Learning a model for mean-field turbulence dynamics

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Introduction: can a computer do plasma physics?

- Calculating turbulent fluxes is an important challenge for the plasma physicist
- Introduce new **machine learning technique** for studying fluxes
- Can the algorithm pick up physics we missed?

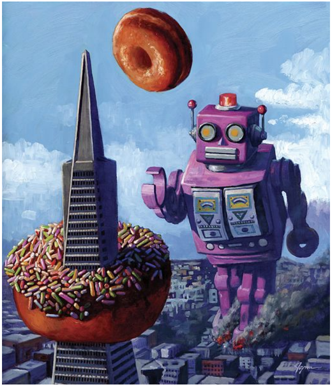

Figure A computer studies tokamak physics

Introduction: this talk

- Apply new method to resistive DW turbulence via 2D Hasegawa-Wakatani system
- Reproduce analytic result for particle flux, including often-overlooked term induced by ZF
- Discuss implications of ZF term, future directions

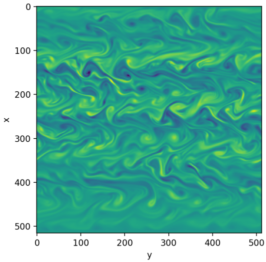

Figure Snapshot of vorticity field from simulation of 2D HW

Background

Drift-wave turbulence

- Drift-wave turbulence features complex interaction between mean profile, ZF, and turbulence
- Dynamics controlled by cross-correlations between fluctuating quantities (turbulent fluxes). E.g. particle flux $\Gamma = \langle \tilde{n}\tilde{v}_x \rangle$ and Reynolds stress $\Pi = \langle \tilde{v}_x \tilde{v}_y \rangle$
- Difficult to calculate! Not many approaches beyond quasilinear theory

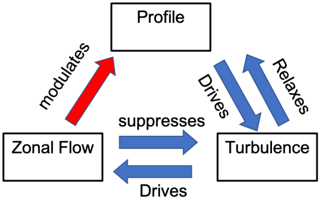

Figure Feedback loop illustrating interaction of mean fields in DW turbulence

Machine learning approach

Basic formalism: pushing MFT to the limit

- Seek maps f_q which send local mean fields to local fluxes, i.e. $f_q : (n, \partial_x n, \ldots, \phi, \partial_x \phi, \zeta, \partial_x \zeta, \ldots, \varepsilon, \partial_x \varepsilon, \ldots) \mapsto \langle \tilde{q} \tilde{v}_x \rangle$. Here $\varepsilon = (\tilde{n} - \tilde{\zeta})^2$ is turb. PE, $\zeta = \partial_x^2 \phi$ is vorticity
- Idea: rather than attempt direct calculation or fitting parameters, use **supervised learning** to train a neural network on numerical simulations
- Essentially nonlinear, model-free regression. Could capture physics missed by human?

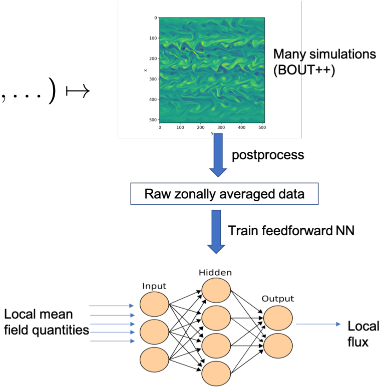

Figure Schematic of machine learning method

Detailed methods

- As proof of concept, learn particle flux Γ from simulations of 2D HW (at fixed $\alpha = 2$). Advantages: fast simulations, captures full feedback loop, yet simple to treat analytically
- Constrain problem using symmetries of HW:
  1. Invariance under uniform shifts $n \to n + n_0$ and $\phi \to \phi + \phi_0$ eliminate dependence on n, ϕ
  2. Invariance under boosts in y

$$\begin{cases} \phi & \to \phi + v_0 x \\ y & \to y - v_0 t \end{cases}$$

     eliminates dependence on ZF speed $\partial_x \phi$
  3. Reflection symmetries $x \to -x, y \to -y$ and $\phi \to -\phi, n \to -n, x \to -x$ and $\phi \to -\phi, n \to -n, x \to -x$ which are enforced by duplicating and transforming data

Results

Particle flux learned by NN

NN learns a model roughly of the form

$$\Gamma = -D_n \varepsilon \partial_x n - D_\zeta \varepsilon \partial_x \zeta$$

Usual QL flux plus an "off-diagonal" term driven by vorticity! (no clear dependence on other quantities)

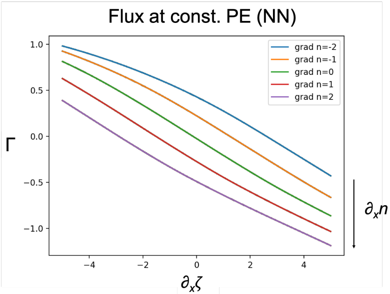

Figure Curves (at fixed $\varepsilon = 10$, $\zeta = 1$, $\partial_x \varepsilon = 0$, and various $\partial_x n$) of Γ vs vorticity gradient. Appears to be simple linear combination of $\partial_x n$ term and $\partial_x \zeta$ term

Derivation of off-diagonal term

Careful analytic treatment in adiabatic limit reproduces off-diagonal term. Need include frequency shift due to ZF!

$$\omega_{\mathbf{k}} = \frac{k_y}{1+k^2}(\partial_x n - \partial_x \zeta) + O(\alpha^{-2})$$

$$\gamma_{\mathbf{k}} = \frac{k_y^2}{\alpha(1+k^2)^3}(\partial_x n - \partial_x \zeta)(k^2 \partial_x n + \partial_x \zeta) + O(\alpha^{-2})$$

$$\Gamma = \mathrm{Re} \sum_{\mathbf{k}} -i k_y \tilde{n}_{\mathbf{k}} \tilde{\phi}^*_{\mathbf{k}}$$

$$= \sum_{\mathbf{k}} \frac{-k_y^2 \partial_x n(\gamma_{\mathbf{k}} + \alpha) + \alpha k_y \omega_{r,\mathbf{k}}}{\omega_{r,\mathbf{k}}^2 + (\gamma_{\mathbf{k}} + \alpha)^2} |\tilde{\phi}_{\mathbf{k}}|^2$$

$$= \frac{1}{\alpha} \sum_{\mathbf{k}} -\frac{k_y^2}{1+k^2} \left(k^2 \partial_x n + \partial_x \zeta\right) |\tilde{\phi}_{\mathbf{k}}|^2 + O(\alpha^{-2})$$

Comparison to QLT (diagonal term)

Compare NN result to QLT result using spectrum centered at most unstable $\mathbf{k}$ for $\partial_x \zeta = 0$

$$\varepsilon_{\mathbf{k}} = \frac{\langle \varepsilon \rangle}{4\pi \Delta k_x \Delta k_y} e^{-k_x^2/2\Delta k_x^2} \left(e^{-(k_y - \sqrt{2})^2/2\Delta k_y^2} + e^{-(k_y + \sqrt{2})^2/2\Delta k_y^2} \right)$$

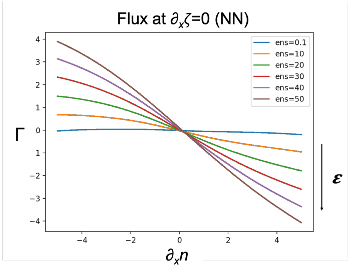

Figure Curves (at fixed $\zeta = 1$, $\partial_x \zeta = \partial_x \varepsilon = 0$, and various ε) of Γ vs density gradient from NN

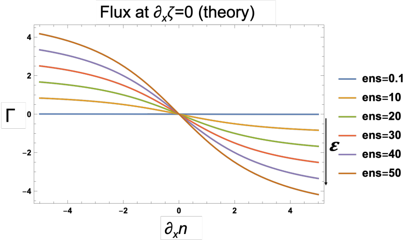

Figure Corresponding curves from QLT with $\Delta k_x = \Delta k_y = 1.5$

Comparison to QLT (off-diagonal term)

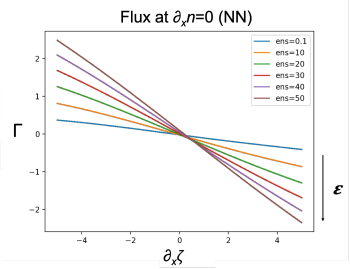

Figure Curves (at fixed $\zeta = 1$, $\partial_x n = \partial_x \varepsilon = 0$, and various ε) of Γ vs vorticity gradient from NN

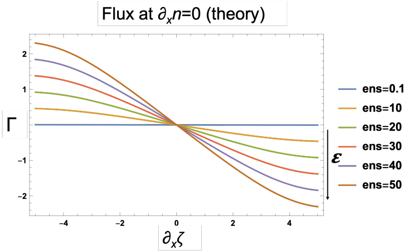

Figure Corresponding curves from QLT with $\Delta k_x = \Delta k_y = 1.5$

Discussion

Implications of off-diagonal term

- Off-diagonal often dismissed, but coupling same order of magnitude (~ 0.5) as that of usual $\partial_x n$ term. Machine picks it out very clearly!
- Consequence: ZF can induce staircase pattern on profile. If $V_y = V_0 \sin(qx)$, $\partial_x \zeta$ term will contribute

$$\partial_t \langle n \rangle \sim -\frac{k_y^2 q^3 V_0 \langle \varepsilon \rangle}{\alpha (1+k^2)^3} \cos(qx)$$

- Alternate mechanism independent of usual shear suppression, bistability. Could explain DIII-D pedestal staircase (Ashourvan et al. PRL 2019)?

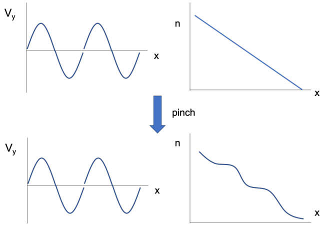

Figure Cartoon indicating how ZF can induce profile staircase via pinch

Conclusions and future work

- Machine learning technique teaches us that shear-induced off-diagonal flux is **significant** effect
- Eventually, ML method may be applicable to more complex systems that resist analytic treatment
- In progress:
  1. Develop more sophisticated ML methods, e.g. spatially nonlocal model. Goal: Reynolds stress
  2. Complete analytic study of effects of ZF frequency shift
- How important is off-diagonal flux relative to other staircasing mechanisms?

Hasegawa-Wakatani system

- Simplest realistic model for drift-wave turbulence which captures full feedback loop

$$\partial_t n + \mathbf{v}_E \cdot \nabla_\perp n = \alpha(\tilde{\phi} - \tilde{n}) + \mathrm{dissipation}$$

$$\partial_t \zeta + \mathbf{v}_E \cdot \nabla_\perp \zeta = \alpha(\tilde{\phi} - \tilde{n}) + \mathrm{dissipation}$$

with $v_E = \hat{z} \times \nabla_\perp \phi$, $\zeta = \nabla_\perp^2 \phi$ and $\alpha = \eta k_\parallel^2$ the adiabatic operator (representing parallel electron response)

- Averaging over symmetry directions ($\langle \cdots \rangle$) yields

$$\partial_t \langle n \rangle + \partial_x \Gamma = \mathrm{dissipation}$$

$$\partial_t \langle \zeta \rangle - \partial_x^2 \Pi = \mathrm{dissipation}$$

where $\Gamma = \langle \tilde{n} \tilde{v}_x \rangle$ and $\Pi = \langle \tilde{v}_x \tilde{v}_y \rangle$

- How to calculate Γ, Π?

Extra slide: error bars

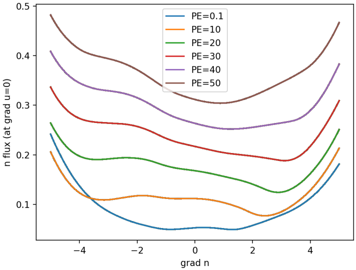

Figure Error bar estimates for NN results for $\partial_x \zeta = 0$

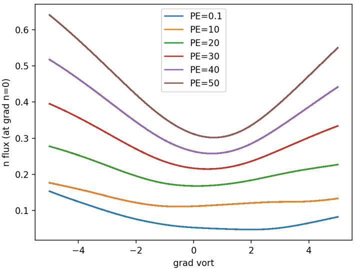

Figure Error bar estimates for NN results for $\partial_x n = 0$