Revisiting the Suppression of Turbulent Diffusion in 2D MHD: Quenching Occurs in Intermittent Transport Barriers

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Sherwood 2019

This research was supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, under Award Number DE-FG02-04ER54738 and CMTFO.

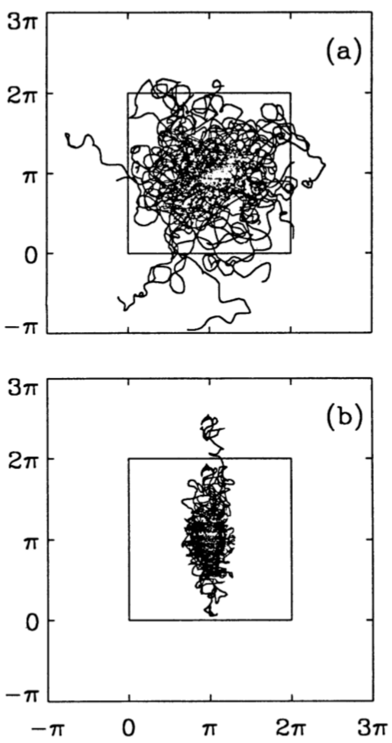
Overview

- This work studies the suppression of turbulent transport in 2D MHD.
- Conventional wisdoms:
  - [Cattaneo and Vainshtein 1991]. Physics underpinning?
  - [Gruzinov and Diamond 1994, 1996] and [Diamond, Hughes, and Kim 2005]. Imposed external B_0.
- What's new:
  - The B field is highly intermittent.
  - Spontaneous formation of transport barriers.
  - Quench is not uniform.
- η_T expression when $\langle B \rangle$ absent. $\langle B \rangle$ is significant in the barriers, but $\nabla^2 \langle A^2 \rangle$ is what is left in the blobs.
- Barrier formation: negative diffusion $(\langle v^2 \rangle - \langle B^2 \rangle)$.
- Analogy with staircase.

Introduction

- Virtually all models of drift-Alfven, EM ITG, etc. turbulence are based upon a vorticity equation, Ohm's Law and (usually multiple) scalar advection equations. The appearance of the Alfven wave introduces a crucial element of *memory* to the dynamics. Such Alfvenization-induced-memory can significantly impact structure formation and transport in turbulence.
- 2D MHD is the simplest model with these features.
- Kinematic expectation (passive scalar): $\eta_K \sim ul$
- Actual result: turbulent transport is suppressed $\eta_T < \eta_K$
- Note: 2D quench problem lead to Rm-dependent α quenching, nonlinear $\langle B \rangle$ feedback in dynamo.

Conventional Wisdom (1)



- [Cattaneo and Vainshtein 1991]: turbulent transport is suppressed even when a weak large scale magnetic field is present.
- Starting point: $\partial_t \langle A^2 \rangle = -2\eta \langle B^2 \rangle$
- Assumptions:
  - Energy equipartition: $\frac{1}{\mu_0 \rho} \langle B^2 \rangle \sim \langle v^2 \rangle$
  - Average B can be estimated by: $|\langle \mathbf{B} \rangle| \sim \sqrt{\langle A^2 \rangle}/L_0$
- Define Mach number as: $M^2 \equiv \langle v^2 \rangle / v_A^2 = \langle v^2 \rangle / (\frac{1}{\mu_0 \rho} \langle \mathbf{B} \rangle^2)$
- Result for suppression stage: $\eta_T \sim \eta M^2$
- Combine with kinematic stage result: $\eta_T \sim \frac{ul}{1 + \mathrm{Rm}/M^2}$
- Lack physics interpretation of the origin of η_T.

Conventional Wisdom (2)

- [Gruzinov and Diamond 1994, 1996] and [Diamond, Hughes, and Kim 2005] derived η_T from dynamics.
- With an external imposed B_0 (i.e. $\frac{\partial\langle A\rangle}{\partial x}$).
- The key of this approach is to calculate the flux $\Gamma_A \equiv \langle v_x A\rangle$
- Standard closure methods yield:

$$\begin{aligned}\Gamma_A &= \sum_{\mathbf{k}}[v_x(-\mathbf{k})\delta A(\mathbf{k}) - B_x(-\mathbf{k})\delta\phi(\mathbf{k})] \\ &= -\sum_{\mathbf{k}}[\tau_c^{\phi}(\mathbf{k})\langle v^2\rangle_{\mathbf{k}} - \frac{1}{\mu_0\rho}\tau_c^{A}(\mathbf{k})\langle B^2\rangle_{\mathbf{k}}]\frac{\partial\langle A\rangle}{\partial x} \\ &= -\sum_{\mathbf{k}}\tau_c[\langle v^2\rangle_{\mathbf{k}} - \frac{1}{\mu_0\rho}\langle B^2\rangle_{\mathbf{k}}]\frac{\partial\langle A\rangle}{\partial x}\end{aligned}$$

- Therefore: $\Gamma_A = -\eta_T\frac{\partial\langle A\rangle}{\partial x}$ with $\eta_T = \sum_{\mathbf{k}}\tau_c[\langle v^2\rangle_{\mathbf{k}} - \frac{1}{\mu_0\rho}\langle B^2\rangle_{\mathbf{k}}]$

Conventional Wisdom (2) Cont'd

- Then calculate $\langle B^2 \rangle$ in terms of $\langle v^2 \rangle$. From:

$$\partial_t A + \mathbf{v} \cdot \nabla A = -v_x \frac{\partial \langle A \rangle}{\partial x} + \eta \nabla^2 A$$

- Multiplying by A and sum over all modes:

$$\frac{1}{2}[\partial_t \langle A^2 \rangle + \langle \nabla \cdot \langle \mathbf{v} A^2 \rangle \rangle] = -\Gamma_A \frac{\partial \langle A \rangle}{\partial x} - \eta \langle B^2 \rangle$$

Dropped stationary case

Dropped periodic boundary

- Therefore: $$\langle B^2 \rangle = -\frac{\Gamma_A}{\eta} \frac{\partial \langle A \rangle}{\partial x} = \frac{\eta_T}{\eta} B_0^2$$
- Define Mach number as: $$M^2 \equiv \langle v^2 \rangle / v_{A0}^2 = \langle v^2 \rangle / (\frac{1}{\mu_0 \rho} B_0^2)$$
- Result: $$\eta_T = \frac{\sum_{\mathbf{k}} \tau_c \langle v^2 \rangle_{\mathbf{k}}}{1 + \mathrm{Rm}/M^2} = \frac{ul}{1 + \mathrm{Rm}/M^2}$$
- This theory is not able to describe the system with no B_0, though can be extended.

Simulation Setup

- PIXIE2D: a DNS code solving 2D MHD equations in real space:

$$\partial_t A + \mathbf{v} \cdot \nabla A = \eta \nabla^2 A$$
$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = \frac{1}{\mu_0 \rho} \mathbf{B} \cdot \nabla \nabla^2 A + \nu \nabla^2 \omega + f$$

- 1024^2 resolution.
- External forcing f is isotropic homogeneous.
- Periodic boundary condition.
- Initial conditions:
  - (1) bimodal: $A_I(x, y) = A_0 \cos 2\pi x$
  - (2) unimodal: $A_I(x, y) = A_0 * \begin{cases} -(x - 0.25)^3 & 0 <= x < 1/2 \\ (x - 0.75)^3 & 1/2 <= x < 1 \end{cases}$

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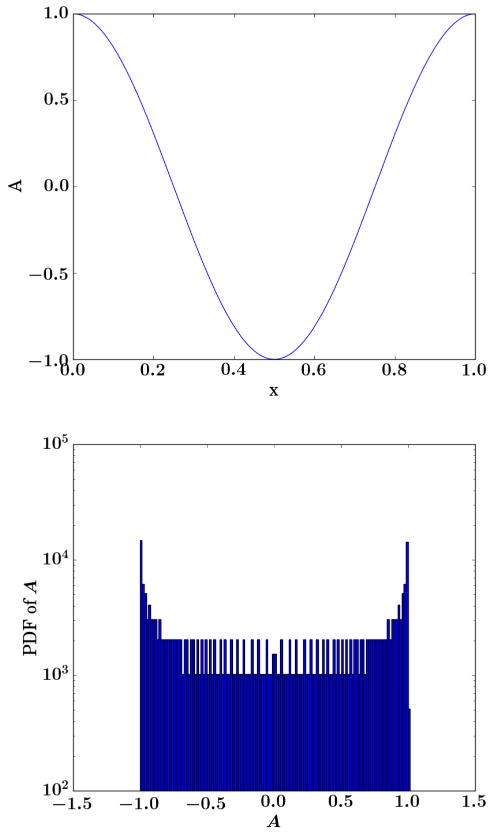
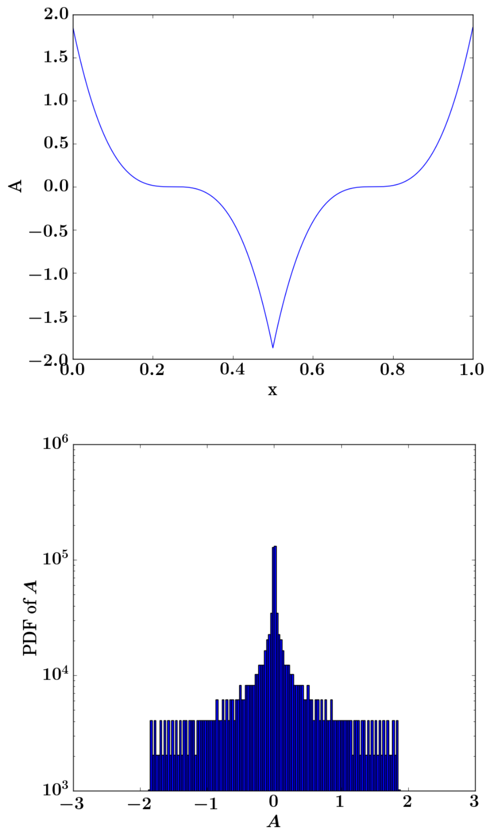
Initial Conditions

Bimodal



Unimodal



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Conserved Quantities

1. Energy

$$E = E_K + E_B = \int (\frac{v^2}{2} + \frac{B^2}{2\mu_0}) d^2x$$

2. Mean Square Magnetic Potential

$$H_A = \int A^2\, d^2x$$

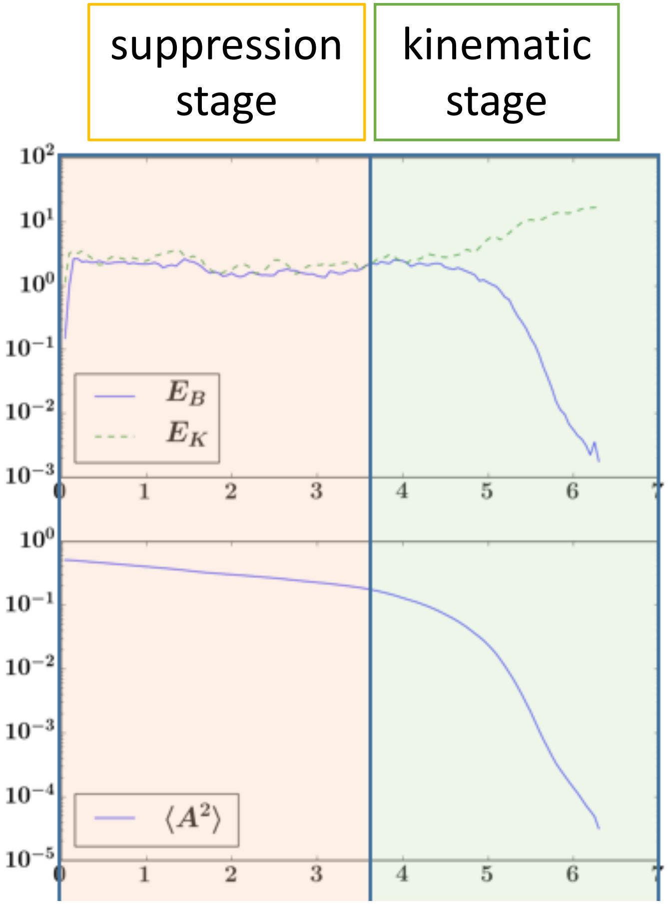
This is why A^2 plays an important role

3. Cross Helicity

$$H_C = \int \vec{v} \cdot \vec{B} d^2x$$

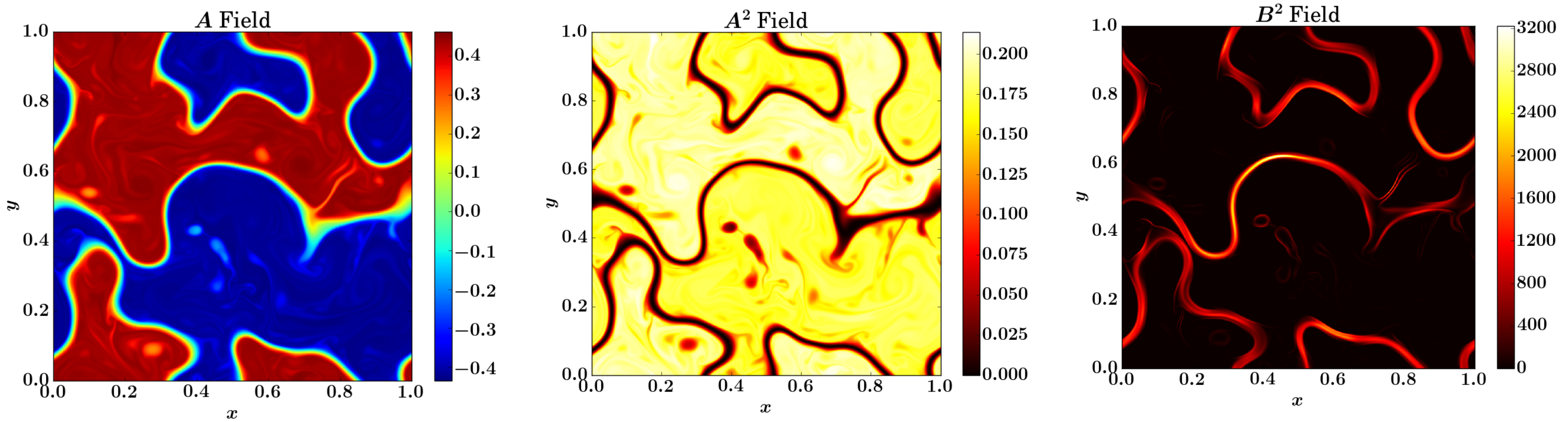
Two Stages

- 1. The suppression stage: the large scale magnetic field is sufficiently strong so that the diffusion is suppressed.
- 2. The kinematic decay stage: the magnetic field is dissipated enough so that the diffusion rate is back to the kinetic rate.
- The suppression is due to the memory provided by the magnetic field.

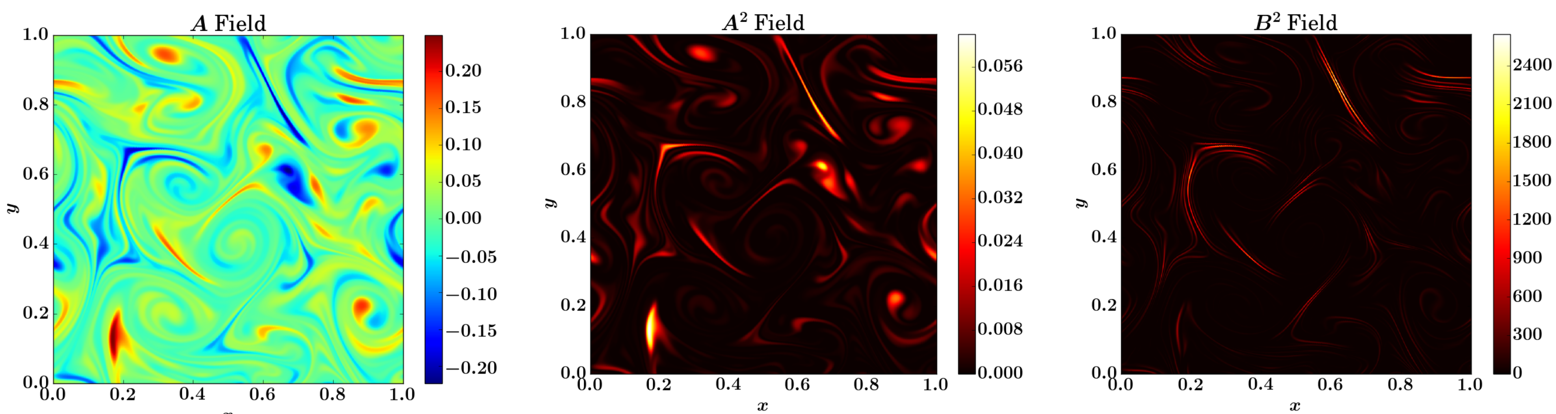


New Observations

- With no imposed B_0, in suppression stage:



- v.s. same run, in kinematic stage (trivial):

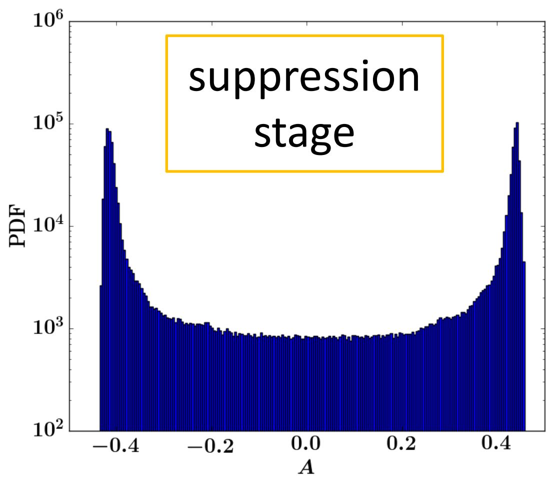


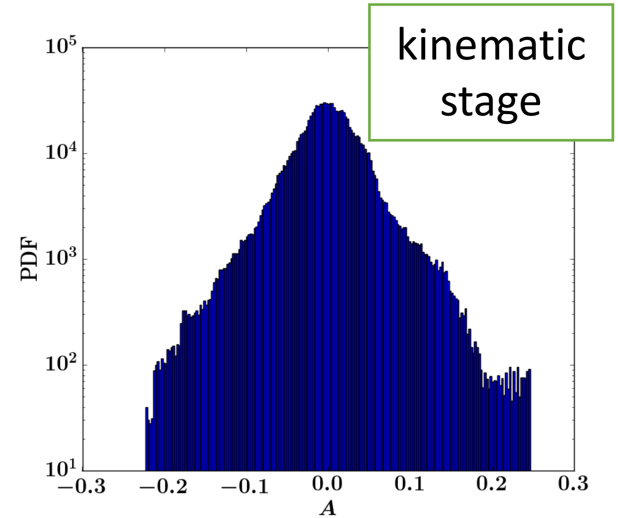
New Observations Cont'd

- Nontrivial structure formed in real space in the suppression stage.
- A field is evidently composed of "blobs".
- The low A^2 regions have a clear 1-dimensional shape.
- The high B^2 regions are strongly correlated with low A^2 regions, and also have a 1-dimensional shape.
- We call these 1-dimensional high B^2 regions ``barriers'', because these are the regions where transport is reduced, relative to η_K.

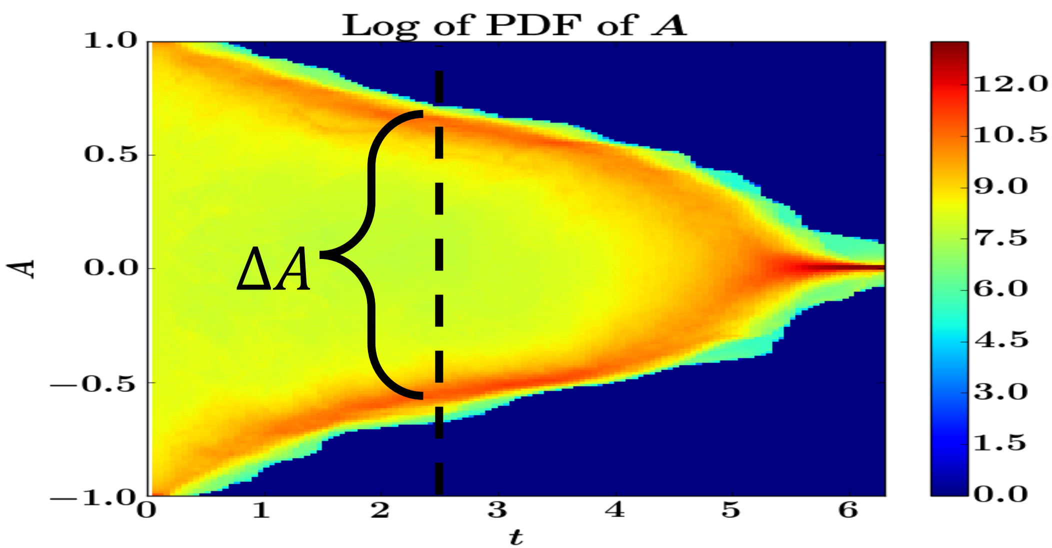
Evolution of PDF of A

- Probability Density Function (PDF) in two stage:





- Time evolution: horizontal “Y”.

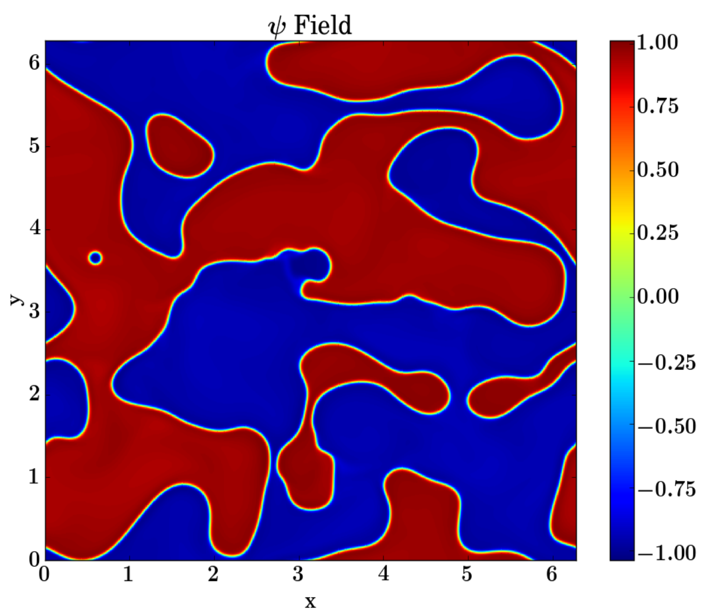


- The PDF changes from double peak to single peak as the system changes from the suppression stage to the kinematic stage.

2D CHNS and 2D MHD

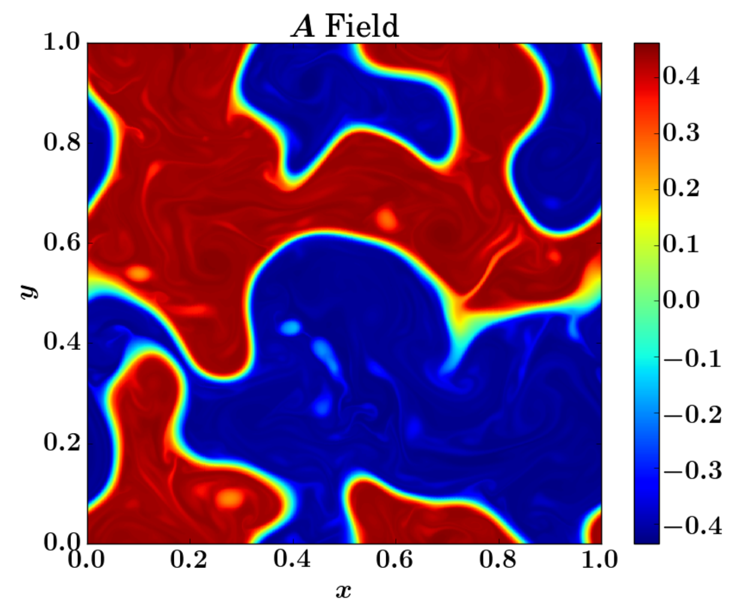
- The A field in 2D MHD in suppression stage is strikingly similar to the ψ field in 2D CHNS (Cahn-Hilliard Navier-Stokes) system:

ψ field in 2D CHNS



v.s.

A field in 2D MHD



2D CHNS and 2D MHD

	2D MHD	2D CHNS
Magnetic Potential	A	ψ
Magnetic Field	$\mathbf{B}$	$\mathbf{B}_\psi$
Current	j	j_ψ
Diffusivity	η	D
Interaction strength	$\frac{1}{\mu_0}$	ξ^2

- 2D CHNS Equations:

$$\partial_t \psi + \vec{v} \cdot \nabla \psi = D\nabla^2(-\psi + \psi^3 - \xi^2 \nabla^2 \psi)$$

$$\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{\xi^2}{\rho} \vec{B}_\psi \cdot \nabla \nabla^2 \psi + \nu \nabla^2 \omega$$

$-\psi$: Negative diffusion term
ψ^3: Self nonlinear term
$-\xi^2 \nabla^2 \psi$: Hyper-diffusion term

With $\vec{v} = \hat{z} \times \nabla \phi$, $\omega = \nabla^2 \phi$, $\vec{B}_\psi = \hat{z} \times \nabla \psi$, $j_\psi = \xi^2 \nabla^2 \psi$. $\psi \in [-1,1]$.

- 2D MHD Equations:

$$\partial_t A + \vec{v} \cdot \nabla A = \eta \nabla^2 A$$

$$\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{1}{\mu_0 \rho} \vec{B} \cdot \nabla \nabla^2 A + \nu \nabla^2 \omega$$

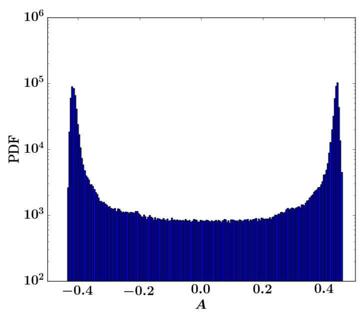
A: Simple diffusion term

With $\vec{v} = \hat{z} \times \nabla \phi$, $\omega = \nabla^2 \phi$, $\vec{B} = \hat{z} \times \nabla A$, $j = \frac{1}{\mu_0} \nabla^2 A$

See [Fan et.al. 2016] for more about CHNS.

Unimodal Initial Condition

- One may question whether the bimodal PDF feature is purely due to the initial condition. The answer is no.
- Two peaks away from 0 on PDF of A still rise, even if the initial condition is unimodal.



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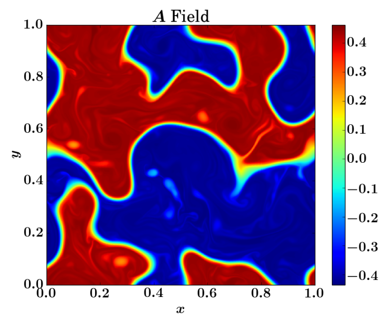

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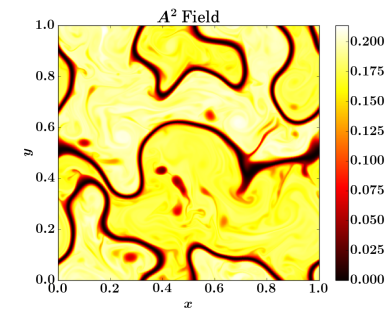
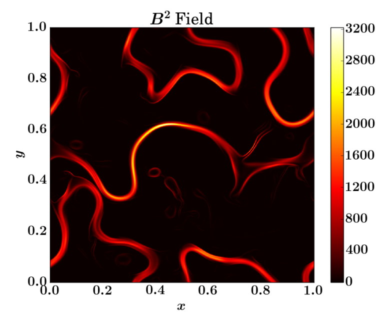


(a3)



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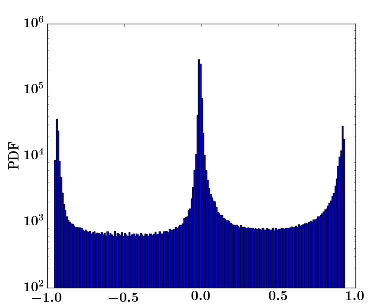
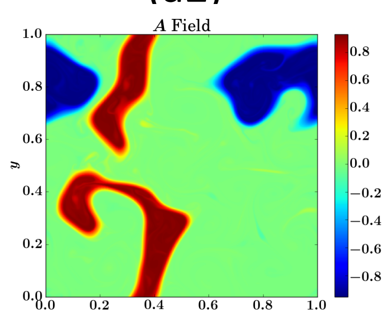
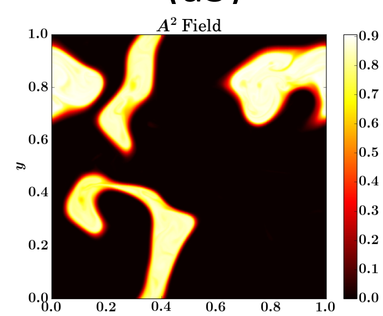


(b1)



(b2)



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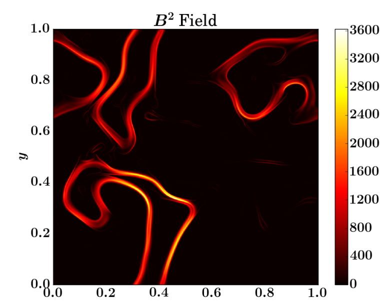
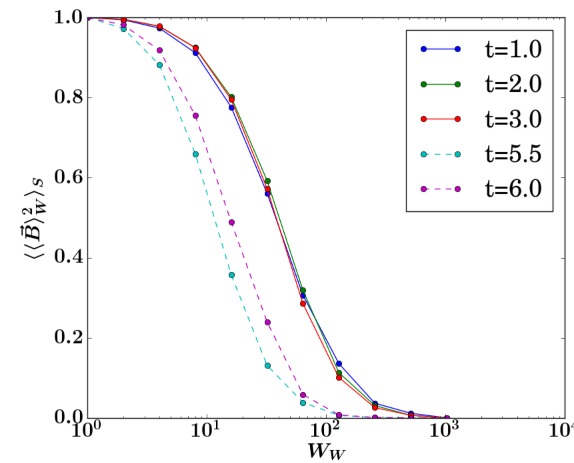
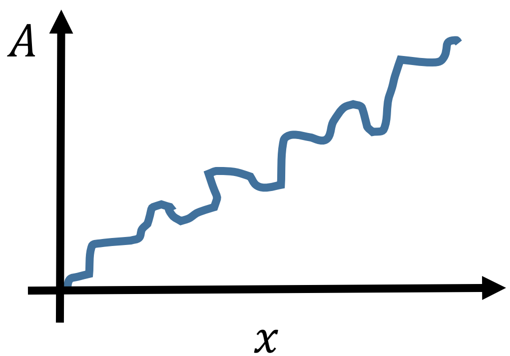


(b4)

The problem of the mean field $\langle \boldsymbol{B} \rangle$

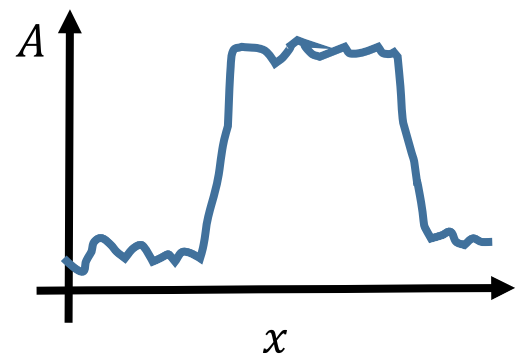
- $\langle B \rangle$ depends on the averaging window.
- With no imposed external field, B is highly intermittent, therefore the $\langle B \rangle$ is not well defined.





$|\langle B \rangle| \sim \sqrt{\langle A^2 \rangle}/L_0$ ✓

v.s.



$\langle B \rangle$ not well defined

Reality

New Understanding

- From $\partial_t \langle A^2 \rangle = -\langle \mathbf{v} A \rangle \cdot \nabla \langle A \rangle - \nabla \cdot \langle \mathbf{v} A^2 \rangle - \eta \langle B^2 \rangle$
- Do not drop 2nd term on RHS. Average taken over an envelope.
- Define diffusion coefficients (closure):

$$\langle \mathbf{v} A \rangle = -\eta_{T1} \nabla \langle A \rangle$$
$$\langle \mathbf{v} A^2 \rangle = -\eta_{T2} \nabla \langle A^2 \rangle$$

- Plugging in: $\partial_t \langle A^2 \rangle = \eta_{T1} (\nabla \langle A \rangle)^2 + \nabla \eta_{T2} \cdot \nabla \langle A^2 \rangle - \eta \langle B^2 \rangle$
- For simplicity: $\langle B^2 \rangle \sim \frac{\eta_T}{\eta} (\langle B \rangle^2 + \langle A^2 \rangle / L_{env}^2)$
- where L_{env} is the envelope size. Scale of $\nabla^2 \langle A^2 \rangle$.
- Define new strength parameter: $M'^2 \equiv \langle v^2 \rangle / (\frac{1}{\mu_0 \rho} \langle A^2 \rangle / L_{env}^2)$
- Result: $\eta_T = \frac{ul}{1 + \mathrm{Rm}/M^2 + \mathrm{Rm}/M'^2} = \frac{ul}{1 + \mathrm{Rm} \frac{1}{\mu_0 \rho} \langle \mathbf{B} \rangle^2 / \langle v^2 \rangle + \mathrm{Rm} \frac{1}{\mu_0 \rho} \langle A^2 \rangle / L_{env}^2 \langle v^2 \rangle}$

New Understanding Cont'd

- Quench is not uniform. Transport coefficient is different in different regions.
- In the regions where magnetic fields are strong, Rm/M^2 is dominant. They are regions of ***barriers***.
- In other regions, i.e. inside blobs, Rm/M'^2 is what remains.
- Summary of important length scales: $l < L_{stir} < L_{env} < L_0$
  - System size L_0
  - Envelope size L_{env}
  - Stirring length scale L_{stir}
  - Turbulence length scale l, here we use Taylor microscale λ
  - Barrier width W

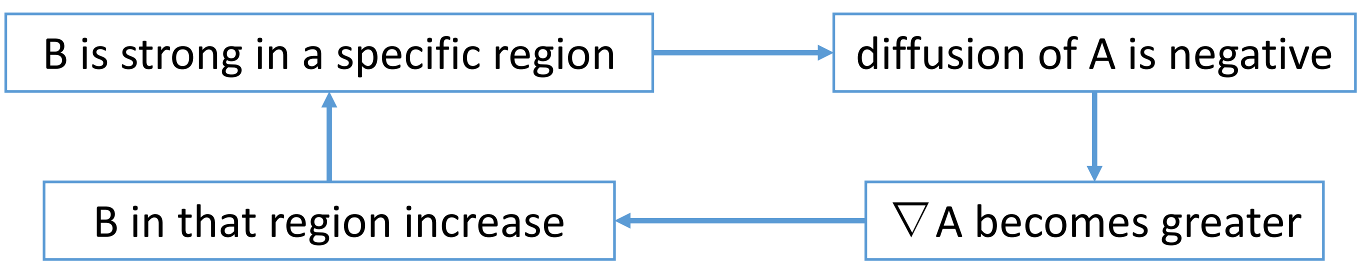
Formation of Barriers

- How do the barriers form?

flux coalescence

$$\eta_T = \sum_{\mathbf{k}} \tau_c [\langle v^2 \rangle_{\mathbf{k}} - \frac{1}{\mu_0 \rho} \langle B^2 \rangle_{\mathbf{k}}]$$

- From above expression, it is possible for some strong B regions to have negative resistivity, while the resistivity is always positive when averaged over the whole system.
- Positive feedback:



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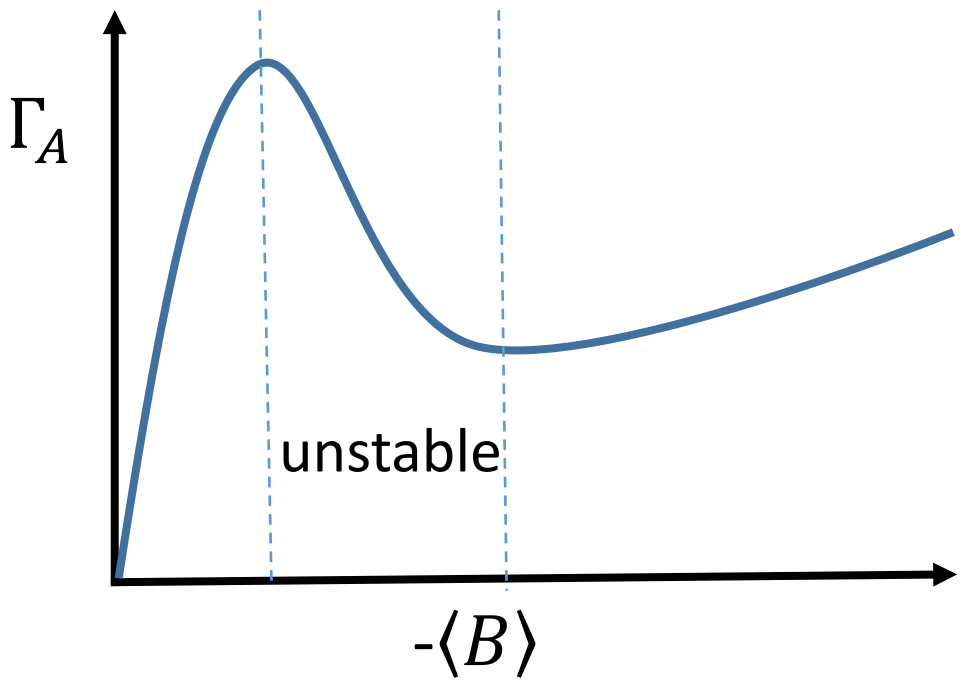
Formation of Barriers Cont'd

- Negative resistivity leads to barrier formation.
- The S-curve is due to the dependence of B on Γ_A.
- When slope is negative, it is negative resistivity.

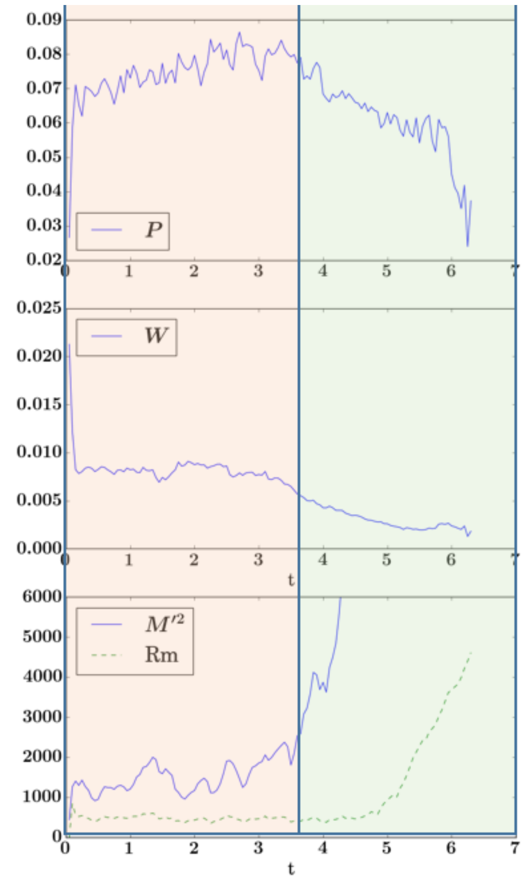


Describing the Barriers

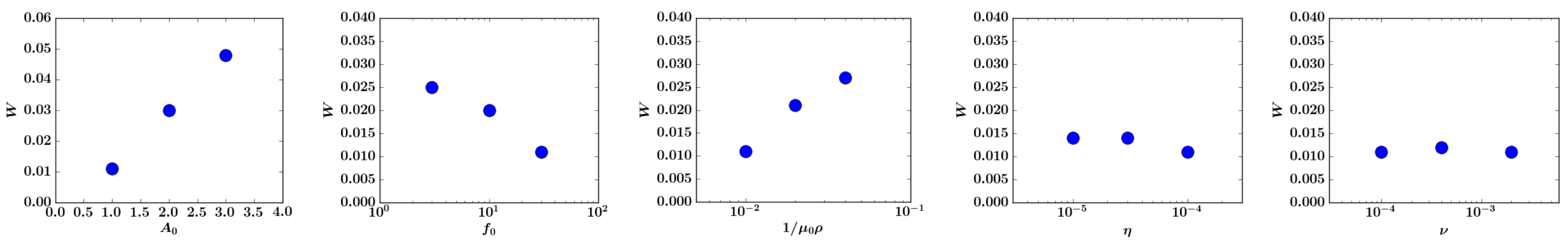
- Measure the barrier width W.
- Starting point: $W \sim \Delta A / B_b$
- Use $\sqrt{\langle A^2 \rangle}$ to represent ΔA
- Define the barrier regions to be: $B(x,y) > \sqrt{\langle B^2 \rangle} * 2$
- Define packing fraction: $P \equiv \frac{\#\text{ of grid points for barrier regions}}{\#\text{ of total grid points}}$
- Use use the magnetic fields in the barrier regions to represent the whole magnetic energy: $\sum_{\text{barriers}} B_b^2 \sim \sum_{\text{system}} B^2$
- Thus $\langle B_b^2 \rangle \sim \langle B^2 \rangle / P$
- So barrier width can be estimated by: $W^2 \equiv \langle A^2 \rangle / (\langle B^2 \rangle / P)$

Describing the Barriers

- Time evolution of P and W:



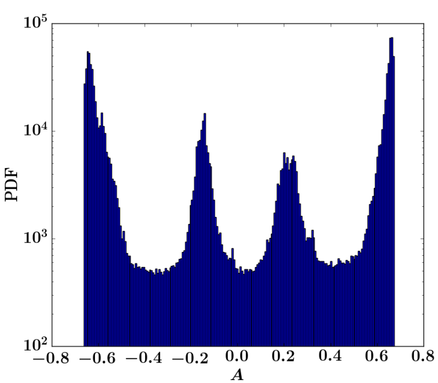
- What determines W:
  - A_0 or $1/\mu_0\rho$ greater, W greater;
  - f_0 greater, W smaller;
  - W not sensitive to η or ν.



(a) (b) (c) (d) (e)

Staircase

- Staircases emerge spontaneously!
- Initial condition is the usual cos function (bimodal)
- The only major different parameter from runs above is the forcing scale is k=32 (for all runs above k=5).
- Resembles the staircase studies in fusion research.



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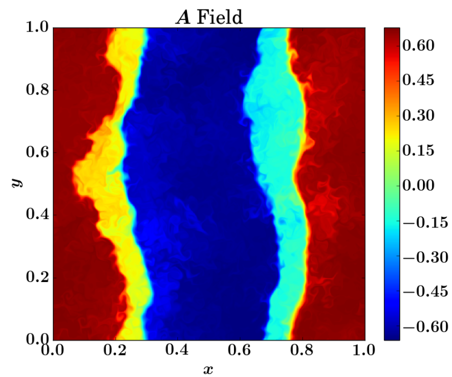

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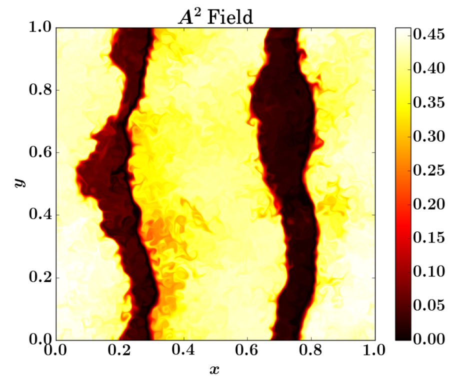

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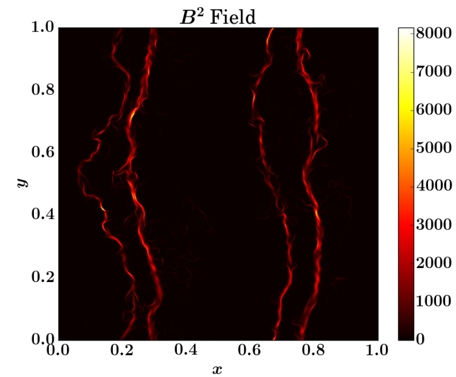


(4)

Conclusions

- Magnetic fields suppress turbulent diffusion in 2D MHD by: formation of intermittent ***transport barriers***.
- Magnetic structures: { Barriers – thin, 1D strong field; Blobs – 2D, weak field }
- Quench not uniform:

$$\eta_T = \frac{ul}{1 + \mathrm{Rm}\frac{1}{\mu_0 \rho}\langle \mathbf{B} \rangle^2 / \langle v^2 \rangle + \mathrm{Rm}\frac{1}{\mu_0 \rho}\langle A^2 \rangle / L_{env}^2 \langle v^2 \rangle}$$

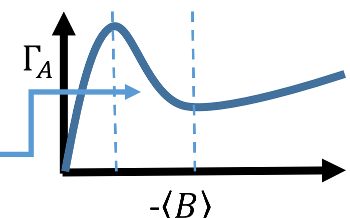
barriers, strong B

blobs, weak B, $\nabla^2 \langle A^2 \rangle$ remains

- Barriers form due to negative resistivity:

$$\eta_T = \sum_{\mathbf{k}} \tau_c [\langle v^2 \rangle_{\mathbf{k}} - \frac{1}{\mu_0 \rho} \langle B^2 \rangle_{\mathbf{k}}]$$

flux coalescence



- Formation of "magnetic staircases" observed for some i.c.
- Do barriers regulate magnetic helicity transport in 3D? Implications for α quenching?