Subcritical turbulence spreading and avalanche birth

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Introduction I

- In magnetic fusion plasma, turbulence driven by linear instability
- However, turbulence is still found to be present in linearly stable regions
- Explanation: **turbulence can spread**
- Basic example of nonlocality

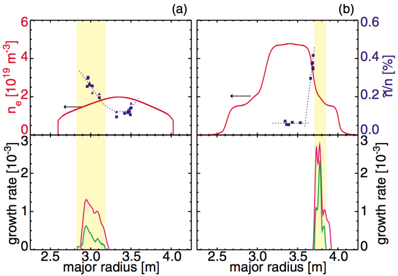


Figure: Experiment [Nazikian et al., 2005] clearly showing fluctuations in stable zone

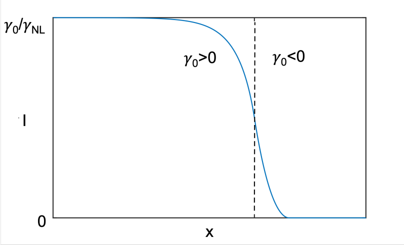
Introduction II

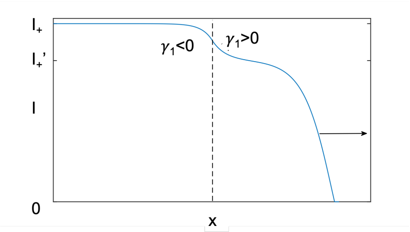
- Turbulence spreading: old news?
- Challenge the conventional wisdom on spreading (supercritical Fisher model)
- Suggest a new model based on subcritical turbulence, which testably differs from old story
- Will see that new model also serves as basic framework for avalanching

Figure: Conventional wisdom on turbulence spreading

For the impatient: preview of results

- New model accounts for robust penetration of turbulence into stable regions via **ballistic propagation**, whereas old model features weak, evanescent penetration $\ell \sim \Delta_c$
- New model features threshold for propagation of a puff of turbulence, akin to an avalanche
- Power law threshold for puff size vs. intensity





Penetration into stable zone in Fisher model (left) and new model (right)

Outline

Background: turbulence spreading

What the Fick?: turbulence spreading

- Turbulence can radially self-propagate via **nonlinear coupling**. Intensity profile gradient $\rightarrow$ intensity flux
- Can penetrate linearly stable zones
- Decouples flux-gradient relation: local turbulence intensity now depends on global properties of the profiles
- Spells doom for local Fickian transport models i.e. $Q \propto \partial_x T$

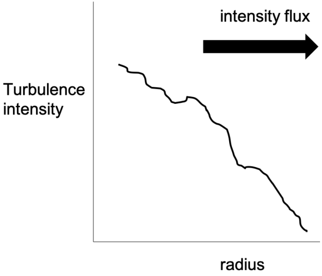


Figure: Mesoscale gradient in intensity envelope generates turbulence flux

Depiction of spreading

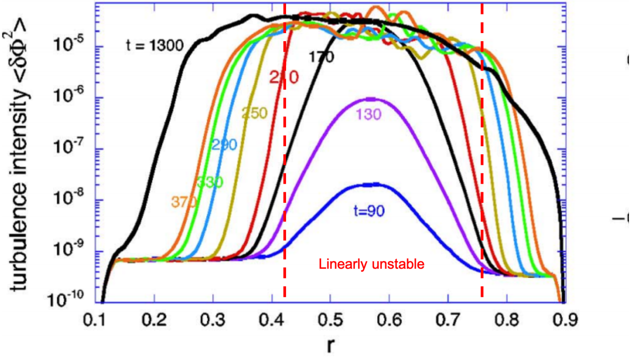


Figure: Spatiotemporal evolution of flux-surface-averaged turbulence intensity in toroidal GK simulation. Linearly unstable region is $0.42 < r < 0.76$; profiles are fixed. From [Wang et al., 2006]

Fisher model

Conventional wisdom: Fisher model

- Conventional wisdom [Gürcan and Diamond, 2005, Hahm et al., 2004, Naulin et al., 2005] for spreading is Fisher-type equation for turbulence intensity:

$$\partial_t I = \underbrace{\gamma_0 I}_{\text{local lin. growth/decay}} - \underbrace{\gamma_{nl} I^2}_{\text{local nonlin. coupling to dissipation}} + \underbrace{\partial_x (D_0 I \partial_x I)}_{\text{nonlin. diffusion of turb. energy}}$$

- When $\gamma_0 > 0$, uniform fixed points are "laminar" $I = 0$ and "saturated turbulence" $I = \gamma_0/\gamma_{nl}$
- Dynamics characterized by traveling fronts connecting roots, with speed $c = \sqrt{\frac{D_0 \gamma_0^2}{2\gamma_{nl}}}$

Depiction of Fisher evolution

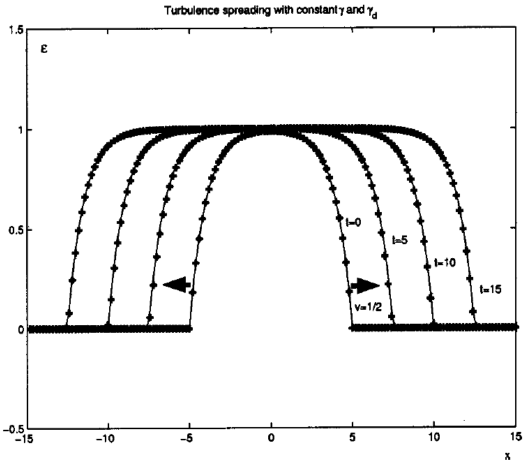


Figure: Evolution of traveling turbulence front in Fisher model. From [Gürcan and Diamond, 2006]

Penetration into stable zone: Fisher

- Consider spreading of turbulence from linearly unstable to linearly stable zone
- Simple model: $\gamma_0 > 0$ for $x < 0$, $\gamma_0 < 0$ for $x > 0$
- Allow turbulent front to form in lefthand region and propagate
- Penetration is **weak**: forms stationary, exponentially-decaying profile with $\lambda \sim \sqrt{D_0/\gamma_{nl}} \sim \Delta_c$. Puny!

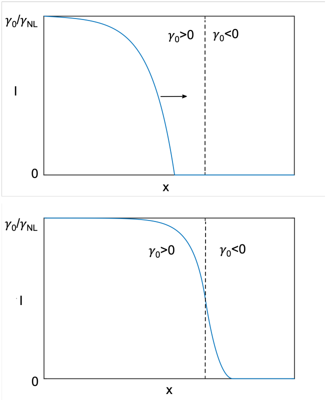


Figure: A front of turbulence crosses into stable zone and penetrates a finite depth

Does Fisher-type spreading make sense?

- No
- Fisher model purports to describe spreading of a patch of turbulence in linearly unstable zone
- Begs the question: *why didn't noise already excite the whole system to turbulence?*
- Only relevant if $\gamma_0 \ll c/\Delta x$ i.e. $\Delta x^2 \gamma_{nl} \ll D_0$
- Otherwise, physical fronts separating laminar/turbulent domains generally require *bistability* à la [Pomeau, 1986]

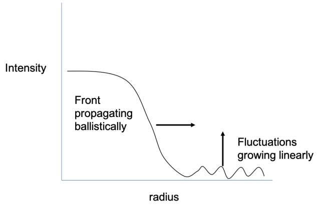


Figure: Fisher spreading only makes sense if front propagation rate beats linear growth

Bistability

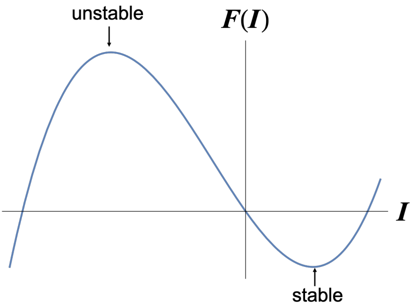


Figure: Free energy of unistable system, corresponding to Fisher

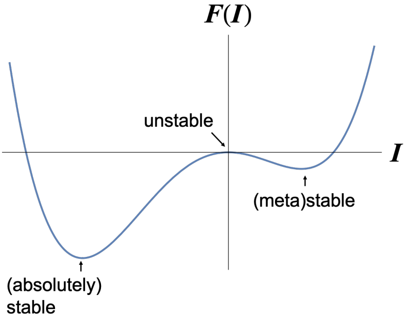


Figure: Free energy of bistable system

Bistable model

A new model is born

- Heinonen and Diamond 2019: propose phenomenological model of form

$$\partial_t I = \underbrace{\gamma_1 I}_{\text{local lin. growth/decay}} + \underbrace{\gamma_2 I^2}_{\text{nonlin. instability}} - \underbrace{\gamma_3 I^3}_{\text{nonlin. coupling to dissipation}} + \underbrace{\partial_x (D_0 I \partial_x I)}_{\text{nonlin. diffusion of turb. energy}}$$

- Simplest extension of Fisher-like model with bistability
- New physics: nonlinear turbulence drive $\propto I^2$. Can sustain sufficiently large fluctuations even when linearly damped
- *Bistable* in weak damping regime
- Estimate $\gamma_1 \sim \epsilon\omega_*$, $\gamma_{2,3} \sim \omega_*$, $D_0 \sim \chi_{GB}$ (drift-wave/Gyro-Bohm scaling)

Evidence for bistability/subcriticality

- [Inagaki et al., 2013]: experiments demonstrate hysteresis between fluctuation intensity and driving gradient (no TB present). Suggests bistable S-curve relation?
- Turbulence subcritical in presence of strong perpendicular flow shear [Barnes et al., 2011] or in the presence of magnetic shear [Drake et al., 1995]
- Profile corrugations [Guo and Diamond, 2017] and phase space structures [Lesur and Diamond, 2013] can drive nonlinear instability

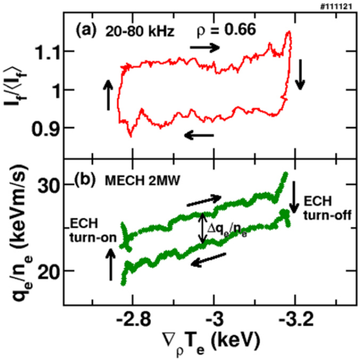


Figure: Hysteresis between intensity and gradient, flux and gradient

Bistable regime

- Qualitatively similar to Fisher EXCEPT in bistable/weak damping case
- Can then transform to Zel'dovich/Nagumo equation

$$\partial_t I = f(I) + \partial_x (D I \partial_x I)$$
$$f(I) \equiv \gamma I (I - \alpha)(1 - I)$$

- Unlike Fisher, traveling fronts admitted (even though damped)! $c \sim \sqrt{D\gamma}$ (depends on α), $\ell \sim \sqrt{D/\gamma}$

$\alpha \equiv I_- / I_+,\ \gamma \equiv I_+^2 \gamma_3,\ D \equiv I_+ D_0,\ I_\pm \equiv (\gamma_2 \pm \sqrt{\gamma_2^2 - 4|\gamma_1|\gamma_3})/2\gamma_3$

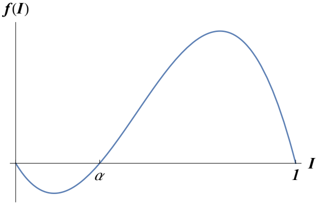
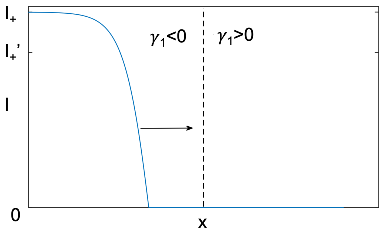
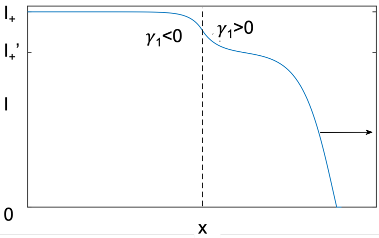


Figure: Reaction function has stable nodes at $I = 0, 1$ and unstable node at $I = \alpha$

Penetration into stable zone: new model

- Take $\gamma_1 = \gamma_g > 0$ for $x < 0$, $\gamma_1 = -\gamma_d < 0$ for $x > 0$
- In contrast to Fisher, a new front with reduced speed/amplitude forms in second region if weakly damped $(\gamma_d < \frac{15\gamma_2^2}{64\gamma_3})$
- Hence: can have **ballistic propagation into stable zone**!
- Much stronger penetration than possible in Fisher—resolves issue of feeble, evanescent penetration





Penetration into stable zone: simulation

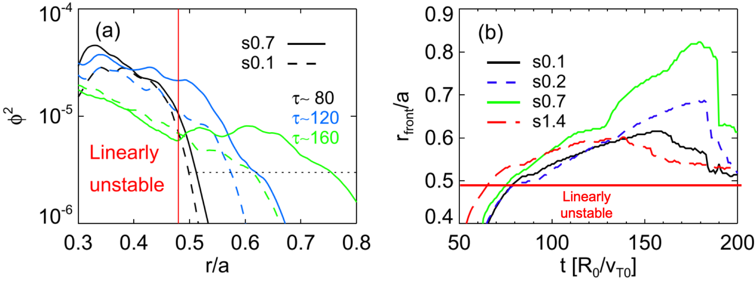


Figure: Spreading into stable zone in GK simulation with magnetic shear [Yi et al., 2014]. Evidence of ballistic propagation? More careful study needed!

Avalanche threshold

Avalanches

- Bursty, intermittent transport events associated with SOC
- Accounts for a large percentage of total flux
- Initially localized fluctuation cascades through neighboring regions via **gradient coupling**, simultaneous firing of many cells
- What does this have to do with spreading?

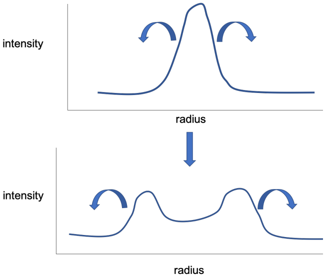


Figure: Cartoon depicting generic avalanche process via overturning of fluctuation into neighboring cells

Spreading vs. avalanching

- Fast, mesoscopic turb front propagation
- Interaction of a small scale (DW, cell) with a mesoscale (envelope, avalanche)
- Turbulence intrinsic to avalanching $\rightarrow$ drives spreading
- Unified model?

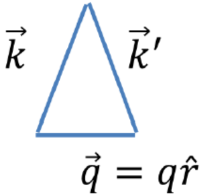


Figure: Spreading and avalanching both result from coupling of small scale k with mesoscale q ($q \ll k$)

Depiction of avalanching

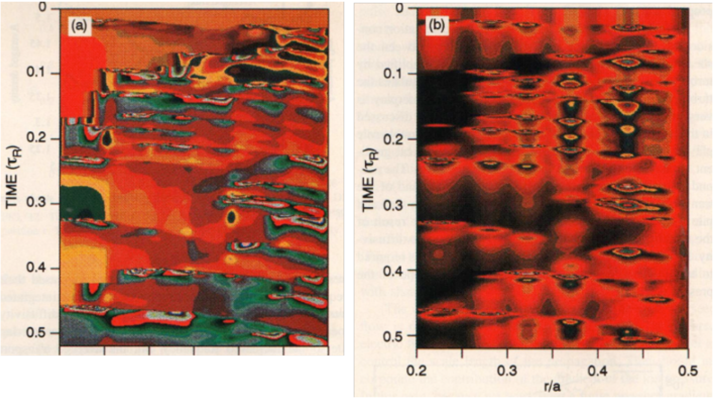
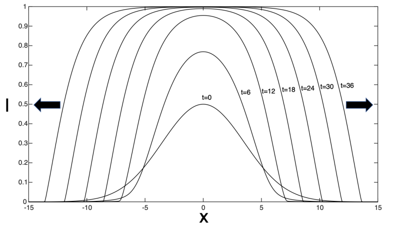
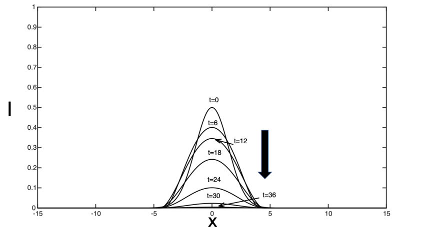


Figure: Pressure (left) and potential (right) contours for simulations of resistive drift interchange turbulence [Carreras et al., 1996]. Diagonal lines → propagating transport events

Local threshold behavior

- In contrast to Fisher, sufficiently large localized puff of turbulence will grow into front and spread. Suggestive of an avalanche triggered by sufficiently strong initial seed
- How to determine threshold?





Two puffs differing only in spatial size are initialized; one grows and spreads, other collapses

Avalanche threshold

- Obviously puff amplitude must exceed $I_0 = \alpha$ or else $\gamma_{eff} = (I - \alpha)(1 - I) < 0$
- Consider "cap" of puff (part exceeding $I = \alpha$)
- Competition between diffusion of turbulence out of cap and total nonlinear growth in cap
- Sets threshold lengthscale $\sqrt{D/\gamma}$

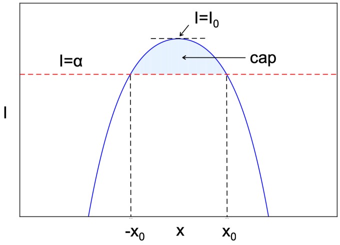


Figure: "Cap" of initial data. There is a competition between nonlinear growth and turbulence diffusion here.

Avalanche threshold II

- Analytic result: puff grows if

$$L > L_{\min} \sim (I_0 - \alpha)^{-1/2}$$

- Near linear marginality, threshold is weak:

$$I_- \sim \frac{|\gamma_1|}{\gamma_2} \ll 1, \; L_{\min} \sim \left(\frac{\chi_{GB}}{\omega_*}\right)^{1/2} \sim \Delta_c$$

- Thus, avalanche could be triggered by noise. Another possibility: corrugation

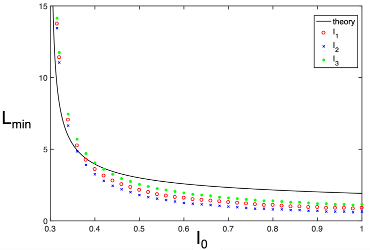


Figure: Numerical result for threshold at $\alpha = 0.3$ for three types of initial data (Gaussian (I_1), Lorentzian (I_2), parabola (I_3)), compared with analytical estimate

Conclusions

Fisher vs. new model

	Fisher	new model
Spreading possible above lin. marginal	✓	✓
Spreading possible below lin. marginal	✗	✓
Threshold behavior	✗	✓
Penetration into stable zone	evanescent	ballistic or evanescent

Bistability in the wild: testing the model

Two key tests:

- To investigate avalanches: perturb plasma locally, observe spatiotemporal response à la [Van Compernolle et al., 2015]. Need distinguish from linear mode response!
- Can we see ballistic penetration of stable region in numerical experiments? More careful study à la [Yi et al., 2014]

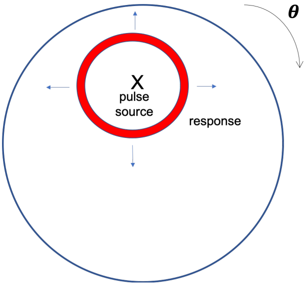
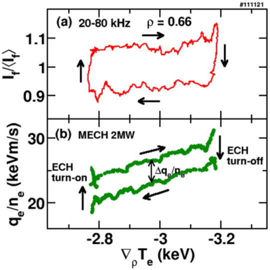


Figure: Cartoon (poloidal cross section) depicting basic setup for avalanching experiment observing response to local pulse.

Variations on a theme by Inagaki

[Inagaki et al., 2013] is interesting but not the last word. We suggest:

- More basic experiments exploring $\tilde{n}/n$ vs ∇T hysteresis
- Better resolution of dependence of fluctuation intensity on the input power
- More careful study of relaxation after ECH is turned off
- More information on fluctuation field (e.g. spatial correlations)
- Simultaneous measurement of zonal flow pattern



Spreading in context

- How does spreading affect profiles in a real system?
- Spreading will be most important when profiles force sharp ∇I
- Basic example: NML. Spreading reduces turbulence intensity, leading to increased pedestal height/width — spreading can be "good" for confinement
- More details: see Rameswar Singh's talk, NO4.2 "When does turbulence spreading matter?"

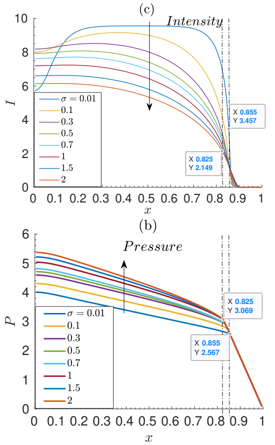


Figure: Intensity and pressure profiles; σ =spreading strength

Conclusions

- Update to Fisher model that allows for **physical** fronts separating laminar/turbulent domains and robust penetration of stable regions
- Supported by substantial evidence for subcritical turbulence
- Provides simple framework for understanding avalanching: local exceedance of nonlinear instability threshold by turbulent puffs
- Key testable predictions: ballistic spreading into weakly linearly damped regions, power-law threshold for spreading of puffs
- Need more experiments in the vein of Inagaki to study bistability

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Isn't this just quasilinear theory?!

- Quasilinear theory describes spreading of active region in phase space
- Related concept but there are key differences
- TS: active region remains **fixed**
- Real/phase space distinction important. We can compute propagation speeds
- QL spreading more similar to avalanching (gradient propagation). Realistic model should incorporate both effects

Cousin models

- Compare to bistable models for subcritical transition to fluid turbulence [Barkley et al., 2015, Pomeau, 2015].
- Compare to [Gil and Sornette, 1996] model for sandpile avalanches

$$\partial_t S = \gamma\left(|\partial_x h|/g_c - 1\right) S + \beta S^2 - S^3 + \partial_x(D_S S \partial_x S)$$
$$\partial_t h = \partial_x(D_h S \partial_x h).$$

- $S \leftrightarrow I$, $h \leftrightarrow p$
- Weak gradient coupling limit $D_h \ll D_S \Rightarrow$ our model
- Strong gradient coupling limit: S slaved to h. $\partial_x h \propto S^{-1} \Rightarrow$ linear term is $c - \gamma S$, where c is a constant which depends on BCs. Bistable again!