Rossby Wave-Zonal Flow Turbulence in a Tangled Magnetic field

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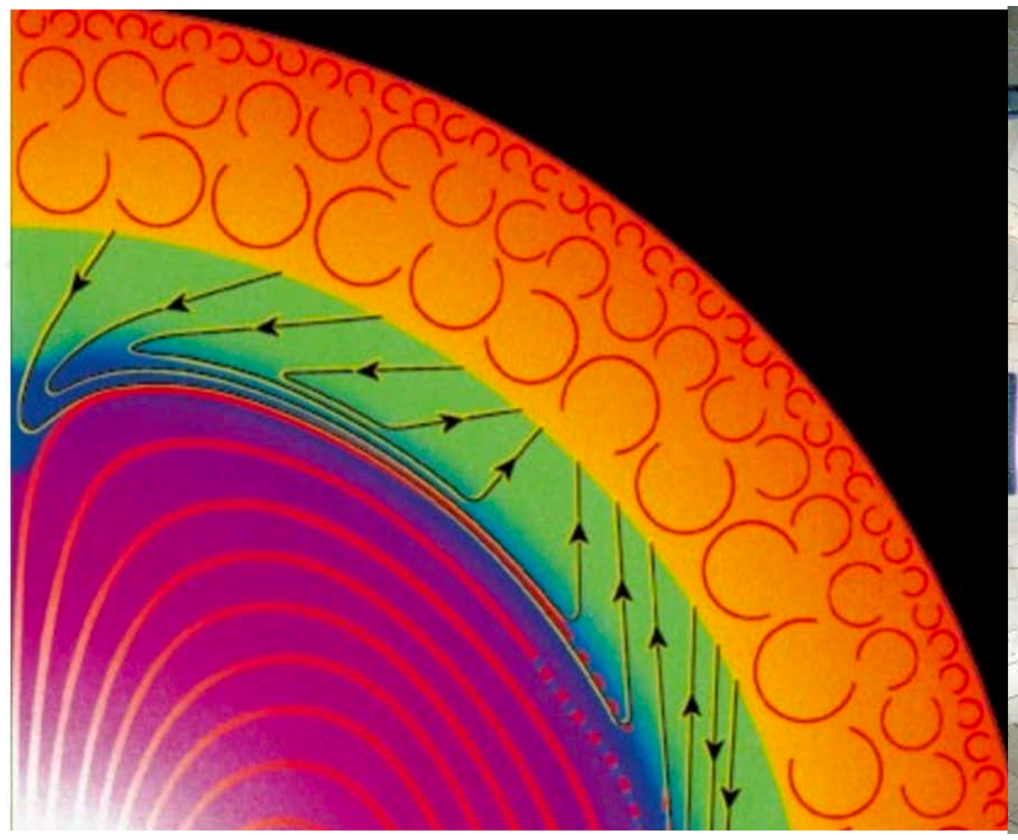
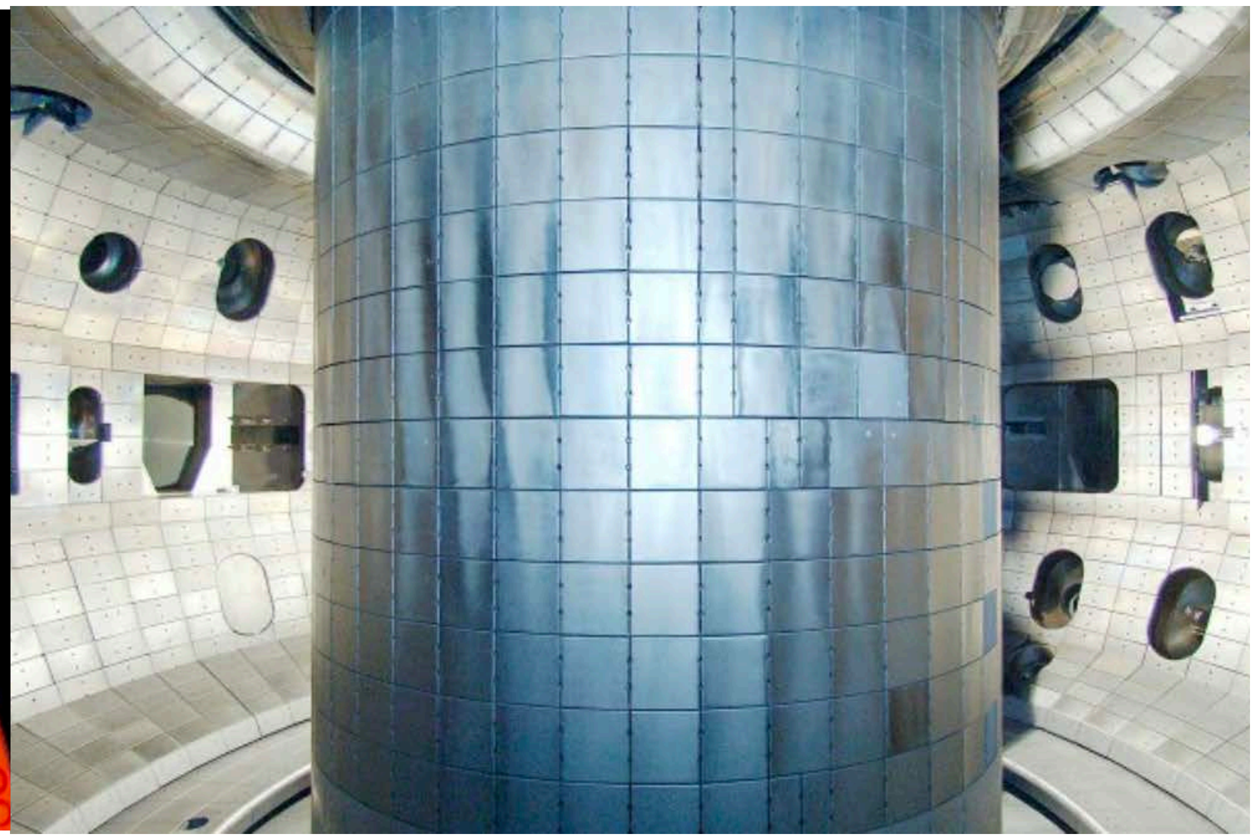
Introduction

- **Why we study tangled magnetic fields?**
  1. Astro: Interstellar Medium and Solar tachocline.
  2. Fusion: energy and momentum transport problem in L-H transition at edge of tokamaks with resonant magnetic perturbation (RMP).

  Turbulent transport processes still poorly understood.

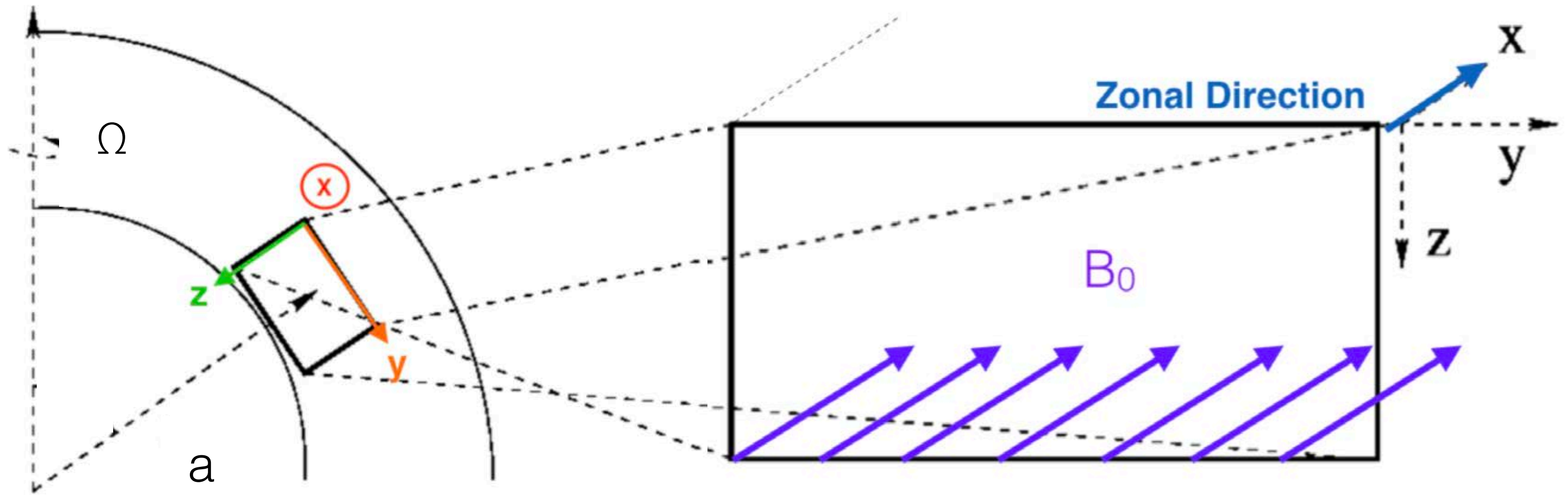




- **We focus on the solar tachocline:**
  1. Between the convective and radiative zone.
  2. Strongly stratified/Pancake-like structures.
     Incompressible rotating fluid in 2D layers— β-plane model
  3. Zonal Flow and Rossby Waves— as in the Jovian Atmosphere.
  4. A weak mean field— large magnetic Kubo number:

$$\beta = \frac{df}{dy}\Big|_{\phi_0} = 2\Omega cos(\phi_0)/a$$

Derivative of angular frequency f (Coriolis parameter); rotation; latitude; radius



$$B = B_0 + \widetilde{B}$$

$$Ku_{mag} \equiv \frac{\delta_l}{\Delta_{eddy}} = \frac{l_{ac}|\widetilde{\mathbf{B}}|}{\Delta_{eddy}B_0}$$

Quasi-Linear Theory (QLT) fails.

Need a model beyond QLT.

Outline

- **A model for PV transport in Strong mean magnetic fields**

- A Model for PV Transport in Random, small-scale Magnetic fields

Model— Strong mean field & The Quasi-Linear Method

Notations we have:

- Stream Function $\psi = \psi(x,y,z)$
- Velocity field $\boldsymbol{u} = (\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x}, 0)$
- Fluid Vorticity $\boldsymbol{\zeta} = (0,0,\zeta)$
- Potential Field $\boldsymbol{A} = (0,0,A)$
- Magnetic Field $\boldsymbol{B} = (\frac{\partial A}{\partial y}, -\frac{\partial A}{\partial x}, 0)$,

Two main equations:

➤ **QL closure**

➤ **Linear response of perturbations**

$$(\frac{\partial}{\partial t} + \mathbf{u}_\perp \cdot \nabla_\perp)\zeta - \beta\frac{\partial\psi}{\partial x} = -\frac{(\mathbf{B}\nabla)(\nabla^2 A_z)}{\mu_0\rho} + \nu\nabla^2\zeta$$

$$(\frac{\partial}{\partial t} + \mathbf{u}_\perp \cdot \nabla_\perp)A = B_0\frac{\partial\psi}{\partial x} + \eta\nabla^2 A,$$

Quasi-Linear Approximation:

$$\zeta = \langle\zeta\rangle + \tilde{\zeta}$$
$$\psi = \langle\psi\rangle + \tilde{\psi}$$
$$A = \langle A\rangle + \tilde{A}$$

Perturbations produced by turbulences

, where $\langle\ \rangle = \frac{1}{L}\int dx \frac{1}{T}\int dt$

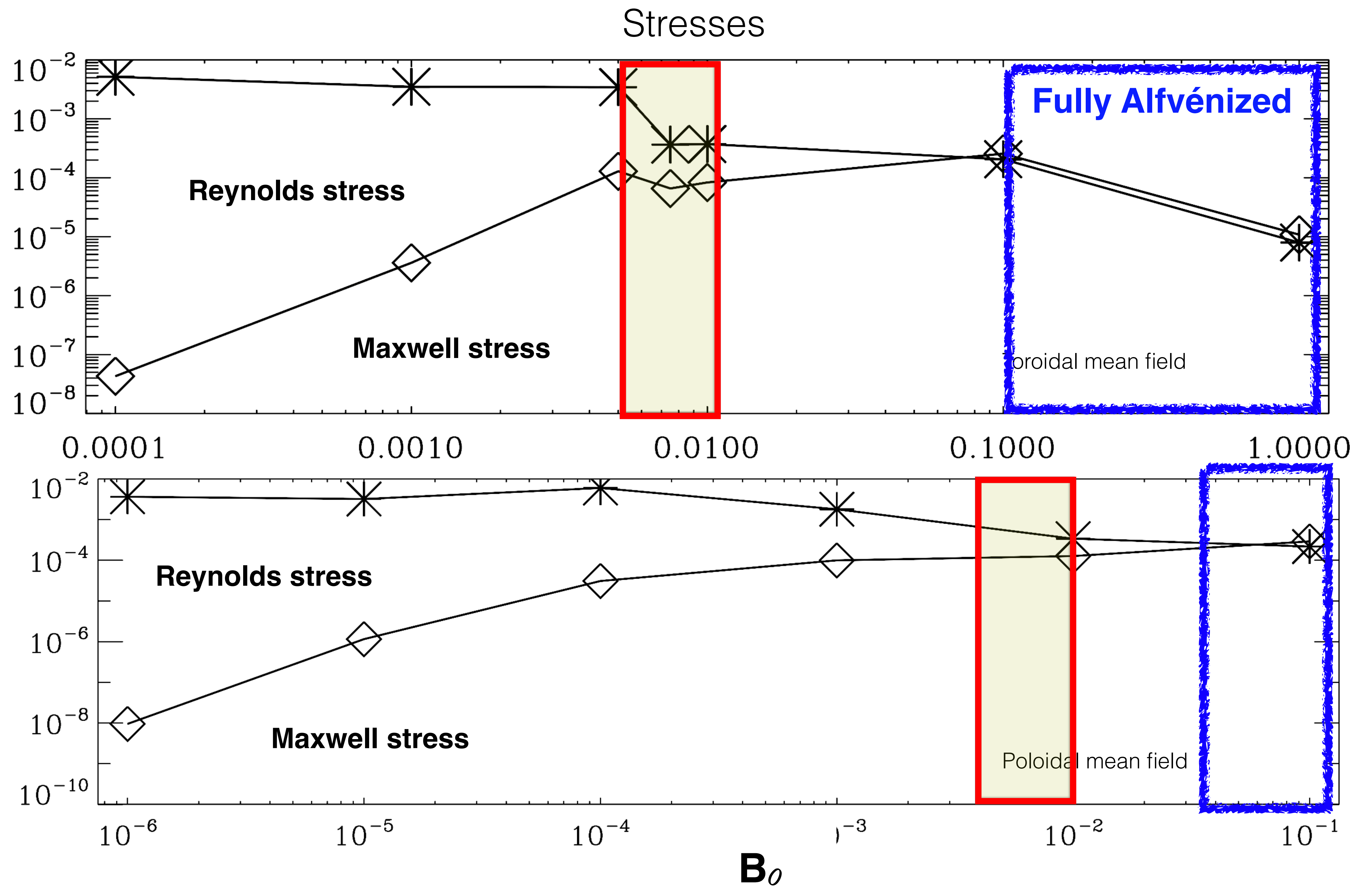
ensemble average over the zonal scales

PV flux:

➤ **By using Taylor Identities we have the Reynolds and Maxwell stress.**

➤ **Express the PV flux with two diffusivities**

$$\begin{cases} D_{fluid} = \sum_k |\tilde{u}_{y,k}|^2 \dfrac{\nu k^2 + \omega_A^2 \frac{\eta k^2}{\omega^2+\eta^2k^4}}{\left(\omega - \omega_A^2\frac{\omega}{\omega^2+\eta^2k^4}\right)^2 + \left(\nu k^2 + \omega_A^2\frac{\eta k^2}{\omega^2+\eta^2k^4}\right)^2} \\ D_{mag} = \sum_k |\tilde{u}_{y,k}|^2 \dfrac{\omega_A^2\left(\nu k^2(\omega^2+\eta^2k^4) + \omega_A^2\eta k^2\right)}{\omega^2\left(\omega^2+\eta^2k^4-\omega_A^2\right)^2 + \left(\nu k^2(\omega^2+\eta^2k^4)+\omega_A^2\eta k^2\right)^2}. \end{cases}$$



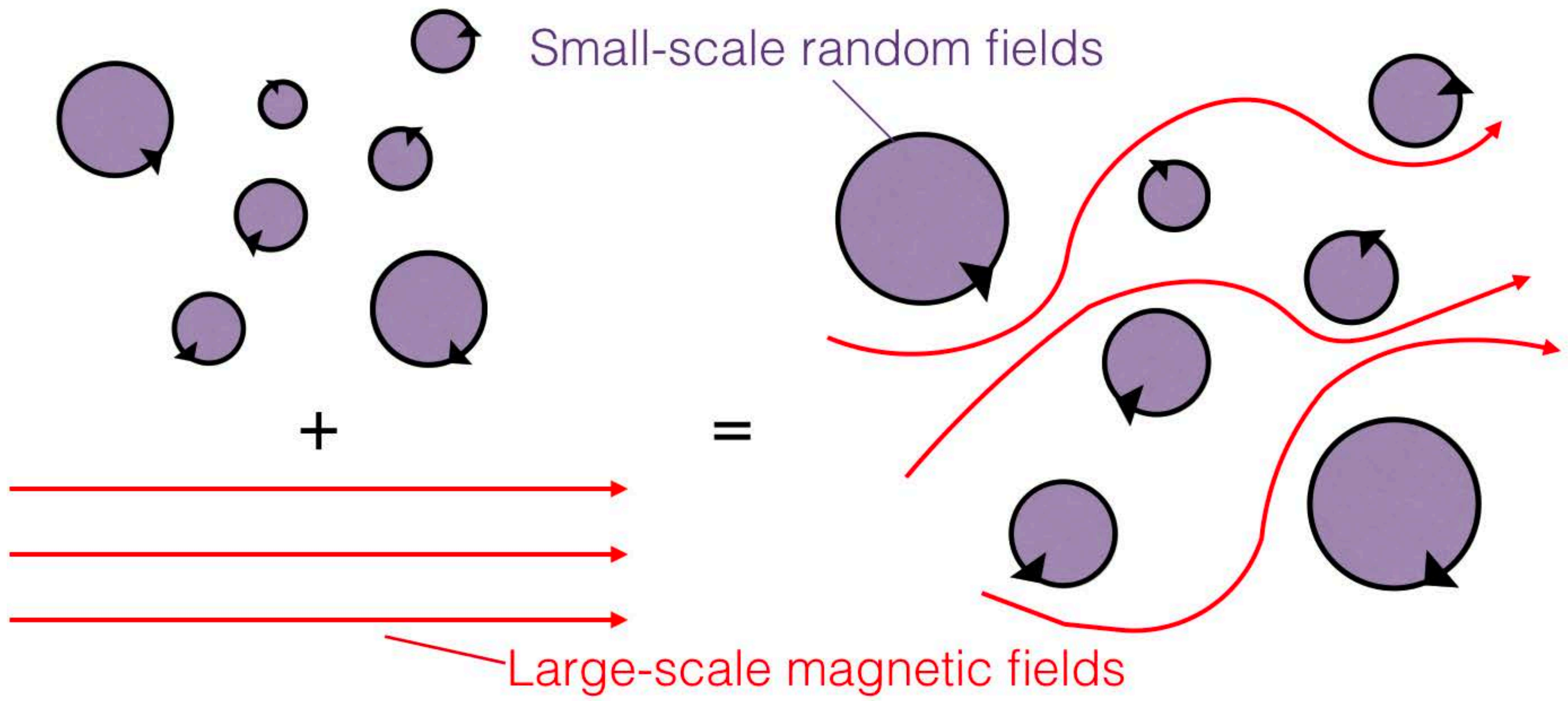
(Tobias et al. in preparation)

The Reynolds stress drops occurs at weaker B_0 than that for which the system is fully Alfvènized! → **The cross phase effect suppresses the Reynolds stress when mean field is weak!**

Outline

- A model for PV transport in Strong mean magnetic fields
- **A Model for PV Transport in Random, small-scale Magnetic fields**

Model— Random fields: Order of Scales

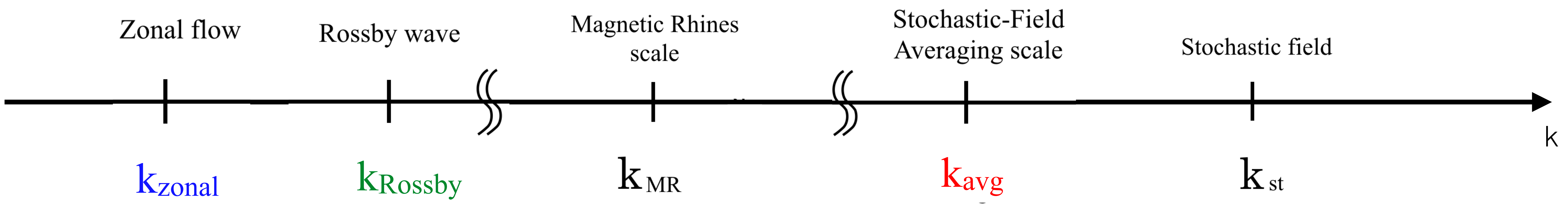


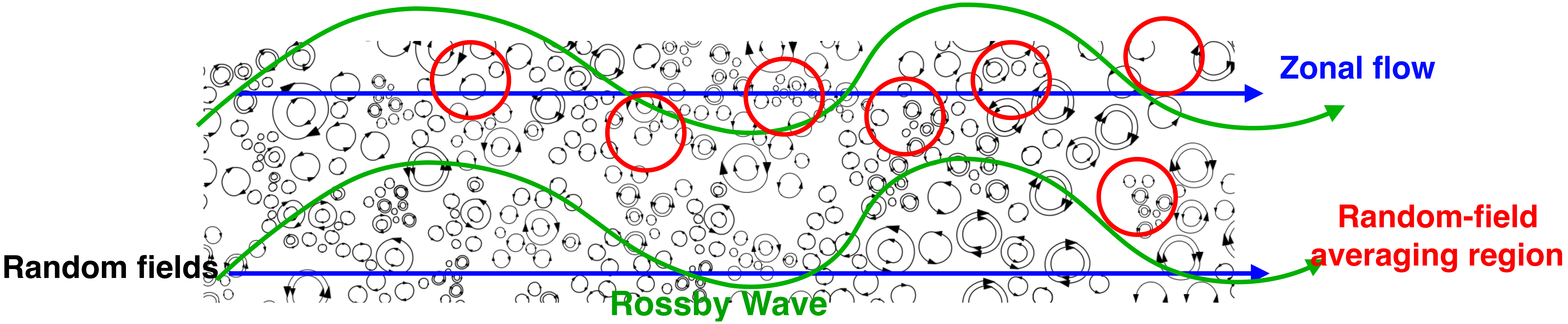
The large-scale magnetic field is distorted by the small-scale fields. The system thus is the 'soup' of cells threaded by sinews of open field line (Zeldovich, 1957).

- **Properties of random fields are (Rechester & Rosenbluth 1978):**
  1. Smaller scale, Static
  2. Randomness in space
  3. Auto-correlation length of fields is small. ⟶ $Ku_{mag} \equiv \frac{\tilde{u}\tau_{ac}}{\Delta_{eddy}} = \frac{l_{\parallel}|\tilde{\mathbf{B}}|}{\Delta_{\perp}B_l} < 1$
  4. Amplitudes of random fields distributed statistically. (assumption: PDF Gaussian)

- **Two-average Method:**
  1. $\overline{F} = \int dR^2 \int dB_{st} \cdot P_{(B_{st,x},B_{st,y})} F$
  2. $\langle\!\langle\ \rangle\!\rangle = \frac{1}{L}\int dx \frac{1}{T}\int dt$ ensemble average over the zonal scales

- **Order of Scales:** Function of fields $\mathbf{F} = \mathbf{F_0} + \widetilde{\mathbf{F}} + \mathbf{F_{st}}$





Model— Random fields: Assumption, Derivation & Results

- **Assumptions:**

1. The collective field at Rossby-scale is NOT large enough to alter the structure of the random fields: $\widetilde{B_{st}} \rightarrow 0$

2. We approximate the correlation matrix as diagonal: $\overline{B_{r,x}B_{r,y}} = 0$

- **Derivation:**

Two main equations:

$$\begin{cases} \frac{\partial}{\partial t}\overline{\zeta} - \beta\frac{\partial\overline{\psi}}{\partial x} = \frac{\overline{(\mathbf{B}\cdot\nabla)J}}{\mu_0\rho} + \nu\nabla^2\overline{\zeta} \\ \frac{\partial}{\partial t}A = \mathbf{B}\cdot\nabla\psi + \eta\nabla^2 A\,. \end{cases}$$

Key term: Average effect of J x B

Linear response of the vorticity:

$$\widetilde{\zeta_k} = \left(\frac{i}{\omega + i\nu k^2 + \frac{i\overline{B^2_{st,j}}k_j^2}{\mu_0\rho\eta k^2} + \frac{i}{\mu_0\rho}\frac{B_0^2k_x^2}{\eta k^2 - i\omega}}\right)\widetilde{u}_{y,k}\left(-\frac{\partial}{\partial y}\overline{\zeta} - \beta\right)$$

- **Dispersion relation of the Rossby-Alfvén wave in random magnetic fields:**

$$\left(\omega - \omega_R + \frac{i\overline{B^2_{st,y}}k_y^2}{\mu_0\rho\eta k^2} + i\nu k^2\right)\left(\omega + i\eta k^2\right) = \frac{B^2_{0,x}k_x^2}{\mu_0\rho}$$

(mean square) (square mean)

Dissipative response to Random magnetic fields

AW of the large-scale magnetic field

Random fields: Result I— Cross-Phase Modification

- **Mean PV Flux (Γ) and PV diffusivity (D_{PV}):**

$$\bar{\Gamma} = -\frac{\partial}{\partial y}\langle \widetilde{u}_x \widetilde{u}_y \rangle$$ (Reynolds stress force)

$$\bar{\Gamma} = -\sum_k |\widetilde{u}_{y,k}|^2 \frac{\nu k^2 + (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\eta k^2}{\omega^2 + \eta^2 k^4} + \frac{\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho \eta k^2}}{\left(\omega - (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\omega}{\omega^2 + \eta^2 k^4}\right)^2 + \left(\nu k^2 + (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\eta k^2}{\omega^2 + \eta^2 k^4} + \frac{\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho \eta k^2}\right)^2}\left(\frac{\partial}{\partial y}\bar{\zeta} + \beta\right)$$

$$\bar{\Gamma} = -D_{PV}\left(\frac{\partial \bar{\zeta}}{\partial y} + \beta\right)$$

PV Diffusivity $\mathbf{D_{pv}}$

Large-scale field

small-scale random fields

The large- and **small-scale magnetic fields** have a **synergistic effect on the cross-phase** in the Reynolds stress.

In a certain limit where $B_0^2 \ll \overline{B_{st,y}^2}$:

$$D_{PV} = \sum_k |\widetilde{u}_{y,k}|^2 \frac{\nu k^2 + \frac{\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho \eta k^2}}{\omega^2 + \left(\nu k^2 + \frac{\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho \eta k^2}\right)^2}$$

Cross-phase effect in the Reynolds stress **decreases** for stronger random fields.

Random fields: Result II— Resisto-Elastic Medium

Evolution of Zonal Flow

➤ Random magnetic fields suppress the Reynolds stress and increase the drag.

$$\frac{\partial}{\partial t}\langle u_x\rangle = \langle\overline{\Gamma}\rangle - \frac{1}{\eta\mu_0\rho}\langle\overline{B_{st,y}^2}\rangle\langle u_x\rangle + \nu\nabla^2\langle u_x\rangle$$

Cross-phase effect on Reynolds Stress Force

Magnetic drag force $(J_{st} \times B_{st})$

Random magnetic fields have an effect on both the **PV flux** and the **magnetic drag.**

Two effects!

Resisto-Elastic medium:

If we turnoff the Rossby frequency, we have a 2D non-rotating plane:

$$\omega^2 + i(\alpha + \eta k^2)\omega - (\alpha\eta k^2 + \chi) = 0$$

drag + resistivity dissipation

effective spring constants

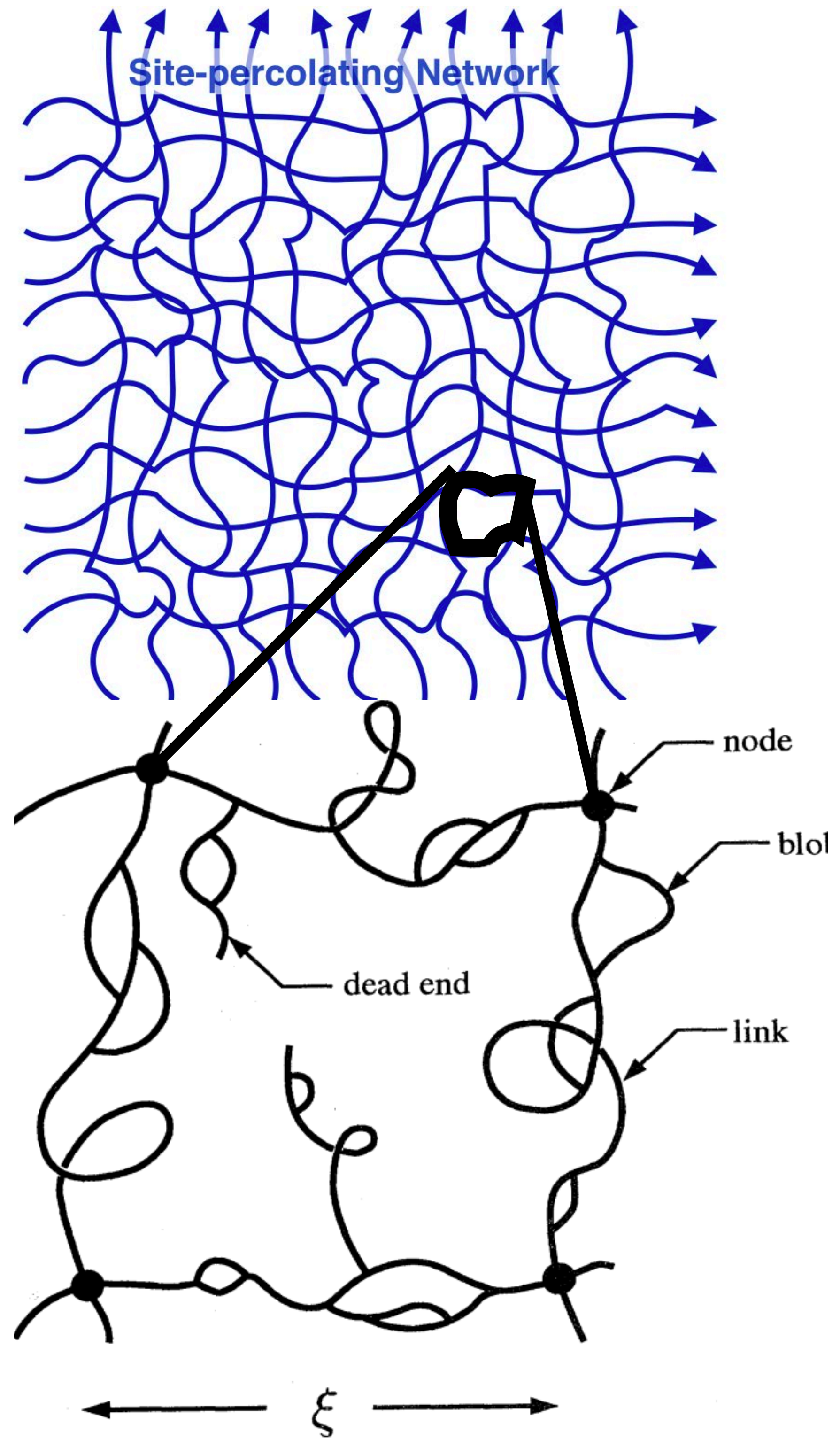
$$\alpha \equiv \frac{\overline{B_{st,j}^2}k_j^2}{\mu_0\rho\eta k^2} \propto \frac{\text{spring constant}}{\text{dissipation}}$$

$$\chi \equiv \frac{B_0^2 k_x^2}{\mu_0\rho}$$

→ **Alfvèn waves propagate in a resisto-elastic** medium. Energy dissipates due to the coupling of the drag and the resistivity.

More can be done:

Fractal Network (Site-percolating) —
Calculate the effective spring constant, effective Young's Modulus of elasticity, and effective "conductivity" of vorticity (such as encountered in amorphous solids).



Schematic of the nodes-links-blobs model (Nakayama & Yakubo 1994).

Takeaways I

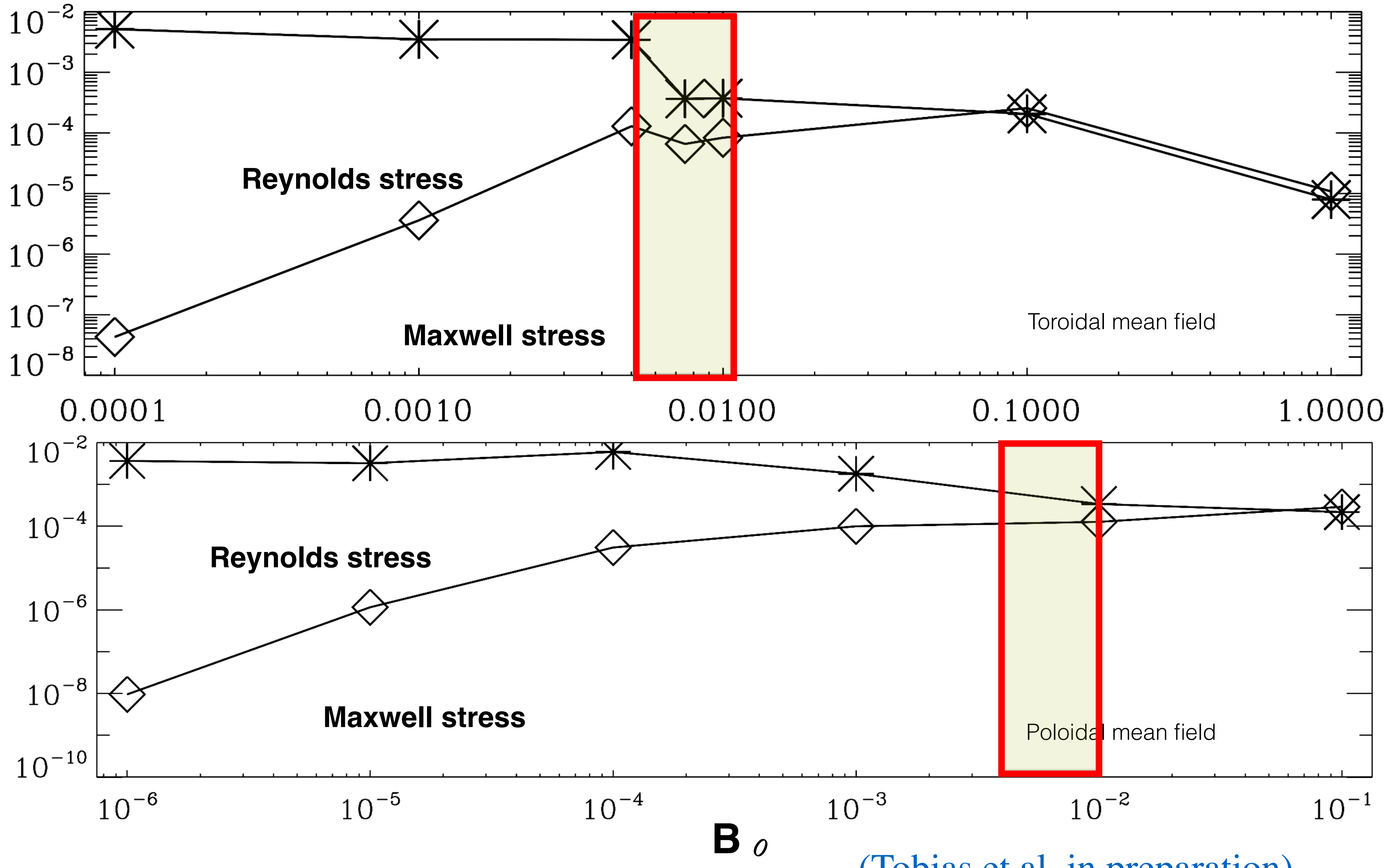
- **What studies have shown and what we obtained:**

Reynolds stress will be suppressed at levels of field intensities **well below that of Alfvènization**, where Maxwell stress balances the Reynolds stress.

The flow generated by PV mixing/Reynolds force are reduced by:

- Coupling of resisto-elastic waves, which is $\overline{B_{st}^2}$ dependence.
- Increase of the magnetic drag.

Cross-phase effect and **the magnetic drag** reduce shear flow generation.



(Tobias et al. in preparation)

Takeaways II— RMP on DIII-D

- **Related: Experiment on edge of DIIID:**

1. Stochastic magnetic fields due to RMP inhibits Reynolds stress and shear suppression.

This work is relevant— close analogy between Rossby-Alfvèn wave and EM drift wave turbulence.

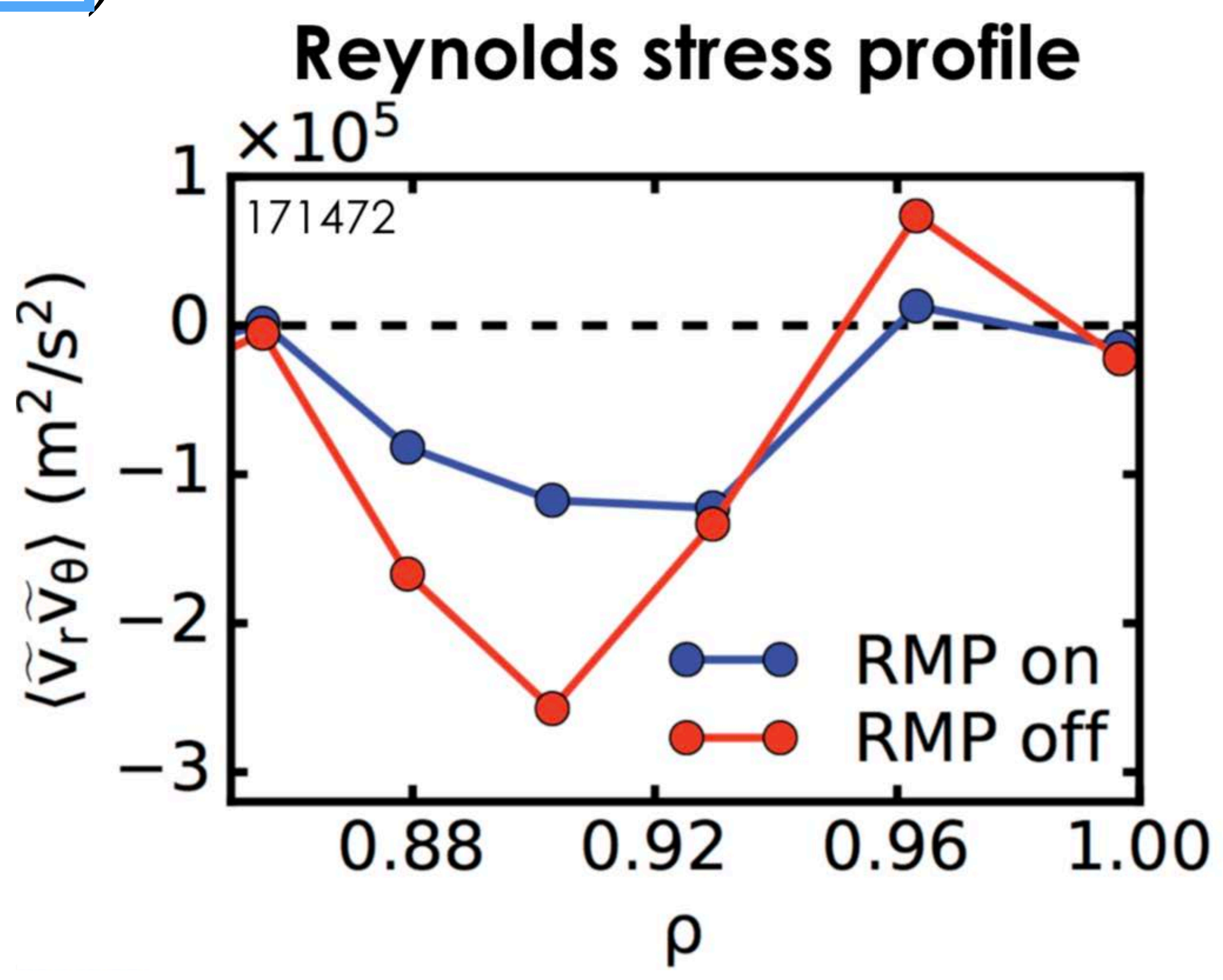
$$\overline{\Gamma} = -\sum_k |\tilde{u}_{y,k}|^2 \frac{\nu k^2 + (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\eta k^2}{\omega^2 + \eta^2 k^4} + \boxed{\frac{\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho \eta k^2}}}{\left(\omega - (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\omega}{\omega^2 + \eta^2 k^4}\right)^2 + \left(\nu k^2 + (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\eta k^2}{\omega^2 + \eta^2 k^4} + \boxed{\frac{\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho \eta k^2}}\right)^2} \left(\frac{\partial}{\partial y}\overline{\zeta} + \beta\right)$$

2. Our results imply: Reduction of Reynolds stress, due to the effect of the stochastic field $\boxed{\overline{B_{st}^2}}$ in cross-phase, inhibits the growth of zonal flow, by coupling energy into elastic waves.

Hence, the **nonlinear energy transfer** from turbulence to flow and thus **shear suppression** at the edge are reduced.

(See experiments from D. M. Kriete et al.)

3. Conjecture: Stochasticity of fields might raise the power threshold, by weakening Reynolds stress trigger.



(D. M. Kriete et al., TTF 2019)

Thank you!