L→H Transition in a Stochastic Magnetic Field

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Re: A. A. Galeev

Tremendous, well known contributions to magnetic confinement theory and basic plasma theory:

- Weak Turbulence Theory: “Sagdeev and Galeev” books
- Neoclassical Transport, Plasma Rotation
- Reconnection and Tearing

What else?

Some personal favorites:

- Galeev, '67: Seminal paper on renormalization and DIA-like approach to Vlasov Turbulence
  - Resonance broadening theory encompassed
  - Noted applicability connected to absence of decay instability (particle vs. wave processes)
  - Cautioned on applicability of $\gamma / k_\perp^2$, dimensional estimates – A lesson ahead of its time...
  - Remarkable intellectual honesty and clarity!

- Galeev, Rosner, Vaiana, ‘76: Hot AGN corona from buoyant flux of loops stretched by V'_θ :
  - End state of magnetic viscosity driven accrection (c.f. Eardley, Lightman); now more significant, as MRI appreciated
  - Concepts of ‘disk flares’
- Galeev, Kuznetsova, Zeleny, ’85 et. sequence
  - ‘Patchy reconnection’ due stochastic fields, magnetic percolation ✱ — micro-macro connection of intermittency
  - Linked to local drift-tearing dynamics

The Rest:

- Changing focus of MFE theory
- ELM and RMP: Benefits and Costs
- L→H Transition in Stochastic Magnetic Field
  - Status
  - Revisiting ZF generation
    - Heuristics
    - Analysis
  - Implications
- Turbulenct transport of momentum and particles in stochastic B-field

Evolution of MFE Theory

Prehistory: 3D

- Beginnings: 60’s ~ 1980

  Trieste
  Micro-stability
  Neoclassical theory
  Disruption models
  Taylor Relaxation

  T3
  Alcator A
  PLT
  TFR

  Galeev still
  active in program

- Understanding Good Confinement: 1980 ~ 2010

  [Self-Organization]

  ExB shear, ZF’s
  Transport Bifurcations
  Gyrokinetics, Simulation
  AE modes
  Intrinsic Rotation

  ASDEX → H-mode
  Alcator C, C-Mod → pellet, n-limit
  TFTR, JET → D-T
  DIII-D → ETBs, ITBs
  JT-60U → ETBs, ITBs

Evolution of MFE Theory

- Good Confinement + Good Power Handling → ITER: 2010 – Present, and beyond

ELMs, Peeling-Ballooning
RMP, QH-mode
Multi-scale problems
Core-Edge coupling,
  Turbulence Spreading
Disruptions (?)
SOL Heat Loads (?)

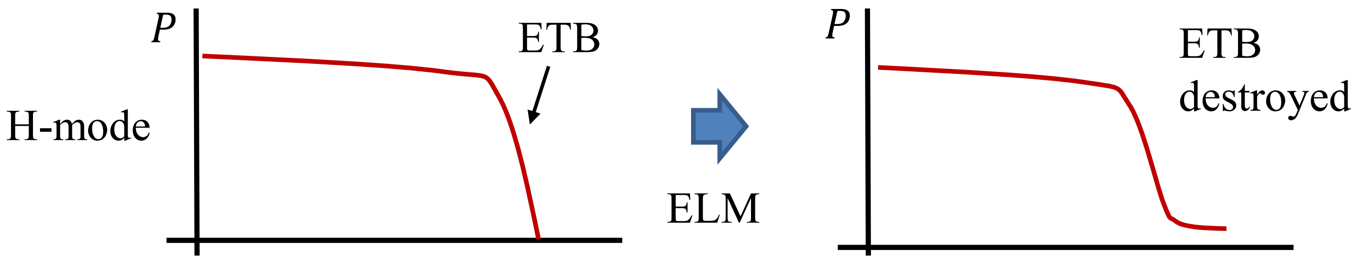
DIII-D, AUG
Alcator C-Mod
LHD
W7X
RFX-QSH *
EAST, KSTAR
...
...

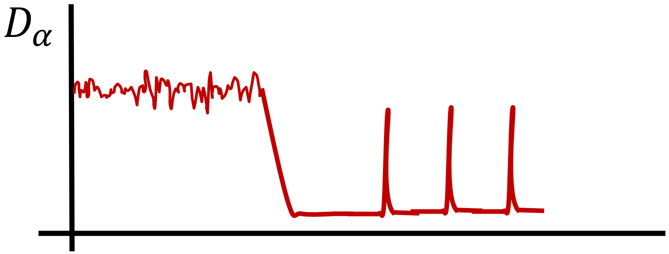
N.B.:
Return to 3D !

→ Theory must address trade-offs

ELMs and RMP – A Primer

- ELM = Edge Localized Mode (Mode ?!)



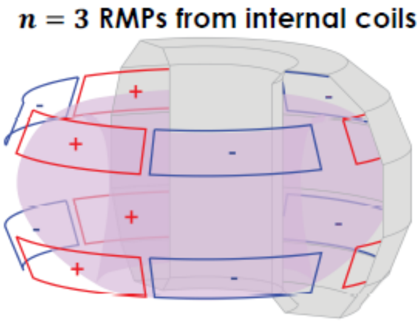


← ELM cycle

ITER: $\Delta W_{ELM} \sim 20\ MJ$!

→ Heat loads on PFC !?

- RMP = Resonant Magnetic Perturbation - δB
  - Stochastic edge
  - Pump out - density
  - Mitigate, suppress ELMs, with good confinement



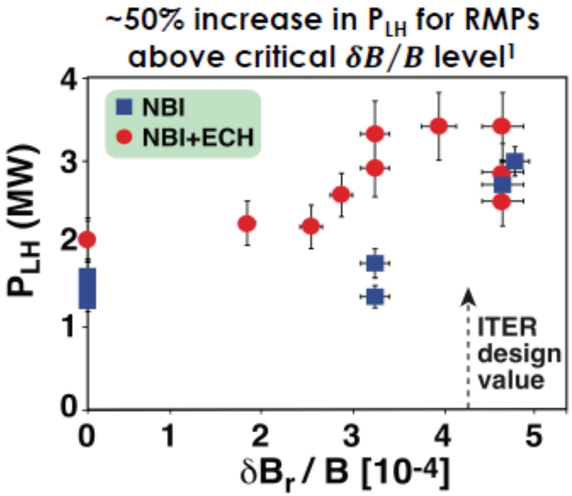
to ITER

Benefit and Cost

- Need make L→H Transition with RMP !
- Increase in P_{th} for L→H !?
  - $(\delta B/B)_{crit}$ for

    L→H Power increase
  - Significant !
- Issues:
  - Why L→H threshold ↑ due RMP
  - What physics defines $(\delta B/B)_{crit}$?
  - What Else?

"First ELM the largest"

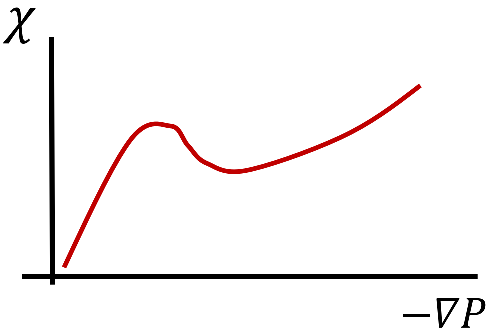

~50% increase in P_{LH} for RMPs above critical $\delta B/B$ level[1]

(resonant vs. non-resonant)

Kriete et. al.
DIII-D

Theoretical Problem: L→H Transition in a Stochastic Magnetic Field

- What of L→H ? → Converging, though still somewhat controversial



- Fundamentals:
  - Transport bifurcation
  - Bistability essential – S curve
  - Robust feeadback channel – ExB shear flows
  - Insulation layer at the edge...

$$\chi_T = \chi_T(V'_{E\times B}/\omega)$$

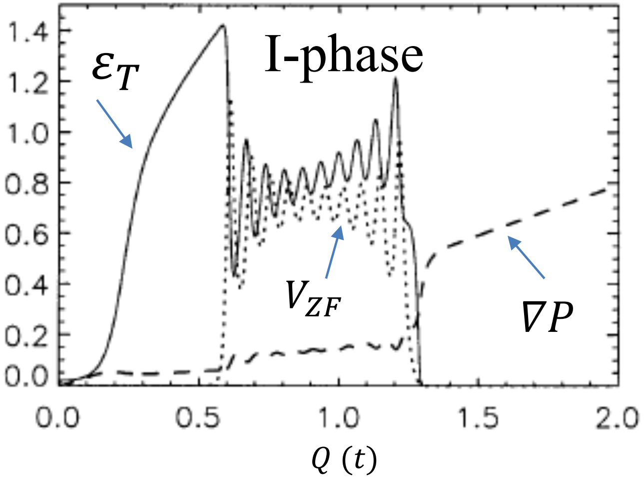
$\chi_T \downarrow$ for $V'_{E\times B}/\omega$ > crit.

$$V_{E\times B} = \nabla P/n + \cdots$$

- Subtleties
  - What is the “trigger”? → i.e.,
  - What physics allows ∇P to steepen?
- Coupling of energy to edge zonal flow
  - Interplay of $\varepsilon_T, V_{ZF}, \nabla P$
  - $P_{Reynolds}$ crit. needed,

    measured (Tynan)
  - Crucial to note <u>$E \times B$ flow</u>

Kim, PD, PRL’03



Implications

(c.f. Samantha Chen, P.D., et al)

- Modest stochastic field $\delta B/B \geq 10^{-3} \sim 10^{-4}$ unlikely to have much effect on $\nabla \mathrm{P_i}$ so...
- Effect on L→H transition via interference with trigger mechanism
  - Prevent / Retard ZF excitation! , i.e. reduce Reynolds power
  - Relate $(\delta B/B)_{crit}$ to ZF physics !?
  - What is physics of ZF dynamics in stochastic field ?!
  - Fundamental question...

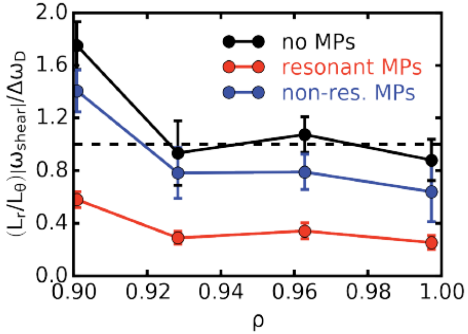
Resonant Magnetic Perturbations Disrupt Shear Suppression of Turbulence, Increasing the L-H Power Threshold

- **RMPs reduce flow shear rates ω_{shear} and raise turbulence decorrelation rates $\Delta\omega_D$ in L-mode**

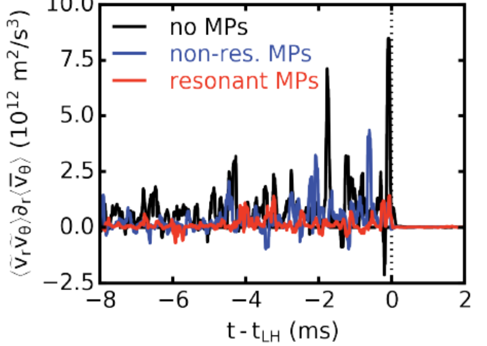
* **Shear suppression parameter $\omega_{shear}/\Delta\omega_D$ is reduced significantly below 1**
  - More shear flow must be driven to access H-mode

* **RMPs disrupt nonlinear energy transfer from turbulence to flows that can trigger L-H transition**

Shear suppression parameter



nonlinear energy transfer bursts



- <u>Reynolds Stress Decoherence</u>
  - N.B. Tobias et. al. simulations of β-plane MHD ZF evolution indicate drop in $\langle V_y V_x \rangle$ prior to Reynolds vs Maxwell balance, c.f. Chen, P.D., 2020, ApJ.
- Reynolds Stress <u>Decoherence</u> ➔ <u>Dephase</u> V_r, V_θ
- N.B. Amplitude change negligible.

Reynolds Stress Decoherence - Heuristics

- ZF Mechanism – Basics ➔ Modulation, etc

$$\langle \tilde{V}_x \tilde{V}_y \rangle = \sum_k c^2 \frac{|\phi_k|^2}{B_0^2} \overset{?}{\langle k_x k_y \rangle}$$

"Tilting Instability" – IKI classic

- What 'correlates' k_x and k_y? ➔ Shear Flow!

$$\frac{d}{dt} k_x = -\frac{\partial}{\partial x} \left(\omega_{0,k} + k_y \langle V_y \rangle \right) \approx -k_y \langle V_y \rangle'$$

$$\therefore \underset{\uparrow}{\langle k_x k_y \rangle} \approx -k_y^2 \, \underset{\uparrow}{\langle V_y \rangle'} \tau_c$$

Growing shear reinforces correlation

➔ Shear Growth

What of Magnetic Chaos Effects?

- Recall Classic: $\omega^2 - \omega_D \omega - k_\parallel^2 V_A^2 = 0$

  $\omega_D = \omega_* / (1 + k_\perp^2 \rho^2)$

  Drift-Alfven
  Dispersion
  Relation

- Now: $k_\parallel = k_\parallel^{(0)} + \vec{b} \cdot \vec{k}_\perp$ → static, stochastic field

  $\omega = \omega_0 + \delta\omega$

  (after MNR)

$$\langle \delta\omega \rangle \approx \frac{V_A^2}{\omega_0} \langle 2 k_\parallel \vec{b} \cdot \vec{k}_\perp + \left(\vec{b} \cdot \vec{k}_\perp \right)^2 \rangle \approx \frac{V_A^2}{\omega_0} \langle \left(\vec{b} \cdot \vec{k}_\perp \right)^2 \rangle$$

Ensemble avg. frequency shift

N.B.: Independent $B_0 \leftrightarrow \langle \tilde{B}^2 \rangle$ only

Magnetic Chaos, Cont'd

- Then: $\omega = \omega_0 + \frac{V_A^2}{\omega_0}\langle(\vec{b}\cdot\vec{k}_\perp)^2\rangle$ averaged frequency shift
- Revisit $\langle k_x k_y\rangle$ correlator:

$$\frac{d}{dt}k_x = -\frac{\partial}{\partial x}\left(\omega_0 + \frac{1}{2}\frac{V_A^2}{\omega_0}\, k_\perp^2 b^2 + k_y\langle V_y\rangle\right)$$

- Thus tilting:

$$\frac{d}{dt}k_x = -k_y\overset{\substack{\text{shear flow}\\ \swarrow}}{\langle V_y\rangle'} - \frac{V_A^2}{2\omega_0}k_\perp^2\partial_x\overset{\substack{\text{stochastic field}\\ \swarrow}}{[b^2]}$$

➔ $|b|^2$ enters tilting mechanism

- Shear amplification feedback disrupted if $[b^2]$ controls tilt process

Magnetic Chaos, Cont'd

$$L_b^{-1} \equiv \frac{\partial_r |b|^2}{|b|^2}$$

$$L_E^{-1} \equiv \frac{\partial_r E_r}{E_r}$$

- Tilting disrupted for: $\boxed{|b|^2 > \beta \left(\frac{\rho_s}{L_n}\right)^2 \frac{L_b}{L_E} L_n \frac{|e|E_r}{T}}$
  - Standard parameters:

$|b|^2 > 10^{-7} \frac{L_b}{L_E}$ → achievable for weak, modest field

- Critical field emerges from dephasing condition
- Higher power → increased V_E' → re-access tilting feedback

Proper Analysis – Schematic

- $\nabla \cdot J = 0 \ \sim \ V_A D_M$ characterizes mixing, D_M - RSTZ, R.R.

  ➔ V_A is signal speed along stochastic magnetic field

- $\partial_x \langle \tilde{V}_r \tilde{V}_\theta \rangle = \langle \tilde{V}_r \nabla^2 \tilde{\phi} \rangle$ Taylor Identity

  Vorticity Perturbation

- $\nabla^2 \tilde{\phi} \ = \ (\ ) \ \partial_x \langle \nabla^2 \phi \rangle + (\ ) \, k \, \nabla_y \tilde{P}$

  diagonal

  residual
  ∇P etc. → flow energy

- $\tilde{P}$ → Acoustic coupling - $c_s D_M$, slower

Outcome

$$\partial_x\langle\tilde{V}_x\tilde{V}_y\rangle = -D_{PV}\frac{\partial}{\partial x}\langle\nabla^2\phi\rangle + F_{res}k\,\partial_x\langle P\rangle$$

$$D_{PV} \approx \sum_{k,\omega}\left|\tilde{V}_{r;k,\omega}\right|^2\left[\frac{V_A b^2\, l_{ac}k^2}{\overline{\omega}^2+(V_A b^2 l_{ac}k^2)^2}\right]$$

$$b^2 = \frac{\langle\tilde{B}^2\rangle}{B_0^2}$$

l_{ac} = field autocorrelation

$$F_{res} \sim -\sum_{k,\omega}\frac{2k_y}{\omega}D_{PV;k,\omega}$$

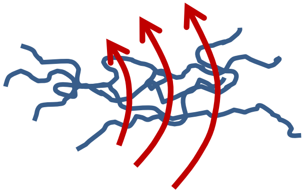
- Onset: $\Delta\omega_k \sim k_\perp^2 V_A D_M$ → Stochastic field decorrelation must beat ambient limits on Reynolds stress phase

spectral linewidth

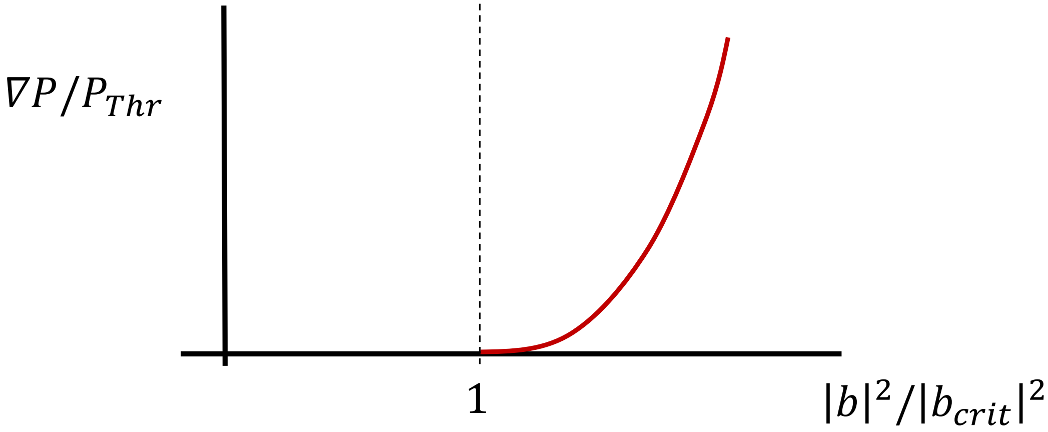
- In practice: $\boxed{Ku \sim 1}$ for effect, a challenge to predictions...

Outcome, cont'd

- Stochastic fields dephase Reynolds stress via Alfvenic radiation → inhibit energy transfer to flow



- Key onset condition: $\Delta\omega$ vs $k_\perp^2 V_A D_M$ ↔ 'Patchiness' correction !?
- Suggests trigger at higher power:



Related Work (Executive Summary)

- Broad Theme: Turbulence and Transport [especially momentum, PV] in Stochastic Field
- What of intrinsic rotation? → $\langle \tilde{V}_r \tilde{V}_\parallel \rangle$
- N.B. : 'Pedestal Torque' essential to stability in high performance discharges!
  - Parallel Flow ↔ Acoustic Dynamics

  So

  - Scattering effect ~ $c_s D_M$ → modest
  - ν_T and $F_{z,res}$ persist, with modification

Intrinsic Rotation, cont'd

But:

- Broken Symmetry required, for $\langle k_\theta k_\parallel \rangle \neq 0$
- $F_{res} \approx -\frac{k_z}{\omega} \nu_{Turb}$
- How does stochastic field interact with symmetry breaking?

➔ V_E' is leading candidate mechanism

What of direct effects of Stochastic Field?

$$\partial_t\langle V_\parallel\rangle + \partial_r\langle \tilde{V}_r \tilde{V}_\parallel\rangle = -\frac{1}{P}\,\partial_r\,\langle b\tilde{P}\rangle$$

kinetic stress
(W.X. Ding)

and like:

$$\partial_t\langle P\rangle + \partial_r\langle \tilde{V}_r P\rangle = -\frac{\partial}{\partial r}P_0\,\langle b\tilde{V}_\parallel\rangle$$

- FGC ‘92 → $c_s D_M \leftrightarrow$ rate: $c_s D_M/l^2$
- But fluxes non-diffusive - surprise!

i.e. for static stochastic field

$$\left.\begin{array}{l} B\cdot\nabla V_\parallel = 0 \\ B\cdot\nabla P = 0 \end{array}\right. \quad \Longrightarrow \quad \begin{array}{lll} -c_s D_M \nabla\langle P\rangle & \to & \pi_{res} \\ -c_s D_M\,\nabla\langle V_\parallel\rangle & \to & \text{convection} \end{array}$$

Cont'd

- But: turbulence co-exists with stochastic field!
- Time scales: $k_\perp^2 D_T$ vs $k_\parallel c_s$

turbulent scattering

- Resonance: $\delta(k_\parallel) \rightarrow 1/\left[k_\parallel^2 c_s^2 + (1/\tau_c)^2\right]$

shift, contrast

resonance broadening

- What balances $b_r \, \partial\langle P\rangle/\partial r$?
  - $c_s \nabla_\parallel \tilde{P}$ → weak turbulence → residual stress

    b only, as previous
  - $k_\perp^2 D_T$ [illegible] $\tilde{v}_\parallel$ → strong turbulence → magnetic viscosity

    $b, v_\perp$ interplay

$$\nu_T \approx \sum_k |b_k|^2 c_s^2 / k_\perp^2 D_T$$

Cont'd

- Structure of flux, 'Fick's law' changes !
- Interesting new direction...

- Correlations?! (M. Cao, P.D. 2020) [Dynamics of Instability in stochastic field]
  - Are $\tilde{b}$, turbulence uncorrelated?
  - No → interaction develops $\langle b\phi \rangle$ correlation
  - ala' Kadomtsev, Pogutse, impose $\nabla \cdot J = 0$ to all orders
  - Novel small scale convective cell, $\tilde{b}$ structure develops.

Conclusion

- Demands of “Good Confinement” + “Good Power Handling” define more severe demands upon our understanding of turbulence, transport physics
- L→H Transition in stochasic magnetic field one such $Ku \sim 1$
- “Turbulence in Stochastic Field” re-appears as an interesting theoretical problem
- A.A. Galeev’s contributions will be of importance on the ITER era

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