Subcritical turbulence spreading and avalanche birth

R.A. Heinonen and P.H. Diamond

CASS and Department of Physics
University of California, San Diego

IUTAM Vortex Dynamics 2019

Supported by the Department of Energy under Award Number DE-FG02-04ER54738

Summary slide

- We present a model for turbulence front propagation (spreading) based on subcritical turbulence:

$$\partial_t I = \gamma_1 I + \gamma_2 I^2 - \gamma_3 I^3 + \partial_x (D_0 I \partial_x I)$$

- Familiar to fluids community (c.f. Pomeau, Barkley) — describes spreading of a vortex patch by entrainment — but new to plasma.
- We calculate a threshold size, generated by a competition between turbulence diffusion and nonlinear turbulence production, for a patch of turbulence to spread

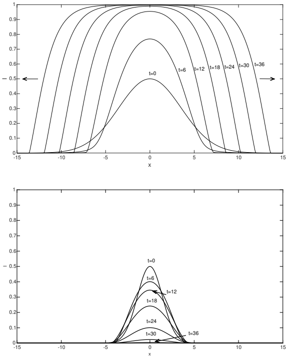

Figure An initially localized path of turbulence may either spread (above) or collapse (below)

Introduction: spreading and subcritical transition

- Certain fluid flows exhibit a subcritical $(\mathrm{Re} < \mathrm{Re}_{crit})$ transition to turbulence where laminar and turbulent domains coexist
- As Re increases, localized puffs evolve into spreading slugs of turbulence
- How to characterize the evolution and spreading of localized patches of turbulence?

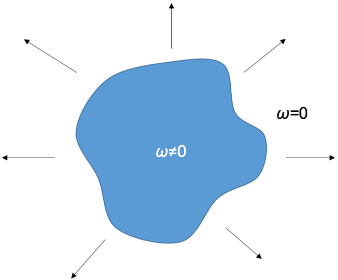

Figure A vortex patch entrains and expands into the surrounding irrotational fluid.

Turbulence spreading in plasma

- Analogously, turbulent fluctuations in confined plasma can propagate radially via pulses, fronts [Garbet et al., 1994, Diamond and Hahm, 1995]
- Fluctuations can penetrate into linearly stable zones and excite turbulence there [Hahm et al., 2004, Naulin et al., 2005]
- Closely related conceptually to avalanching: both are mesoscale, nonlinear turbulent front propagation phenomena

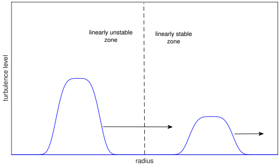

Figure Cartoon depicting a turbulence pulse propagating into the stable zone and exciting turbulence there.

Why does the plasma physicist care?

- Magnetic fusion people want to understand and control fluxes of heat and particles
- Spreading results in the fluctuation intensity being influenced by dynamics outside of the turbulence correlation length
- Result: fluctuation level, turbulent fluxes have *nonlocal dependence* on driving gradient, e.g.

$$Q(r) = -\chi \nabla T(r) \;\longrightarrow\; Q(r) = -\chi \int dr'\, K(r, r') \nabla T(r')$$

- Spreading also believed to be involved in (a) observed breakdown of gyro-Bohm transport scaling [Lin and Hahm, 2004], (b) transport barrier formation, (c) staircase formation

Avalanches

- Bursty, intermittent transport events. Should be thought of as a kind of spreading
- Account for a large percentage of total flux
- "Domino effect": localized fluctuation cascades through neighboring regions via local gradient coupling
- Exhibits features of self-organized criticality, e.g. long tails, $1/f$ spectra, profile stiffness, near-marginal.
- Prototypical model is the sandpile
- Interact with PV staircase

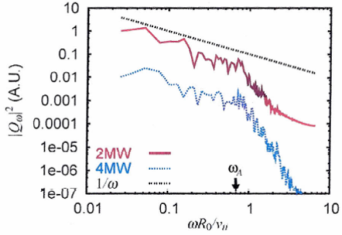

Figure Heat flux spectrum from GK simulation

Conventional wisdom: Fisher equation

- Simplest, most common model:

$$\partial_t I = \underbrace{\gamma_0 I}_{\substack{\text{local lin.}\\ \text{growth/decay}}} - \underbrace{\gamma_{nl} I^2}_{\substack{\text{local nonlin.}\\ \text{coupling to}\\ \text{dissipation}}} + \underbrace{\partial_x (D_0 I \partial_x I)}_{\text{nonlin. diffusion of turb. energy}}$$

- Spreading occurs in supercrit. ($\gamma_0 > 0$): traveling waves connecting the laminar and turbulent fixed points with constant speed

$$c = \sqrt{\frac{D_0 \gamma_0^2}{2\gamma_{nl}}}$$

- No marginal or subcrit. spreading

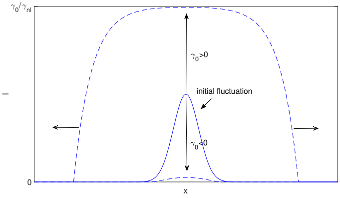

Figure Fisher equation dynamics

Does it make sense?

- While partly successful (e.g. propagation speed), supercritical spreading is a cheat! Noise should excite the system in the first place
- For turbulent/laminar phase coexistence, a *subcritical bifurcation* is necessary [Pomeau, 1986, Pomeau, 2015]
- Also, penetration into stable zone is *weak*. Turbulence level decays exponentially to finite depth $\sim \sqrt{D_0/\gamma_{nl}}$, i.e. just a few correlation length [Gürcan and Diamond, 2005] — suggests Fisher may be insufficient to explain nonlocality!

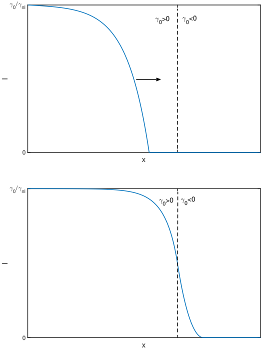

Figure Penetration of Fisher front into stable zone

A new model is born

- We propose (Heinonen and Diamond PoP (2019)) a new model:
$$\partial_t I = \gamma_1 I + \gamma_2 I^2 - \gamma_3 I^3 + \partial_x (D_0 I \partial_x I) \qquad (*)$$
- Roughly anticipate $\gamma_i \sim \omega_*$, $D_0 \sim \chi_{GB} \sim c_s \rho_i^2 / a$
- Motivation: simplest, generic 1D model with subcritical bifurcation. Other forms possible, but qualitative features should be the same!
- Similar to [Barkley et al., 2015, Pomeau, 2015] models for onset of turbulence in pipe flow
- But is MF plasma turbulence actually subcritical?

Evidence for subcritical turbulence

- Experiments have clearly demonstrated hysteresis between fluctuation intensity and gradient in the L-mode (no ITB) [Inagaki et al., 2013] $\rightarrow$ bistable S-curve relation??
- In simulation, subcritical turbulence observed in the presence of magnetic shear damping [Biskamp and Walter, 1985, Scott, 1990] or strong perpendicular sheared flows [Barnes et al., 2011, van Wyk et al., 2016]

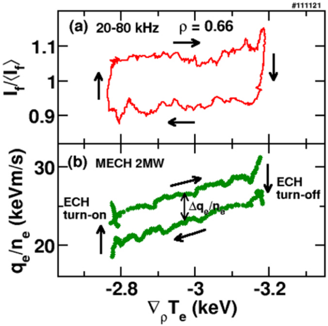

Figure Inagaki *et al.* 2013

Summary of model regimes

regime	stable roots	unstable roots	waves	comments
$\gamma_1 > 0$	I_+	0	forward-propagating	unistable similar to Fisher
$\gamma_1 < 0$ $\lvert\gamma_1\rvert\gamma_3/\gamma_2^2 < 15/64$	$0, I_+$	I_-	foward-propagating	$\alpha < \alpha^*$ turbulent root abs. stable
$\gamma_1 < 0$ $15/64 < \lvert\gamma_1\rvert\gamma_3/\gamma_2^2 < 1/4$	$0, I_+$	I_-	receding	$\alpha > \alpha^*$ turbulent root metastable
$\gamma_1 < 0$ $\lvert\gamma_1\rvert\gamma_3/\gamma_2^2 > 1/4$	0	none	none	"strong damping"

Table Summary of features of the various parameter regimes in cubic model. Here $I_\pm = (\gamma_2 \pm \sqrt{\gamma^2 + \gamma_1\gamma_3})/2\gamma_3$.

In bistable case can transform to FitzHugh-Nagumo form

$$\partial_t I = f(I) + \partial_x(DI\partial_x I)$$

with $f(I) = \gamma I(I-\alpha)(1-I), \gamma = \gamma_3 I_+^2, D = I_+ D_0, \alpha = I_-/I_+$

Free energy picture

- Dynamics governed by dissipation of free energy: can rewrite

$$D(I)\partial_t I = -\frac{\delta \mathcal{F}}{\delta I}$$

with free energy functional

$$\mathcal{F} = \int dx\, [\underbrace{\frac{1}{2}(D(I)\partial_x I)^2}_{\text{kinetic/flux}} \underbrace{- \int_0^I dI'\, D(I')f(I')]}_{\text{potential}}$$

and $d\mathcal{F}/dt \leq 0$

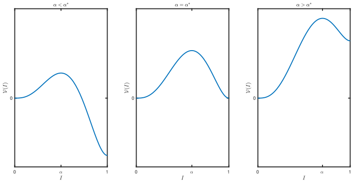

Figure Plot of potential part of free energy $\mathcal{V}(I) = -\int_0^I dI'\, D(I)f(I)$

Key predictions of bistable model

- In marginal and weakly subcritical regime, again have propagating turbulence fronts with speed $\sim \sqrt{D\gamma}$ (coeff. depends on α)
- If turbulence level driven globally above “potential barrier” at α (say by external flux), system relaxes to turbulent root $\rightarrow$ explains hysteresis in Inagaki
- There is also *local* threshold behavior: a sufficiently large slug of turbulence will grow and propagate. How to determine threshold size? Spreading of a turbulent spot is classic problem in turbulence

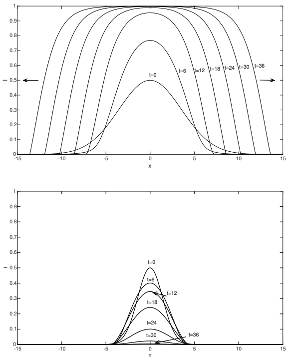

Figure A slug will either grow into a wave (above) or collapse (below)

Threshold for spreading of a slug of turbulence

- Threshold for amplitude is clear: intensity must exceed $I = \alpha$ somewhere
- Otherwise effective linear growth $\gamma_{eff} = (I - \alpha)(1 - I)$ is negative everywhere
- What about threshold in spatial extent? Question seems largely unexplored in literature!

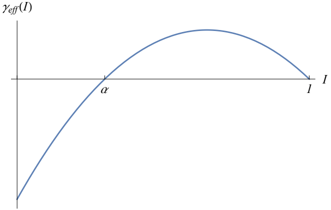

Figure Plot of effective local linear growth as function of turbulence intensity

Lengthscale threshold

- Can estimate by assuming initial growth of turbulent mass in "cap" (part $> \alpha$) of slug governs asymptotic spreading
- Threshold then determined by competition between outgoing diffusive flux from cap and local growth in cap
- This competition suggested by form of free energy functional
- Leads to power law $L_{min} \sim (I_0 - \alpha)^{-1/2}$. Excellent agreement with simulation of PDE

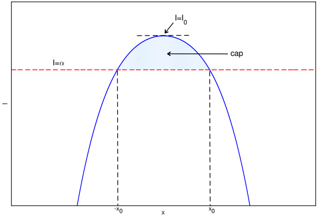

Figure Illustration of slug's "cap"

Lengthscale threshold: analytical vs. simulation

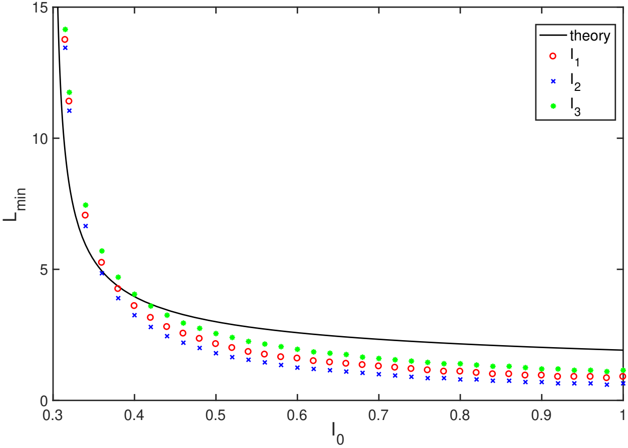

Figure Numerical result for threshold at $\alpha = 0.3$ for three types of initial condition (Gaussian (I_1), Lorentzian (I_2), parabola (I_3)), compared with analytical estimate

From slugs to avalanches

- So: an initially localized turbulent slug with amplitude exceeding I_- and spatial extent exceeding L_{min} will spread and excite the system to turbulence
- This closely resembles an avalanche. Note the similarity of our model to [Gil and Sornette, 1996] model for sandpile avalanching
- Diffusion both provides mechanism for turbulence to topple from one region to next and limits avalanching by setting minimum scale length
- Near marginal linear stability, threshold is "small":

$$I_- \sim \frac{|\gamma_1|}{\gamma_2} \ll 1, \; L_{\min} \sim \left(\frac{\chi_{GB}}{\omega_*}\right)^{1/2} \sim \rho_i$$

- Suggests that noise (e.g. background sub-ion-scale turbulence) can *intermittently* excite turbulence pulses. Related: turbulence transition in fluids is highly intermittent [Pomeau and Manneville, 1980]

Penetration into bistable zone

- Finally, let's revisit the problem of spreading from weakly supercritical into weakly subcritical ($\alpha < \alpha^*$), now with nonlinear instability
- Amplitude of wave in unstable region always exceeds amplitude threshold in stable region
- Thus, another wave forms in second region! Turbulence front propagates at constant speed (instead of finite depth), as long as weakly subcritical
- Conclude: delocalization effect much stronger than in Fisher

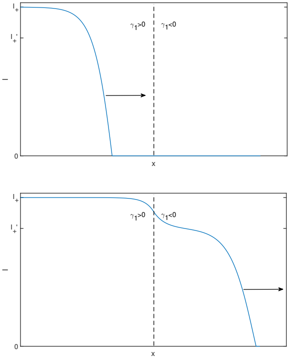

Figure A wave develops in the unstable zone, penetrates into the bistable zone, and forms a new traveling wave with reduced speed and turbulence level.

Conclusions

- Updating the unistable Fisher model to a bistable model simultaneously resolves several issues
  1. Subcritical/marginal spreading properly supported
  2. Can account for hysteresis in fluctuation intensity
  3. Reflects the emerging understanding that MF turbulence is subcritically unstable, at least in certain scenarios
  4. Allows for stronger penetration into stable zone via ballistic spreading
- Also functions as a basic model for avalanching by local excitation
- Future directions:
  1. Full model needs to incorporate coupling to zonal flow and/or profiles
  2. Avalanche threshold can be tested by initializing seed fluctuations in simulation and observing response
  3. Should also test for ballistic spreading into stable zone numerically. Possible inspiration: [Yi et al., 2014]

Experimental ideas

- Extensions of Inagaki
  - Better resolution of dependence of fluctuation intensity on the input power. Are there any jumps?
  - More careful study of relaxation after ECH is turned off. How does relaxation time compare to other timescales of interest?
  - More information on fluctuation field. E.g. spatial correlations?
  - Include spatiotemporal measurement of zonal flow pattern via Thomson scattering, heavy ion beam probe, etc.
- To investigate avalanches: perturb plasma locally, observe spatiotemporal response à la [Van Compernolle et al., 2015]. Compare near-marginal, far above stability threshold to rule out possibility of linear mode response

References I

Barkley, D., Song, B., Mukund, V., Lemoult, G., Avila, M., and Hof, B. (2015).
The rise of fully turbulent flow.
Nature, 526:550–553.

Barnes, M., Parra, F. I., Highcock, E. G., Schekochihin, A. A., Cowley, S. C., and Roach, C. M. (2011).
Turbulent transport in tokamak plasmas with rotational shear.
Phys. Rev. Lett., 106:175004.

Biskamp, D. and Walter, M. (1985).
Suppression of shear damping in drift wave turbulence.
Phys. Lett., 109A(1,2).

Diamond, P. H. and Hahm, T. S. (1995).
On the dynamics of turbulent transport near marginal stability.
Physics of Plasmas, 2(10):3640–3649.

Dif-Pradalier, G., Diamond, P. H., Grandgirard, V., Sarazin, Y., Abiteboul, J., Garbet, X., Ghendrih, P., Strugarek, A., Ku, S., and Chang, C. S. (2010).
On the validity of the local diffusive paradigm in turbulent plasma transport.
Phys. Rev. E, 82:025401.

FitzHugh, R. (1961).
Impulses and physiological states in theoretical models of nerve membrane.
Biophysical Journal, 1(6):445–466.

Garbet, X., Laurent, L., Samain, A., and Chinardet, J. (1994).
Radial propagation of turbulence in tokamaks.
Nuclear Fusion, 34(7):963.

References II

Gil, L. and Sornette, D. (1996).
Landau-Ginzburg theory of self-organized criticality.
Phys. Rev. Lett., 76:3991–3994.

Guo, Z. B. and Diamond, P. H. (2017).
Bistable dynamics of turbulence spreading in a corrugated temperature profile.
Physics of Plasmas, 24(10).

Gürcan, O. and Diamond, P. (2005).
Dynamics of turbulence spreading in magnetically confined plasmas.
Physics of Plasmas, 12(3):032303.

Hahm, T. S., Diamond, P. H., Lin, Z., Itoh, K., and Itoh, S.-I. (2004).
Turbulence spreading into the linearly stable zone and transport scaling.
Plasma Physics and Controlled Fusion, 46(5A):A323.

Ida, K., Shi, Z., Sun, H., Inagaki, S., Kamiya, K., Rice, J., Tamura, N., Diamond, P., Dif-Pradalier, G., Zou, X., et al. (2015).
Towards an emerging understanding of non-locality phenomena and non-local transport.
Nuclear Fusion, 55(1):013022.

Inagaki, S., Tokuzawa, T., Tamura, N., Itoh, S.-I., Kobayashi, T., Ida, K., Shimozuma, T., Kubo, S., Tanaka, K., Ido, T., et al. (2013).
How is turbulence intensity determined by macroscopic variables in a toroidal plasma?
Nuclear Fusion, 53(11):113006.

Lin, Z. and Hahm, T. S. (2004).
Turbulence spreading and transport scaling in global gyrokinetic particle simulations.
Physics of Plasmas, 11(3):1099–1108.

References III

Nagumo, J., Arimoto, S., and Yoshizawa, S. (1962).
An active pulse transmission line simulating nerve axon.
Proceedings of the IRE, 50(10):2061–2070.

Naulin, V., Nielsen, A. H., and Rasmussen, J. J. (2005).
Turbulence spreading, anomalous transport, and pinch effect.
Physics of Plasmas, 12(12):122306.

Politzer, P. A. (2000).
Observation of avalanchelike phenomena in a magnetically confined plasma.
Phys. Rev. Lett., 84:1192–1195.

Pomeau, Y. (1986).
Front motion, metastability and subcritical bifurcations in hydrodynamics.
Physica D: Nonlinear Phenomena, 23(1):3 – 11.

Pomeau, Y. (2015).
The transition to turbulence in parallel flows: A personal view.
Comptes Rendus Mecanique, 343:210–218.

Pomeau, Y. and Manneville, P. (1980).
Intermittent transition to turbulence in dissipative dynamical systems.
Comm. Math. Phys., 74(2):189–197.

Scott, B. D. (1990).
Self-sustained collisional drift-wave turbulence in a sheared magnetic field.
Phys. Rev. Lett., 65:3289–3292.

References IV

Van Compernolle, B., Morales, G. J., Maggs, J. E., and Sydora, R. D. (2015).
Laboratory study of avalanches in magnetized plasmas.
Phys. Rev. E, 91:031102.

van Wyk, F., Highcock, E. G., Schekochihin, A. A., Roach, C. M., Field, A. R., and Dorland, W. (2016).
Transition to subcritical turbulence in a tokamak plasma.
Journal of Plasma Physics, 82(6):905820609.

Waltz, R. E., Austin, M. E., Burrell, K. H., and Candy, J. (2006).
Gyrokinetic simulations of off-axis minimum-q profile corrugations.
Physics of Plasmas, 13(5):052301.

Yi, S., Kwon, J. M., Diamond, P. H., and Hahm, T. S. (2014).
Effects of q-profile structure on turbulence spreading: A fluctuation intensity transport analysis.
Physics of Plasmas, 21(9):092509.

Avalanching 101

- Observed in MFE plasma [Politzer, 2000]
- Basic picture: a sufficiently large, localized increase in the turbulence level radially cascades into neighboring regions, ultimately causing a sudden burst of transport
- Closely related to turbulence spreading: avalanching and (subcritical) spreading essentially two ways of looking at same phenomenon
- Associated with self-organized criticality (occurs near marginal, $1/f$ spectra)
- Intermittent (long tails)

Bistable case: reduction to FitzHugh-Nagumo

- $(*)$ is bistable for weak damping $\gamma_1 < 0$ and $\gamma_2^2 > 4|\gamma_1|\gamma_3$
- Roots: $I = 0$, $I_\pm = (\gamma_2 \pm \sqrt{\gamma_2^2 - 4|\gamma_1|\gamma_3})/2\gamma_3$. $0, I_+$ stable (note: nonzero for marginal γ_1), I_- unstable
- If $\gamma_1 < 0$ and γ_2 sufficiently large, can be written

$$\partial_t I = f(I) + \partial_x(D(I)\partial_x I)$$

with $f(I) = \gamma I(I - \alpha)(1 - I)$ by defining

$$|\gamma_3|{I_+}^2 \to \gamma,\ \frac{I_-}{I_+} \to \alpha,\ I_+ D_0 \to D$$

- This is a version of the Nagumo equation, a simplification of the FitzHugh-Nagumo model for excitable media [FitzHugh, 1961, Nagumo et al., 1962]

Lengthscale threshold (details)

- Strategy: assume initial slug is even, has single max at I_0 and single lengthscale L
- Expand intensity curve about max to quadratic order, plug into dynamical equation, integrate over extent of cap
- Result: growth if

$$L > L_{\min} = \sqrt{\frac{\lambda D(\alpha) I_0}{f(I_0) - \frac{1}{3}(I_0 - \alpha) f'(I_0)}} = \sqrt{\frac{3\lambda D \alpha I_0}{\gamma(I_0 - \alpha)((1 - 2\alpha) I_0 + \alpha)}}$$

$E \times B$ staircase

- $E \times B$ staircase: quasiperiodic shear flow pattern observed in GK simulation [Waltz et al., 2006]
- [Guo and Diamond, 2017] showed that in mean field approx., result is additional nonlinear drive term, equation of the type $(*)$ $\rightarrow$ global bistability
- Basic physics: inhomogeneous turbulence mixing. Shear suppression of turb. heat flux $\rightarrow$ effective negative turbulent heat diffusion $\rightarrow$ temperature corrugations $\rightarrow$ critical gradient locally exceeded $\rightarrow$ turbulence growth $\rightarrow$ further profile roughening

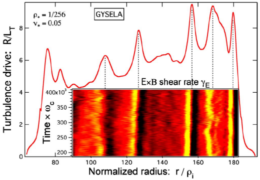

Figure: Profile corrugations correlate with $E \times B$ shear (from [Dif-Pradalier et al., 2010])