Decoherence of Vorticity Flux by Stochastic B-Fields Quenches Zonal Flows

— with Application to L-H transition

Chang-Chun Chen[1], Patrick H. Diamond[1], Rameswar Singh, and Steven M. Tobias[2]

[1]University of California, San Diego, US
[2]University of Leeds, Leeds LS2 9JT, UK

This work is supported by the U.S. Department of Energy under Award No. DE-FG02-04ER54738

AAPPS-DPP, Remote e-conference, October 27th 2020

Outline

1. Introduction

- Why tangled magnetic fields are important? A system with stochastic fields is a generic problem.
- In weak mean magnetization: The solar tachocline.

  In strong mean magnetization: L-H Transition Experimental results — with RMP.
- **General Ideas:**

  Evolution of momentum transport and PV mixing.

2. Solar Tachochline

3. L-H transition in tokamak

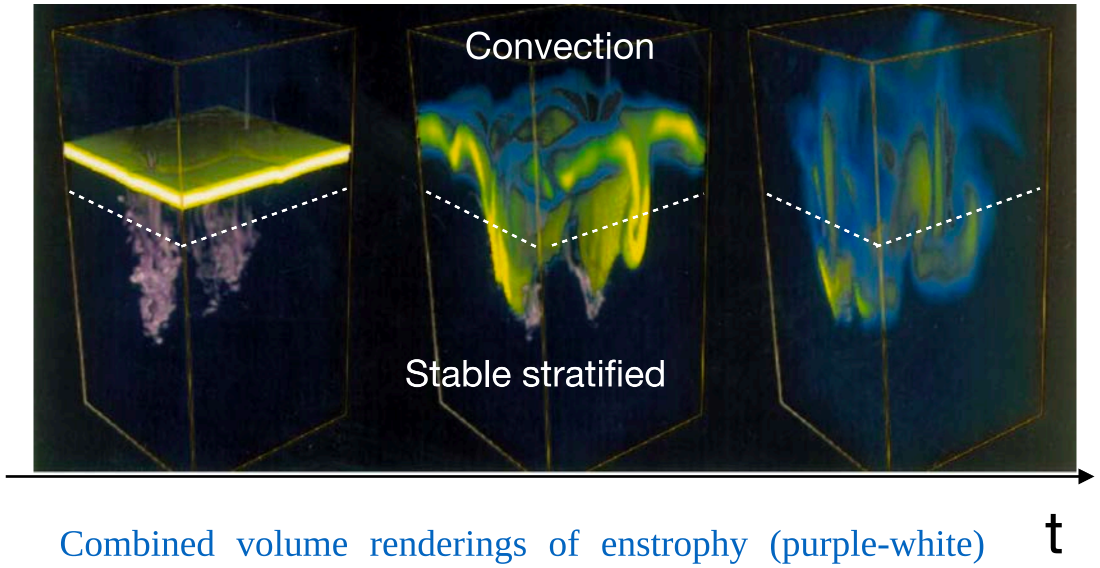
4. Conclusion

Introduction— Why

- **Why study disordered magnetic fields?**
  Disordered magnetic fields is frequently encountered.

The solar Tachocline

Weak mean magnetization

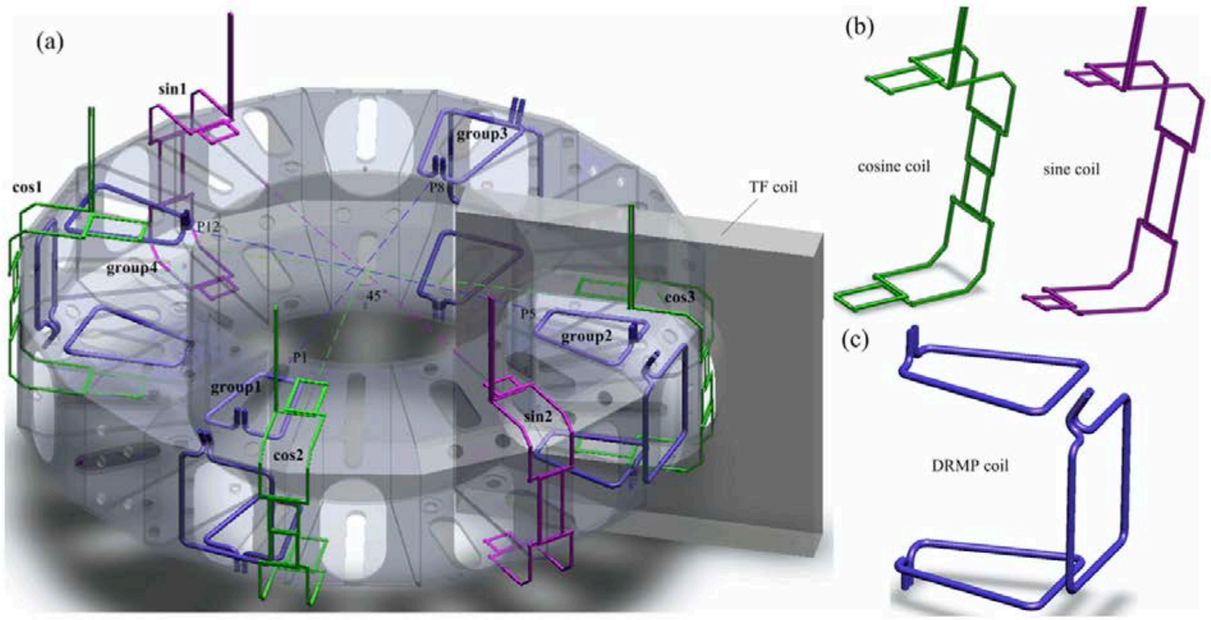


Combined volume renderings of enstrophy (purple-white) and of magnetic energy (blue-green-yellow), in which high values appear as opaque and bright (Tobias & Brummell 2008).

Simulation: the stochastic magnetic field has been "pumped" from the convection zone into the stably stratified region.

The tokamak

Strong mean magnetization



(J-TEXT)

The resonant magnetic perturbation (RMP) raises L-H transition power threshold.

PV mixing in a disordered field **is a generic problem!**

Introduction

What is Potential Vorticity (PV)?

1. Potential Vorticity is a generalized vorticity.

$PV \equiv \zeta \equiv \nabla \times \mathbf{v}$ **(pure 2D fluid)**

$PV \equiv \zeta + 2\Omega \sin \phi_0 + \beta y$ **(on the β-plane)**

$PV \equiv (1-\rho_s^2\nabla^2)\frac{|e|\phi}{T} + \frac{X}{L_n}$ **(Hasegawa-Mima eq. for tokamak)**

2. It is conserved along the fluid — acts as conserved phase space density. Magnetic fields will break the PV conservation:

$$\frac{D}{Dt}\langle\zeta\rangle = \frac{\partial}{\partial y}\frac{\langle \widetilde{J}_z \widetilde{B}_y\rangle}{\rho} + \nu\nabla^2\langle\zeta\rangle$$

How the zonal flow evolves?

1. Flux of the potential vorticity $\equiv \langle \widetilde{u}\widetilde{\zeta}\rangle$
2. Taylor Identity and the evolution of zonal flow.

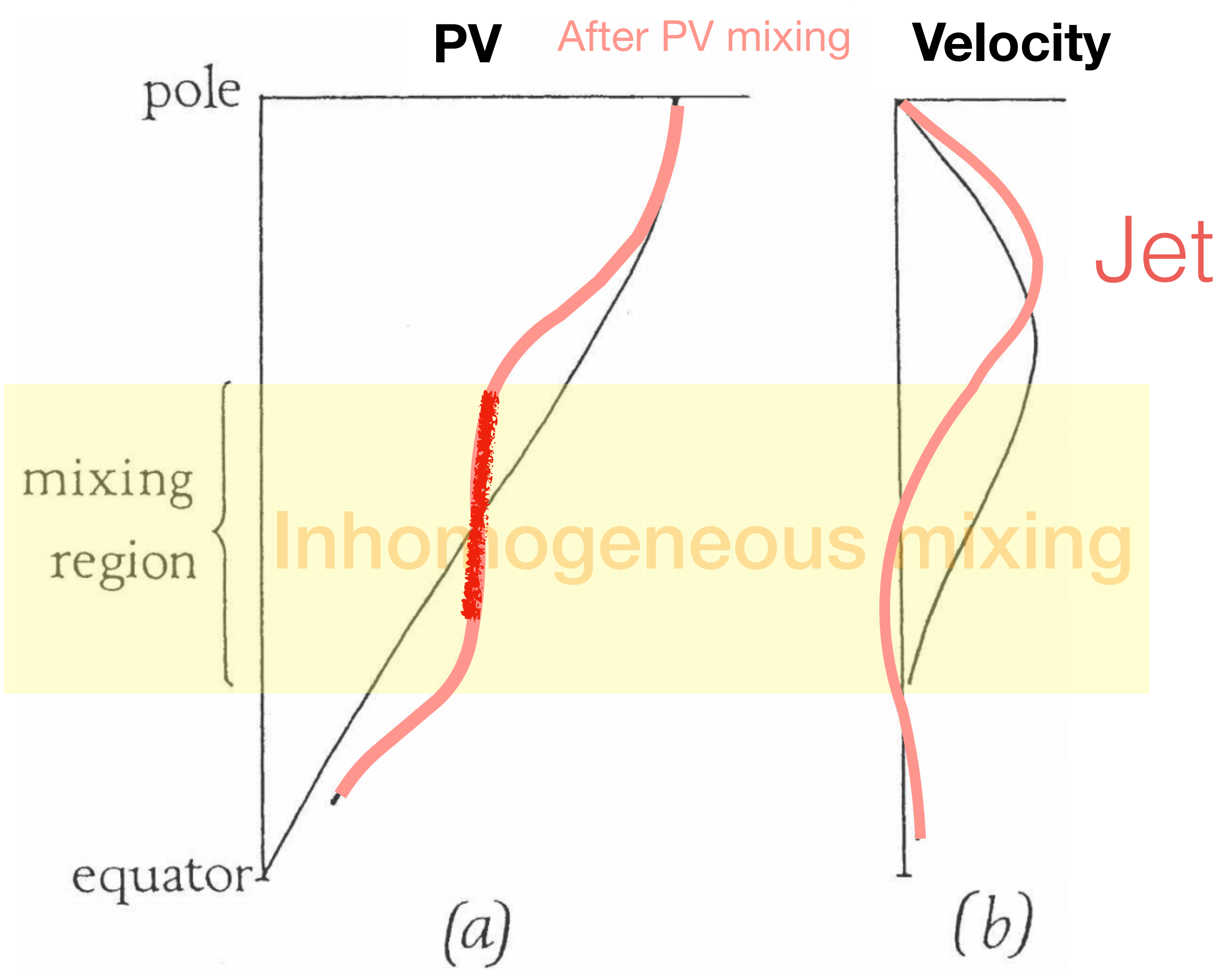
Taylor Identity: $\underbrace{\langle \widetilde{u}_y\widetilde{\zeta}\rangle}_{PV\ flux} = \underbrace{-\frac{\partial}{\partial y}\langle \widetilde{u}_y\widetilde{u}_x\rangle}_{Reynolds\ force}$

Evol. of zonal flow: $\boxed{\frac{\partial}{\partial t}\langle u_x\rangle = \langle \widetilde{u}_y\widetilde{\zeta}\rangle = -\frac{\partial}{\partial y}\langle \widetilde{u}_y\widetilde{u}_x\rangle\,.}$

What is inhomogeneous PV mixing?

Local PV mixing causes changes in flow structure.

$$\textbf{PV flux} \equiv \underbrace{\langle \widetilde{u}_y\widetilde{\zeta}\rangle}_{phase\ correlation\ \ between\ \ u\ and\ \zeta} \neq 0$$



Outline

1. Introduction

2. Solar Tachocline

- Stochastic B-Model — stochastic fields and Kubo number.
- Order of scales, assumptions, mean field theory, and Reynolds stress.
- **Tachocline results**:

   1. Suppression of zonal flow before fully system is Alfvénized— suppression of PV diffusivity

   2. Large- and small-scale have synergetic effect on dephasing Reynolds stress (multi-scale dephasing).

   4. Resisto-elastic Network: wave coupling to resisto-elastic medium, magnetic drag.

3. L-H transition in tokamak

4. Conclusion

How we describe the stochastic magnetic field

Magnetic field = mean field + stochastic field $B = B_0 + \widetilde{B}$

- **Fluid Kubo number:** $Ku_f \equiv \frac{\delta_l}{\Delta_\perp} \sim \frac{\widetilde{v}\tau_{ac}}{\Delta_\perp} \sim \frac{\tau_{ac}}{\tau_{eddy}} < 1,$

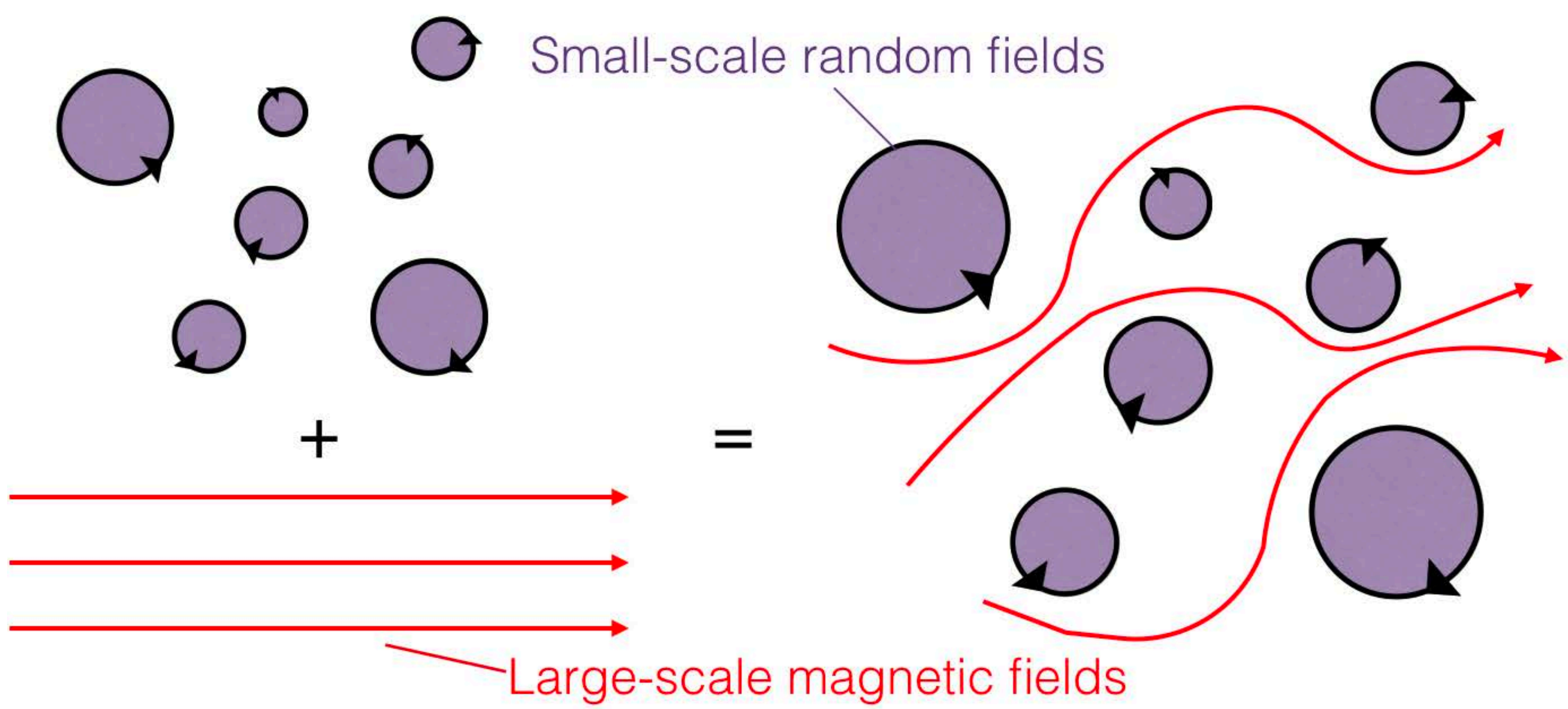
  τ_{ac}: Auto correlation time

  τ_{eddy}: Eddv turnover time

- **Magnetic Kubo number:** $Ku_{mag} \equiv \frac{\delta_l}{\Delta_{eddy}} = \frac{l_{ac}|\widetilde{\mathbf{B}}|}{\Delta_{eddy} B_0}$

$$Ku = \begin{cases} < 1, & \textbf{Quasi-linear theory} \\ > 1, & \textbf{Quasi-linear theory fails} \end{cases}$$

A weak mean field— large magnetic **Kubo number,** if $|\widetilde{B}^2|/B_0^2 \gg 1$



Simple quasi-linear theory might fail.
Need a model beyond quasi-linear Theory.

The large-scale magnetic field is distorted by the small-scale fields. The system thus is the ‘soup’ of cells threaded by sinews of open field line (Zel’dovich, 1957).

How we describe the stochastic magnetic field

Truth in Advertising

The system is **strongly nonlinear** and **simple quasi-linear method fails**.

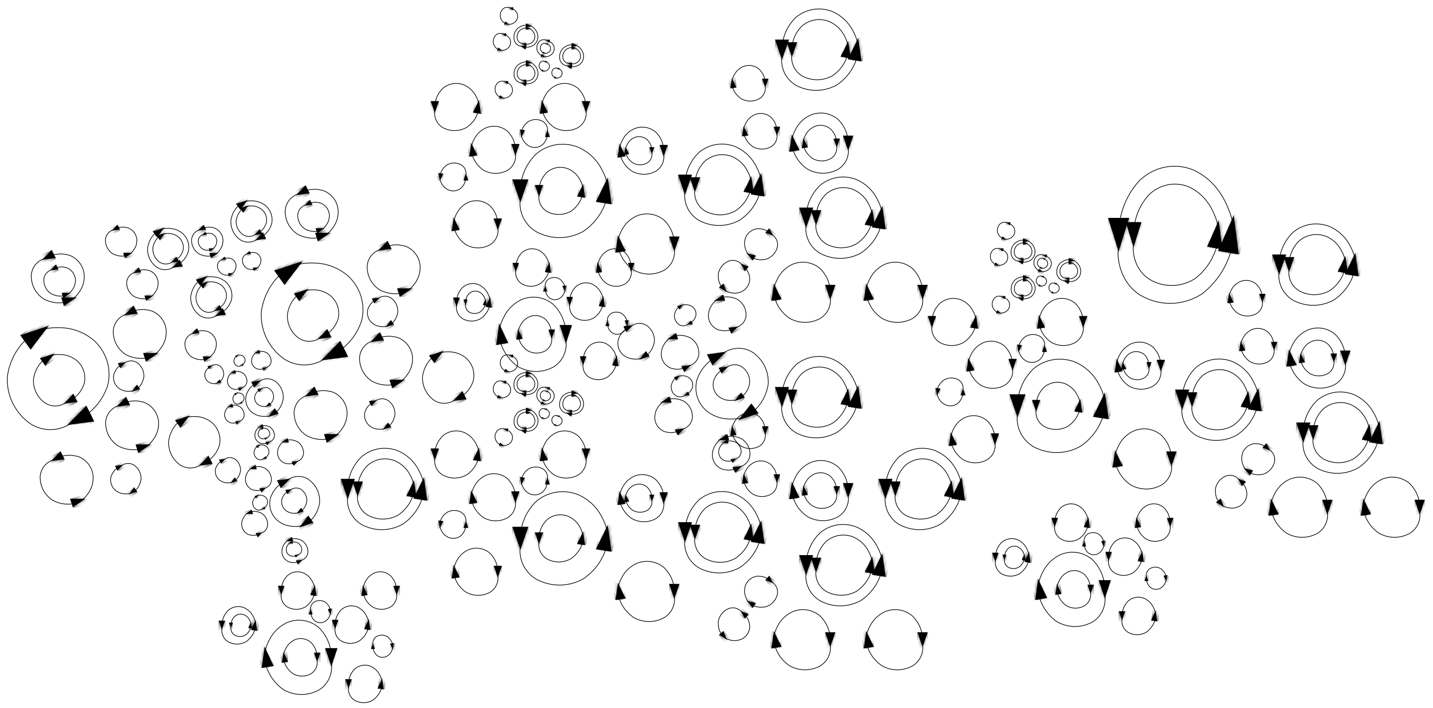
A" frontal assault" on calculating PV transport in an ensemble of tangled magnetic fields is a daunting task.

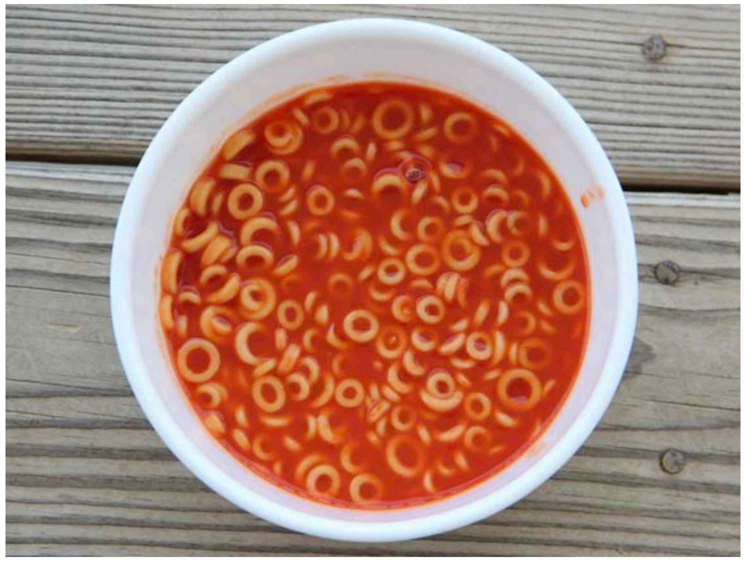
Rechester & Rosenbluth (1978) suggested replacing the "full" problem with one where waves, instabilities, and transport are studied in the presence of **an ensemble of prescribed, static, stochastic fields.**

Assumptions:

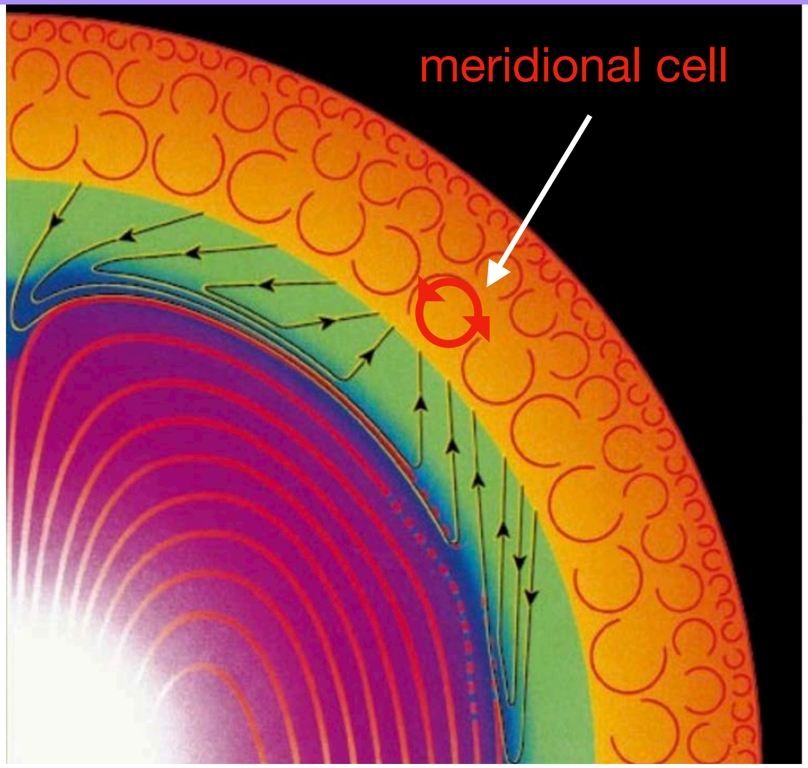
1. Amplitudes of random fields distributed statistically.
2. Auto-correlation length of fields is small ($l_{ac} \to 0$, such that $Ku_{mag} \equiv \frac{\tilde{u}\tau_{ac}}{\Delta_{eddy}} = \frac{l_{ac}|\widetilde{\mathbf{B}}|}{\Delta_{\perp}B_0} < 1$)

➤ Quasi-linear closure .





Model: the solar tachocline



Properties:

1. Strongly Stratified (β-plane model)
2. Zonal Flow and Rossby wave — as in the Jovian Atmosphere.
3. Large magnetic perturbation — large magnetic Kubo number.
4. Meridional cells forms tachocline but will make it spread inward.

The tachocline formation:

- Spiegel & Zahn (1992) —
  Spreading of the tachocline is opposed by turbulent viscous diffusion of momentum in latitude.

- Gough & McIntyre (1998) —
  Spreading of the tachocline is opposed by a hypothetical fossil field in the radiational zone.

***"At the heart of this argument, therefore, is the role of the fast turbulent processes in redistributing angular momentum on a long timescale."* — (Tobias et al. 2007)**

These two models ignore the "likely" strong stochasticity of the tachocline magnetic field.

Basic Equations in β-plane MHD

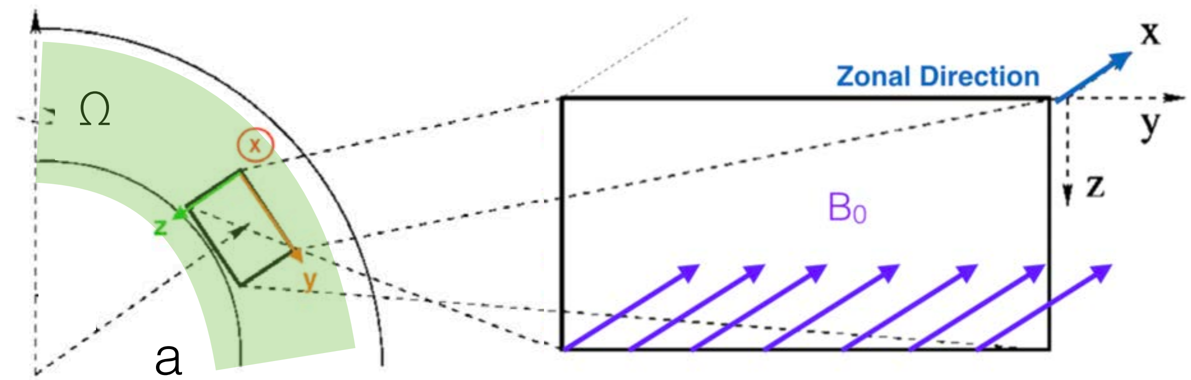
Rossby Parameter: $\beta = \frac{df}{dy}|_{\phi_0} = 2\Omega cos(\phi_0)/a$

Derivative of angular frequency f (Coriolis parameter)

latitude

rotation

radius



Methods:

$$
\begin{cases}
\text{Stream Function} & \psi = \psi(x, y, z) \\
\text{Velocity field} & \boldsymbol{u} = (\frac{\partial \psi}{\partial y}, -\frac{\partial \psi}{\partial x}, 0) \\
\text{Fluid Vorticity} & \boldsymbol{\zeta} = (0, 0, \zeta) \\
\text{Potential Field} & \boldsymbol{A} = (0, 0, A) \\
\text{Magnetic Field} & \boldsymbol{B} = (\frac{\partial A}{\partial y}, -\frac{\partial A}{\partial x}, 0),
\end{cases}
$$

Quasi-Linear Approximation:

$$
\zeta = \langle\zeta\rangle + \boxed{\tilde{\zeta}} \qquad \psi = \langle\psi\rangle + \boxed{\tilde{\psi}} \qquad A = \langle A\rangle + \boxed{\tilde{A}}
$$

Perturbations produced by turbulences

$$, where \ \langle\ \rangle = \frac{1}{L}\int dx \frac{1}{T}\int dt$$

ensemble average over the zonal scales

Two main equations:

➤ **Quasi-linear closure**

Navier-Stoke Eq:

$$(\frac{\partial}{\partial t} + \mathbf{u}_\perp \cdot \nabla_\perp)\zeta - \beta\frac{\partial \psi}{\partial x} = -\frac{(\mathbf{B}\cdot\nabla)(\nabla^2 A_z)}{\mu_0\rho} + \nu(\nabla \times \nabla^2 \mathbf{u})$$

Induction Eq:

$$(\frac{\partial}{\partial t} + \mathbf{u}_\perp \cdot \nabla_\perp)A = B_l\frac{\partial \psi}{\partial x} + \eta\nabla^2 A,$$

➤ **linear response of perturbations**

$$
\begin{cases}
\tilde{\zeta}_k = \left(\frac{i}{\omega + i\nu k^2 + (\frac{B_0^2}{\mu_0\rho})\frac{k_x^2}{-\omega - i\eta k^2}}\right)(\tilde{u}_y\frac{-\partial}{\partial y}\langle\zeta\rangle - \beta\tilde{u}_y) \\
\tilde{A}_k = \frac{\tilde{\zeta}_k}{k^2}(\frac{B_0 k_x}{-\omega - i\eta k^2})
\end{cases}
$$

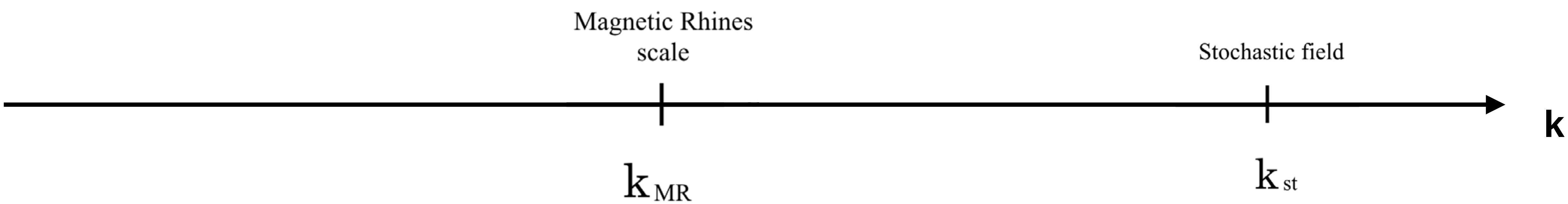
Order of Scales — Tachocline

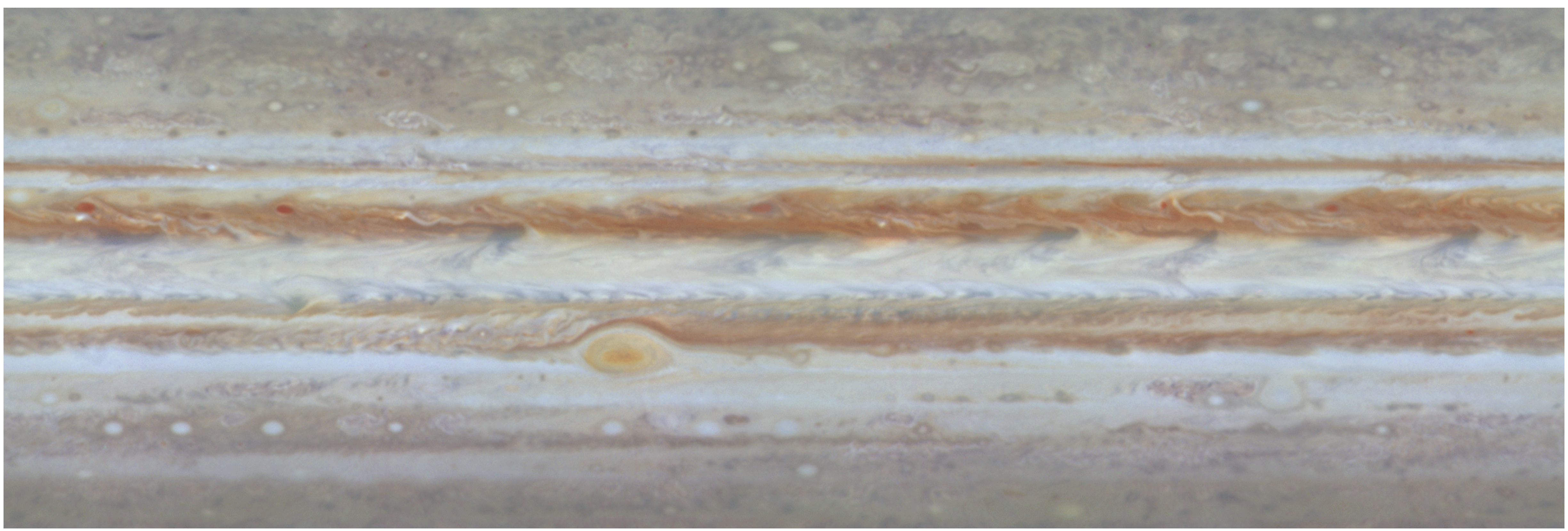
- **Two-average Method:**

1. $\bar{F} = \int dR^2 \int dB_{st} \cdot P_{(B_{st,x}, B_{st,y})} F$

2. $\langle \, \rangle = \frac{1}{L} \int dx \frac{1}{T} \int dt$ ensemble average over the zonal scales

- **Order of scales** Function of fields $\mathbf{F} = \mathbf{F_0} + \widetilde{\mathbf{F}} + \mathbf{F_{st}}$





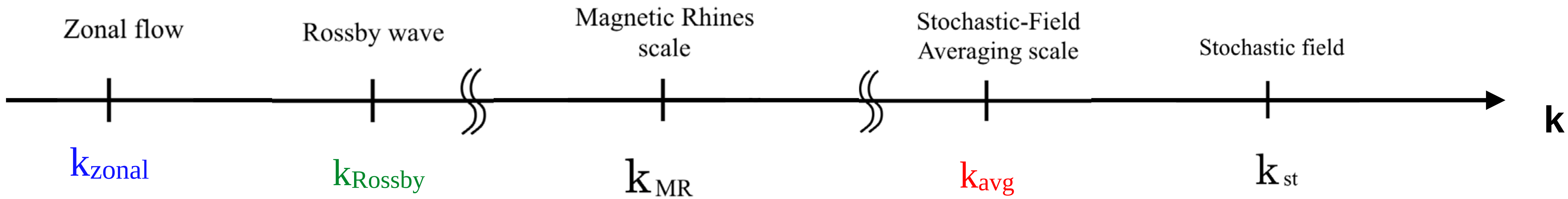
Order of Scales — Tachocline

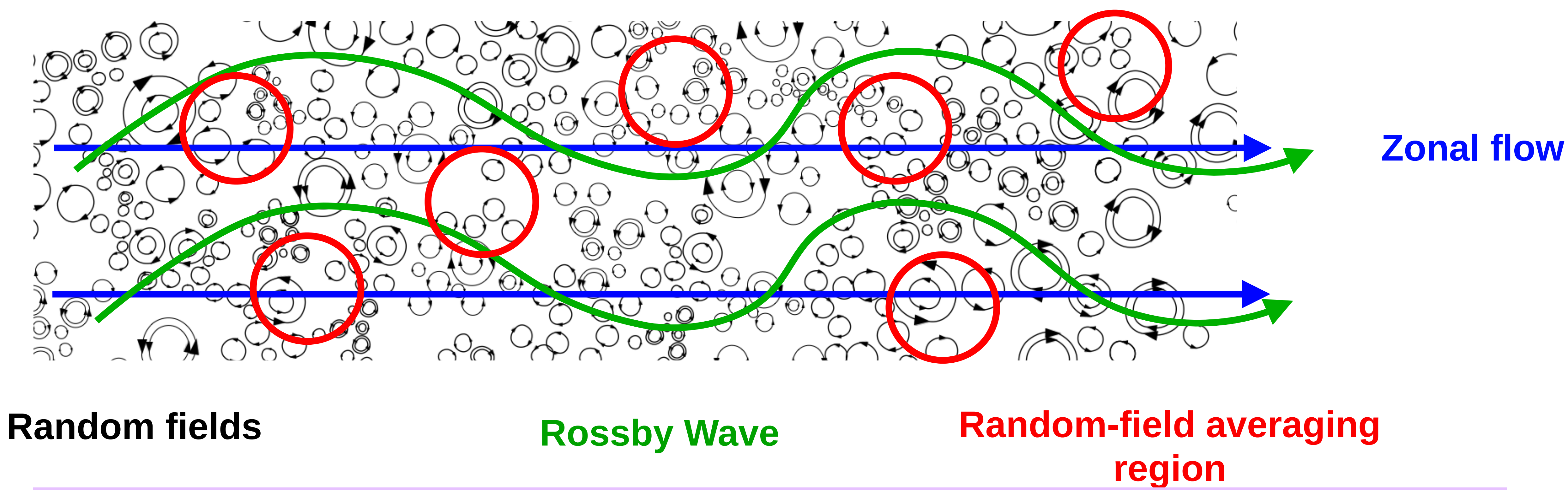
- **Two-average Method:**

1. $$\overline{F} = \int dR^2 \int dB_{st} \cdot P_{(B_{st,x},B_{st,y})} F$$

2. $$\langle \; \rangle = \frac{1}{L}\int dx \frac{1}{T}\int dt$$ ensemble average over the zonal scales

- **Order of scales** Function of fields $\mathbf{F} = \mathbf{F_0} + \widetilde{\mathbf{F}} + \mathbf{F_{st}}$





Results— the solar tachocline

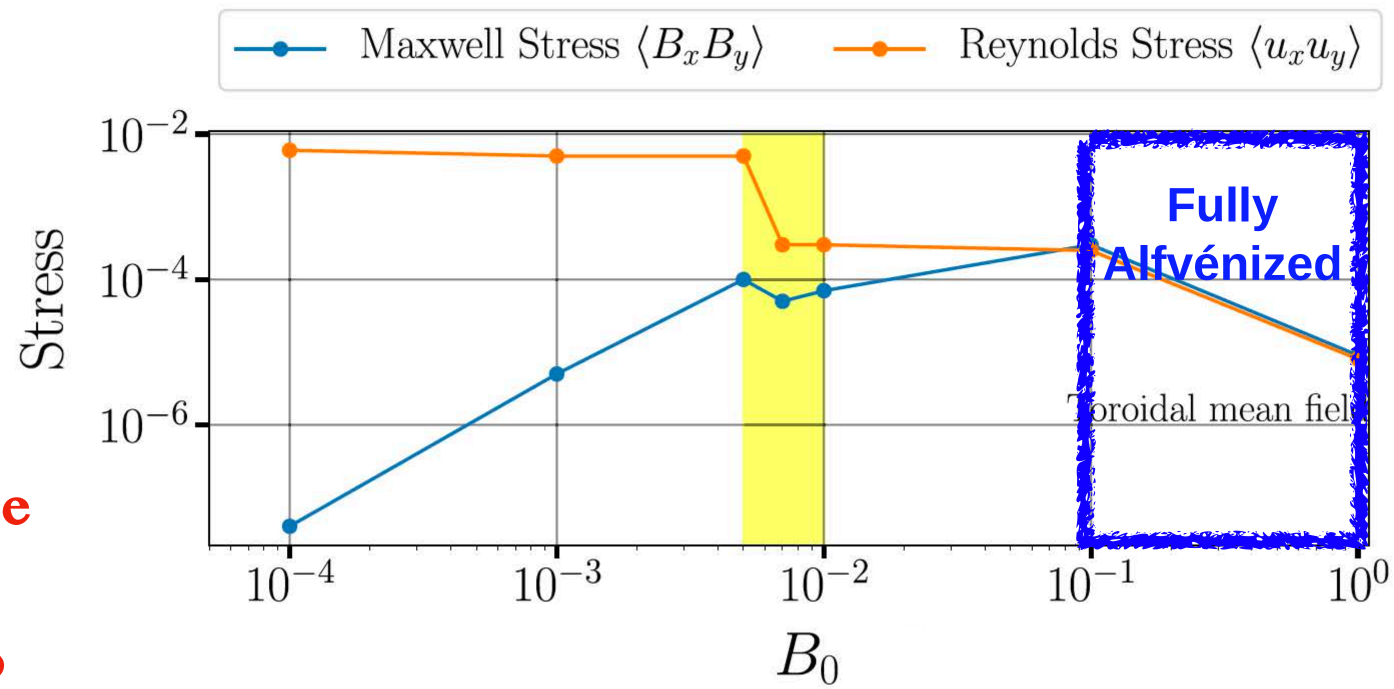
What is “fully Alfvénization” means?

1. All the wave energy transferred into Alfvén wave (dominated mode of the wave is Alfvén frequency).
2. The wave and magnetic energy reach equi-partition.
3. Then the Maxwell stress cancels the Reynolds stress.

Before full Alfvénization…

Conventional wisdom: Maxwell/ Reynolds stress balance when the system is Alfvénized.

The cross phase scattering suppresses the Reynolds stress when mean field is weak, before the mean field is strong enough to fully Alfvénize the system!



(Chen & Diamond, ApJ 892 24, (2020))

Results— the solar tachocline

Multi-scale dephasing:

Mean PV Flux (Γ) and PV diffusivity (D_{PV}).

Mean field $\boxed{B_0^2 < B_{st}^2}$ small-scale random fields

PV Diffusivity $\mathbf{D_{pv}}$

$$\overline{\Gamma} = -\boxed{\sum_k |\tilde{u}_{y,k}|^2 \frac{\nu k^2 + (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\eta k^2}{\omega^2 + \eta^2 k^4} + \frac{\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho \eta k^2}}{\left(\omega - (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\omega}{\omega^2 + \eta^2 k^4}\right)^2 + \left(\nu k^2 + (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\eta k^2}{\omega^2 + \eta^2 k^4} + \frac{\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho \eta k^2}\right)^2}} \left(\frac{\partial}{\partial y}\overline{\zeta} + \beta\right)$$

➤ **The large-** and **small-scale magnetic fields** have a **synergistic effect on the cross-phase** in the Reynolds stress.

Suppression of zonal flow

$$\frac{\partial}{\partial t}\langle u_x \rangle = \langle \overline{\Gamma} \rangle - \frac{1}{\eta \mu_0 \rho}\langle \overline{B_{st,y}^2} \rangle \langle u_x \rangle + \nu \nabla^2 \langle u_x \rangle$$

Cross-phase effect on Reynolds Stress Force

Magnetic drag force $(J_{st} \times B_{st})$

➤ Random magnetic fields modify the **PV flux** and induce **magnetic drag**.

Results— the solar tachocline

$\overline{B_{st}^2}$ - **Resisto-elastic Medium:** **Rossby frequency** $\omega_R \equiv -\beta k_x/$

Dispersion relation of the Rossby-Alfvén wave

$$\omega^2 + i(\alpha + \eta k^2)\omega - \boxed{\left(\frac{\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho} + \frac{B_0^2 k_x^2}{\mu_0 \rho}\right)} = 0,$$

Spring constant

(mean square)

(square mean)

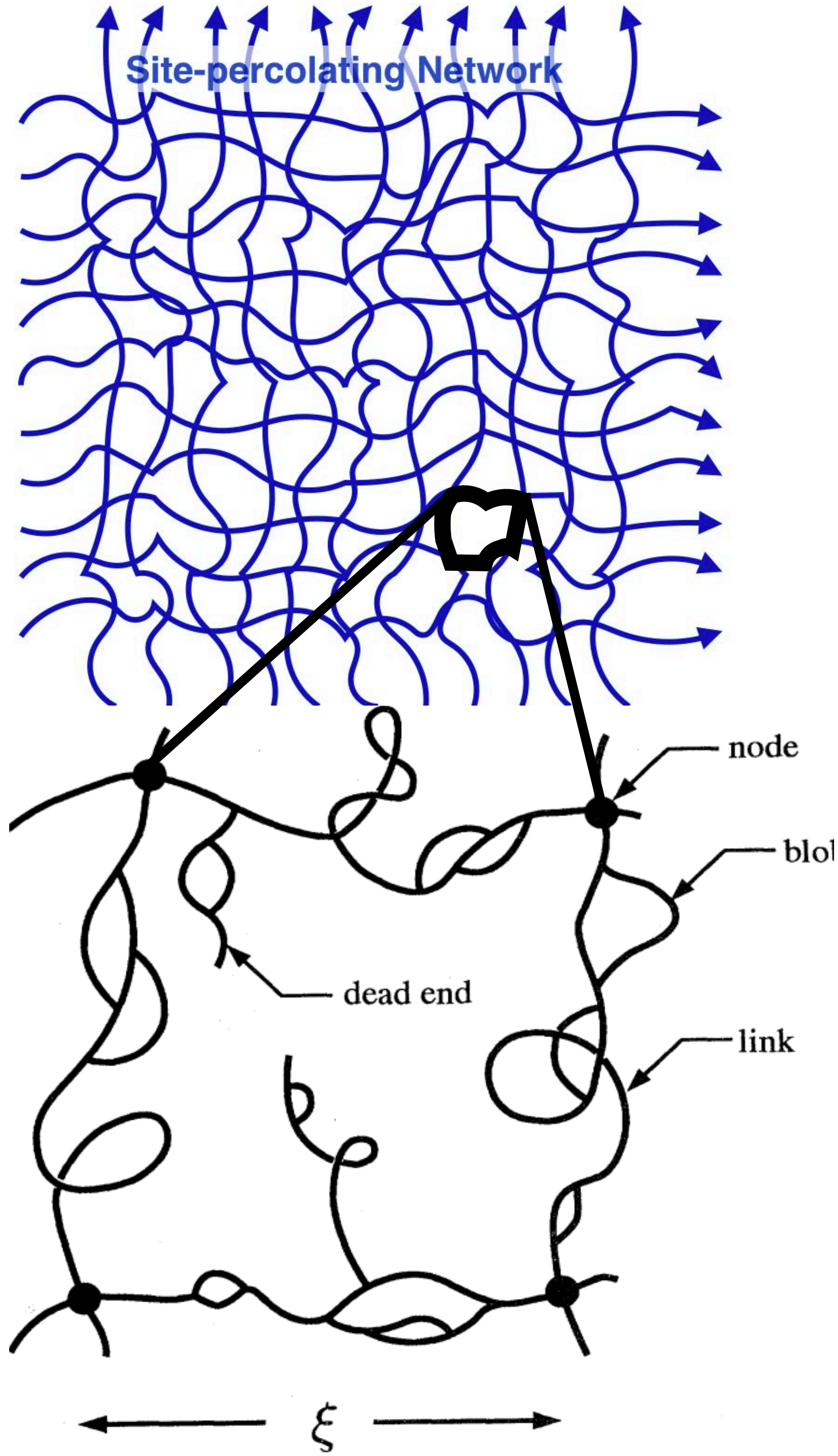
$$\left(\omega - \omega_R + \boxed{\frac{i\overline{B_{st,y}^2} k_y^2}{\mu_0 \rho \eta k^2}} + i\nu k^2\right)\left(\omega + i\eta k^2\right) = \frac{B_{0,x}^2 k_x^2}{\mu_0 \rho}$$

Dissipative response to Random magnetic fields

AW of the large-scale magnetic field

➤ **This network can be fractal and intermittent.**

➤ **Fluids couple to network elastic modes.**



Schematic of the nodes-links-blobs model (Nakayama & Yakubo 1994).

Conclusions for the tachocline

- **What studies have shown and what we obtained:**

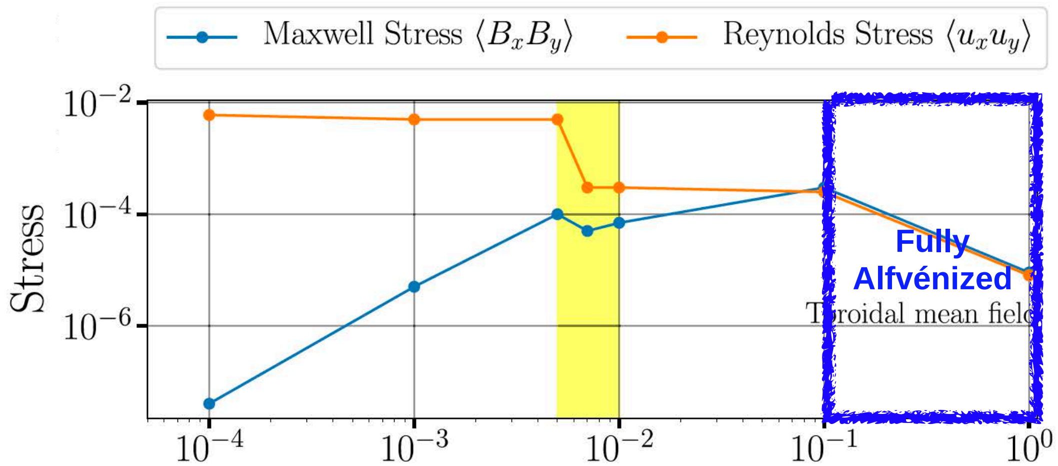
Reynolds stress will be suppressed at levels of field intensities **well below that of Alfvénization**, where Maxwell stress balances the Reynolds stress.

The flow generated by PV mixing/Reynolds force are reduced by:

$$\frac{\partial}{\partial t}\langle u_x\rangle = \langle\Gamma\rangle - \frac{1}{\eta\mu_0\rho}\langle\overline{B_{st,y}^2}\rangle\langle u_x\rangle + \nu\nabla^2\langle u_x\rangle$$

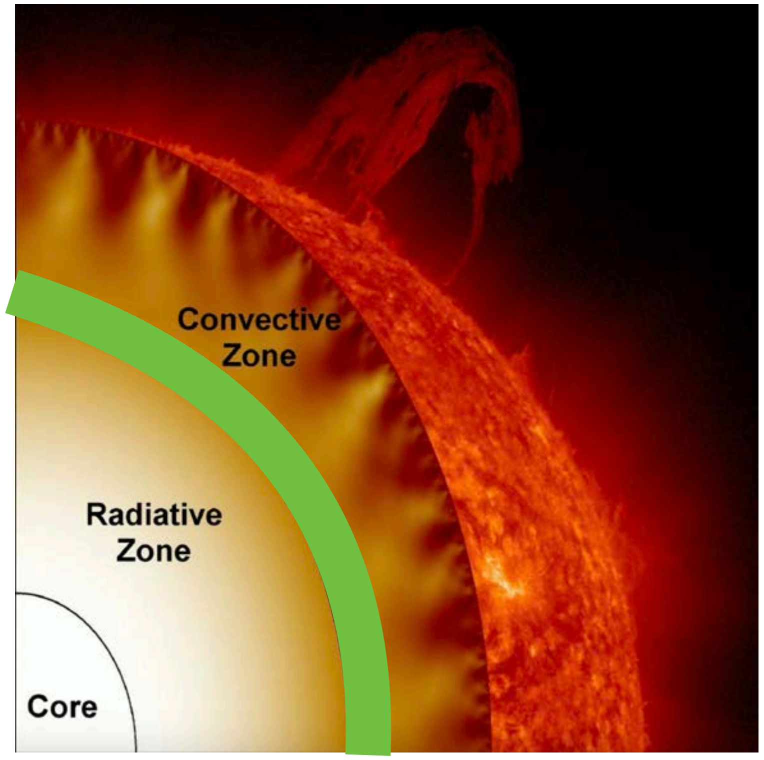
1. Coupling of resisto-elastic waves, which is $\overline{B_{st}^2}$ dependence.

2. Increase of the magnetic drag.



(Chen & Diamond, ApJ 892 24, (2020))

Solar Tachocline



- Spiegel & Zahn (1992) and Gough McIntyre (1998) Models for the solar Tachocline are not fully correct. The truth here is **'neither pure nor simple'** (apologies to Oscar Wilde).

S&Z: burrowing is opposed by turbulent viscous diffusion of momentum.
G&M: burrowing is opposed by PV mixing and by a hypothetical fossil magnetic field.

These two models ignore the strong stochasticity of the tachocline.

Outline

1. Introduction

2. Solar Tachocline

3. L-H transition in tokamak

- Order of scales, Decoherence effect of stochastic fields.

  Expectation: increment of power threshold due to the increment of stocastic fields.

- **L-H Transition in DIII-D:**

  1. DIII-D: Burst of Reynolds stress (Kriete 2020).

  2. Suppression of poloidal Reynolds stress, phase tilting that dephases of D_PV and turbulent diffusivity

  3. **Other effect:** Implication to L-H Transition — Extending Kim-Diamond Model.

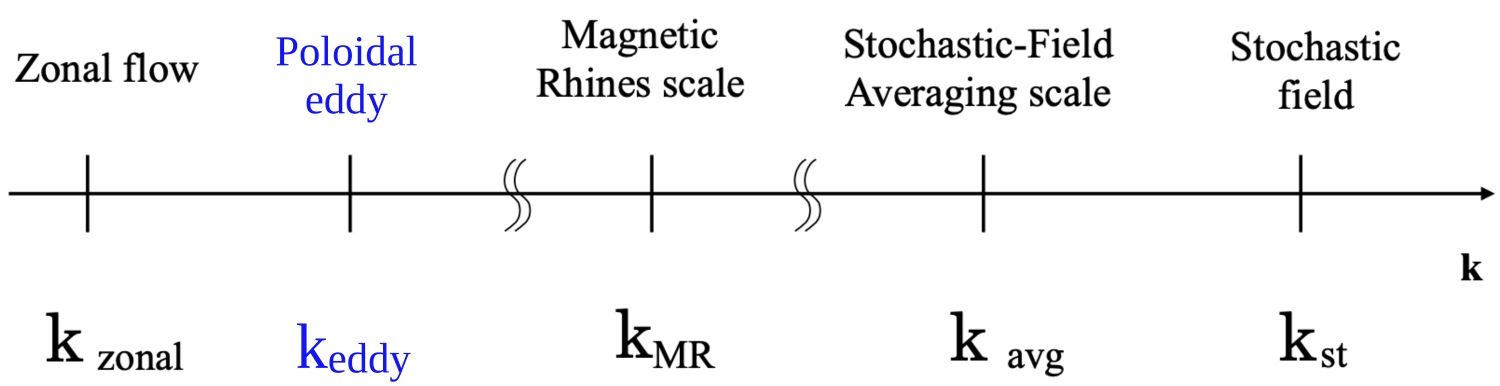
  Linewidth broadening gives a critical dimensionless parameter: $\alpha \equiv \frac{b^2}{\sqrt{\beta}}(\frac{L_n}{\rho_s})^2\frac{q}{\epsilon}$

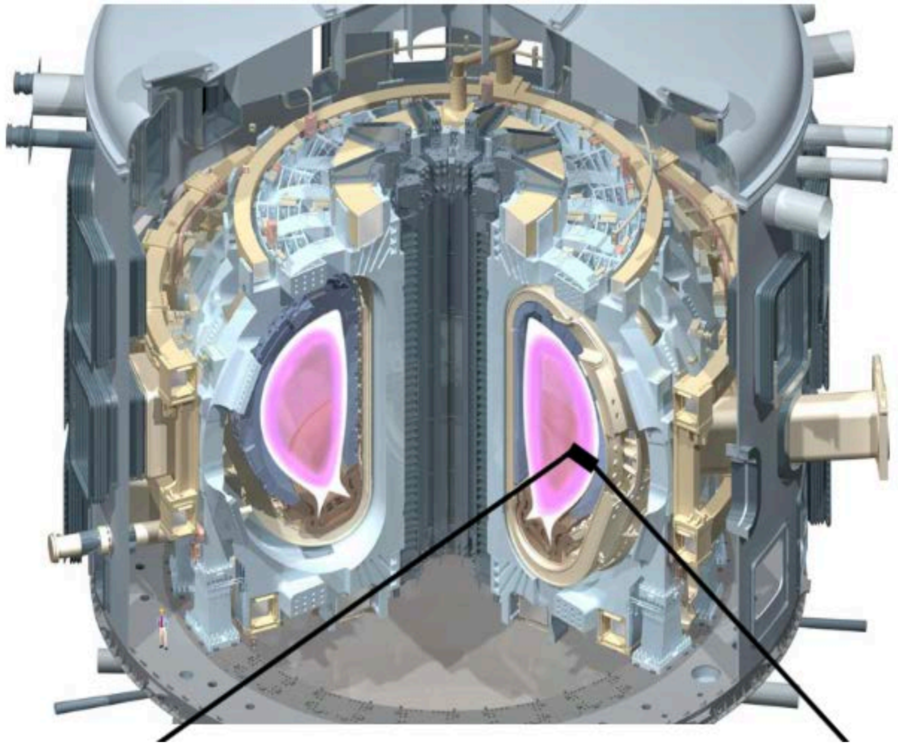
  The power threshold to onset of L-H transition increases: Input power increment (Figures).

4. Conclusion

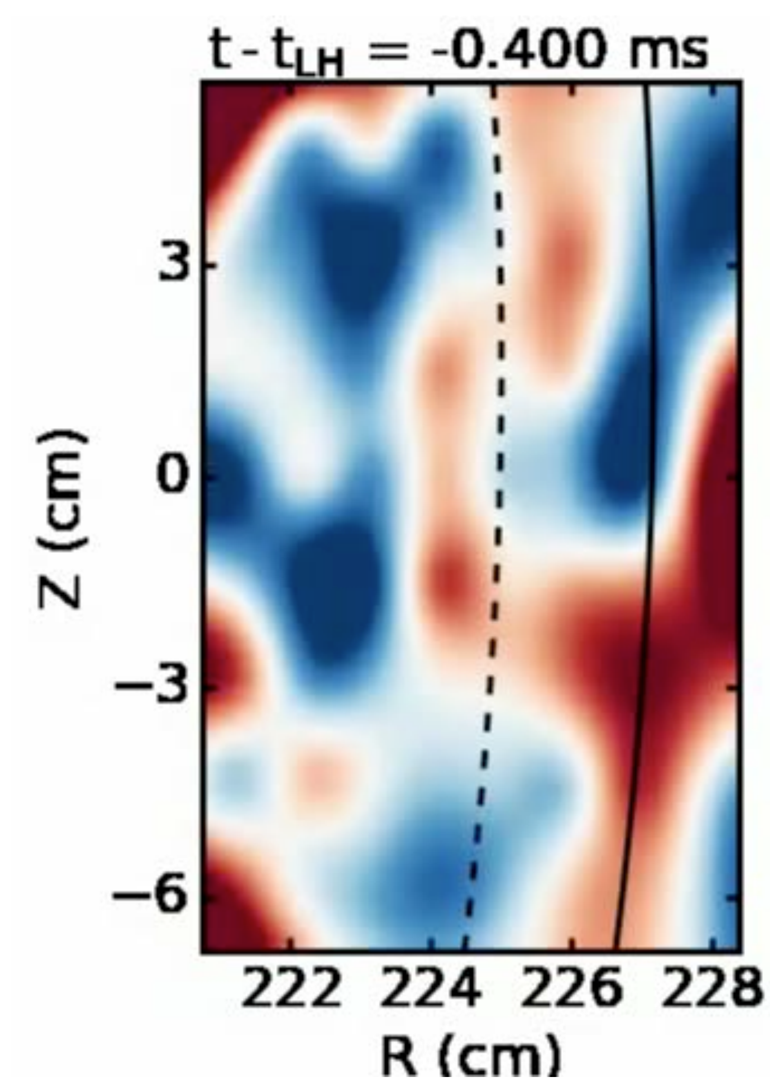
Order of Scales — Edge of Tokamak

- **Use the same stochastic fields to study suppression poloidal eddy at the edge of tokamak:**





(Source: NASA)



(D. Kriete et al, PoP **27** 062507 (2020))

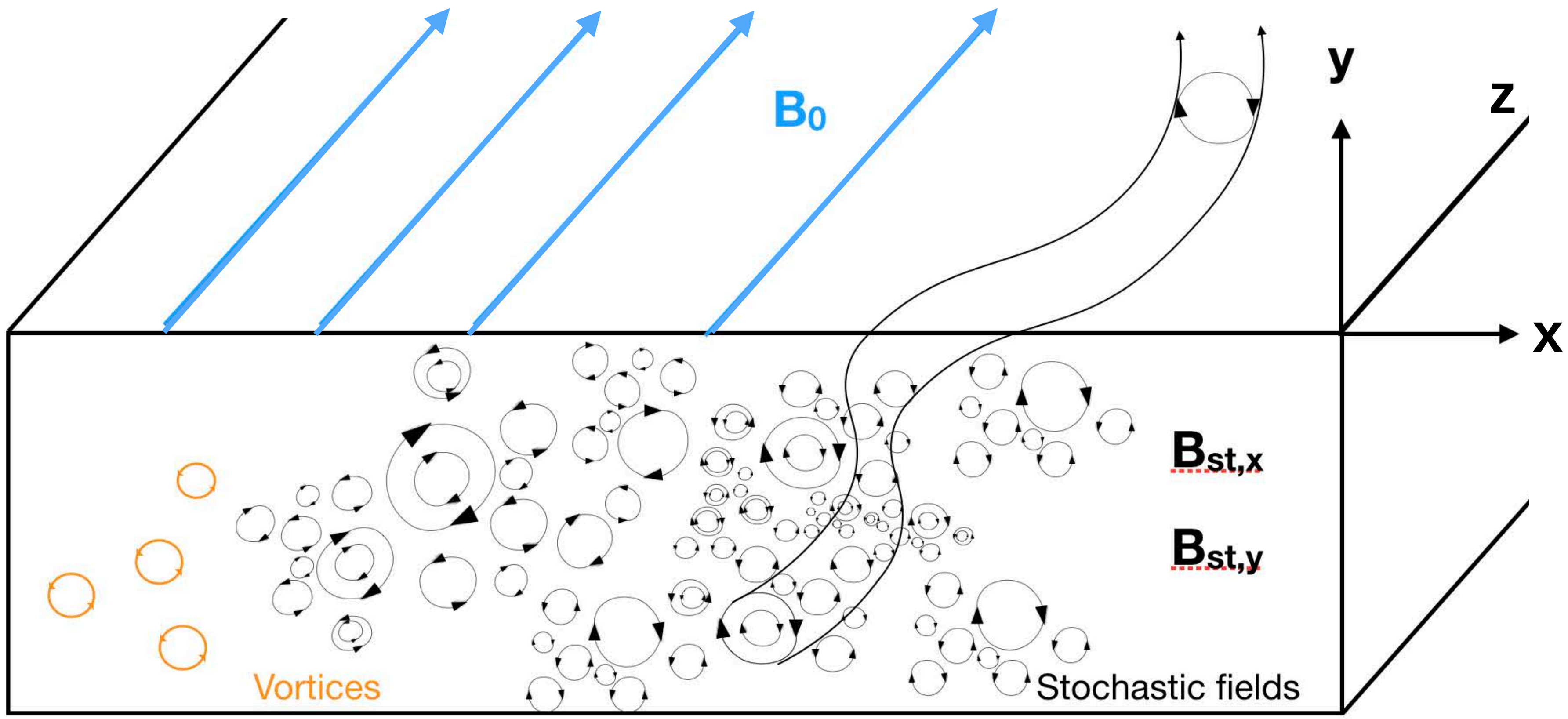
Order of Scales — Edge of Tokamak

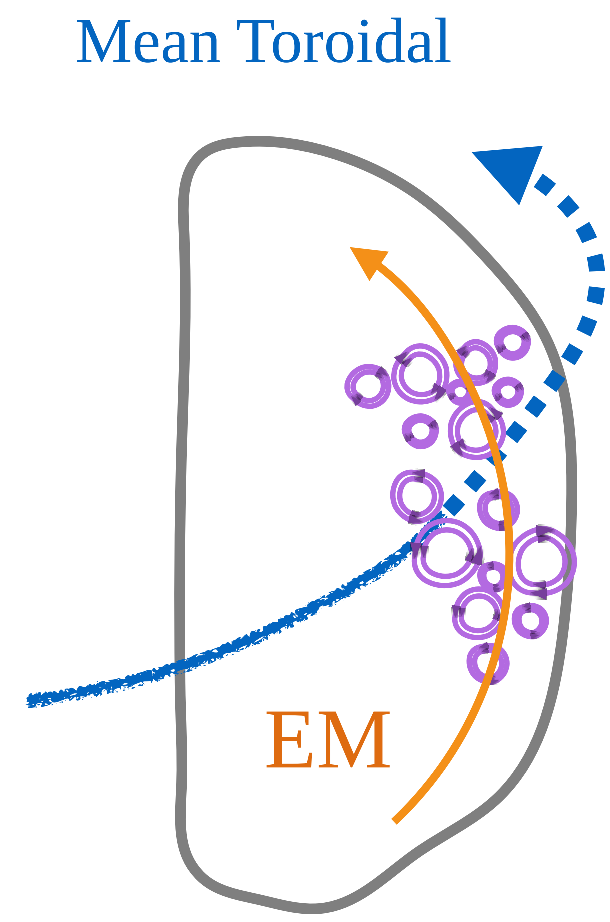
- **The model (Cartesian coordinate):**

$\underline{k} \cdot \underline{B}$ resonant at rational surface has third direction — $\omega \to \omega \pm v_A k_z$.

Four-field equations —

1. Vorticity equation
2. Induction equation
3. Pressure equation
4. Parallel flow equation





Order of Scales — Edge of Tokamak

When stochastic Fields dephasing becomes noticeable?

Stochastic field decoherence beats self-decoherence

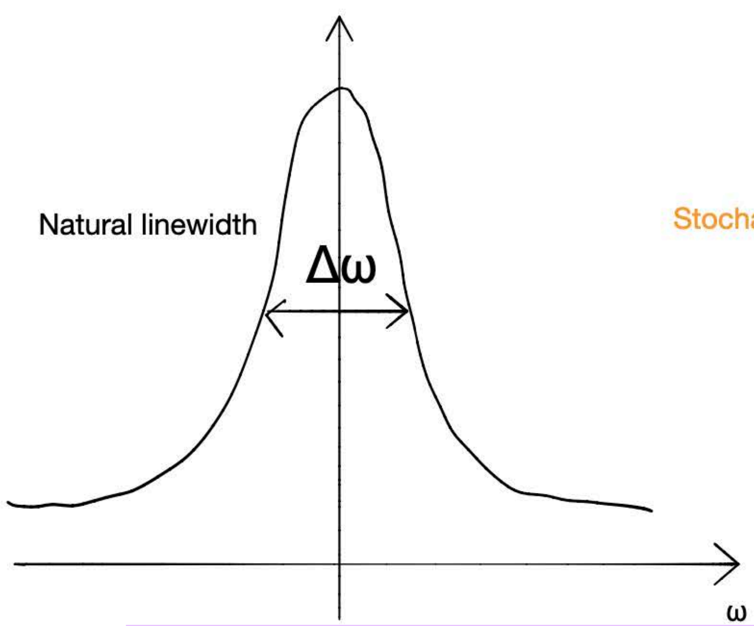
$$k_\theta \Delta x \frac{\partial}{\partial x} u_y < v_A |\Delta k_\parallel| < \boxed{\Delta\omega \leq k_\perp^2 D}$$

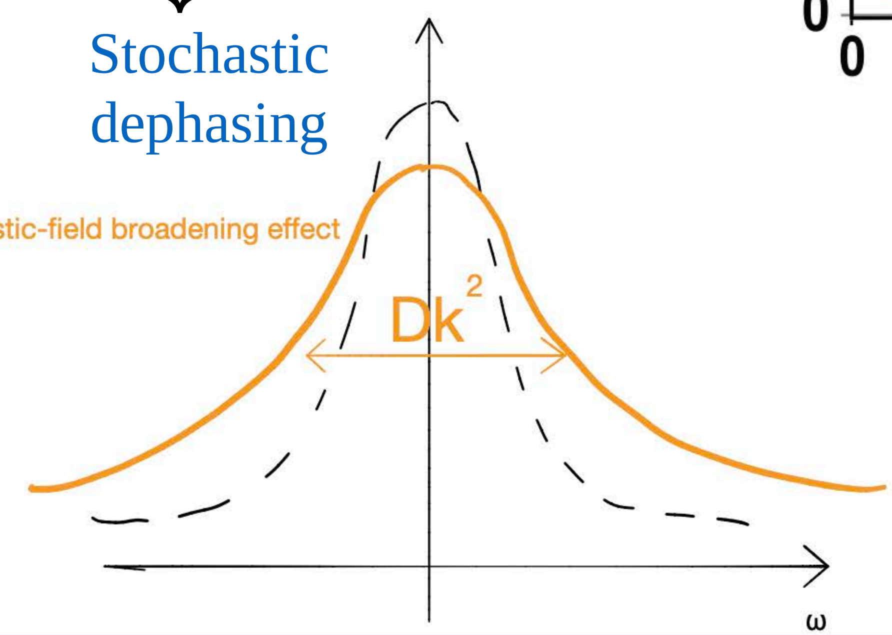
$$b^2 \equiv \left(\frac{\delta B_r}{B_0}\right)^2 > \sqrt{\beta}\rho_*^2 \frac{\epsilon}{q} \sim 10^{-7}$$

$$\begin{cases} l_{ac} \simeq Rq \\ \epsilon \equiv L_n/R \sim 10^{-2} \\ \beta \simeq 10^{-2\sim-3} \\ \rho_* \equiv \frac{\rho_s}{L_n} \simeq 10^{-2\sim-3} \end{cases}$$

Shear flow rate	Alfvénic Dispersion	Natural linewidth	Stochastic broadening	
$k_\theta \Delta x \frac{\partial}{\partial x} u_y$	$v_A \lvert \Delta k_\parallel \rvert$	$\Delta\omega$	$D k_\perp^2$	ω

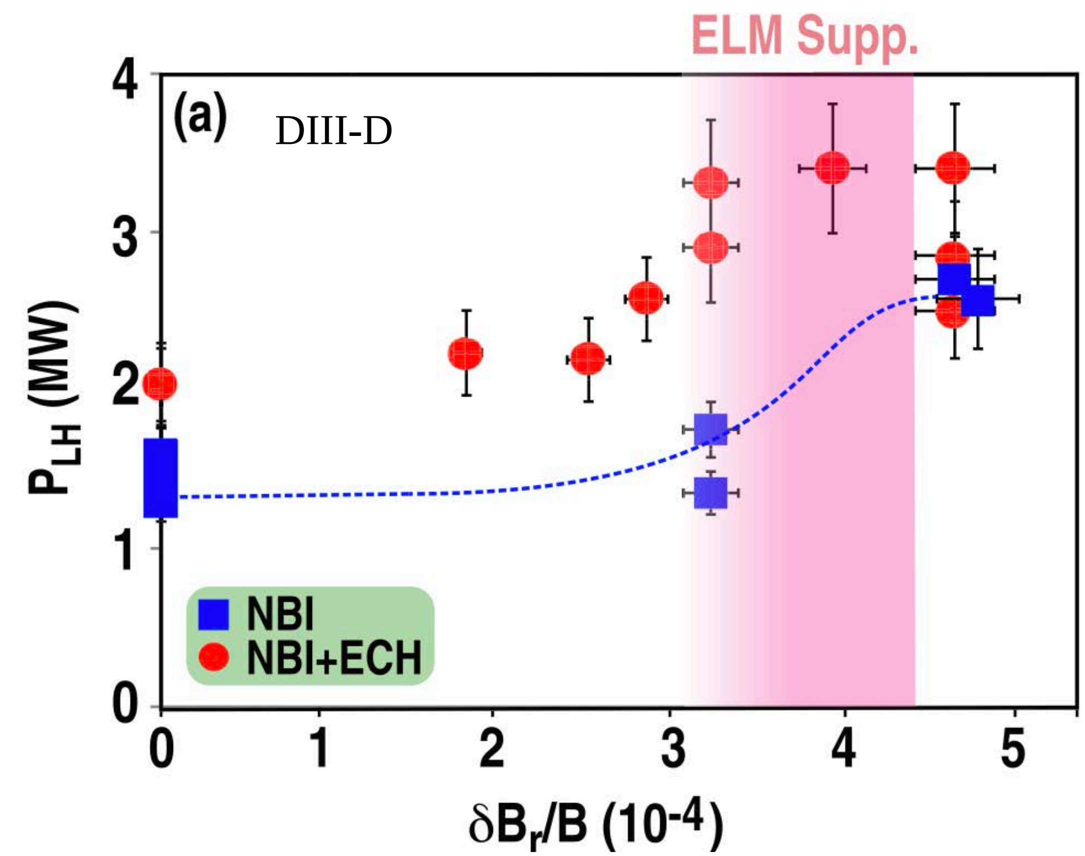
Stochastic dephasing





Expect the increment of power threshold in L-H transition:

Increment of power threshold requires $\Delta\omega < k_\perp^2 D$.



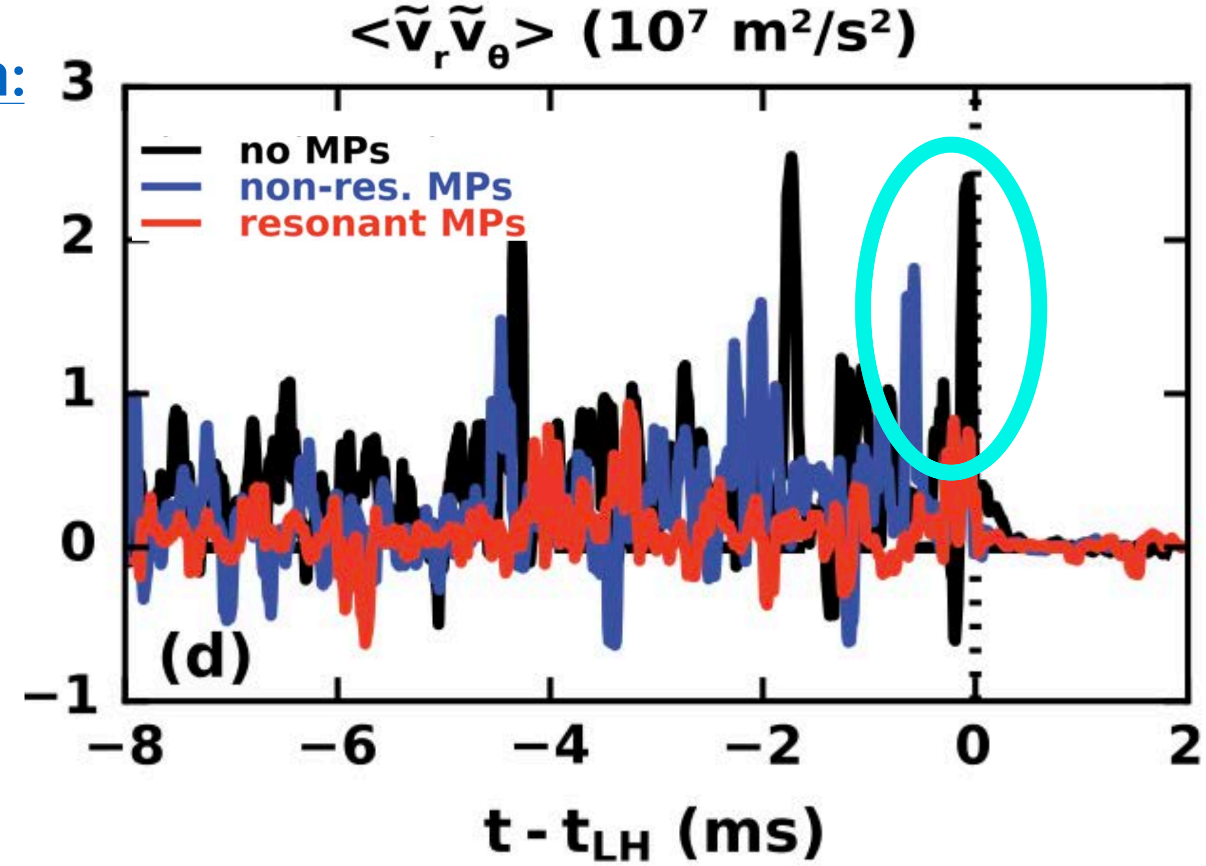
(L. Schmitz et al, NF **59** 126010 (2019))

DIII-D NATIONAL FUSION FACILITY SAN DIEGO

Results— L-H Transition

Rynolds stress burst before L-H transition:

Burst of the Reynolds stress is suppressed by stochastic fields from resonant RMP.



(D. Kriete et al, PoP **27** 062507 (2020))

Suppression of poloidal Reynolds stress:

$$\langle \widetilde{u}_x \widetilde{u}_y \rangle = -\boxed{D_{PV}} \frac{\partial}{\partial x} \langle u_y \rangle + \boxed{F_{res}} \kappa \langle p \rangle$$

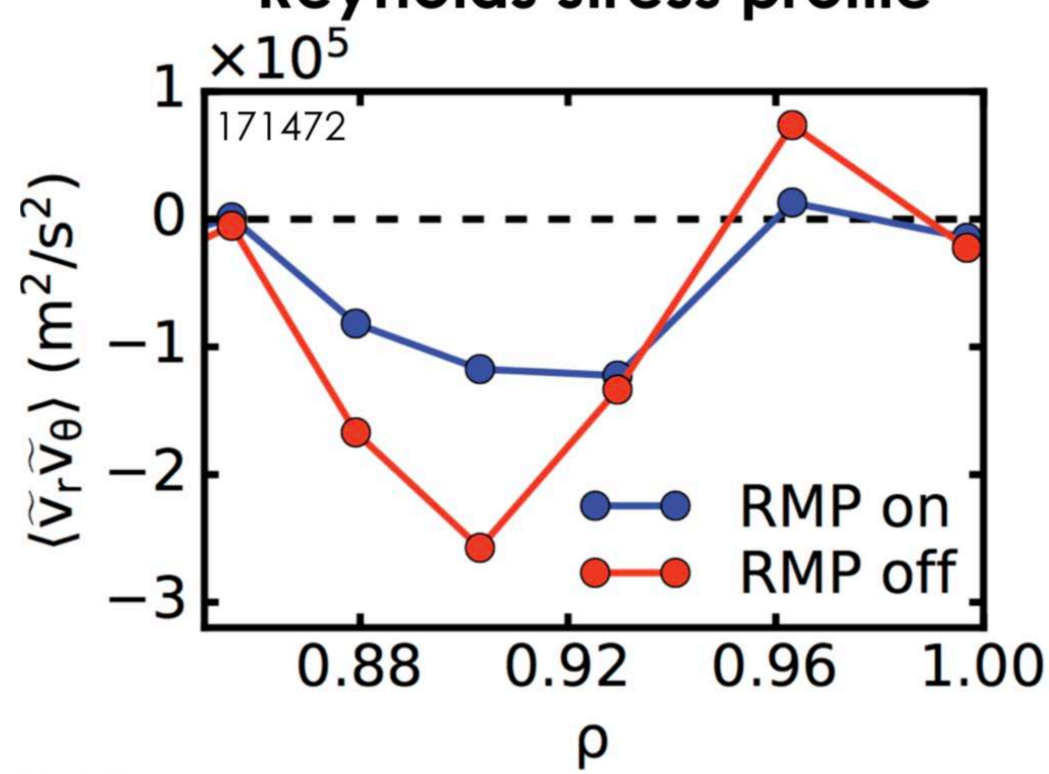
Residual Stress

Suppressed by stochastic fields

$$D_{PV} = \sum_{k\omega} |\widetilde{u}_{x,k\omega}|^2 \frac{v_A (\delta B/B_0)^2 l_{ac} k^2}{\bar{\omega}^2 + \left(v_A (\delta B/B_0)^2 l_{ac} k^2 \right)^2}$$

➤ This stochastic dephasing is insensitive to turbulence modes (e.g. ITG, ETG,…etc.).

Reynolds stress profile



(D. Kriete et al, PoP **27** 062507 (2020))

Physical Picture — Eddy tilting feedback

- **Self-feedback of Reynolds stress:**

Strong Shear enhances the Reynolds stress

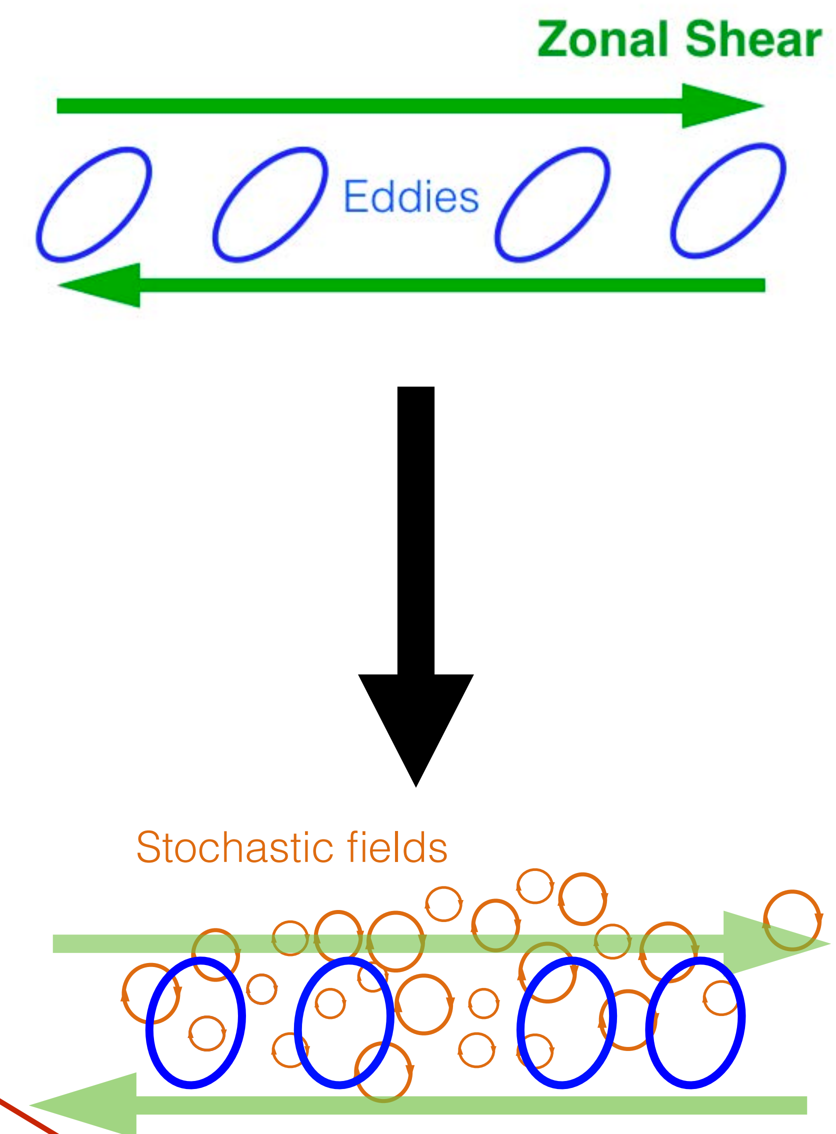
The $E \times B$ shear generates the $\langle k_x k_y \rangle$ correlation and hence support the non-zero Reynolds stress.

$$\langle \tilde{u}_x \tilde{u}_y \rangle \simeq -\sum_k \frac{|\tilde{\phi}_k|^2}{B_0^2}\left(k_y^2 \frac{\partial u_y}{\partial x} \tau_c\right)$$

Reynolds stress support the shear flow

The Reynold stress will modify the shear via momentum transport.

➤ **The shear flow reenforce its self-tilting.**



- **Stochastic fields dephase the self-feedback loop of Reynolds stress:**

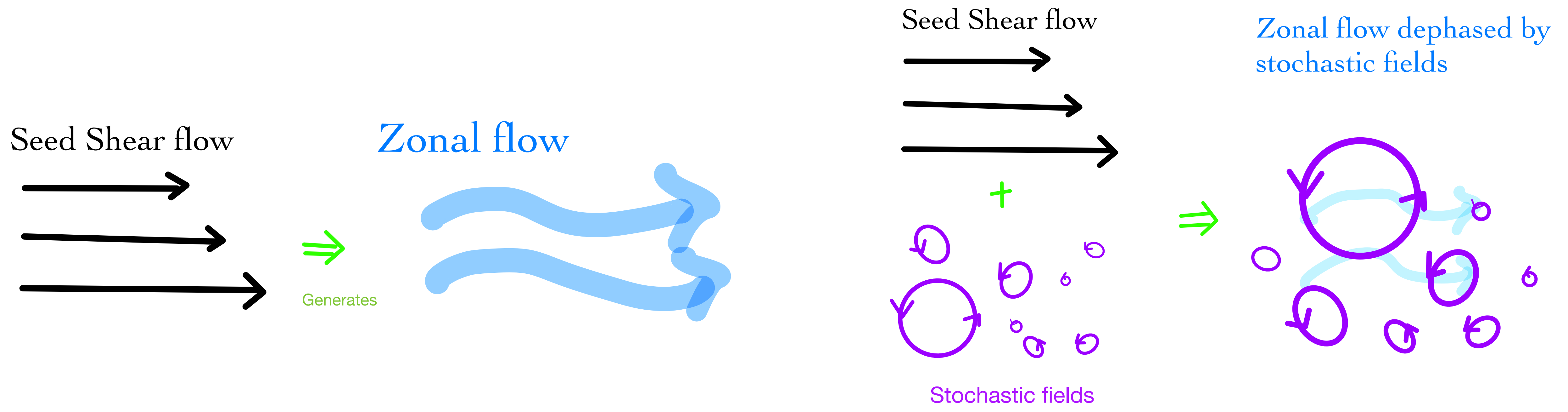
$$\langle \tilde{u}_x \tilde{u}_y \rangle \simeq -\sum_k \frac{|\tilde{\phi}_k|^2}{B_0^2}\left(k_y^2 \frac{\partial u_y}{\partial x} \tau_c - \frac{1}{2} \frac{v_A^2 k_\perp^2}{\omega_0} \frac{\partial |b|^2}{\partial x} \tau_c\right)$$

Stochastic dephasing

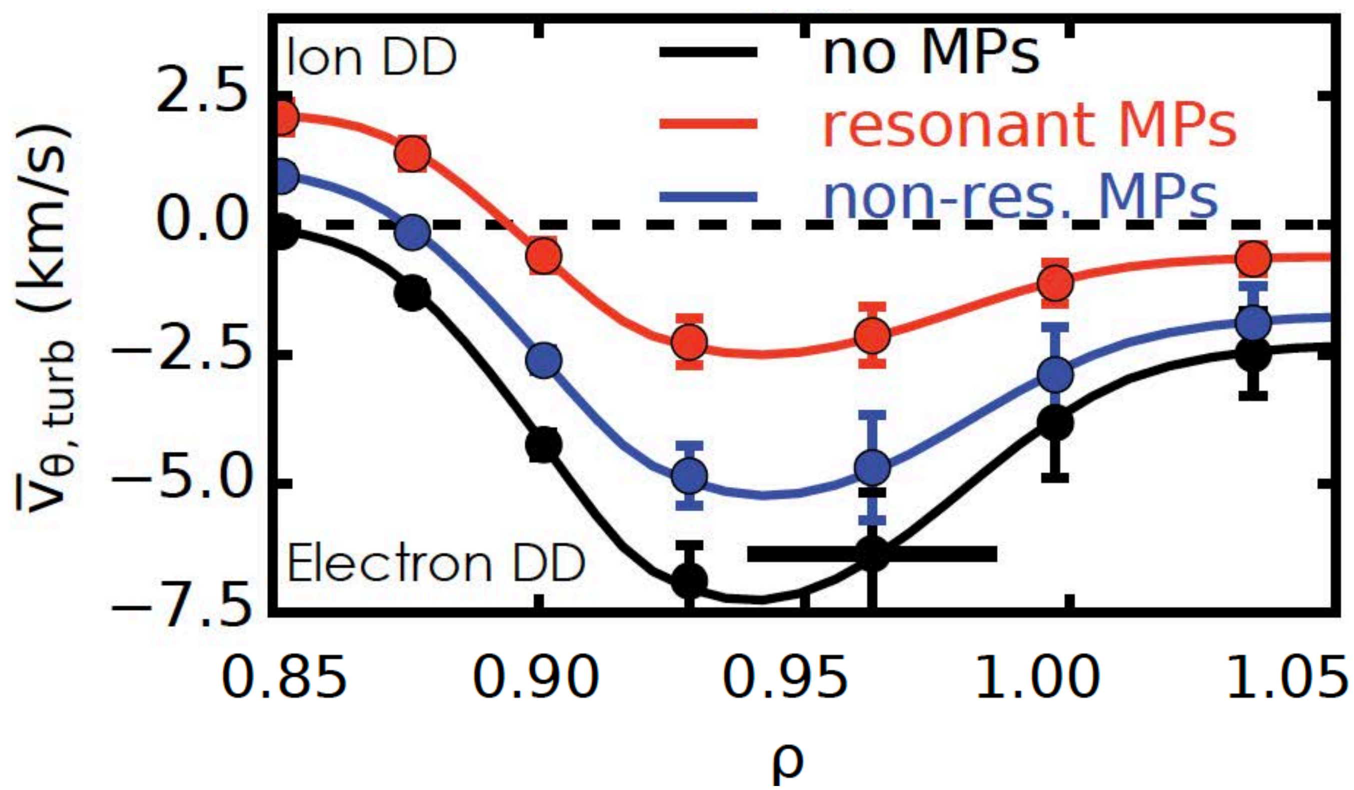
➤ **Shear-tilt feedback loop is broken by stochastic fields.**

Results— L-H Transition

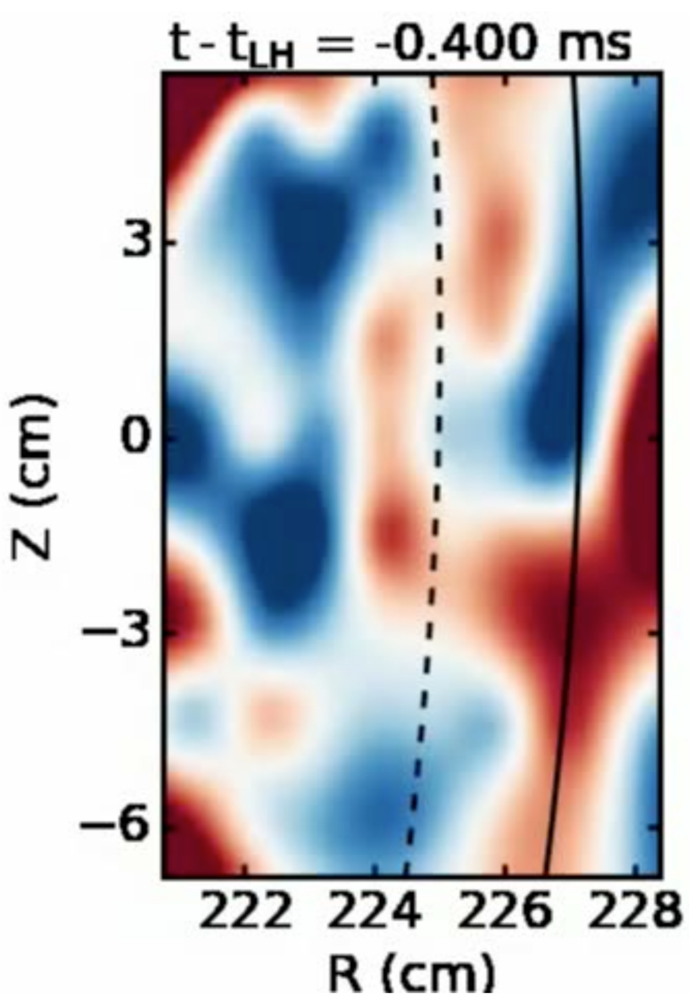
Zonal flow suppressed by stochastic dephasing:



Experimental result:



(D. Kriete et al, PoP **27** 062507 (2020))



(D. Kriete et al, PoP **27** 062507 (2020))

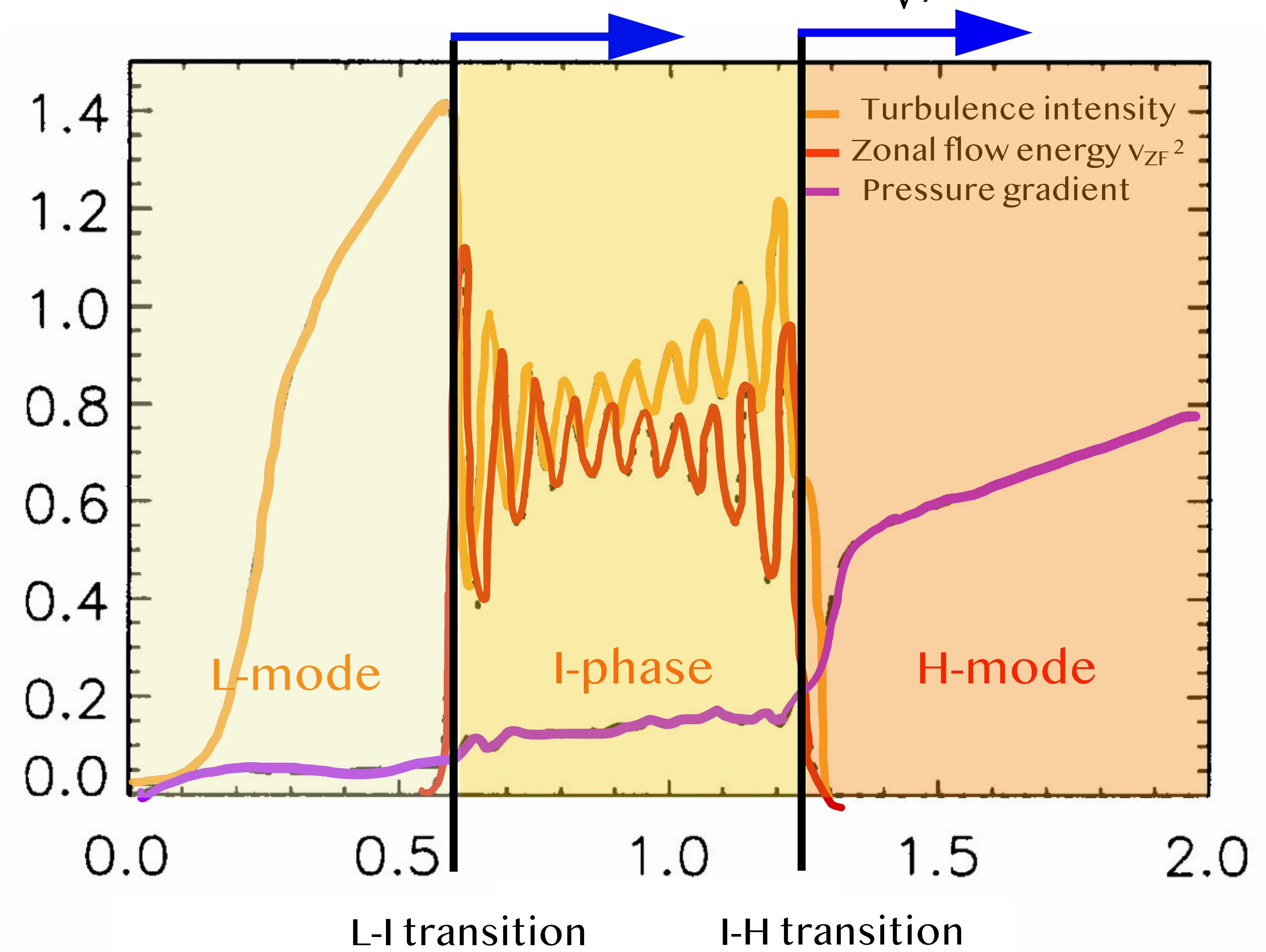
Results— L-H Transition

Macroscopic Impact

- **Extended Kim-Diamond Model:**

  Stochastic fields broadening effect requires: $\Delta\omega \leq k_\perp^2 D$

  This gives dimensionless parameter (α): $\alpha \equiv \frac{b^2}{\sqrt{\beta}}\rho_*^2\frac{q}{\epsilon} > 1$

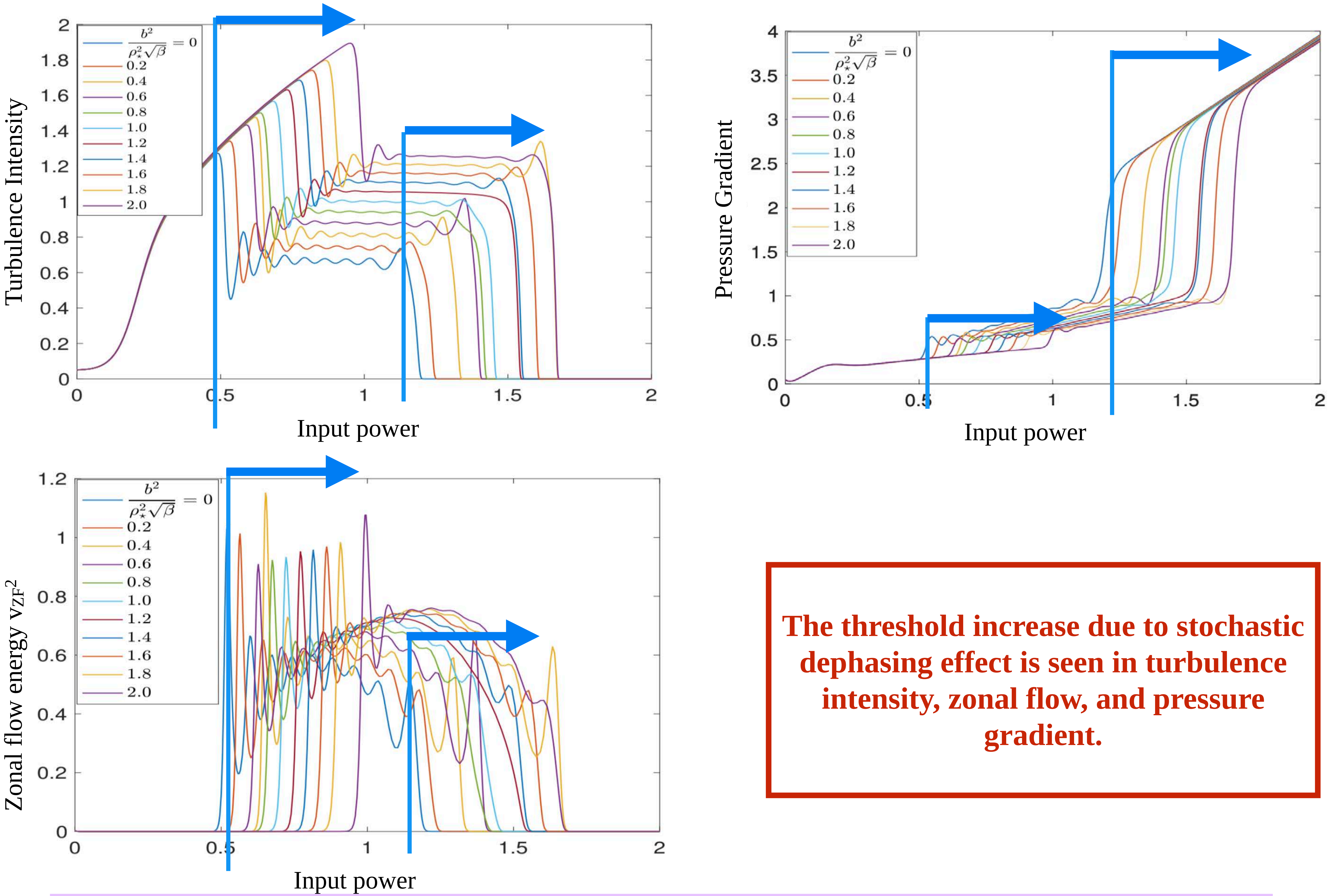


➤ **We expect stochastic fields to raise transition thresholds.**

α quantifies the strength of stochastic dephasing.

Results— Transitions in DIII-D

Extended Kim-Diamond Model



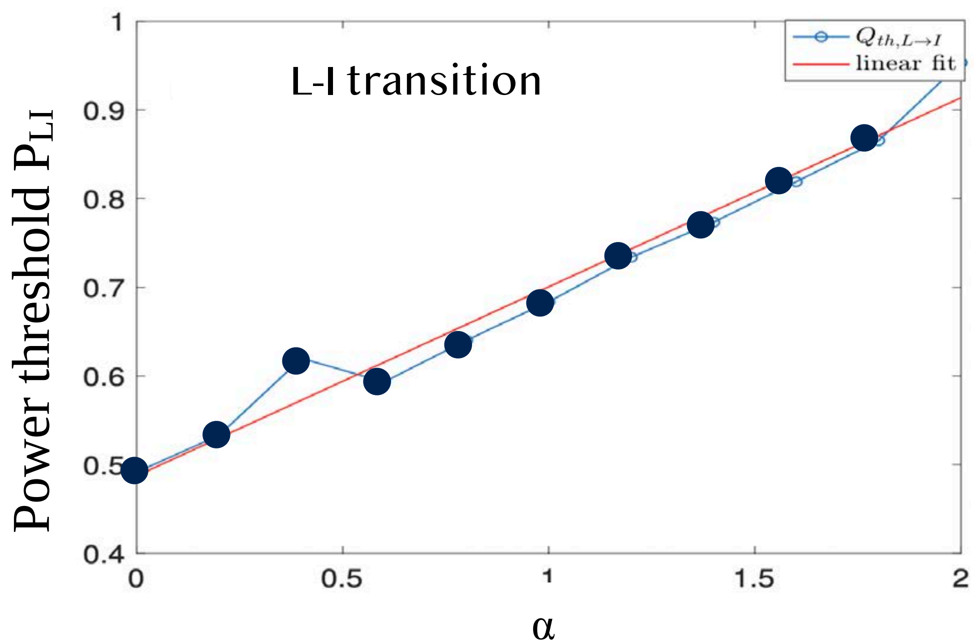
The threshold increase due to stochastic dephasing effect is seen in turbulence intensity, zonal flow, and pressure gradient.

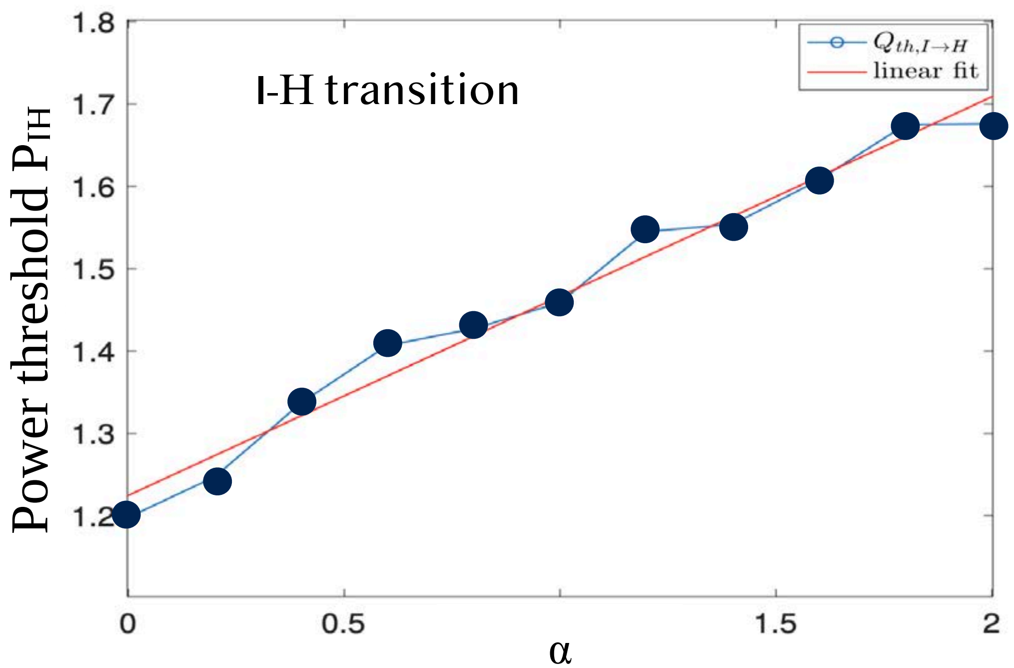
Increment of Power threshold:

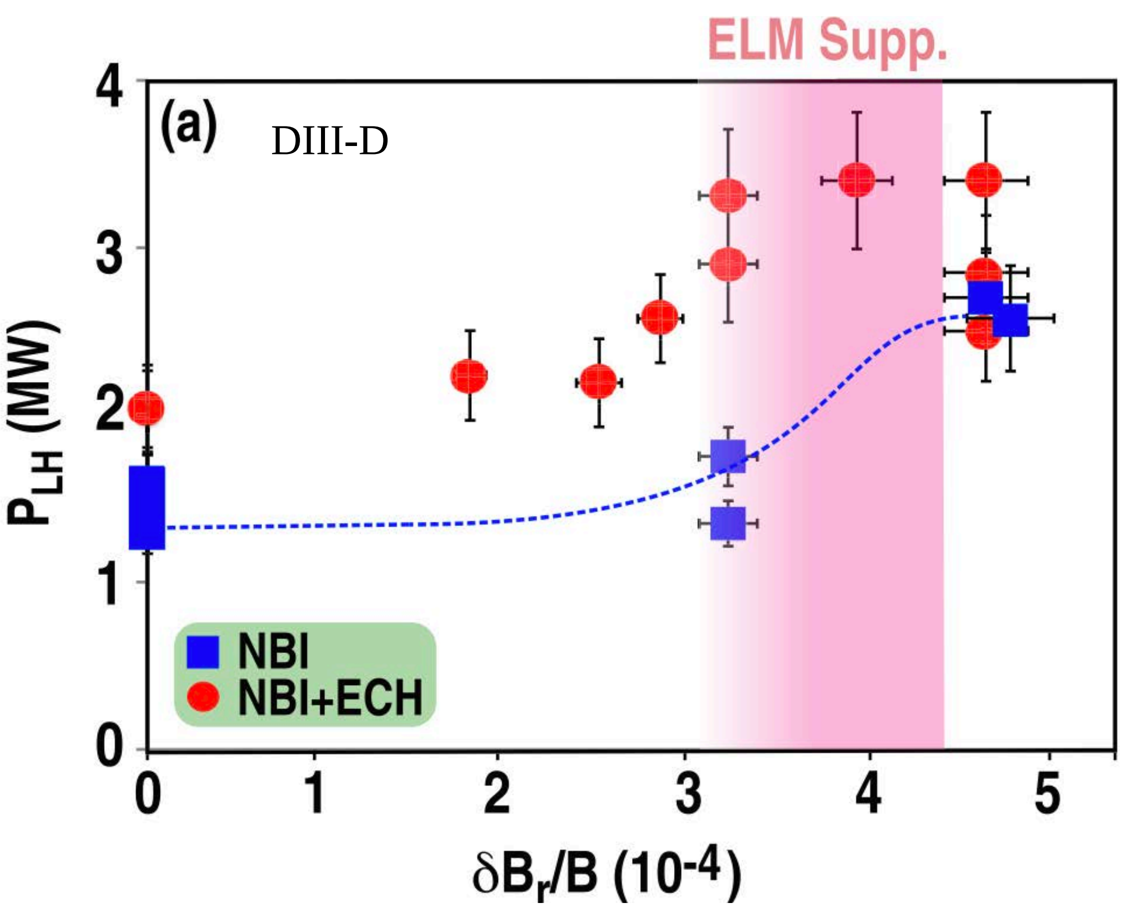
The power threshold increases with the increment of stochastic fields.

$$\alpha \equiv \frac{b^2}{\sqrt{\beta}} \rho_*^2 \frac{q}{\epsilon} > 1$$

$\overline{b^2}$ shift L-H, I-H thresholds to higher power, in promotion to α.



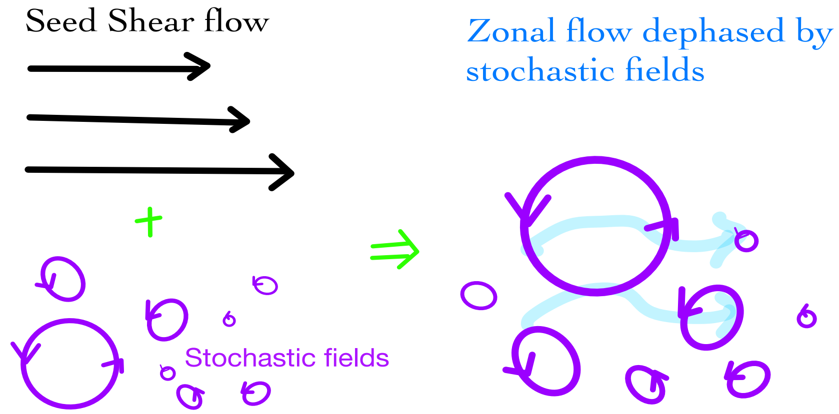


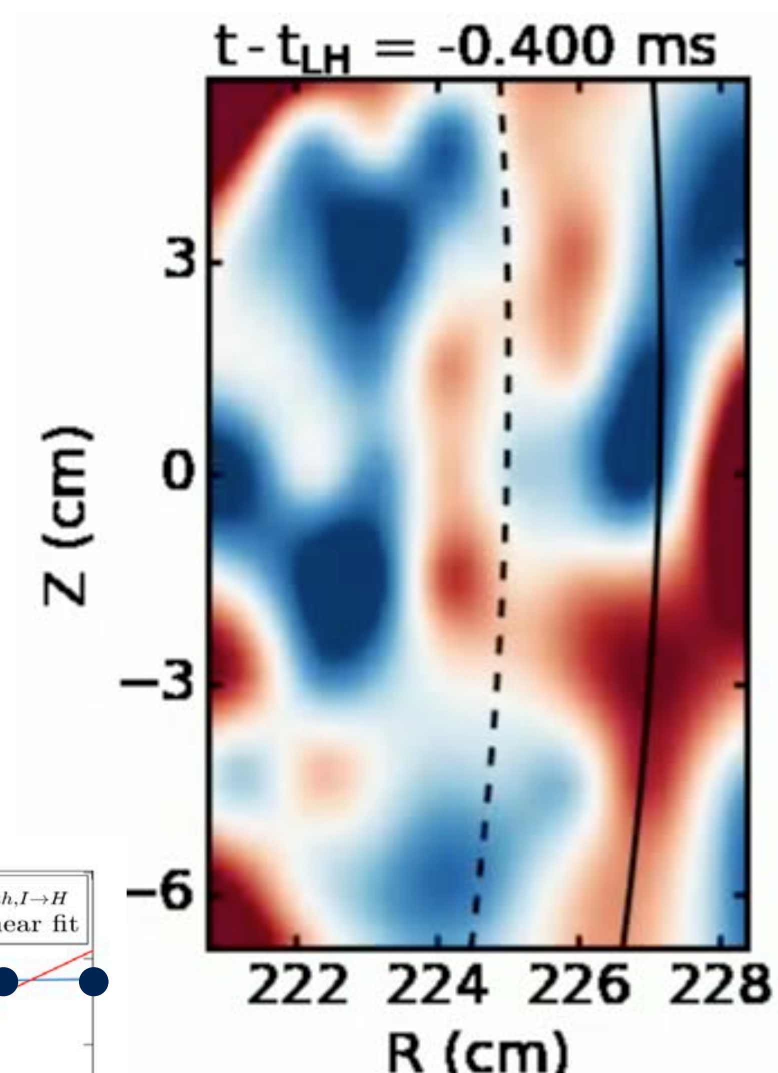


(L. Schmitz et al, NF **59** 126010 (2019))

Conclusions for L-H transition

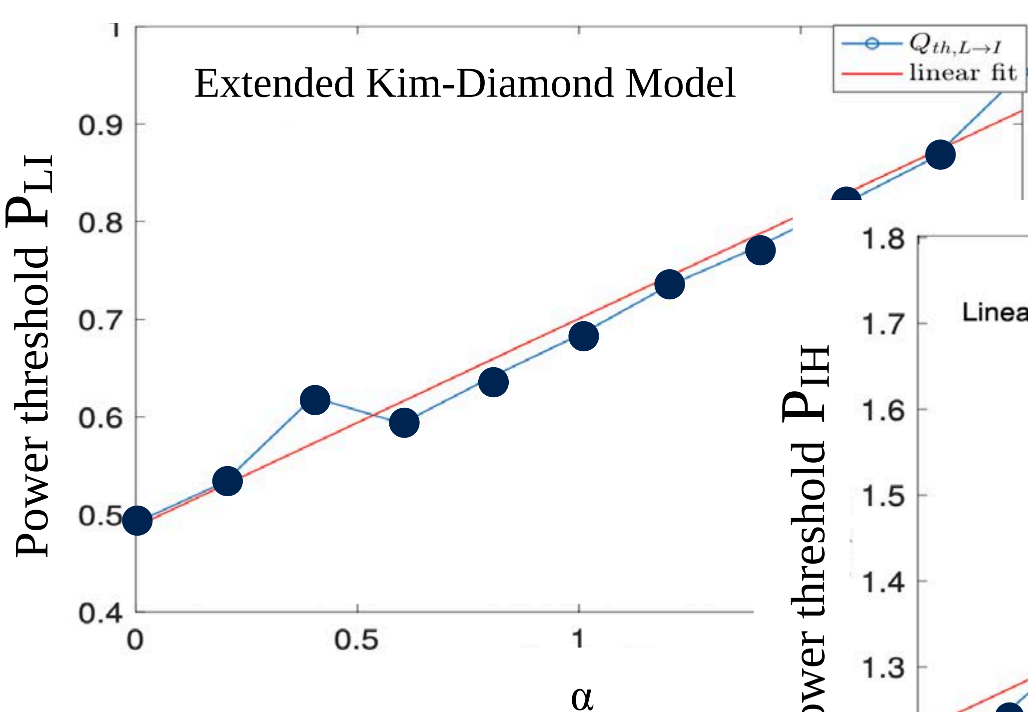
- **Stochastic fields dephase the Reynolds stress, hence suppresses the poloidal zonal flow.**

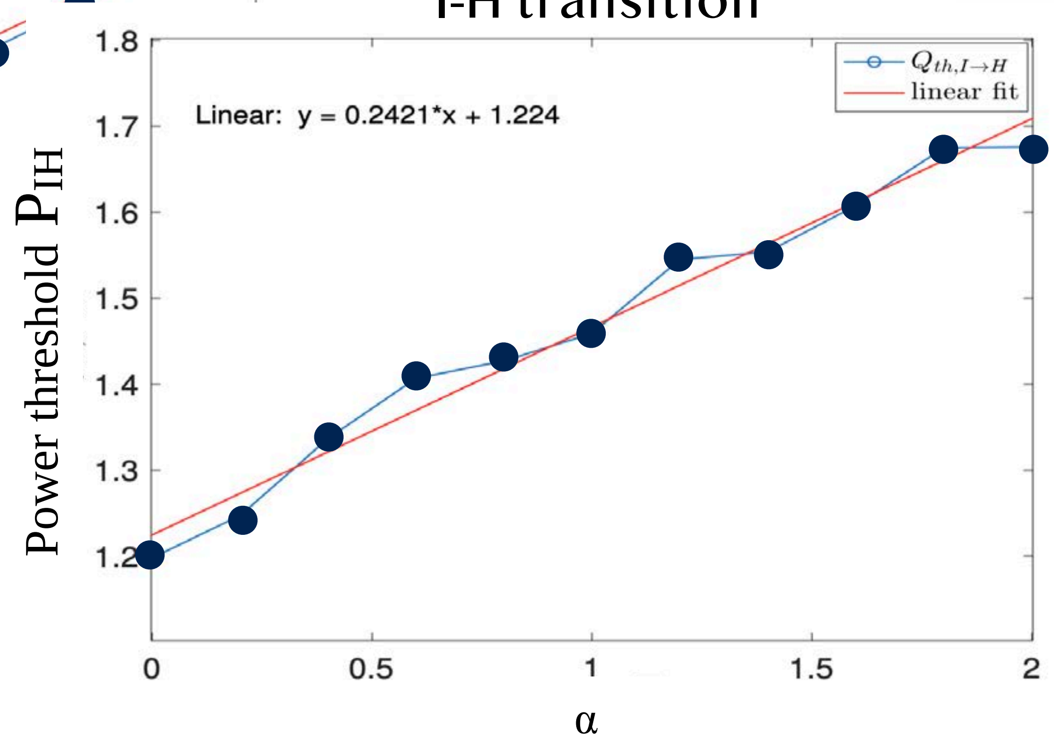




(D. Kriete et al, PoP **27** 062507 (2020))

- **Stochastic fields increase the power threshold for L-H transition, by Reynolds stress decoherence.**





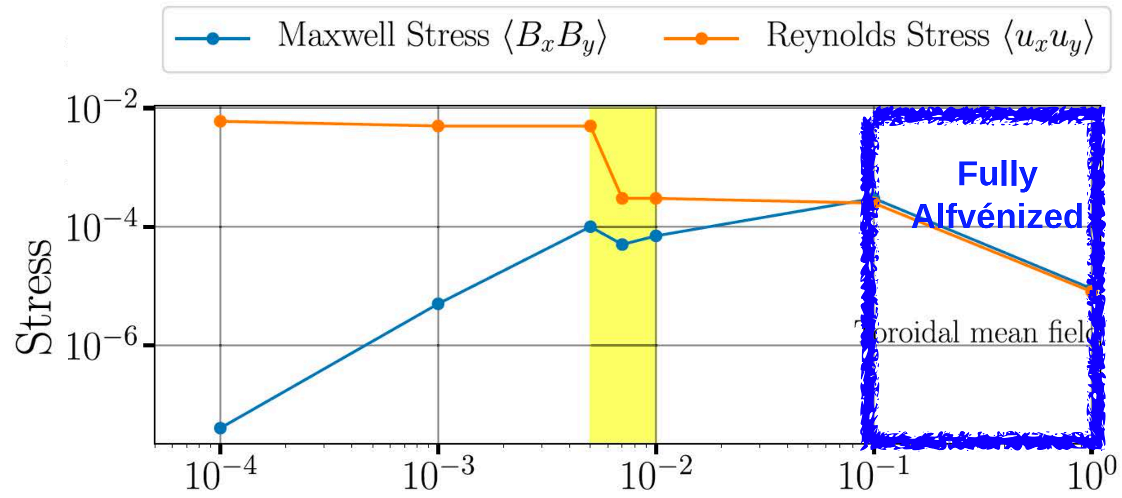
DIII-D NATIONAL FUSION FACILITY SAN DIEGO

UNIVERSITY OF WISCONSIN

Outline

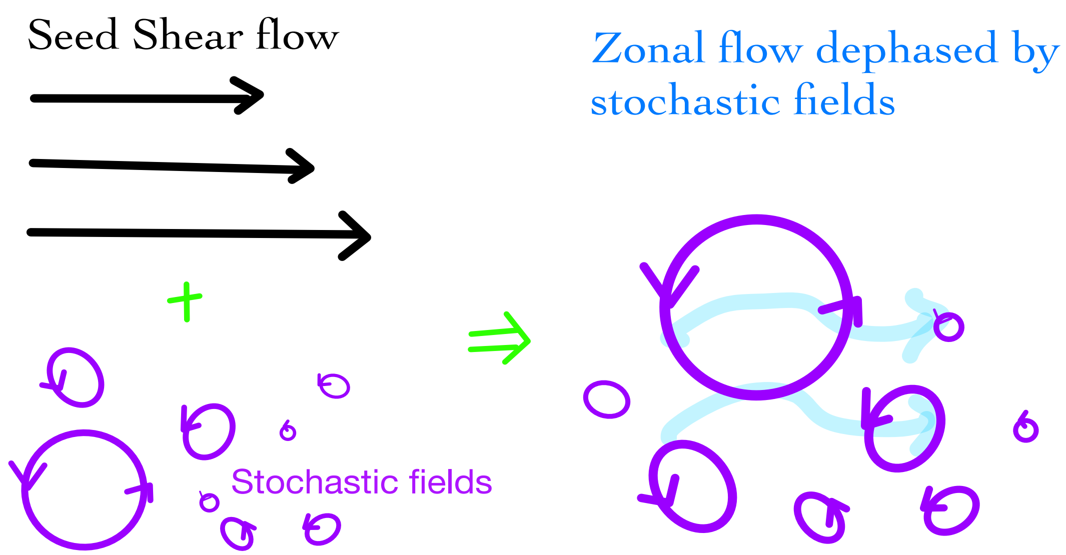
Conclusions — General Ideas

- **Reynolds stress** will be suppressed at levels of B_0 intensity **well below that of Alfvénization**, where Maxwell stress balances the Reynolds stress.

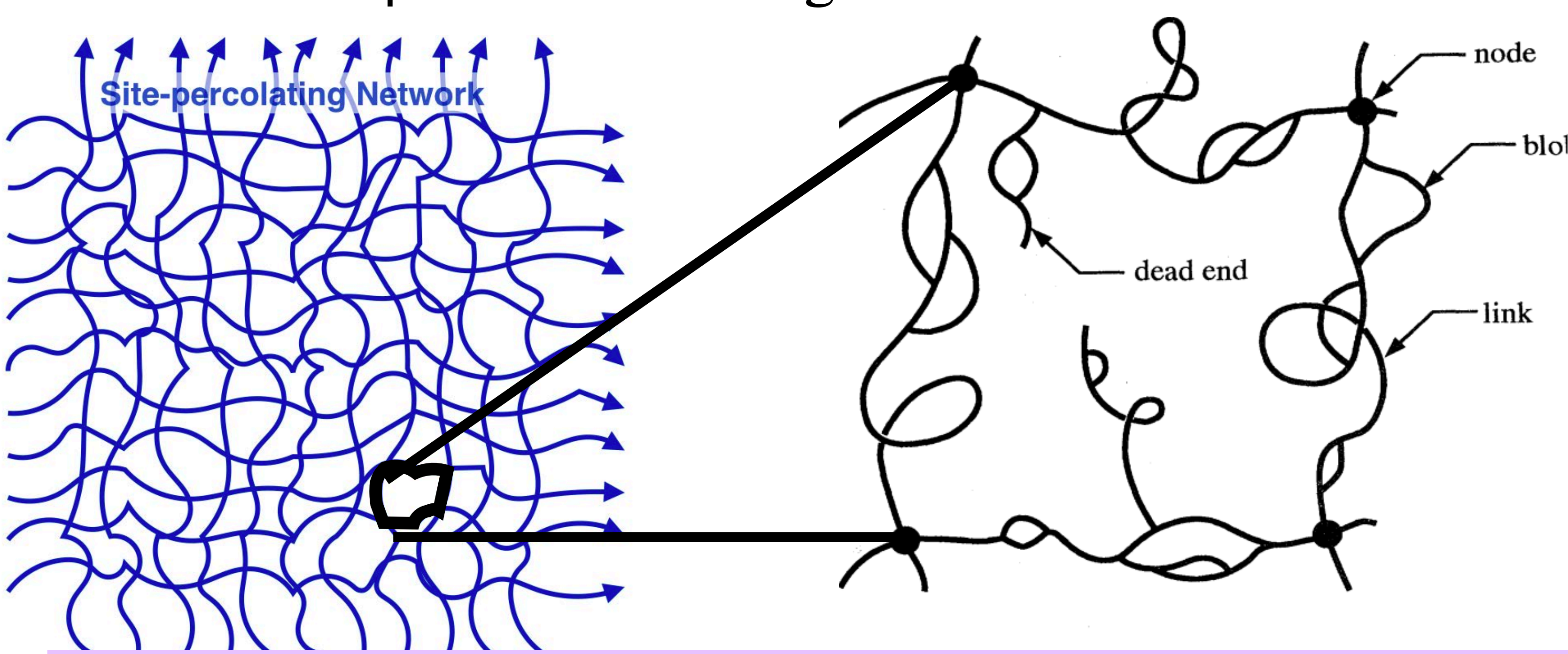


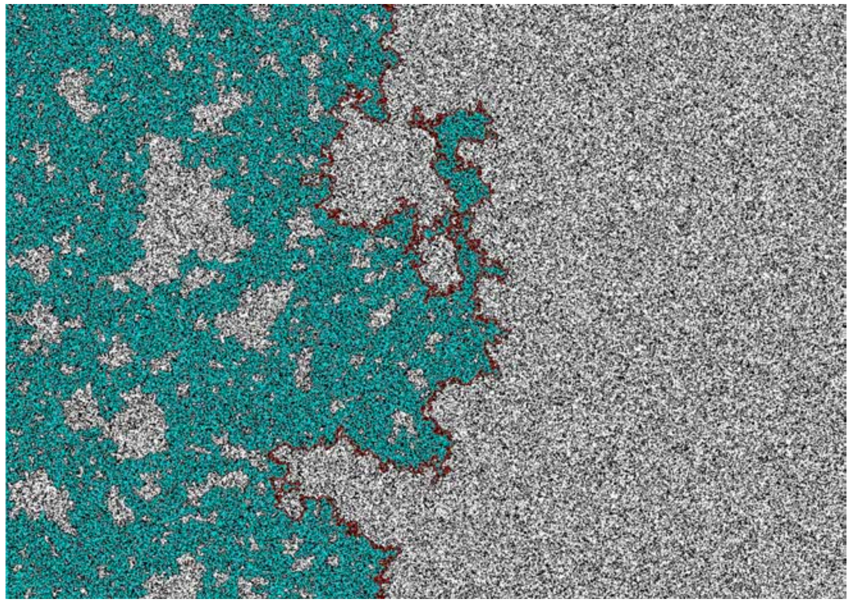
(Chen & Diamond, ApJ 892 24, (2020))

- **Dephasing effect** caused by stochastic fields quenches zonal flow generation.



- Stochastic fields forms a **fractal, elastic network**. Strong coupling of flow turbulence to the fractal network prevents PV mixing and hence zonal flow formation.





Thank you!