When does turbulence spreading matter?

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Outline

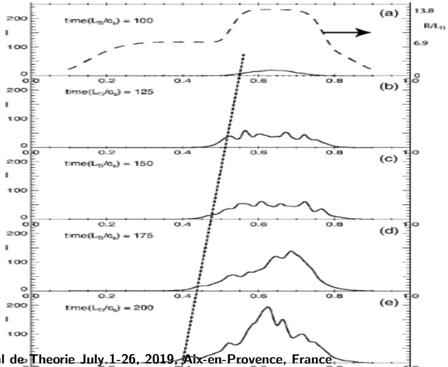
- Take aways
- Introduction: what is spreading and why worry about it?
- Motivation: Impact on profiles and confinement
- Spreading effect on L mode profiles
- Spreading effect on H mode profiles
- Conclusions
- Future directions

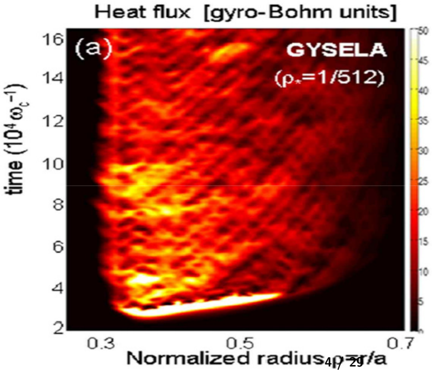
Preview of the bottom line

- Spreading matters whenever there is strong intensity gradient.
- Particular interesting situations are :
  - L mode edge with edge localized source of intensity/ Turbulence invasion from SOL.
  - No Man's Land (NML) in H mode. [NML connects core to the pedestal]
- Spreading effect on L mode profiles is weak due to weak intensity gradient.
- H mode profiles are strongly affected by turbulence spreading due strong intensity gradient in NML. Turbulence in NML is reduced and pedestal height and width increases in response to spreading.
- Spreading may be actually good for H mode confinement.
- We argue that predictive models of pedestal structure must address NML turbulence and spreading effects.

What is turbulence spreading and avalanching ?

- Turbulence spreading is self-scattering and expansion/ redistribition of turbulence energy from excitation zone to stable zone. Phenomenon of inhomogeneous turbulence, outside of usual realm of $K41$ paradigm.
- Avalanches are space-time localized large transport events.
- Avalanches are often observed in nature, gyrokinetic simulations, sand pile models and experiments.





Why worry about turbulence spreading and avalanching ?

- Nonlocality[Ida et al., 2015], breakdown of gyroBohm transport scaling[Lin and Hahm, 2004], breakdown of Ficks law [Hahm and Diamond, 2018].
    - mixing length estimate of effective diffusivity $D \sim v_\star l$, where l is charecteristic scale, $v_\star$ is diamagnetic velocity. For $\rho_i < l < a$ yields D in between gyro-Bohm and Bohm $\rho_\star D_B < D < D_B$
    - However experiments indicate $D = \rho_\star^\alpha D_B$ with $\alpha < 1 (\alpha \sim 0.6 - 0.7) \implies$ gyro-Bohm scaling is broken !
- The “short fall problem” i.e., failure of G-K simulations to predict turbulence and transport in 'no man's land' and the dynamics of core edge coupling[T S hahm etal., 2005, 2018].
- Turbulence spreading can invade regions with magnetic islands, impact on physics of $3d$ systems and neoclassical tearing modes. [K. Ida etal 2018]
- Any effect of spreading on avalanching ?
- **Most important: what is spreading effect on steady state profiles ?**

Usual story

A simple nonlinear reaction diffusion model for local turbulence intensity $I(x,t)$.

$$\frac{\partial I}{\partial t} = f(I) + \underbrace{\sigma \frac{\partial}{\partial x} I \frac{\partial I}{\partial x}}_{turb.\, spreading}$$

- $f(I) = \begin{cases} \underbrace{\gamma(x) I}_{lin.\, growth} - \underbrace{\gamma_{NL}(x) I^2}_{nonlin.\, damping} & TSH, PHD, ODG, XG, KI \\ \underbrace{\gamma_1(x) I}_{lin.\, growth} + \underbrace{\gamma_2 I^2}_{nonlin.\, growth} - \underbrace{\gamma_3(x) I^3}_{nonlin.\, damping} & RH, PHD, ... \end{cases}$
- $\gamma(x)$ **is prescribed, i.e., frozen background profiles.**
- **Gives space time evolution of an initial slug of turbulence. But who cares about transient pulses?**
- **Missed/Ignored/cheated on one of the most important question in fusion: what is the effect of spreading on steady state plasma profiles?**

Lock them up ...

Repeated crimes over past decades...

What about profiles ?(The new story)

Physical plasma systems are flux driven. So in steady state, **total flux(r)=input flux(r)** . Total flux

$$Q(r) \quad = \quad \underbrace{-D_c \frac{\partial P}{\partial x}}_{collisional} - \underbrace{\alpha_P I(r) \frac{\partial P}{\partial x}}_{turbulent}$$

So profiles (P) **must** evolve with intensity (I) spreading.
From fusion point of view, anything affecting profiles is of immediate interest. So we ask

- How does turbulence spreading affect profiles?
- In the cases where it does, what are the distinguishing features?

Spreading in L mode: 2-field model

Model eqautions for intensity (I) and pressure (P):

$$\frac{\partial I}{\partial t} = \chi\left[\mu I + 2\beta I^2 - I^3\right] + \underbrace{\sigma \frac{\partial}{\partial x} I \frac{\partial I}{\partial x}}_{spreading} + \underbrace{\delta(x-a)\frac{I_0}{\tau}}_{edge\,source} \qquad (1)$$

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial x}\alpha I \frac{\partial P}{\partial x} + \underbrace{\phi_p(x)}_{heat\,source}, \qquad \phi_p = \phi_{p0} e^{-wx^2} \qquad (2)$$

Growth rate: $\mu = \left(\frac{\partial P}{\partial x}\right)^2 - \mu_c^2$. Boundary conditions:
$\frac{\partial I}{\partial x}(x=0) = \frac{\partial I}{\partial x}(x=1) = 0$, $\frac{\partial P}{\partial x}(x=0) = 0$, $P(1) = 0$.
For SOL invasion: $I_0 = 0$, and $\frac{\partial I}{\partial x}(x=1) \neq 0$.
To excite avalanches: Add white noise in pressure source.

Spreading effect w/o edge localized source or SOL invasion

NO effect of spreading on pressure profile !

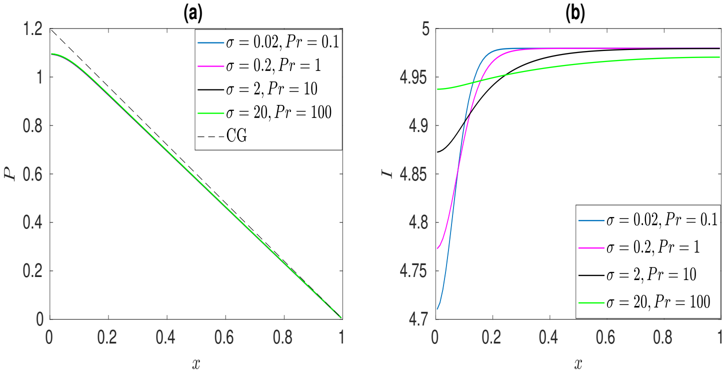


Figure: σ scans of pressure (a) and intensity (b) profiles at $\beta = 2.5$, $\alpha = 0.2$, $\phi = 13e^{-100x^2}$ without edge source $I_0/\tau = 0$.

Spreading in presence of edge localized source

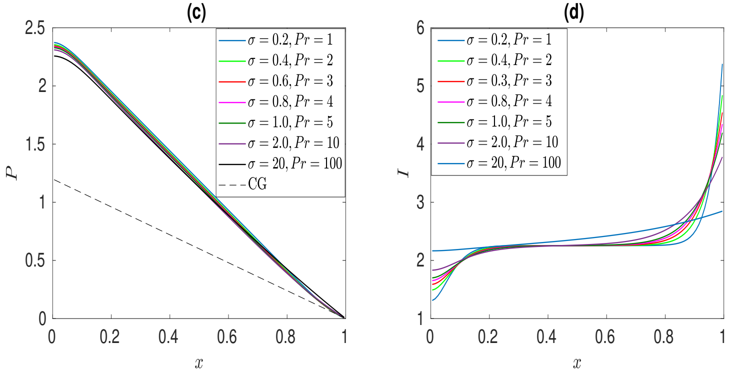


Figure: σ scans of pressure (c) and intensity (d) profiles at $\beta = 0$, $\alpha = 0.2$, $\phi = 13e^{-100x^2}$ with edge source $I_0/\tau = 10^4$

- I_{edge} and ∇I_{edge} increases with $I_0 \rightarrow \nabla P_{edge}$ softens
- Both I_{edge} and ∇I_{edge} decreases with $\sigma \rightarrow \nabla P_{edge}$ steepens

Spreading with SOL turbulence invasion

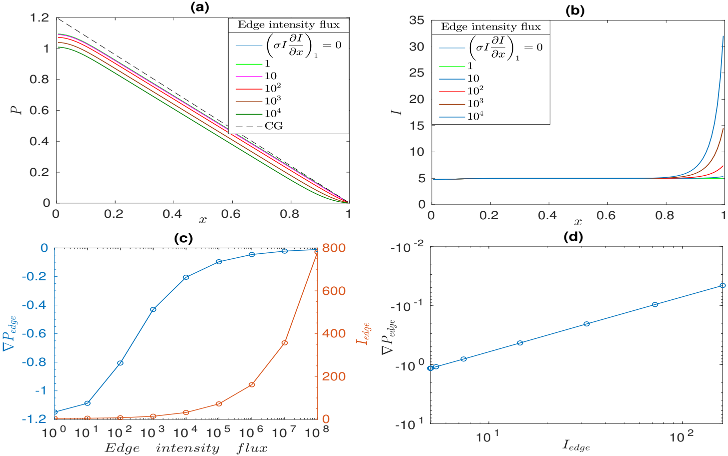
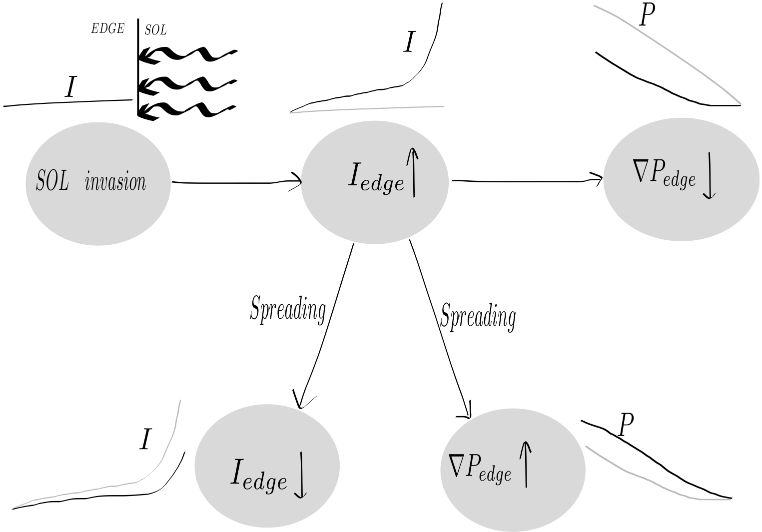


Figure: Pressure (a) and intensity (b) profiles with edge intensity flux. (c) Edge intensity flux scan of edge intensity and edge pressure gradient at $Pr = \frac{\sigma}{\alpha} = 1$. (d) Edge intensity and edge pressure gradient relation. Parameters $\beta = 2.5$, $\alpha = 0.2$, $\sigma = 0.1$, $\phi = 13e^{-100x^2}$.

Summary of spreading in L mode



Interaction of spreading and avalanching

Avalanche distributions are weakly affected by spreading !

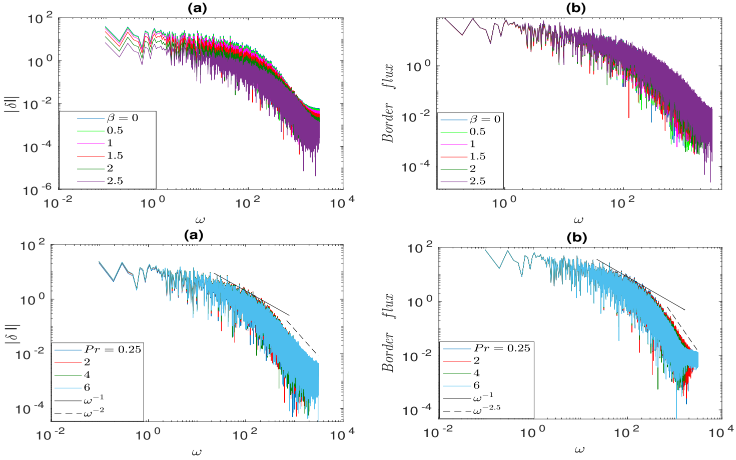


Figure: β scan (Top) and P_r scan (Bottom) of frequency spectra of intensity (a) and edge heat flux (b)

Intensity auto-correlation function (2d)

Asymmetry in speed of in-coming and out-going avalanches !

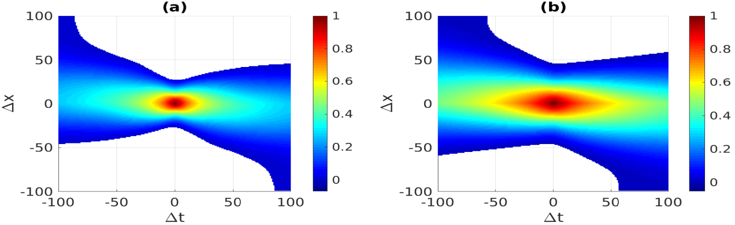


Figure: 2d auto-correlation of intensity at $Pr = 1$. (b) 2d auto-correlation of intensity at $Pr = 6$.

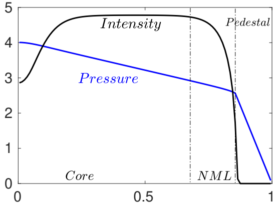
In-out velocity asymmetry increases with P_r !
Correlation length and correlation time increases with P_r.

Profile issues in H mode

What are the effects of spreading on H mode profile?

- Conventional wisdom: Pedestal height and width impact global confinement. The limiting stable height and width are believed to be set by P-B mode.
- At pedestal top: pressure gradient changes rapidly; flux continuous.
- Sharp variation in turbulence intensity across pedestal "corner".
- Strong intensity grdient in NML helps maintain flux continuity.
- Strong intensity near top of pedestal $\rightarrow$ pedestal performance?

What is turbulence spreading doing before pedestal hits P-B? $\rightarrow$Spreading effect on pedestal seems to be more important in P-B stable QH mode?



Spreading in H mode: 3 field model

We consider the following 3 field model consisting of intensity I, pressure P and density $n$none

$$\frac{\partial I}{\partial t} = \chi\left[\left(\left|\frac{\partial p}{\partial x}\right| - \mu_c\right)\Theta\left(\left|\frac{\partial p}{\partial x}\right| - \mu_c\right) - \lambda V_E'^2\right] I - \beta I^2 + \sigma \frac{\partial}{\partial x} I \frac{\partial I}{\partial x}$$

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\alpha_P I}{1+\epsilon V_E'^2} + D_{cP}\right)\frac{\partial P}{\partial x} + \phi_p$$

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\alpha_n I}{1+\epsilon V_E'^2} + D_{cn}\right)\frac{\partial n}{\partial x} + \phi_n$$

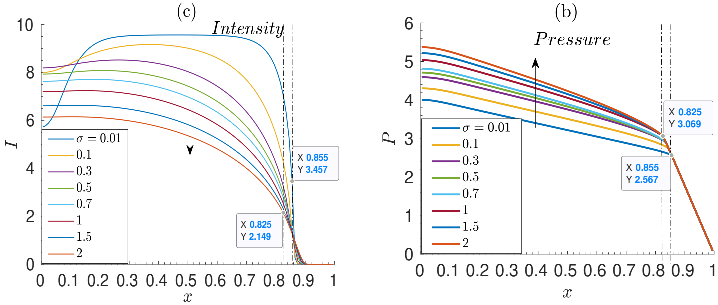
$E \times B$ velocity shear is obtained from the radial force balance without toroidal and poloidal flows

$$V_E' = -\frac{1}{eBn^2}\frac{dp}{dx}\frac{dn}{dx}$$

Pressure source is core localized $\phi_p = \phi_{0p} e^{-w_p x^2}$ and particle source is edge localized $\phi_n = \phi_{0n} e^{-(x-x_0)^2}$. Spreading effect is studied by varying σ.

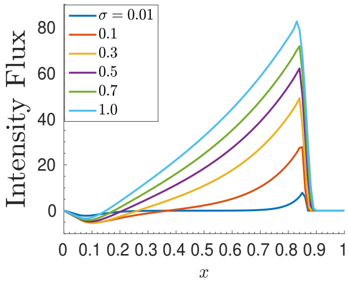
H mode results I

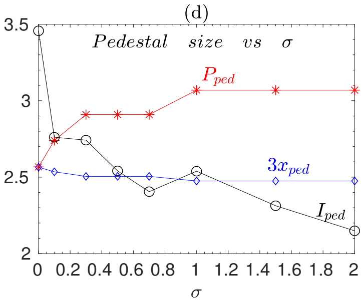
- Turbulence intensity is strongest in NML, when spreading is weakest.
- Intensity flux is radially outward in NML and inward in core.
- Outward spreading from NML→Pedestal increases and inward spreading in core decreases with σ.
- Decrease of intensity in NML→ increase of pedestal height and width.



H mode results II

- Turbulence spreads from NML$\rightarrow$ pedestal, where it is killed by strong $E \times B$ shear. Pedestal works as a sink of turbulence coming from NML.
- Pedestal height grows with turbulence reduction at NML.
- Width and height of pressure pedestal increase maintaining the pressure gradient.





Effect of additional non-diamagnetic shear (V'_ϕ) at NML

- Shear due to toroidal rotation added to diamagnetic shear at NML elevates the pedestal by reducing turbulence at NML !
- This appears consistent with wide pedestal QH mode transition in torque ramp down in DIII-D !

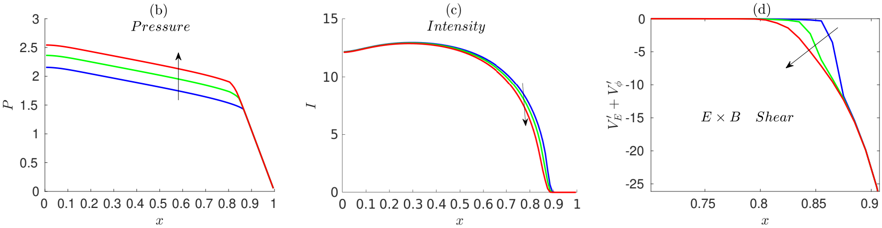
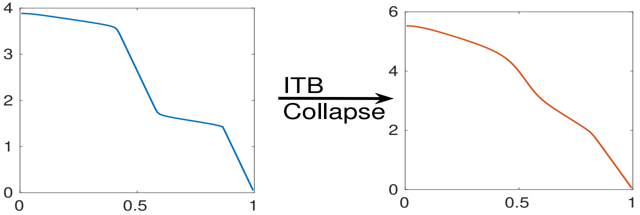


Figure: Radial profiles with $V'_\phi = V'_{\phi 0} \left[\Theta\left(x - 0.8\right) - \Theta\left(x - 0.86\right)\right]$ where $V'_{\phi 0} = 0, -1, -2$

Proposed Experimental Tests

- Spreading effect on pedestal can be seen in transient response of pedestal interacting with an intensity front.
- ITB collapse in a double barrier discharge (DBD) can be used to probe spreading effects on pedestal.
- Preliminary numerical simulations of spreading effect in DBD show that pedestal size increases at fixed pressure gradient after ITB shrinks..
- Hence following turbulence front and pressure profile evolution just after ITB collapse can elucidate the effect of spreading on pedestal.



Conclusions and Discussions

- Focus: Profiles.
- Spreading affects profiles weakly in L mode, due to weak intensity gradients.
- Avalanche distributions are weakly affected by spreading.
- H mode profiles are strongly affected by turbulence spreading due strong intensity gradient at interface connecting barrier and core. Turbulence in NML is reduced and pedestal height and width increases in response to spreading.
- Spreading is good for H mode confinement.
- Extremely hard to test spreading effect in G-K simulations and experiments as there is no external knob to controll spreading.
- Following transient response of pedestal after ITB collapse may elucidate spreading effect on pedestal height and width.
- Finally we argue that predictive models of pedestal structure must address NML turbulence and spreading effects.

Looking ahead: CTRW model of spreading

- Fokker Planck assumes finite drift and variance, time steps fixed. (Gaussian step size pdf p)
- Fluctuations pdf are often non-Gaussian with fat tails i.e., variance $\rightarrow \infty$.
- In CTRW times steps evolves as the walker position does. (waiting time pdf ψ)
- One can construct a **reaction-transport** equation for separable joint pdf $\xi(x-x', x'; t-t', t) = p(x-x', x'; t)\psi(x'; t-t')$

$$\frac{\partial I(x,t)}{\partial t} = f(I) + \int_0^t dt' \int dx' \phi(x'; t-t') p(x-x', x'; t) I(x', t') - \int_0^t dt' \phi(x; t-t') I(x, t')$$

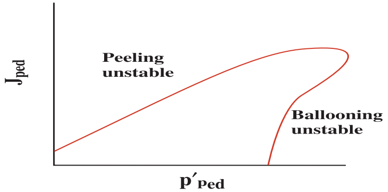
- Laplace transforms of the memory function ϕ the waiting time pdf ψ are related as $\phi(x; s) = s\psi(x,s)\left[1-\psi(x,s)\right]^{-1}$

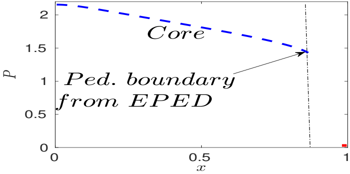
Finally I would say ...

MSGA: Make spreading great again !

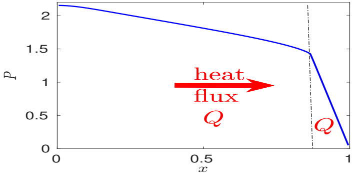
Back up slides

Boundary Condition vs Flux matching





- Pedestal parameters from EPED → BCs in GK codes for core turbulence simulation.
- But no spreading in EPED !



- Flux is continuous at pedestal top.
- Flux continuity takes care of turbulence spreading and vice-versa.
- Flux continuity is more accurate than fixing BC for core simulation with EPED.

Evidences of turbulence spreading

In experiments:

- Anomalous transport in linearly stable zone of JT-60U reversed shear plasma.[Nazikian etal 2005]
- Anomalous transport in the core of NSTX. [Kaye etal 2007]
- Drop of core intensity after H mode transition in DIII-D in time much shorter than transport time scale.[McKee etal 2007]
- Density fluction rise on H-L back transition in TJ-2. [Estrada etal 2011]
- Turbulence spreading through magnetic island.[Choi etal 2017, Ida etal 2018]

In simulations:

- Garbet etal 1994, Sydora etal 1996, Parker etal 1996, kishimoto etal 1996, Lee etal 1997, Lin and Hahm 2004, Hahm etal 2004.
- Spreading through ITB. [Yagi etal 2006]
- Spreading through magnetic island. [Poli etal 2009, 2010]

Looking back: Theoretical approaches II

A Fokker Planck theory [Gurcan etal 2005]

$$
\begin{aligned}
I(x, t+\Delta t) &= \left[\gamma(x) I(x) - \gamma_{NL}(x) I^2(x)\right] \\
&\quad + \int d\Delta x T(x-\Delta x, \Delta x, \Delta t) I(x-\Delta x, t)
\end{aligned}
$$

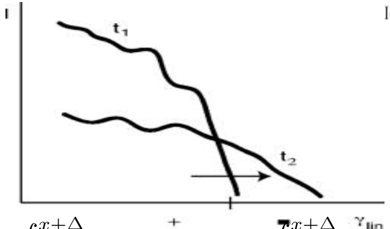
Assuming $\Delta I'/I < 1$, the integrand can be expanded and noting that $\int d\Delta x T = 1$, $\int d\Delta x T \Delta x = \langle \Delta x \rangle$ and $\int d\Delta x T \Delta x^2 = \left\langle \Delta x^2 \right\rangle$

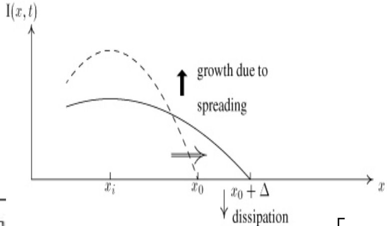
$$
\frac{\partial I(x,t)}{\partial t} = \left[\gamma(x) I(x) - \gamma_{NL}(x) I^2(x)\right] - \frac{\partial}{\partial x}\left[\frac{\langle \Delta x \rangle}{\Delta t} I\right] + \frac{\partial^2}{\partial x^2}\left[\frac{\left\langle \Delta x^2 \right\rangle}{2\Delta t} I\right]
$$

Now drift: $V_I = \frac{\langle \Delta x \rangle}{\Delta t}$ and diffusion $D_I = \frac{\left\langle \Delta x^2 \right\rangle}{\Delta t}$ and using $V_I = \frac{1}{2}\frac{\partial D_I}{\partial x}$ yields

$$
\frac{\partial I(x,t)}{\partial t} = \left[\gamma(x) I(x) - \gamma_{NL}(x) I^2(x)\right] + \frac{\partial}{\partial x}\left[D_I \frac{\partial I}{\partial x}\right]
$$

Intensity front dynamics



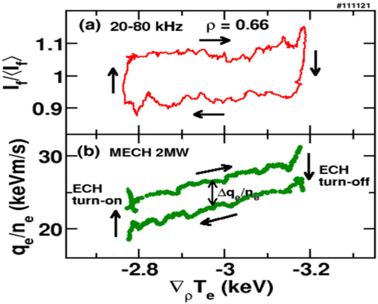


$$\frac{\partial}{\partial t}\int_{x-\Delta}^{x+\Delta} dx I(x,t) \quad = \quad \int_{x-\Delta}^{x+\Delta} dx\left[\gamma(x)I(x)-\gamma_{NL}(x)I^2(x)\right]+\left[D_I\frac{\partial I}{\partial x}\right]_{x-\Delta}^{x+\Delta}$$

- Total turbulence intensity in a layer of width 2Δ can **grow(reduce)**, **even for negative (positive)** $\gamma(x)$, as long as $\left[D_I\frac{\partial I}{\partial x}\right]_{x-\Delta}^{x+\Delta}$ is **sufficiently large (negative).**
- **Profile of turbulence intensity is crucial for its spatio-temporal evolution.**
- **No propagation in linearly damped region.** Front stops when growth due to spreading balances dissipation due to damping. **Spreading length** $\lambda \approx \sqrt{D/\gamma_{nl}}$ **is a few correlation lengths.**

Subcritical turbulence spreading

Hysteresis between flux-gradient and fluctuation-gradient in L-mode suggests bistability [Inagaki etal 2013].



$$\frac{\partial I(x,t)}{\partial t} = \chi\left[\alpha I(x) + 2\beta I^2(x) - I^3\right] + \frac{\partial}{\partial x}\left[D(I)\frac{\partial I}{\partial x}\right]$$

- **Supports propagating solutions in linearly damped region with reduced intensity and speed.**