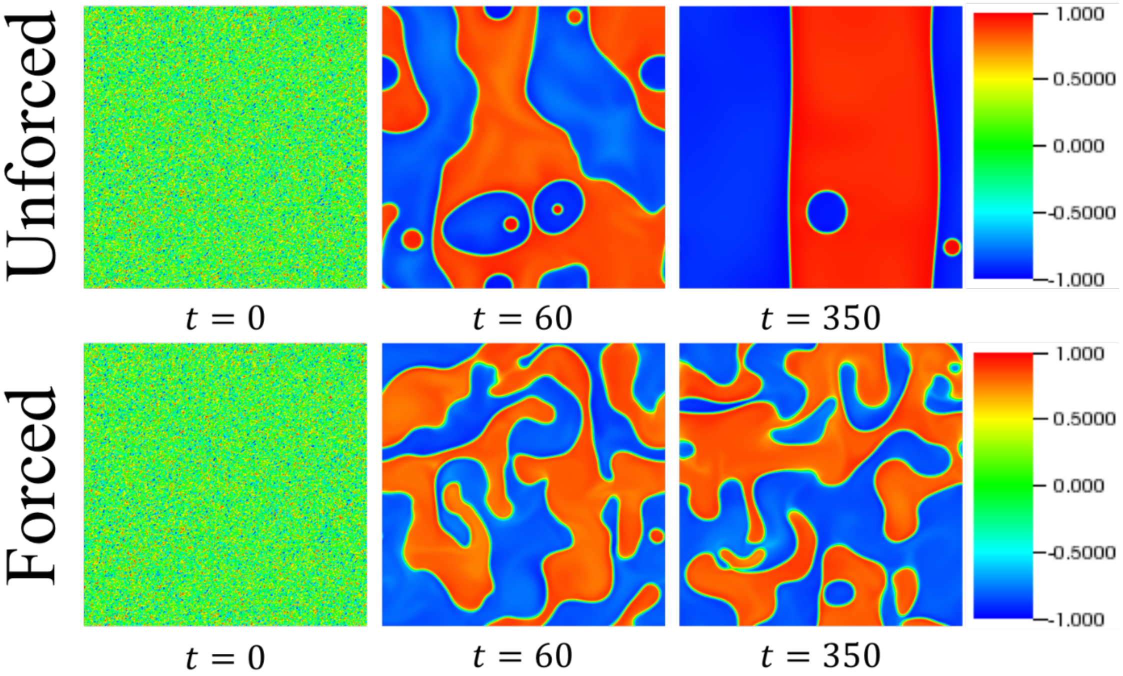
2. Mean Square Concentration

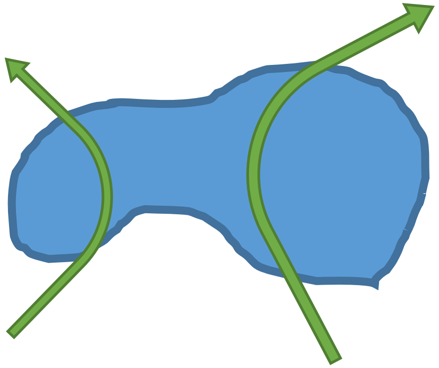
$$H^\psi = \int \psi^2 \, d^2x$$

3. Cross Helicity

$$H^C = \int \vec{v} \cdot \vec{B}_\psi \, d^2x$$

Scales, Ranges, Trends





How big is a raindrop?

- Turbulent straining vs capillarity.
- ρv^2 vs σ / l.

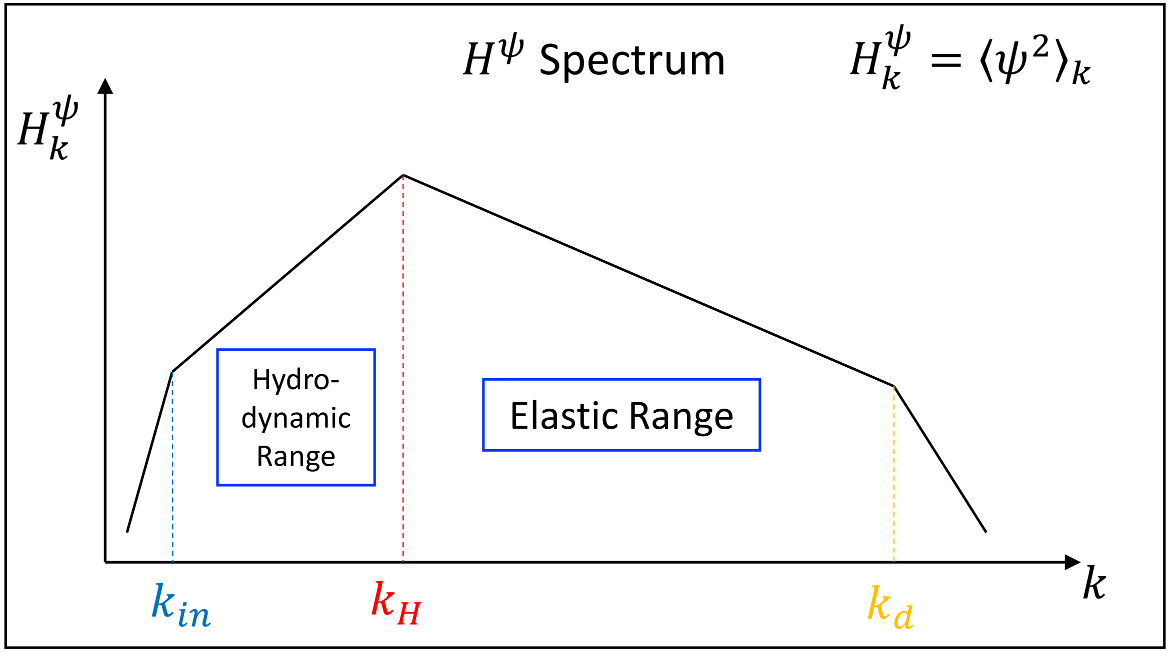
(Hinze 1955)

- Fluid forcing → Fluid straining vs Blob coalescence
- Scale where turbulent straining ~ elastic restoring force (due surface tension): Hinze Scale

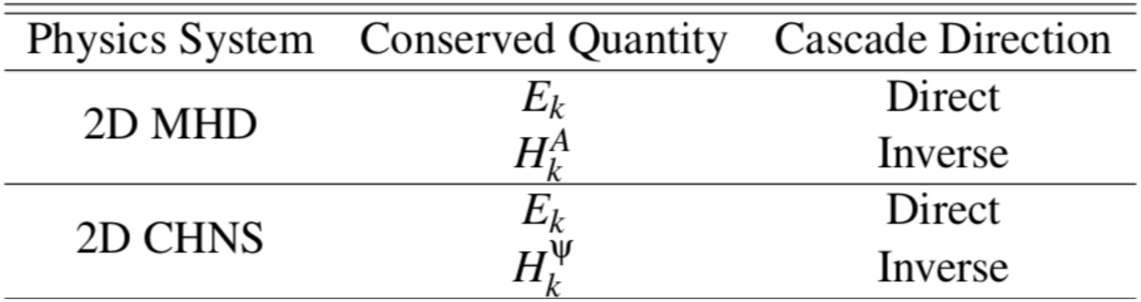
$$L_H \sim \left(\frac{\rho}{\xi}\right)^{-1/3} \epsilon_{\Omega}^{-2/9}$$

Scales, Ranges, Trends

- Elastic range: $L_H < l < L_d$: where elastic effects matter.
- $L_H/L_d \sim (\frac{\rho}{\xi})^{-1/3} \nu^{-1/2} \epsilon_{\Omega}^{-1/18}$ → Extent of the elastic range
- $L_H \gg L_d$ required for large elastic range → case of interest



Cascades

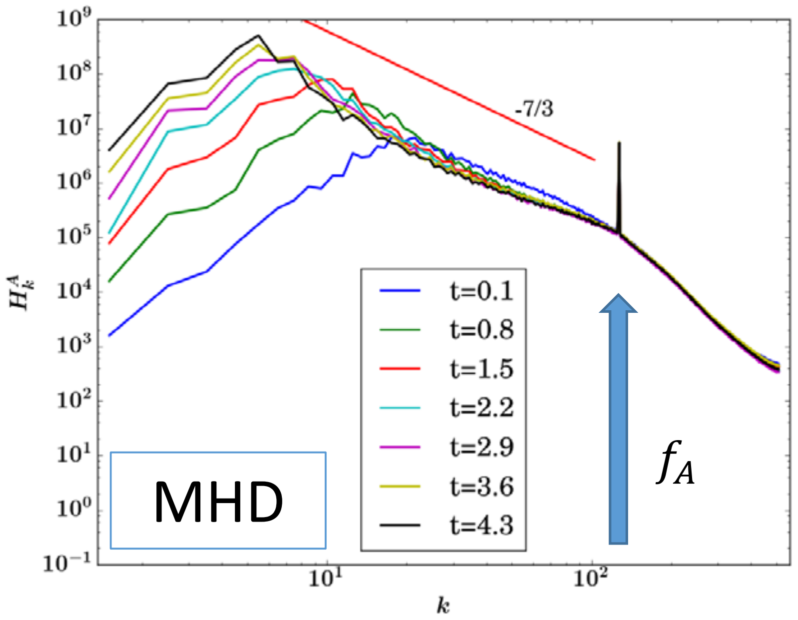


Physics System	Conserved Quantity	Cascade Direction
2D MHD	E_k H_k^A	Direct Inverse
2D CHNS	E_k H_k^Ψ	Direct Inverse

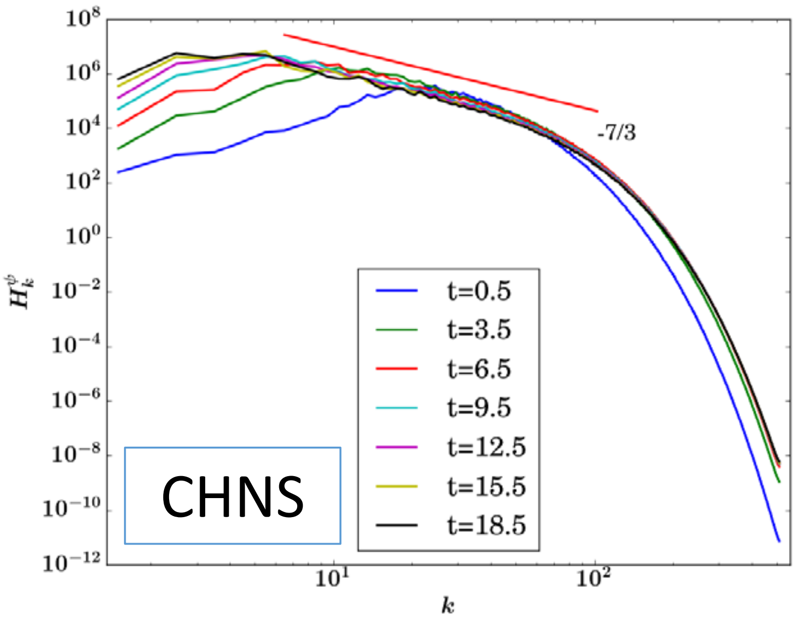
- By statistical mechanics studies (absolute equilibrium distributions) → dual cascade:
  - ***<u>Inverse</u>*** cascade of $\langle\psi^2\rangle$⍰ ⍰
  - ***<u>Forward</u>*** cascade of E⍰ ⍰
- Blob coalescence in the elastic range of CHNS ←→ flux coalescence in MHD.
- Inverse cascade of $\langle\psi^2\rangle$ is formal expression of blob coalescence process → generate larger scale structures till limited by straining
- Forward cascade of E as usual, as elastic force breaks enstrophy conservation

Power Laws

- $\langle A^2 \rangle$ spectrum:

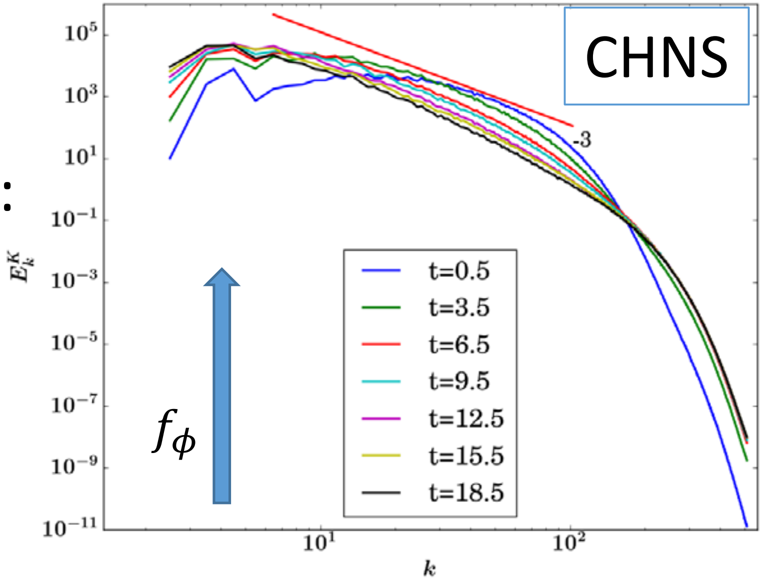


$\langle \psi^2 \rangle$ spectrum:



- Both systems exhibit $k^{-7/3}$ spectra.
- Inverse cascade of $\langle \psi^2 \rangle$ exhibits same power law scaling, so long as $L_H \gg L_d$, maintaining elastic range: Robust process.

More Power Laws



- Kinetic energy spectrum (Surprise!):
- 2D CHNS: $E_k^K \sim k^{-3}$;
- 2D MHD: $E_k^K \sim k^{-3/2}$.

!

- The -3 power law:
  - Closer to enstrophy cascade range scaling, in 2D Hydro turbulence.
  - Remarkable departure from expected -3/2 for MHD. ***Why?***
- Why does CHNS ←→ MHD correspondence hold well for $\langle \psi^2 \rangle_k \sim \langle A^2 \rangle_k \sim k^{-7/3}$, yet break down drastically for energy?
- ***What physics*** underpins this surprise?

Interface Packing Matters!

- Need to understand ***<u>differences</u>***, as well as similarities, between CHNS and MHD problems.

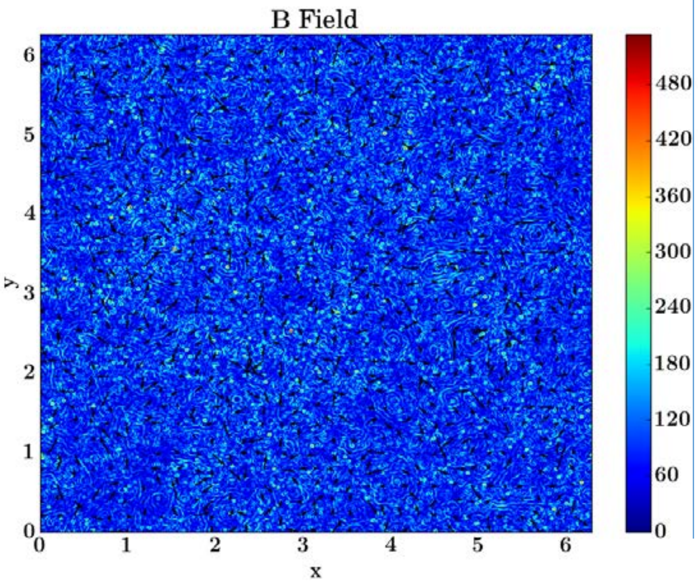
In MHD:

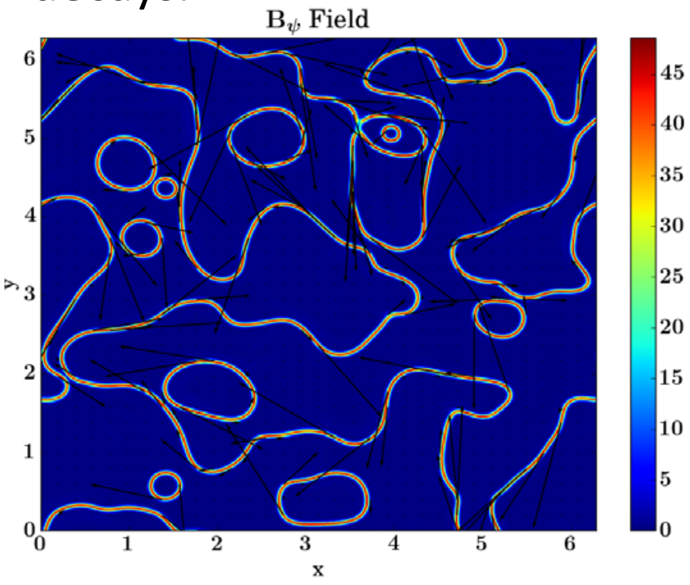
- ➢ Fields pervade system.

In CHNS:

- ➢ Elastic back-reaction is limited to regions of density contrast i.e. $|\vec{B}_\psi| = |\nabla\psi| \neq 0$.
- ➢ As blobs coalesce, interfacial region diminished. 'Active region' of elasticity decays.

MHD



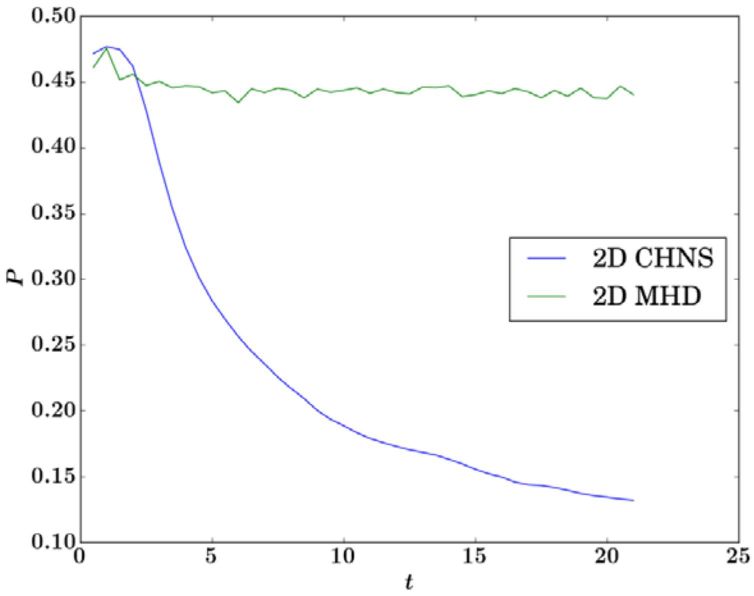


CHNS

Interface Packing Matters!

- Define the ***interface packing fraction*** P:

$$P = \frac{\#\text{ of grid points where } |\vec{B}_\psi| > B_\psi^{rms}}{\#\text{ of total grid points}}$$



- ➢ P for CHNS decays;
- ➢ P for MHD stationary!

- $\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{\xi^2}{\rho} \vec{B}_\psi \cdot \nabla \nabla^2 \psi + \nu \nabla^2 \omega$: small P → local back reaction is weak.
- Weak back reaction → reduce to 2D hydro

Summary

- Avoid power law tunnel vision!
- ***Real space*** realization of the flow is necessary to understand key dynamics. Track interfaces and packing fraction P.
- One player in dual cascade (i.e. $\langle\psi^2\rangle$) can modify or constrain the dynamics of the other (i.e. E).
- Against conventional wisdom, $\langle\psi^2\rangle$ inverse cascade due to blob coalescence is the robust nonlinear transfer process in CHNS turbulence.