Drift Wave-Zonal Flow Turbulence in Tangled Magnetic Fields

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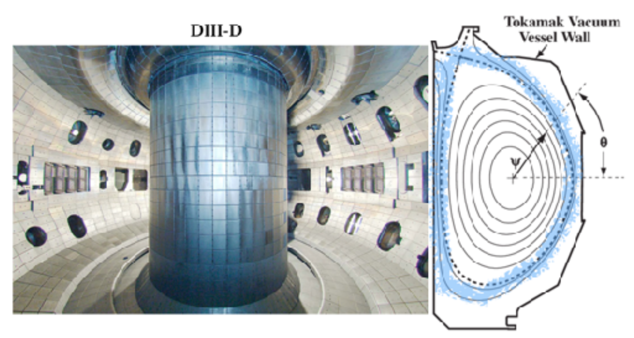
Abstract

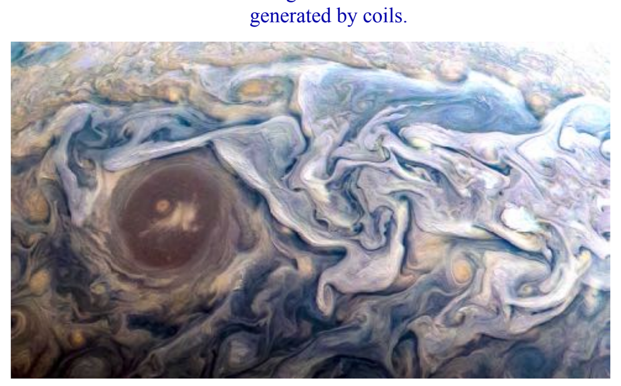
Drift wave-zonal flow turbulence frequently occurs in the presence of a tangled stochastic magnetic field. Tangled fields that coexist with an ordered mean field play a key role in edge plasmas with Resonant Magnetic Perturbations (RMP) and in turbulence in the Solar tachocline. The stochastic field forms an effective viscoelastic medium where the drift waves and zonal flows evolve. We are interested in how tangled small-scale stochastic magnetic fields (B_r) regulate the drift wave. We treat this B_r-dominated (high Kubo number) system beyond quasilinear theory by developing a 'double averaged' theory. Principal results are that the vorticity flux is modified by the cross-phase effect due to the tangled field, which leads to an suppression of zonal flow. This effect occurs at levels that field intensities below that for Alfvènization (i.e. $\langle \tilde{v}_r \tilde{v}_\theta \rangle \sim \langle \overline{B_r B_\theta} \rangle$, where Maxwell stress balances the Reynold stress).

Introduction

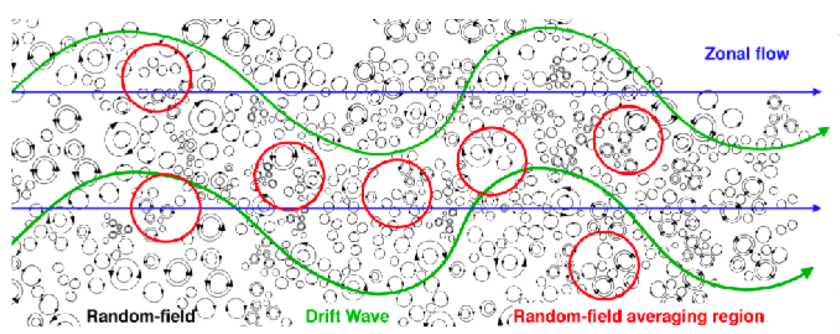
- Random or stochastic magnetic fields can be found in the interstellar medium, the Solar tachocline and the edge of tokamaks.
- The Solar tachocline is strongly stratified. If we could remove the convection zone of the Sun, we'd see the similar pattern of the tachocline as of the Jupiter atmosphere.
- We consider the 2D turbulence for this quasi-geostrophic MHD, instead of spherical shell model, for simplicity.
- We discuss a low-Kubo number problem with Quasi-Linear Theory. **Physics behind the β-plane turbulence does not just merely depend on Aflvén or Rossby state! Key parameter successfully predicts results of the simulation from Tobias et al. (2007).**
- We discuss a high-Kubo number problem with an environment consisting of stochastic fields. The Method we use is the two-averages theory.
- Results: 1. We found the cross-phase of random fields system analytically. 2. The system acts as a "resisto"-elastic medium. 3. The Alfvèn wave can still be under-damped if the mean field is slightly larger than root-mean-square of random fields.



The blue line indicates the edge of a tokamak where random fields is generated by coils.



Atmosphere of Jupiter (NASA 2019).

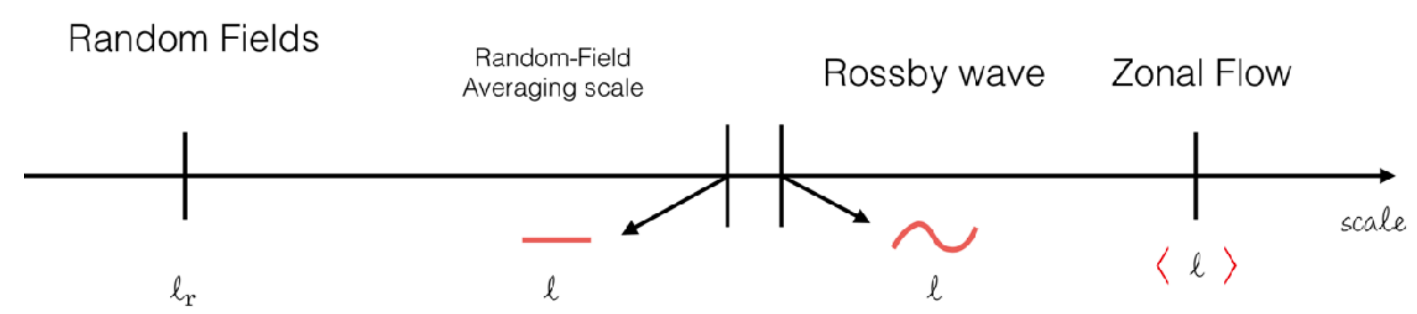


Critical Questions

- **What's the Physics of drift zonal flow in random magnetic fields in edge tokamaks?**
- **What modifies the cross-phase of the transport of mean potential vorticity, hence limits the zonal flow?**
- **How symmetry breaking by the zonal shear affect the cross-phase of stresses, especially magnetic stress?**

Our work— Zonal Flow Evolution in Stochastic Fields

(2-averages theory)



Assumptions: l: zonal scale ~ : Rossby scale r : random-field scale

$$\begin{aligned}
&\text{(potential field)} \quad \boldsymbol{A} = \boldsymbol{A_l} + \overbrace{\widetilde{\boldsymbol{A}} + \boldsymbol{A_r}}^{\textit{large scale}\rightarrow\textit{small scale}} \\
&\text{(magnetic field)} \quad \boldsymbol{B} = \boldsymbol{B_l} + \widetilde{\boldsymbol{B}} + \boldsymbol{B_r} \quad , \\
&\text{(magnetic current)} \quad \boldsymbol{J} = \boldsymbol{0} + \widetilde{\boldsymbol{J}} + \boldsymbol{J_r}
\end{aligned}$$

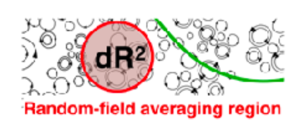
$$k_{B_r} > \bar{k} \sim k_{Rossby} > k_{Zonal}$$

$$\overline{(B_r + \frac{\delta B_r}{\delta \widetilde{B}}\widetilde{B})^2} \simeq \overline{B_r^2}$$

The collective field at Rossby – scale is NOT large enough to alter the structure of the random fields .

Two-averages method:

$$\text{(1).} \quad \bar{F} = \int dR^2 \int dB_r \cdot P_{(B_{r,x}, B_{r,y})} F$$



(average random fields)

$$\text{(2).} \quad \langle\ \rangle = \frac{1}{L}\int dx \frac{1}{T}\int dt,$$

(ensemble average over zonal flows)

$$\text{More Assumptions:} \begin{cases} \overline{B_{r,i}} = 0 & \text{averaged of random fields is zero} \\ \overline{B_{r,x}B_{r,y}} = 0 & \text{No correlation of the random fields (\underline{\textit{see discussion}}),} \\ \overline{L \cdot M} = \overline{L} \cdot \overline{M}, & \text{L, M are arbitrary function,} \end{cases}$$

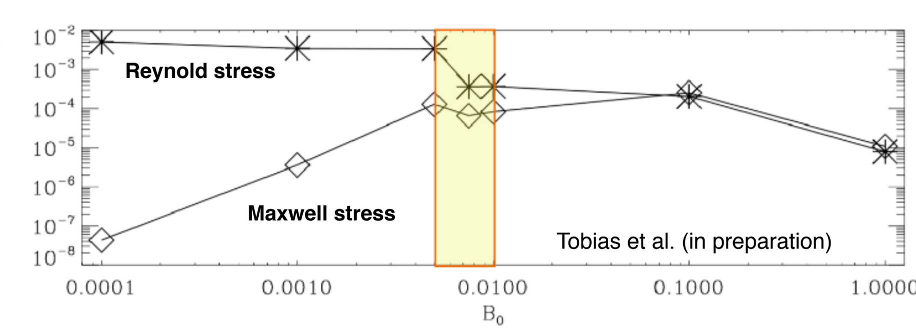
We start with two basic equations— vorticity and induction equation:

$$\text{(1).} \quad \frac{\partial}{\partial t}\bar{\zeta} - \beta\frac{\partial \bar{\psi}}{\partial x} = -\frac{\overline{(\boldsymbol{B}_r \cdot \nabla)\nabla^2 A_r}}{\mu_0 \rho} + \nu \nabla^2 \bar{\zeta} \qquad \text{(2).} \quad \frac{\partial}{\partial t}A = (\boldsymbol{B}\cdot\nabla)\psi + \eta\nabla^2 A$$

Results we have (QL expressions):

	$B_l = 0$	$B_l \neq 0$
Vorticity flux ($\overline{\Gamma_k} \equiv \widetilde{u}^*_{y,k}\widetilde{\zeta_k}$)	$\overline{\Gamma_k} = \left(\frac{i}{\omega + i\nu k^2 + \frac{i\overline{B^2_{r,j}}k_j^2}{\mu_0\rho\eta k^2}}\right)\lvert\widetilde{u}_{y,k}\rvert^2(-\frac{\partial}{\partial y}\bar{\zeta} - \beta)$	$\overline{\Gamma_k} = \left(\frac{i}{\omega + i\nu k^2 + \frac{i\overline{B^2_{r,j}}k_j^2}{\mu_0\rho\eta k^2} + \frac{i}{\mu_0\rho}\frac{B^2_{l,x}k_x^2}{\eta k^2 - i\omega}}\right)\lvert\widetilde{u}_{y,k}\rvert^2(-\frac{\partial}{\partial y}\bar{\zeta} - \beta)$
Dispersion Relation	$(\omega - \omega_R + i\nu k^2)(i\eta k^2) = \frac{\overline{B^2_{r,j}}k_j^2}{\mu_0\rho}$	$\left(\omega - \omega_R + \frac{i\overline{B^2_{r,j}}k_j^2}{\mu_0\rho\eta k^2} + i\nu k^2\right)\left(\omega + i\eta k^2\right) = \frac{B^2_{l,x}k_x^2}{\mu_0\rho}$
Evolution of Mean Flow	$\frac{\partial}{\partial t}\langle v_x\rangle = \langle\overline{\Gamma}\rangle - \boxed{\frac{1}{\eta\mu_0\rho}\langle\overline{B^2_{r,y}}\rangle\langle v_x\rangle}$	

1. The term $\boxed{-\frac{1}{\eta\mu_0\rho}\langle\overline{B^2_{r,y}}\rangle\langle v_x\rangle}$ act as a drag force. It comes from $J_r \times B_r$ force. Two effects here: **Magnetic drag** and the **cross-phase** effects.
2. Both large-and small-scale magnetic fields modify the cross-phase. ✔
3. **Cross-Phase effect** occurs at levels of the field intensities **well below** that of Alfvènization. This result matches simulation from Tobias et al. (in preparation).



How Alfvèn Waves Propagate in small-scale Stochastic Fields

We rewrite the dispersion equation, we have (turnoff Rossby wave: $\omega_R = 0$):

$$(\omega + i\alpha + i\nu k^2)(\omega + i\eta k^2) = \chi\ , \qquad \text{where } \alpha \equiv \frac{\overline{B^2_{r,j}}k_j^2}{\mu_0\rho\eta k^2}, \text{ and } \chi \equiv \frac{B^2_{l,x}k_x^2}{\mu_0\rho}.$$

$$\omega^2 + i\underbrace{(\alpha + \eta k^2)}_{\text{drag + dissipation}}\omega - \underbrace{(\alpha\eta k^2 + \chi)}_{\text{effective spring constant}} = 0$$

	$B^2_{l,x} \ll \overline{B^2_{r,j}}$	$B^2_{l,x} \geq \overline{B^2_{r,j}} \quad (\sqrt{\chi} \geq \alpha \sim \eta k^2)$
Frequency	$\omega^2_{RE} = 0,\ \omega_{IM} = -\alpha = -\eta k^2$	$\omega^2_{RE} = \chi - \frac{1}{4}\alpha^2 + \alpha\eta k^2 \geq 0,\ \ \omega_{IM} = \frac{-1}{2}(\eta k^2 + \alpha)$
Q factor	Always Overdamped	$\lambda^2 \begin{cases} > 1 & \text{Overdamped: } \frac{1}{2}(\alpha^2 + \eta^2 k^4) > \chi > \frac{1}{4}(\alpha^2 + \eta^2 k^4)\,. \\ = 1 & \text{Critical damped: } \chi = \frac{1}{2}(\alpha^2 + \eta^2 k^4) \\ < 1 & \text{Underdamped: } \chi > \frac{1}{2}(\alpha^2 + \eta^2 k^4)\,. \end{cases}$

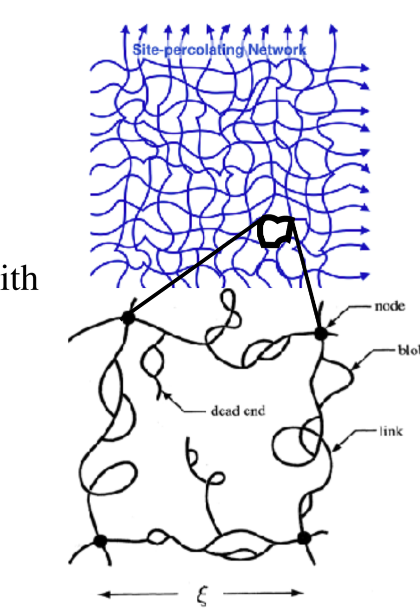
The term $\alpha\eta k^2$ can be viewed as effective spring constant. At this small scale formed by (α), system will dissipate energy via drag and resistivity.

→ energy forward cascade toward small scales! ✔

→ a **resisto-elastic** MHD medium!

Discussion

- **How to calculate the nonzero cross phase $\langle \overline{B_{r,x}B_{r,y}} \rangle$?**
  We are interested in Maxwell stress ➔ **Symmetry breaking** by zonal shear! Shear will induce the correlation even $B_{r,x}$ **and** $B_{r,y}$ **are initially uncorrelated**!
  Starting with $\frac{D}{Dt}k_y = -\frac{\partial}{\partial y}(k_y\langle v_x\rangle)$, and modify cross phase $\langle k_x k_y\rangle$ with $k_y = k_y^{(0)} - k_x\frac{\partial\langle V_x\rangle}{\partial y}\tau_c$.
- Next: Consider weak field where QL approximation fails. We'll recalculate the cross phase in $\langle \widetilde{v}_x\widetilde{v}_y\rangle$ and $\langle \overline{B_{r,y}B_{r,x}}\rangle$.
- **Fractal Network** (Site-percolating):
  Effective spring constant, effective Young's Modulus of elasticity, and effective "conductivity" of vorticity (such as encountered in amorphous solids).



Schematic of the nodes-links-blobs model (Nakayama & Yakubo 1994).

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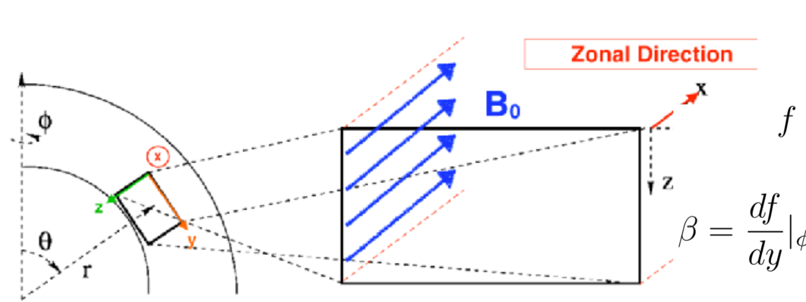
Basic Definitions

1. β-Plane Approximation

Consider a solid sphere– a planet, which is covered by a thin atmosphere. The sphere is rotating in a constant angular velocity. At latitude ϕ_0 the velocity is at the surface is v, and thus the **Coriolis Force** is $2\Omega \times$ v.

$$(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla)\mathbf{v} + 2\Omega \times \mathbf{v} = -\nabla P + \mathbf{F}$$



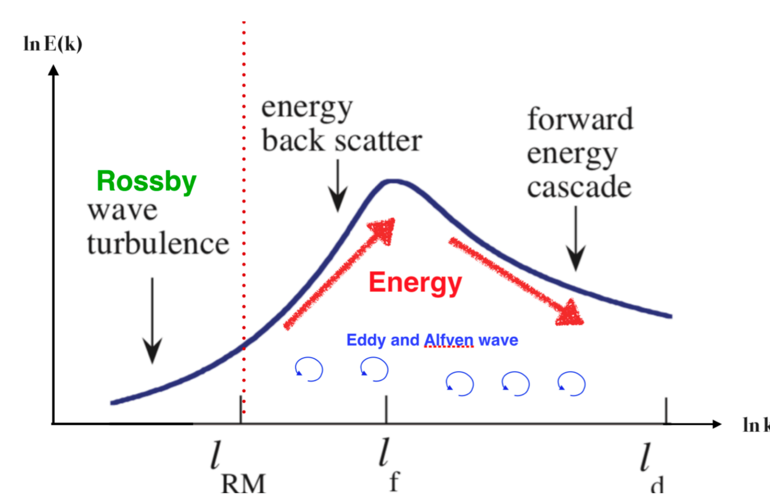
$$f = f_0 + \beta y$$

$$\beta = \frac{df}{dy}|_{\phi_0} = 2\Omega cos(\phi_0)/a$$

β: is the **Rossby parameter**
y: is a meridional distance Ω is angular rotation rate of the planet

ϕ_0 : latitude raising from the equator

2. Rhines scale in 2D MHD



It's widely accepted that the Zel'dovich Theorem for 2D MHD is applicable to β-plane MHD:

$$\frac{\langle B_r^2\rangle}{B_0^2} = \frac{\eta_T}{\eta}, \quad \text{Zel'dovich Theorem}$$

$$l_{RM} = \sqrt{\frac{V_{r,A}}{\beta}}, \quad \text{MHD Rhines scale}$$

3. Kubo Number

Kubo number: $Ku \equiv \frac{\tau_{ac}}{\tau_{turn\ over}} = \frac{l_\parallel |B_r|}{l_\perp |B_l|}$, where $l_\parallel$ is auto-correlation length, which his parallel to the large-scale magnetic field B_l.

$$\text{When } Ku = \begin{cases} Ku > 1, \quad B_0 \gg |B_r| & \text{Quasi-Linear theory fails} \\ Ku < 1, \quad |B_r| \gg B_0 & \text{Quasi-Linear theory is valid} \end{cases}$$