Supplemental Materials

Xiang Fan and P. H. Diamond
University of California at San Diego, La Jolla, California 92093

L. Chacón
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Dated: March 22, 2019)

THE UNIMODAL INITIAL CONDITION

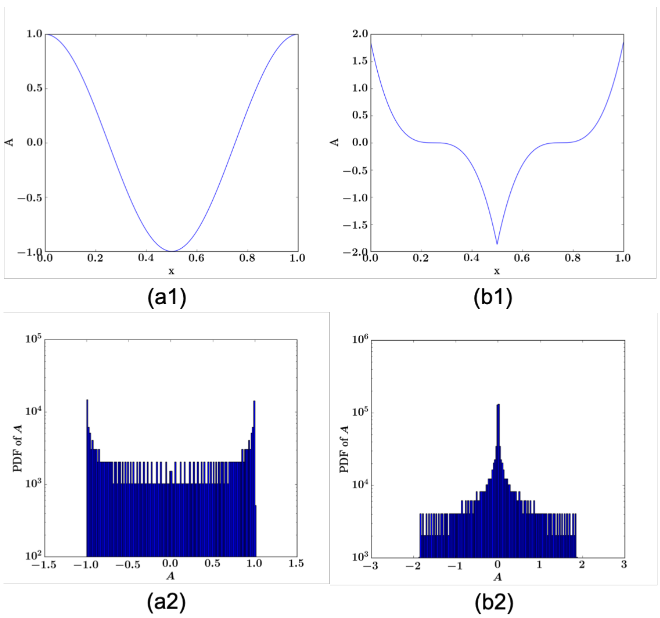

FIG. 1. The initial conditions for A and their PDFs: (a) "Bimodal" for Run1 and Run3; (b) "Unimodal" for Run2.

The unimodal initial condition used in Run2 is

$$A_I(x,y) = A_0 * \begin{cases} -(x-0.25)^3 & 0 <= x < 1/2 \\ (x-0.75)^3 & 1/2 <= x < 1 \end{cases} \tag{1}$$

The PDF of A has one peak at $A = 0$, see Fig. 1 for the comparison between bimodal and unimodal initial condition. To make Run2 and Run1 have the same time duration of the suppression stage, the initial magnitude A_0 in Run2 is chosen such that the initial $\langle A^2 \rangle$ (not E_B!) is the same with Run1.

THE CAHN-HILLIARD NAVIER-STOKES (CHNS) SYSTEM

Some binary fluid transfers from miscible phase to immiscible phase when the temperature dropped to below the corresponding critical temperature, and this second order phase transition is called spinodal decomposition.

TABLE I. The correspondence between 2D MHD and the 2D CHNS system. Reprint from [1].

	2D MHD	2D CHNS
Magnetic Potential	A	ψ
Magnetic Field	$\mathbf{B}$	$\mathbf{B}_\psi$
Current	j	j_ψ
Diffusivity	η	D
Interaction strength	$\frac{1}{\mu_0}$	ξ^2

The Cahn-Hilliard Navier-Stokes (CHNS) equations describe a binary fluid undergoing spinodal decomposition:

$$\partial_t \psi + \mathbf{v} \cdot \nabla \psi = D \nabla^2 (-\psi + \psi^3 - \xi^2 \nabla^2 \psi) \tag{2}$$

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = \frac{\xi^2}{\rho} \mathbf{B}_\psi \cdot \nabla \nabla^2 \psi + \nu \nabla^2 \omega \tag{3}$$

$$\mathbf{v} = \hat{\mathbf{z}} \times \nabla \phi, \ \omega = \nabla^2 \phi \tag{4}$$

$$\mathbf{B}_\psi = \hat{\mathbf{z}} \times \nabla \psi, \ j_\psi = \xi^2 \nabla^2 \psi \tag{5}$$

where $\psi = \frac{\rho_A - \rho_B}{\rho_A + \rho_B}$ is the local relative concentration, D is diffusivity, and ξ is a coefficient describing the strength of the surface tension interaction.

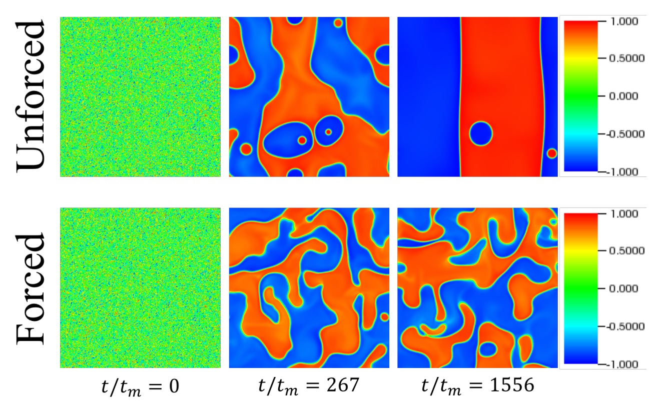

FIG. 2. Some typical screenshots for the ψ field in the 2D CHNS system. Reprint from [1].

2D CHNS and 2D MHD are both active scalar systems. The two systems are analogous, and the correspondence of the physics quantities between the two systems are summarized in Table. I. One of the most important similarities between these two systems is that, both systems support an inverse cascade of potential or, equivalently, progressive blob coalescence. The comparison and contrast of the two systems are summarized in Table. II and Table. III. See Ref. [1–3] for more details about turbulence in 2D CHNS.

[1] X. Fan, P. H. Diamond, L. Chacn, and H. Li, Physical Review Fluids **1**, 054403 (2016).
[2] X. Fan, P. H. Diamond, and L. Chacn, Physical Review E **96**, 041101 (2017).
[3] X. Fan, P. H. Diamond, and L. Chacn, Physics of Plasmas **25**, 055702 (2018).

TABLE II. Comparison of 2D MHD and the 2D CHNS system. Reprint from [1].

	2D MHD	2D CHNS
Ideal Quadratic Conserved Quantities	Conservation of E, H^A and H^C	Conservation of E, H^ψ and H^C
Role of elastic waves	Alfven wave couples $\mathbf{v}$ with $\mathbf{B}$	CHNS linear elastic wave couples $\mathbf{v}$ with $\mathbf{B}_\psi$
Origin of elasticity	Magnetic field induces elasticity	Surface tension induces elasticity
Origin of the inverse cascades	The coalescence of magnetic flux blobs	The coalescence of blobs of the same species
The inverse cascades	Inverse cascade of H^A	Inverse cascade of H^ψ
Power law of spectra	$H^A_k \sim k^{-7/3}$	$H^\psi_k \sim k^{-7/3}$

TABLE III. Contrast of 2D MHD and the 2D CHNS system. Reprint from [1].

	2D MHD	2D CHNS
Diffusion	A simple positive diffusion term	A negative, a self nonlinear, and a hyper-diffusion term
Range of potential	No restriction for range of A	$\psi \in [-1, 1]$
Interface Packing Fraction	Not far from 50%	Small
Back reaction	$\mathbf{j} \times \mathbf{B}$ force can be significant	Back reaction is apparently limited
Kinetic energy spectrum	$E^K_k \sim k^{-3/2}$	$E^K_k \sim k^{-3}$
Suggestive cascade by E^K_k	Suggestive of direct energy cascade	Suggestive of direct enstrophy cascade