Learning mean field dynamics of drift-wave turbulence

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Introduction

- Drift-wave plasma features rich nonlinear (and nonlocal) phenomena
- Of particular interest are zonal flows, turbulence spreading, and subcritical turbulence, each of which have profound implications for turbulent transport
- Problem: how can we realistically model these phenomena? How do they relate to each other and interact?
- This poster presents research on this problem in two parts:
    1. A recently published model for subcritical turbulence spreading
    2. Progress on a new technique that uses machine learning to infer features of the full mean-field dynamics from simulation

Background: turbulence spreading

- Radial propagation of turbulence, characterized by traveling fronts and pulses in turbulence intensity
- Can be thought of as driven by nonlinear scattering of ballooning modes. In an envelope approximation, this yields a term $\partial_t I \sim \partial_x I \partial_x I$ for turbulence intensity I
- Leads to nonlocality: the turbulent flux cannot be described by a local Fick's law since the turbulence intensity is influenced by dynamics occurring at other spatiotemporal locations

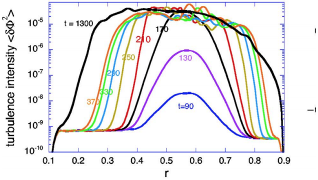

Figure Spatiotemporal evolution of turbulence field from simulation

Background: subcritical turbulence I

- Defined by self-sustaining turbulence *below* the threshold for linear instability
- Differs strongly from the conventional picture that plasma turbulence is strongly suppressed below the critical gradient
- Characterized by coexistence of laminar and turbulent domains. Generic model is *bistable*:

$$\partial_t I \simeq -aI + bI^2 - cI^3$$

as opposed to supercritical case with positive linear coefficient. Note nonlinear instability term ($\propto I^2$)

Background: subcritical turbulence II

- Long known to exist in fluid flows. Increasingly acknowledged to exist in confined plasma, e.g. in presence of magnetic shear [Biskamp and Walter, 1985] or strong perpendicular shear flow [Barnes et al., 2011]
- Experiments [Inagaki et al., 2013] also hint at bistability by demonstrating hysteresis between the fluctuation intensity and the gradient, in the absence of a transport barrier

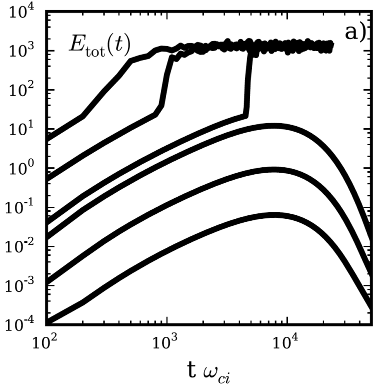

Figure Turbulence energy self-sustains beyond a threshold level, in a subcritical, modified version of 3D HW model [Friedman and Carter, 2015]

Key questions

- Nonlinear diffusion model of spreading really describes the *total* turbulence energy, including ZFs. Not necessarily a good model for the spreading of the fluctuations alone. How to improve?
- ZFs can be expected to spread with fluctuations but to themselves suppress the fluctuations/spreading. How to describe this interplay?
- What determines the saturated ZF profile?
- Physics of subcritical turbulence? When do we expect it, and how does it affect spreading?
- Seek answers with the aid of reduced models

Bistable turbulence spreading: Fisher model

- As a first step, we study turbulence spreading in a subcritical/bistable system
- Simplest, most common model for spreading is supercritical (Fisher equation):

$$\partial_t I = \underbrace{\gamma_0 I}_{\substack{\text{local lin.}\\ \text{growth/decay}}} - \underbrace{\gamma_{nl} I^2}_{\substack{\text{local nonlin.}\\ \text{coupling to}\\ \text{dissipation}}} + \underbrace{\partial_x (D_0 I \partial_x I)}_{\text{nonlin. diffusion of turb. energy}}$$

- If system unstable ($\gamma_0 > 0$), turbulence propagates via fronts connecting laminar and saturated fixed points $I = 0, \gamma_0/\gamma_{nl}$

Bistable turbulence spreading: problems with Fisher

- Fisher doesn't make a whole lot of sense: why do we need spreading if the system is unstable in the first place?
- Also, penetration of turbulence into stable zone is feeble, with a depth on the order of just a few ρ_i. Dubiously consistent with clear experimental observation of fluctuations in stable zones
- Resolution: use subcritical model, which allows for coexistence of laminar and turbulent domains

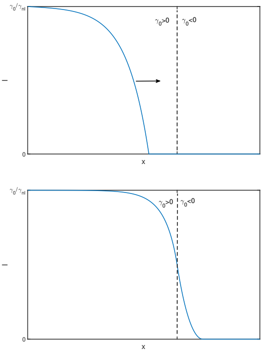

Figure Evanescent penetration of Fisher front into stable zone

Bistable turbulence spreading: new model

- We propose [Heinonen and Diamond, 2019] a new model:

$$\partial_t I = \gamma_1 I + \gamma_2 I^2 - \gamma_3 I^3 + \partial_x (D_0 I \partial_x I) \qquad (*)$$

- Roughly anticipate $\gamma_i \sim \omega_*$, $D_0 \sim \chi_{GB} \sim c_s \rho_i^2 / a$
- Motivation: simplest, generic 1D model with subcritical bifurcation. Other forms possible, but qualitative features should be the same!
- Similar to [Barkley et al., 2015, Pomeau, 2015] models for onset of turbulence in pipe flow
- Supported by aforementioned observations of subcritical turbulence

Bistable turbulence spreading: key predictions

- Supports traveling turbulence waves in (weakly) subcritical regime, unlike Fisher. Speed $\sim \sqrt{D\gamma}$ (coeff. depends on γ_i's)
- Turbulence can then *strongly* penetrate subcritical regions via ballistic propagation
- As a bonus, this model also predicts avalanche-like behavior, due to local threshold behavior: an initially localized seed of turbulence can propagate if it exceeds threshold

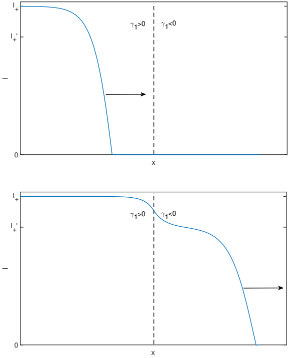

Figure A wave develops in the unstable zone, penetrates into the bistable zone, and forms a new traveling wave with reduced speed and turbulence level.

Bistable turbulence spreading: avalanche threshold I

- First transform to Zeldovich-Frank-Kamenetsky form

  $$\partial_t I = \gamma I(1 - I)(I - \alpha) + \partial_x (D I \partial_x I)$$

- If puff is to spread, intensity must exceed $I = \alpha$ somewhere, otherwise effective linear growth $\gamma_{eff} = (I - \alpha)(1 - I)$ is negative
- How "wide" must the puff be?

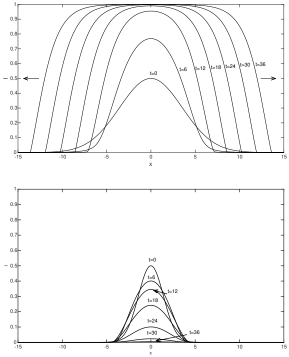

Figure A puff will either grow into a wave (above) or collapse (below) depending on its width

Bistable turbulence spreading: avalanche threshold II

- Can estimate by assuming initial growth of turbulent mass in "cap" (part $> \alpha$) of slug governs asymptotic spreading
- Threshold then determined by competition between outgoing diffusive flux from cap and local growth in cap
- This competition suggested by form of free energy functional
- Leads to power law $L_{min} \sim (I_0 - \alpha)^{-1/2}$. Excellent agreement with simulation of PDE

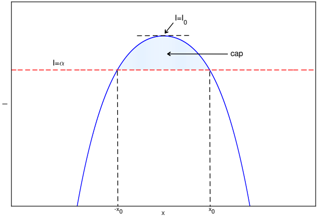

Figure Illustration of slug's "cap"

Bistable turbulence spreading: avalanche threshold III

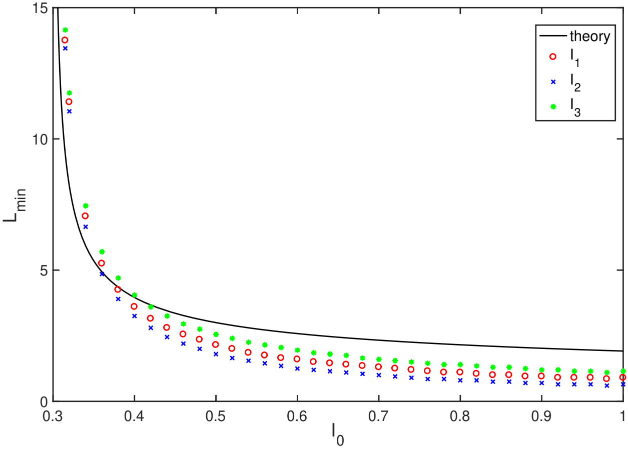

Figure Numerical result for threshold at $\alpha = 0.3$ for three types of initial condition (Gaussian (I_1), Lorentzian (I_2), parabola (I_3)), compared with analytical estimate

Bistable turbulence spreading: avalanche threshold IV

- So: an initially localized turbulent slug with amplitude exceeding $I_- = \alpha$ and spatial extent exceeding L_{min} will spread and excite the system to turbulence
- Near marginal linear stability, threshold is "small":

$$I_- \sim \frac{|\gamma_1|}{\gamma_2} \ll 1,\ L_{\min} \sim \left(\frac{\chi_{GB}}{\omega_*}\right)^{1/2} \sim \rho_i$$

- Thus noise (e.g. background sub-ion-scale turbulence) can *intermittently* excite turbulence pulses. This can be thought of as a simple model for avalanches

Conclusions for bistable model

- Updating the unistable Fisher model to a bistable model simultaneously resolves several issues
  1. Properly allows for coexistence of laminar and turbulent domains so that fronts are physical
  2. Reflects the emerging understanding that MF turbulence is subcritically unstable, at least in certain scenarios
  3. Allows for stronger penetration into stable zone via ballistic spreading
- Also functions as a basic model for avalanching by local excitation
- But:
  1. This model is still *dumb*: realistic model needs to properly treat coupling to zonal flow and profiles, also nonlinear diffusion is a dubious model for spreading term
  2. Can we find a firm justification for the subcritical nonlinearity?

Mean field dynamics I

- Let's try to properly attack the problem, with the density profile, zonal flow field, and turbulence field treated on an equal footing
- Start with Hasegawa-Wakatani (simplest model for drift-wave turbulence):

$$\partial_t n + \{\phi, n\} = \alpha(n - \phi) + \mathrm{dissipation}$$

$$\partial_t \zeta + \{\phi, \zeta\} = \alpha(n - \phi) + \mathrm{dissipation}$$

with $\zeta = \nabla_\perp^2 \phi$ and $\alpha = \eta \partial_\parallel^2$ the adiabatic operator (representing parallel electron response)

Mean field dynamics II

- Averaging over symmetry directions ($\langle \cdots \rangle$) yields

$$\partial_t \langle n \rangle + \partial_x \langle \tilde{n} \tilde{v}_x \rangle = \text{diss.}$$

$$\partial_t \langle \zeta \rangle + \partial_x \langle \tilde{\zeta} \tilde{v}_x \rangle = \text{diss.}$$

$$\partial_t \langle \varepsilon \rangle + \langle (\tilde{n} - \tilde{\zeta}) \tilde{v}_x \rangle \partial_x \langle n - \zeta \rangle + \partial_x \langle \varepsilon \tilde{v}_x \rangle = \text{diss.}$$

- Here $\varepsilon = (\tilde{n} - \tilde{\zeta})^2$ is the turbulent potential enstrophy, a proxy for the turbulence intensity
- Thus the problem is one of modeling the turbulent fluxes $\Gamma_q = \langle \tilde{q} \tilde{v}_x \rangle$ $(q = n, \zeta, \varepsilon)$. Can we write down a mean field theory where Γ_q is a function of local $\langle n \rangle, \langle \zeta \rangle, \langle \varepsilon \rangle$ and derivatives?

Mean field dynamics III

- To proceed, Ashourvan and Diamond (2016) used quasilinear theory with a mixing-length approximation (dropping $\langle \cdots \rangle$ henceforth):

$$\Gamma_n = -D_n \partial_x n$$
$$\Gamma_\zeta = (\chi - D_n)\partial_x n - \chi \partial_x \zeta$$
$$\Gamma_\varepsilon = -D_\varepsilon \partial_x \varepsilon$$

  with

$$D_n \propto \ell^2 \varepsilon,\ \chi \propto \ell^2 \varepsilon / (\alpha + \zeta^2)^{1/2},\ D_\varepsilon \propto \ell^2 \varepsilon^{1/2}$$

- Mixing-length ansatz à la Balmforth, Llewellyn Smith, and Young (1998)

$$\ell = \frac{\ell_0}{(1 + \ell_0^2 [\partial_x (n - \zeta)]^2 / \varepsilon)^{\kappa/2}}$$

  models inhomogeneous mixing of potential vorticity $n - \zeta$

Beyond Ashourvan and Diamond: machine learning

- How to go beyond this model, which is somewhat ad hoc? Can we use a similar model to capture nonlinear instability/subcritical turbulence?
- The latter likely requires going beyond QLT. Very difficult to treat analytically
- Instead, we suggest gleaning the model from full simulation of dynamical equations (here, HW), using machine learning as a form of *nonlinear, model-free regression*
- Can exploit ML techniques to avoid overfitting and local minima

Constraining the model with symmetry

- Formally, we seek maps which send local profiles to local fluxes, i.e. $f_q : (n, n_x, n_{xx}, \ldots, \phi, \phi_x, \zeta, \zeta_x, \ldots, \varepsilon, \varepsilon_x, \ldots) \mapsto \Gamma_q$
- Symmetries of HW constrain the problem considerably:
  1. Invariance under uniform shifts $n \to n + n_0$ and $\phi \to \phi + \phi_0$ eliminate dependence on n, ϕ
  2. Invariance under boosts in y

     $$\begin{cases} \phi & \to \phi + v_0 x \\ y & \to y - v_0 t \end{cases}$$

     eliminates dependence on ZF speed ϕ_x
  3. Also have reflection symmetries $x \to -x, y \to -y$ and $\phi \to -\phi, n \to -n, x \to -x$ which require, roughly speaking, that $\Gamma_q \to -\Gamma_q$ under $\partial_x q \to -\partial_x q$

Initial results I

- As proof of concept, start with simpler 2D HW equations (α is now a constant). To study subcritical turbulence, eventually need to go to 3D!
- Simple NN trained on simulations with varying initial profiles. Data averaged over mesoscopic windows in x to achieve reasonable statistics
- Reflection symmetries enforced by duplicating data with appropriate signs flipped
- Curved density profiles chosen to sample many gradients and allow turbulence to invade linearly stable regions

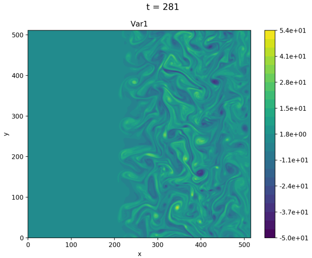

Figure Snapshot of vorticity field in typical simulation, which invades leftward into region of low density gradient

Initial results II

Early results show that Γ_n and Γ_ζ couple to $\partial_x n$ in a way qualitatively consistent with Ashourvan-Diamond, including saturation of flux at large gradient

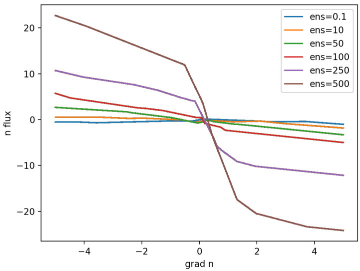

Figure Curves (at fixed $\zeta = 1$, $\zeta_x = \varepsilon_x = 0$, and various ε) of density flux vs density gradient

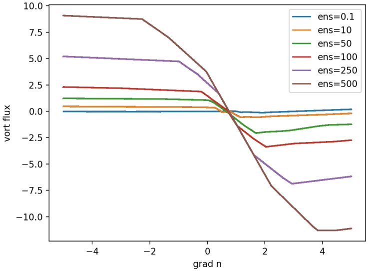

Figure Curves (at fixed $\zeta = 1$, $\zeta_x = \varepsilon_x = 0$, and various ε) of density flux vs density gradient

Conclusions from initial results and further work

- Reflection symmetry not exactly enforced: how to improve?
- Coupling of fluxes to vorticity profile not clear from simulation. Likely because turbulence-driven shear flow here not strong enough (say to drive KH turbulence). Requires further investigation
- More work required to understand enstrophy flux. Highly intermittent, largely carried by point vortices and their interactions! Simple gradient model doesn't appear to work: can we construct a model for spreading based on mutual vortex induction?!
- Beyond mean field theory: fluctuations from mean fluxes appear to scale with enstrophy (unsurprisingly). Suggests addition of multiplicative noise to model. How does this affect dynamics?

For the experimentalist/simulation practitioner

- Bistable model suggests several experiments
  1. Variations on a theme by Inagaki. Better resolution of dependence of fluctuation intensity on the input power? More careful study of relaxation after ECH is turned off? More information on fluctuation field (e.g. spatial correlations)? Simultaneous measurement of zonal flow pattern?
  2. To investigate avalanches: perturb plasma locally, observe spatiotemporal response à la [Van Compernolle et al., 2015]. Compare near-marginal, far above marginal to rule out possibility of linear mode response
  3. Can we see ballistic penetration of stable region in numerical experiments?
- Reduced models require tuning. The ML technique represents a possible route to improved reduced models (say, beyond QLT), by letting the machine do the tuning. There has been some recent work on this, e.g. [Citrin et al., 2015]

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Avalanching 101

- Observed in MFE plasma [Politzer, 2000]
- Basic picture: a sufficiently large, localized increase in the turbulence level radially cascades into neighboring regions, ultimately causing a sudden burst of transport
- Closely related to turbulence spreading: avalanching and (subcritical) spreading essentially two ways of looking at same phenomenon
- Associated with self-organized criticality (occurs near marginal, $1/f$ spectra)
- Intermittent (long tails)

Bistable case: reduction to FitzHugh-Nagumo

- $(*)$ is bistable for weak damping $\gamma_1 < 0$ and $\gamma_2^2 > 4|\gamma_1|\gamma_3$
- Roots: $I = 0$, $I_\pm = (\gamma_2 \pm \sqrt{\gamma_2^2 - 4|\gamma_1|\gamma_3})/2\gamma_3$. $0, I_+$ stable (note: nonzero for marginal γ_1), I_- unstable
- If $\gamma_1 < 0$ and γ_2 sufficiently large, can be written

$$\partial_t I = f(I) + \partial_x(D(I)\partial_x I)$$

with $f(I) = \gamma I(I - \alpha)(1 - I)$ by defining

$$|\gamma_3|{I_+}^2 \to \gamma,\ \frac{I_-}{I_+} \to \alpha,\ I_+ D_0 \to D$$

- This is a version of the Nagumo equation, a simplification of the FitzHugh-Nagumo model for excitable media [FitzHugh, 1961, Nagumo et al., 1962]

Lengthscale threshold (details)

- Strategy: assume initial slug is even, has single max at I_0 and single lengthscale L
- Expand intensity curve about max to quadratic order, plug into dynamical equation, integrate over extent of cap
- Result: growth if

$$L > L_{\min} = \sqrt{\frac{\lambda D(\alpha) I_0}{f(I_0) - \frac{1}{3}(I_0 - \alpha) f'(I_0)}} = \sqrt{\frac{3\lambda D \alpha I_0}{\gamma (I_0 - \alpha)((1 - 2\alpha) I_0 + \alpha)}}$$

$E \times B$ staircase

- $E \times B$ staircase: quasiperiodic shear flow pattern observed in GK simulation [Waltz et al., 2006]
- [Guo and Diamond, 2017] showed that in mean field approx., result is additional nonlinear drive term, equation of the type $(*) \to$ global bistability
- Basic physics: inhomogeneous turbulence mixing. Shear suppression of turb. heat flux $\to$ effective negative turbulent heat diffusion $\to$ temperature corrugations $\to$ critical gradient locally exceeded $\to$ turbulence growth $\to$ further profile roughening

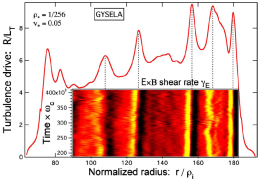

Figure Profile corrugations correlate with $E \times B$ shear (from [Dif-Pradalier et al., 2010])

- For cold ions, described by Hasegawa-Wakatani model:

$$\partial_t n + \{\phi, n\} = \alpha(n - \phi) + \mathrm{diss.}$$

$$\partial_t \nabla_\perp^2 \phi + \{\phi, \nabla_\perp^2 \phi\} = \alpha(n - \phi) + \mathrm{diss.}$$

with $\alpha = \eta \partial_\parallel^2$ the adiabatic operator representing parallel electron response

Zonal flows

- Poloidal, sheared $E \times B$ flows with $n = 0$ and $\omega = 0$
- Robust and ubiquitous, driven by modulational instability on packet of drift waves
- Shear drift wave eddies, leading to a reduction in turbulence transport. ZF formation widely believed to be involved in the L-H transition

Figure Obligatory picture of Jupiter's bands, a classic example of ZF