

A Mean Field Model of the L→H Transition in a Stochastic Magnetic Field

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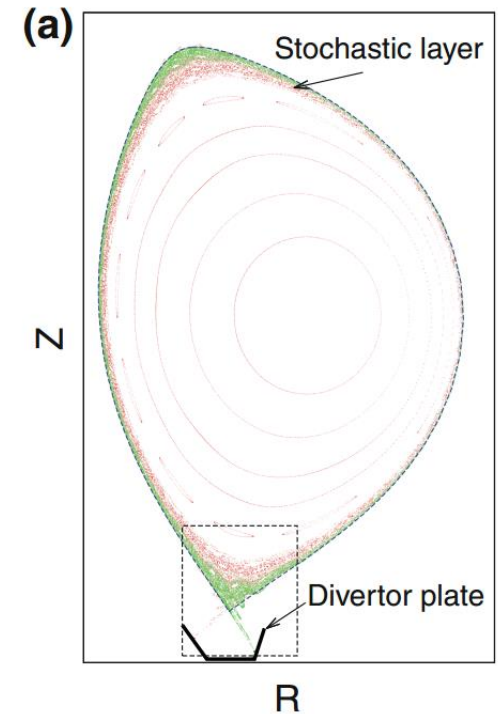
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Outline

- **Motivation and Background**
- **Mean Field Model** (a work in progress)
 - What determines $\langle v'_E \rangle$?
 - Key novelty:** $\langle J_r \rangle$ induced by magnetic perturbation
 - $\langle V_\theta \rangle$ and $\langle V_\phi \rangle$ evolution
 - Particle and heat flux
 - Turbulence intensity
 - Modified 1D “Predator-Prey” model for $\langle E_r \rangle'$
- **Lessons learned, so far**
- **Conclusion and Open Issues**

Stochastic magnetic field

- The phenomenon of **chaos of magnetic field lines** is known as magnetic stochasticity (or magnetic chaos)
- In early studies, magnetic stochasticity is thought to be **bad for confinement** due to the enhanced radial transport of particles and energy along the chaotic field lines.
- However, at end of 1970s, magnetic stochasticity can be used to control the transport of energy and particles.
→ **stochasticity as a positive**: ergodic divertor concept



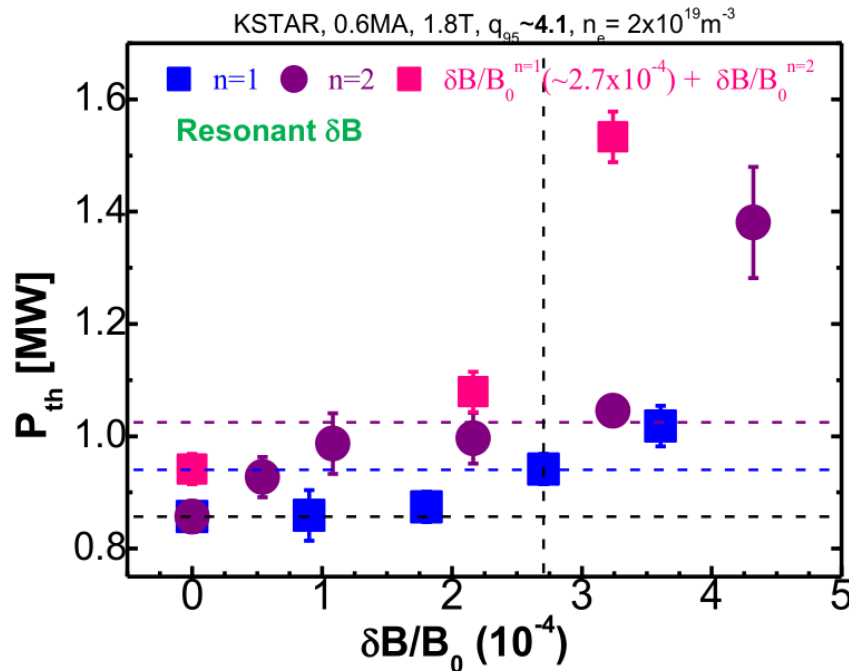
Typical Poincaré section of field lines in DIII-D like plasmas

Stochastic magnetic field induced by RMP

- Most of today's tokamaks need to use **RMP** (**resonant magnetic perturbation**) before L→H transition, in order to **mitigate/suppress** large ELMs, including first
- RMPs are thought to produce **stochastic layer** near separatrix
- The stochastic B field observed to influence
 - threshold and power
 - $V_{E \times B}$ and flows
 - fluctuations

Motivation (why?)

- RMPs increase P_{th} of L→H transition (experiments on DIII-D, MAST, AUG and KSTAR).

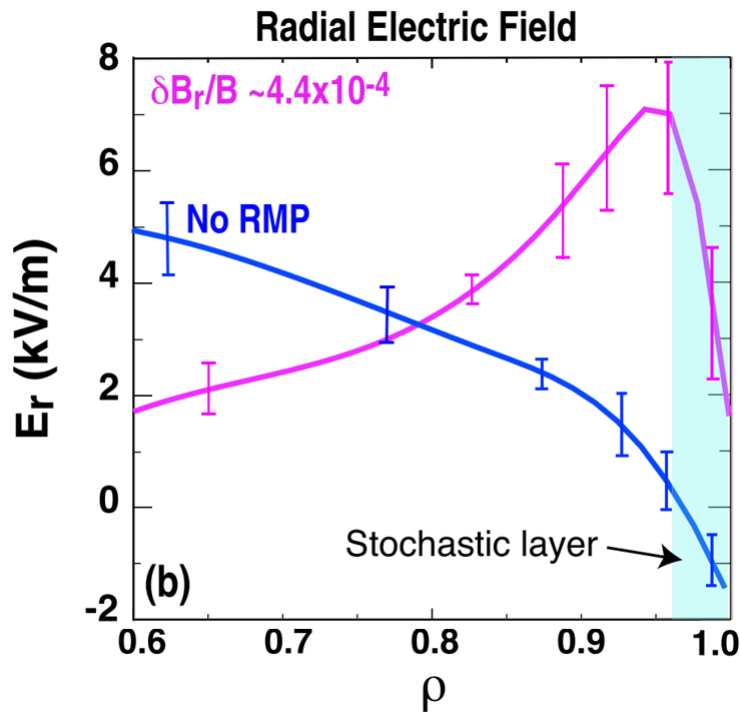


Here, stochastic B-field is thought to be caused by resonant δB .

Need model to elucidate the physical mechanism.

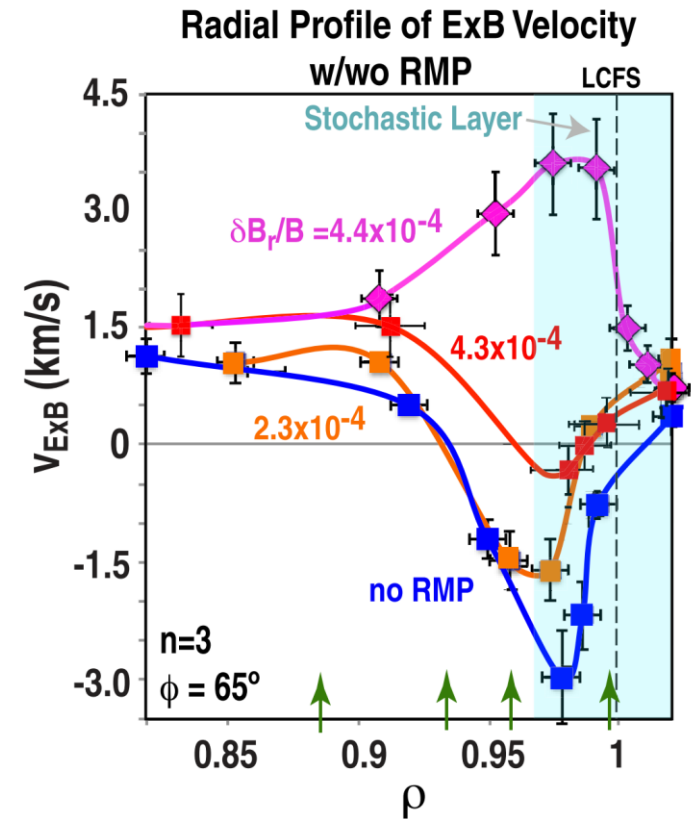
So, how stochastic B-field affects $\langle v'_E \rangle$?

$\langle E_r \rangle$ structure: reversal by RMPs



DIII-D
L-mode

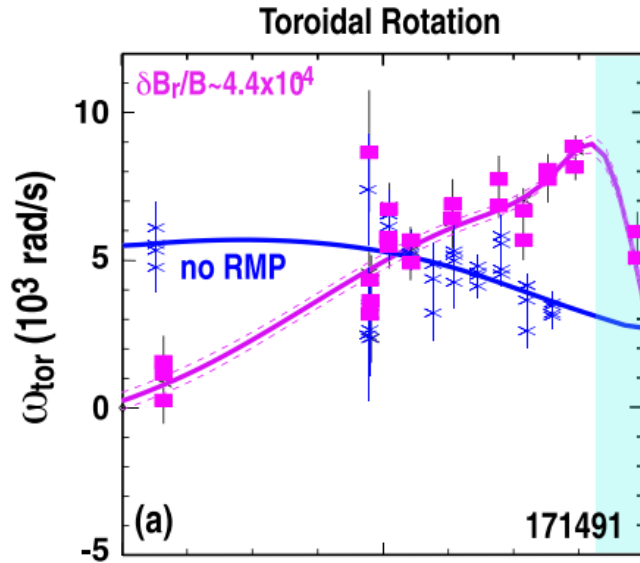
L. Schmitz et al NF 2019



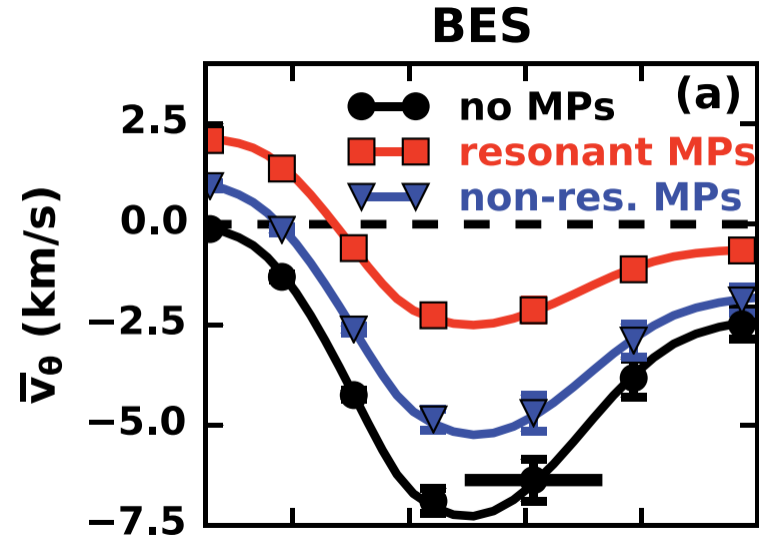
- ✓ E_r reversal or 'bifurcation' : E_r well reduced or inverted to E_r hill with increasing RMP. Edge E_r shear layer sits in stochastic field region.
- ✓ Clear Change in E_r' due to increase of RMP field

Reduced toroidal/poloidal flow by RMPs

L. Schmitz et al NF 2019



Kriete, PoP 2020



- ✓ RMP increased toroidal (co)rotation and shear at separatrix

- ✓ RMP reduces mean turbulence poloidal velocity



Clear effect on flows, and would further affect

$$E_r = \frac{1}{enB} \frac{\partial}{\partial r} P_i + v_\phi B_\theta - v_\theta B_\phi.$$

Fluctuations point: RMPs degrade energy transfer to mean flows prior to L-H transition

Energy balance between turbulence and mean flow^{1,2}

$$\text{Mean flow energy: } E_{\bar{v}} = \frac{1}{2} n_0 m_i \langle \bar{v}_\theta \rangle^2$$

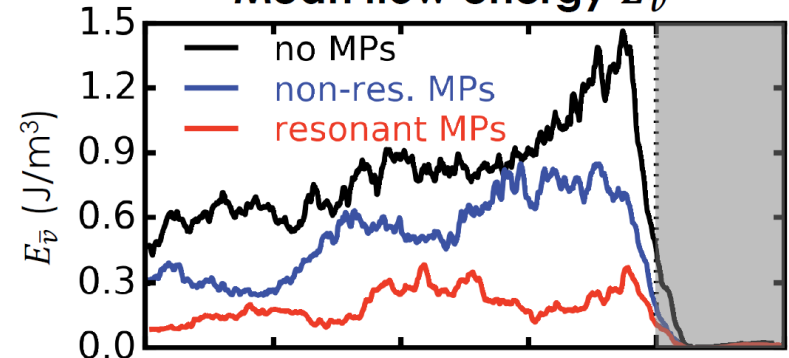
$$\text{Turbulent flow energy: } E_{\tilde{v}} = \frac{1}{2} n_0 m_i (\langle \tilde{v}_r^2 \rangle + \langle \tilde{v}_\theta^2 \rangle)$$

$$\text{Thermal free energy: } E_{\tilde{n}} = \frac{1}{2} n_0 T_{e0} (\tilde{n}_e / n_0)^2$$

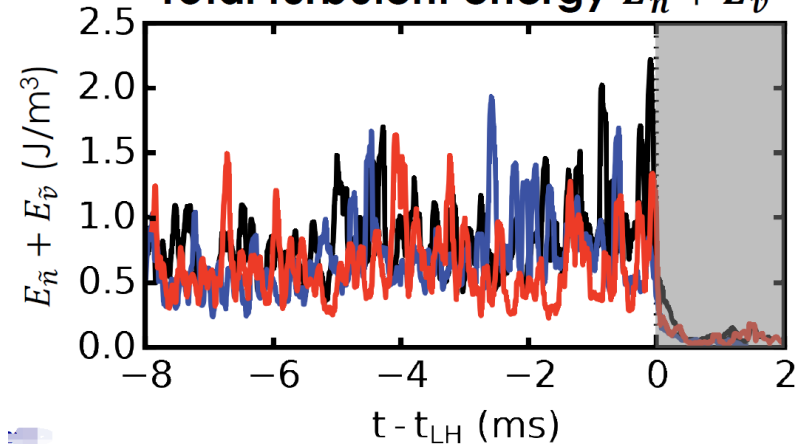
- ✓ $E_{\bar{v}}$ **increases** prior to L-H in axisymmetric case
- ✓ $E_{\bar{v}}$ **decreases** due to RMPs, but total turbulent energy only changes slightly
- ✓ Power transfer to zonal flow decreases

Kriete, PoP 2020

Mean flow energy $E_{\bar{v}}$

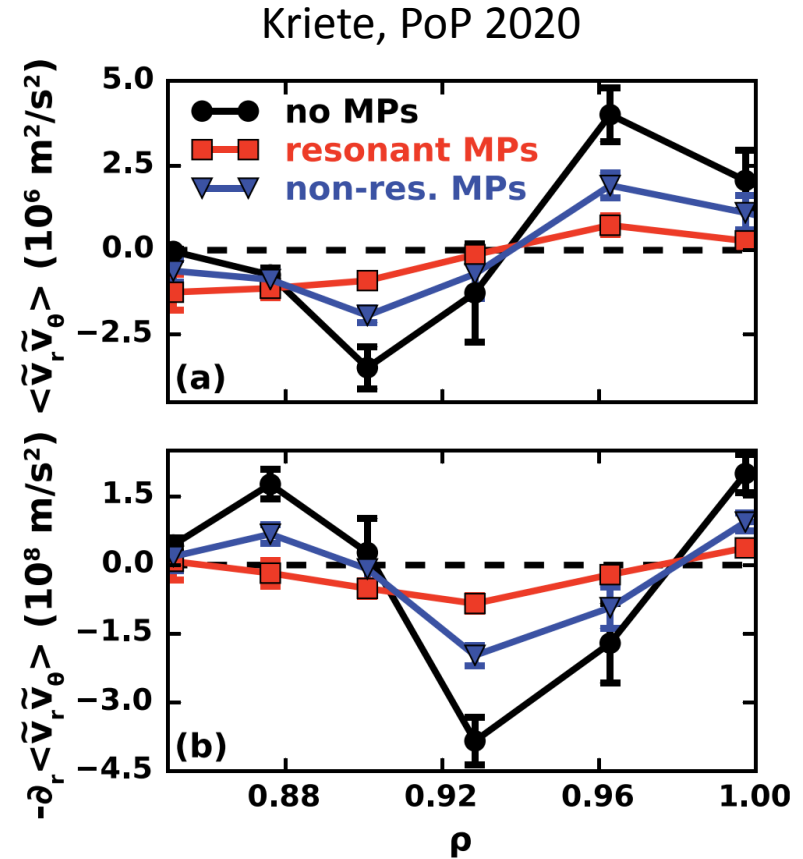
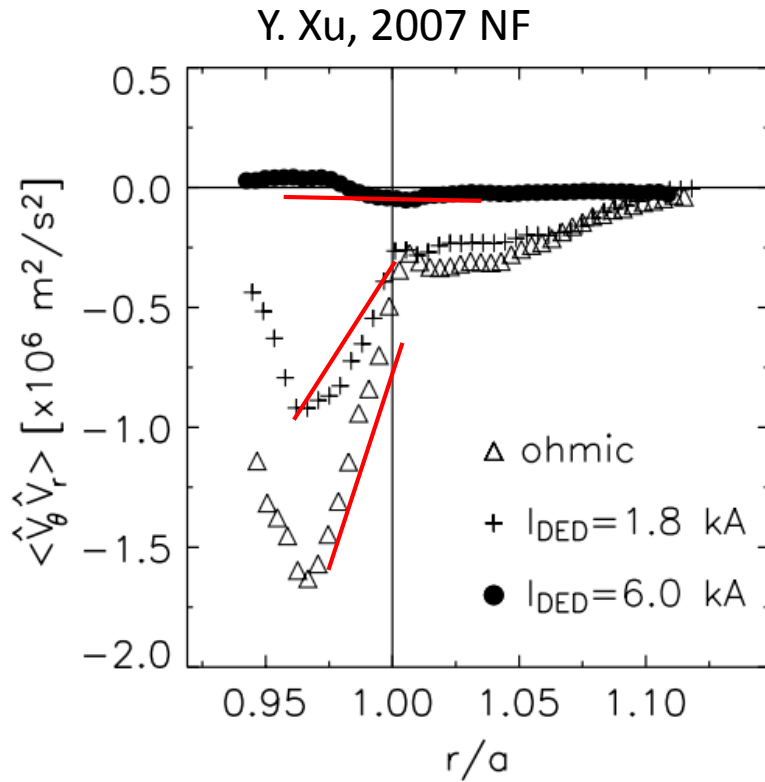


Total turbulent energy $E_{\tilde{n}} + E_{\tilde{v}}$



➔ Need more power to supply sufficient energy to the mean flow

Reduced Reynold stress/force by RMP



✓ Increase I_{DED} reduces **Reynold force**.

✓ RMP reduces Reynold force in L-mode, thus increasing power required for H-mode.



Effects of RMP on transition? \leftrightarrow Model !

Model – $\langle E_r \rangle$ as critical

- **Goal:** ascertain change due to RMP
mean field model for $\langle E_r \rangle$ and $\langle v'_E \rangle$

— Radial force balance equation: $\langle E_r \rangle$ is determined by

$$\langle E_r \rangle = \frac{\langle \nabla P_i \rangle}{ne} - \langle V_\theta \rangle B_\phi + \langle V_\phi \rangle B_\theta$$

Diamagnetic drift Poloidal rotation Toroidal rotation

Γ_e, Q_i

$\langle J_r \rangle \leftrightarrow \langle \tilde{b}_r \tilde{b}_\theta \rangle$

— Direct effect of $|\tilde{b}_r|^2$ on turbulence discussed later. Take turbulence as electrostatic in L-mode.

— Prescribe stochastic B-field

$\langle J_r \rangle$ is key to L→H trigger mechanism

- L→H transition:

V'_E shearing feedback (mainly ∇P)
+ “trigger” due to $\langle J_r \rangle$

- Several candidates for $\langle J_r \rangle$

—Radial flux of polarization charge → turbulence intensity gradient scale

$$\rho_s^2 \langle \tilde{v}_r \nabla_{\perp}^2 \tilde{\phi} \rangle \rightarrow \frac{\partial}{\partial r} \langle \tilde{V}_{\theta} \tilde{V}_r \rangle : \text{Reynolds force}$$

Critical Reynolds work criterion [Rich literatures: Diamond, Tynan et al]

—neoclassical polarization → $\rho_{\theta i}$ scale [S-I. Itoh, et al PRL, 60 (1988) 2276]

—orbit loss → $\rho_{\theta i}, \dots$ [K.C.Shaing, PoF B: Plasma Physics 4 (1992) 171]

— **New player** here: $\langle J_r \rangle = \frac{\langle \tilde{J}_{\parallel} \tilde{B}_r \rangle}{B}$

stochastic field intensity profile scale, how enters?

Common element: $\langle J_r \rangle$ induced by stochastic B-field

- Ambipolarity breaking due to stochastic field $\Rightarrow \langle J_r \rangle$

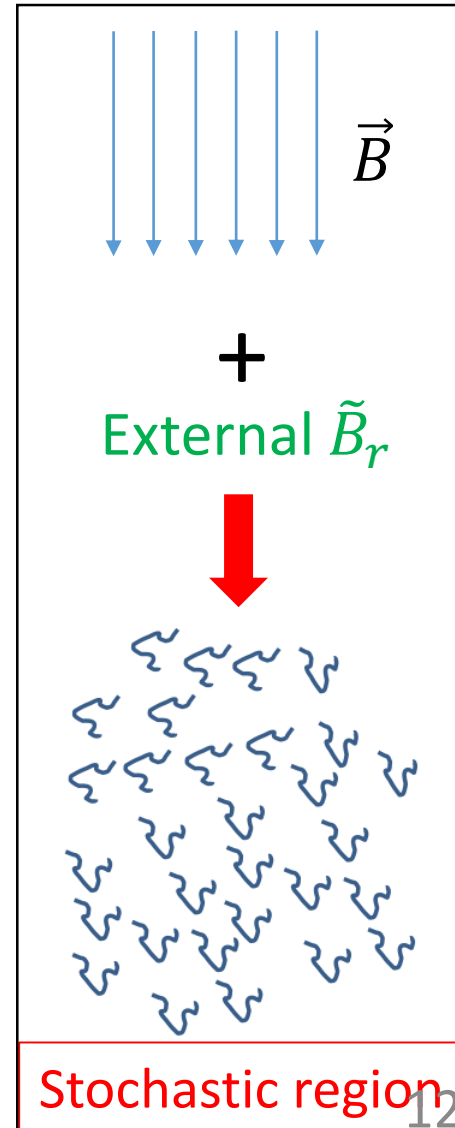
$$\langle J_r \rangle = \left\langle \vec{J}_{\parallel} \cdot \vec{e}_r \right\rangle = \frac{\langle \tilde{J}_{\parallel} \tilde{B}_r \rangle}{B} \quad \langle J_{\parallel} \rangle = \langle J_{\parallel,e} \rangle + \langle J_{\parallel,i} \rangle$$

- From Ampere law: $\tilde{J}_{\parallel} = -\frac{c}{4\pi} \nabla^2 \tilde{A}_{\parallel}$

Stochastic field produces currents in plasmas

$$\begin{aligned} \langle J_r \rangle &= \frac{\langle \tilde{J}_{\parallel} \tilde{B}_r \rangle}{B} = -\frac{c}{4\pi B} \left\langle \frac{\partial}{\partial y} \tilde{A}_{\parallel} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \tilde{A}_{\parallel} \right\rangle \\ &= -\frac{c}{4\pi B} \frac{\partial}{\partial x} \left\langle \left(\frac{\partial}{\partial x} \tilde{A}_{\parallel} \right) \left(\frac{\partial}{\partial y} \tilde{A}_{\parallel} \right) \right\rangle = \frac{c}{4\pi B} \frac{\partial}{\partial x} \langle \tilde{B}_x \tilde{B}_y \rangle \\ &= \frac{cB}{4\pi} \frac{\partial}{\partial x} \langle \tilde{b}_x \tilde{b}_y \rangle \Rightarrow \frac{cB_0}{4\pi} \frac{\partial}{\partial r} \langle \tilde{b}_r \tilde{b}_\theta \rangle \quad \boxed{\text{Maxwell Force}} \end{aligned}$$

Note: $\langle J_r \rangle$ tracks momentum, not heat transport.



Phase in Maxwell stress

- Maxwell stress $\langle \tilde{B}_r \tilde{B}_\theta \rangle = \sum_k |\tilde{A}_k|^2 \langle k_x k_y \rangle$

➤ \tilde{A}_k tilted by developing $E \times B$ flow

$$\frac{\partial A}{\partial t} + V \cdot \nabla A = \eta J$$

$$\frac{\partial A}{\partial t} + V'_E \frac{\partial A}{\partial y} + \tilde{V} \cdot \nabla A = \eta J$$

$$k_x = k_x^{(0)} - k_y V'_E \tau_c$$

$$\tau_c: \left\{ \begin{array}{l} \textit{shear} \\ \textit{fluctuations} \end{array} \right\}$$

➤ Thus, $\langle \tilde{B}_r \tilde{B}_\theta \rangle = - \sum_k |\tilde{A}_k|^2 \langle k_y^2 V'_E \tau'_c \rangle$

- Hence, Reynolds and Maxwell cross-phase closely linked.

Stochastic B-field affects $\langle V_\theta \rangle$

- For V_θ :
Poloidal momentum balance

$$\frac{\partial \langle V_\theta \rangle}{\partial t} = -\mu(\langle V_\theta \rangle - V_{\theta,neo}) - \frac{\partial}{\partial r} \left(\langle \tilde{V}_\theta \tilde{V}_r \rangle - \frac{1}{4\pi\rho} \langle \tilde{B}_r \tilde{B}_\theta \rangle \right)$$

Turbulence
Reynold stress Maxwell stress
of stochastic field
perturbation

$B_\phi \langle J_r \rangle$

- For SS: $\langle V_\theta \rangle = V_{\theta,neo} - \frac{1}{\mu} \frac{\partial}{\partial r} \left(\langle \tilde{V}_\theta \tilde{V}_r \rangle - \frac{1}{4\pi\rho} \langle \tilde{B}_r \tilde{B}_\theta \rangle \right)$
 $= V_{\theta,neo} - \frac{1}{\mu} \frac{\partial}{\partial r} \left(\frac{1}{B^2} \tau_c V_E' \frac{I}{1 + \alpha V_E'^2} - \frac{B^2}{4\pi\rho} \tau_c' V_E' |\tilde{b}_r|^2 \right)$

with

$$\mu = \mu_{00} \left(1 + \frac{v_{CX}}{v_{ii}} \right) \nu_{ii} q^2 R^2 \quad V_{\theta,neo} \approx -1.17 \frac{\partial T_i}{\partial r} \quad \tau_c' = \left(\frac{k_\theta^2 V_E'^2 D_T}{3} \right)^{-1/3}$$

- V_E' phasing via tilt tends to align turbulence and stochastic B-field, which counteracts the spin-up of $\langle V_\theta \rangle$.
- $\frac{\partial}{\partial r} |\tilde{b}_r|^2$ not only $|\tilde{b}_r|^2$, plays a role.

Stochastic B-field affects $\langle V_\phi \rangle$

- For V_ϕ :
$$\frac{\partial \langle V_\phi \rangle}{\partial t} + \nabla \cdot \langle \tilde{V}_r \tilde{V}_\phi \rangle = \frac{1}{\rho c} \langle J_r \rangle B_\theta + S_M$$

$$\langle \tilde{V}_r \tilde{V}_\phi \rangle = -\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle, \quad \chi_\phi = \chi_T = \frac{\rho_s^2 c_s}{L_T}, \quad S_M = S_a \exp\left(-\frac{r^2}{2L_{M,dep}^2}\right)$$

Only consider diffusive term.

G.B.

Momentum source tracks heat (from core).

$$\Rightarrow \frac{\partial \langle V_\phi \rangle}{\partial t} = \frac{\partial}{\partial r} \left(\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle \right) + \frac{1}{4\pi\rho} \frac{B_\theta}{B} \frac{\partial}{\partial r} \langle \tilde{B}_r \tilde{B}_\theta \rangle + S_M$$

- For SS:
$$\frac{\partial}{\partial r} \left(\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle \right) = -\frac{V_{Ti}^2}{\beta} \frac{B_\theta}{B} \frac{\partial}{\partial r} \langle \tilde{b}_r \tilde{b}_\theta \rangle - S_M$$

Stochasticity edge toroidal velocity, shear

$$\Rightarrow \frac{\partial}{\partial r} \langle V_\phi \rangle |_{r_{sep}} = -\frac{1}{\chi_\phi} \int_0^{r_{sep}} S_M dr - \frac{V_{Ti}^2}{\beta \chi_\phi} \frac{B_\theta}{B} \langle \tilde{b}_r \tilde{b}_\theta \rangle |_{r_{sep}} \quad B_\theta \langle J_r \rangle$$

Integrated external torque

With $\langle \tilde{b}_r \tilde{b}_\theta \rangle = k_y^2 V_E' \tau_c' |\tilde{b}_r|^2$

✓ Force through radial current across separatrix.

✓ Shear affected by stochasticity. 15

Stochastic B-field affects electron density flux

- For electron density :
$$\frac{\partial n_e}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_e) = S_p$$


with $\Gamma_e = -(D_{neo} + D_T) \frac{\partial n}{\partial r} + \Gamma_{e, stoch}$ ←


- $D_{neo} = (m_e/m_i)^{1/2} \chi_{i, neo}$
- $D_T \sim b D_{GB}$ with $b < 1$

$$S_p = \Gamma_a \frac{a - r + d_a}{L_{dep}^2} \exp\left(-\frac{(a + d_a - r)^2}{2L_{dep}^2}\right)$$

- The stochastic field can induce particle flux ($n_e = n_i$):

$$\Gamma_{e, stoch} = \frac{c}{4\pi e} \langle \tilde{b}_r \nabla_{\perp}^2 \tilde{A}_{\parallel} \rangle + n \langle \tilde{V}_{\parallel, i} \tilde{b}_r \rangle$$


 $\langle \tilde{b}_r \tilde{J}_{\parallel} \rangle$


 $\langle \tilde{b}_r \tilde{J}_{\parallel, i} \rangle$

with

$$\checkmark \frac{c}{4\pi e} \langle \tilde{b}_r \nabla_{\perp}^2 \tilde{A}_{\parallel} \rangle = -\frac{c}{4\pi e} \frac{\partial}{\partial r} \langle \tilde{b}_r \tilde{b}_{\theta} \rangle = -n \frac{D_B}{\beta} \frac{\partial}{\partial r} \langle \tilde{b}_r \tilde{b}_{\theta} \rangle$$

• $\langle \tilde{b}_r \tilde{b}_{\theta} \rangle$ phasing via V'_E tilt.

✓ $n \langle \tilde{V}_{\parallel, i} \tilde{b}_r \rangle$: parallel ion flow along tilted field lines

Stochastic B-field affects electron density flux

- The stochastic field can induce particle flux :

$$\Gamma_{stoch} = \frac{c}{4\pi e} \langle \tilde{b}_r \nabla_{\perp}^2 \tilde{A}_{\parallel} \rangle + n \langle \tilde{V}_{\parallel,i} \tilde{b}_r \rangle$$

- And from $\nabla_{\parallel} V_{\parallel} \cong 0$, $\tilde{v}_{\parallel,i} \sim -\tilde{b}_r \frac{\partial \langle V_{\parallel,i} \rangle}{\partial r} / (ik_{\parallel})$

$$\rightarrow \langle \tilde{b}_r \tilde{V}_{\parallel,i} \rangle = -D_{M,eff} \frac{\partial \langle V_{\parallel,i} \rangle}{\partial r} = -\sum_k \langle \tilde{b}_r^2 \rangle_k l_{ac,eff} \frac{\partial \langle V_{\parallel,i} \rangle}{\partial r}$$

Modify l_{ac} due to $\mathbf{E} \times \mathbf{B}$ shear

$$l_{ac,eff} = \frac{l_{ac} l_{V'_E}}{l_{ac} + l_{V'_E}}, \quad l_{ac} = qR, \quad l_{V'_E} = \frac{C_s}{|k_{\theta} \Delta V'_E|} \sim \frac{C_s}{|V'_E|}$$

- ✓ V_{pinch} is not diffusive, induced by stochastic B field
- ✓ Flow gradient drives particle flux.

Ion heat flux with stochastic field

- For ion temperature : $n \frac{\partial T_i}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r Q_i) = S_H$

with

$$Q_i = \boxed{-n(\chi_{i,neo} + \chi_{i,T}) \nabla T_i} + Q_{i,stoch}$$

$$S_H = Q_a \exp\left(-\frac{r^2}{2L_{h,dep}^2}\right)$$

$$\left\{ \begin{array}{l} \bullet \chi_{i,neo} = \varepsilon^{-3/2} q^2 \rho_s^2 v_{ii} \\ \bullet v_{ii} = \frac{n_0 Z^4 e^4 \ln \Lambda}{\sqrt{36\pi} \varepsilon_0^2 m_i^{1/2} T_{i0}^{3/2}} \\ \bullet \chi_{i,T} = \left(\frac{c_s^2 \tau_c}{1 + \alpha V_E'^2}\right) * I \\ \quad \sim \chi_{GB} * I \end{array} \right.$$

- The stochastic field affects ion heat flux :

$$Q_{i,stoch} = \int V_{\parallel} \langle \tilde{B}_r \delta f \rangle (V_{\parallel}^2 + V_{\perp}^2) = -\frac{\partial \langle T_i \rangle}{\partial r} \sqrt{\chi_{\parallel,i} \chi_{\perp,i}} \langle \tilde{b}_r^2 \rangle l_{ac} k_{\perp}$$

$$= -v_{th,i} D_{M,eff} \frac{\partial \langle T_i \rangle}{\partial r} \simeq -D_{B,i} D_M \sqrt{\bar{k}_{\perp}^2} \frac{\partial \langle T_i \rangle}{\partial r}$$

- ✓ Potentially important as threshold power is directly related to heat flux.
- ✓ Power uptake determines turbulence and Reynolds force

Turbulence intensity

- For turbulence intensity :

$$\frac{\partial}{\partial t} I = \frac{\gamma_L}{(1+\alpha V_E'^2)^\sigma} I - \beta I^2 \rightarrow I = \frac{\gamma_L}{(1+\alpha V_E'^2)^\sigma} \frac{1}{\beta}$$

(γ_L – growth rate, β – nonlinear decay rate, V_E' – $E \times B$ shear rate)

$$\frac{\gamma_L}{(1+\alpha V_E'^2)^\sigma} \sim \beta \quad \alpha = \frac{1}{\alpha_0 (C_s / (qR))^2}, \quad \alpha_0 \text{ and } \sigma \text{ are adjustable.}$$

$$\gamma_L = \gamma_{L0} \left(\frac{C_s}{R} \right) \sqrt{\frac{R}{L_T} - \underbrace{\left(\frac{R}{L_T} \right)_{crit}}_{\text{threshold}}} \sim \gamma_{L0} \left(\frac{C_s}{R} \right), \quad \gamma_{L0} \approx 0.01$$

So, mean field “Predator-Prey” model

- RFB equation: $\langle V_E \rangle' = \frac{1}{eB} \frac{\partial}{\partial r} \langle \nabla P_i / n \rangle - \frac{\partial}{\partial r} \langle V_\theta \rangle + \frac{B_\theta}{B} \frac{\partial}{\partial r} \langle V_\phi \rangle$

- Five field model:

① $\frac{\partial}{\partial t} I = \frac{\gamma_L}{(1 + \alpha V_E'^2)^\sigma} I - \beta I^2$ Turbulence intensity

② $n \frac{\partial T_i}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r Q_i) = S_H$ Ion temperature

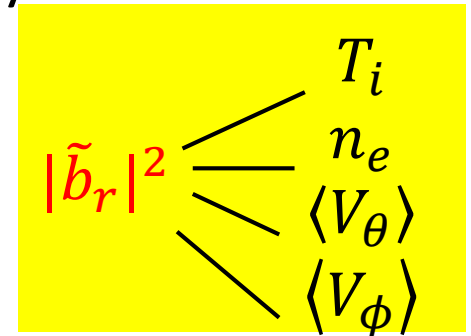
③ $\frac{\partial n_e}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_e) = S_p$ Electron density

④ $\langle V_\theta \rangle = V_{\theta, neo} + \frac{1}{\mu} \frac{\partial}{\partial r} \left(\frac{1}{B^2} \tau_c V_E' \frac{I}{1 + \alpha V_E'^2} - \frac{B^2}{4\pi\rho} \tau_c' V_E' |\tilde{b}_r|^2 \right)$

Poloidal flow

⑤ $\frac{\partial}{\partial r} \langle V_\phi \rangle |_{r_{sep}} = -\frac{1}{\chi_\phi} \int_0^{r_{sep}} S_M dr - \frac{V_{Ti}^2}{\beta \chi_\phi} \frac{B_\theta}{B} \langle \tilde{b}_r \tilde{b}_\theta \rangle |_{r_{sep}}$

Toroidal flow



$\langle J_r \rangle$ directly related to Γ_e and Maxwell stress in flows

The sources

$$S_H = Q_a \exp\left(-\frac{r^2}{2L_{h,dep}^2}\right)$$

$$L_{h,dep} = 0.15a$$

Heat source

$$S_p = \Gamma_a \frac{a - r + d_a}{L_{dep}^2} \exp\left(-\frac{(a + d_a - r)^2}{2L_{dep}^2}\right)$$

$$L_{dep} = 0.1a$$

$$d_a = 0$$

Particle source

$$S_M = S_a \exp\left(-\frac{r^2}{2L_{M,dep}^2}\right)$$

Momentum

$$L_{M,dep} = 0.15a$$

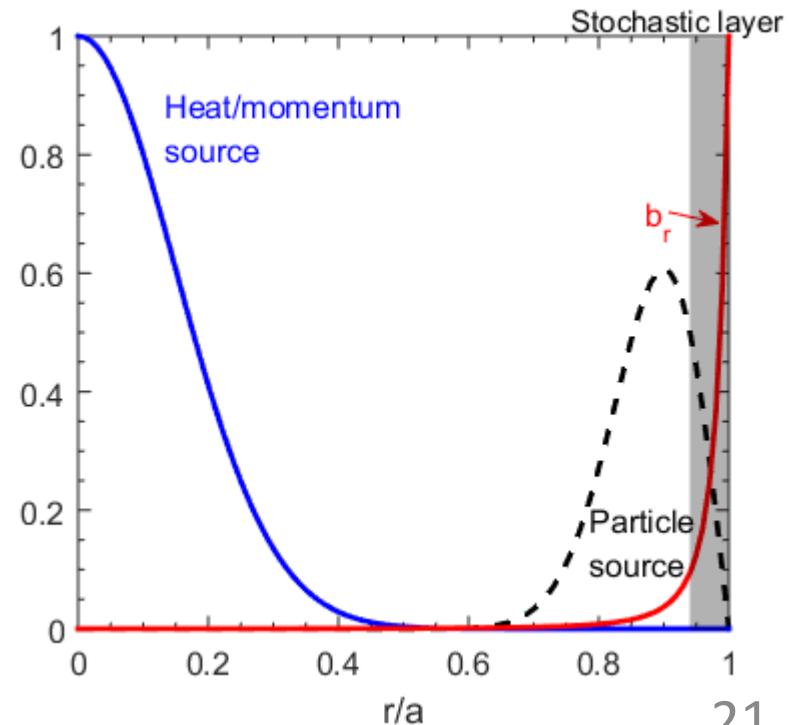
- Now momentum source tracks heat, but with different coefficients
- Try to change the direction of rotation

$$|\tilde{b}_r| = |\tilde{b}_0| \frac{1}{(L_{dep} - r)^3}$$

Perturbed magnetic field

$$L_{dep} = 1.05a$$

- Adjust $|\tilde{b}_0|$, now use $10^{-4} \sim 10^{-3}$, may broaden the width of stochastic layer by changing profile

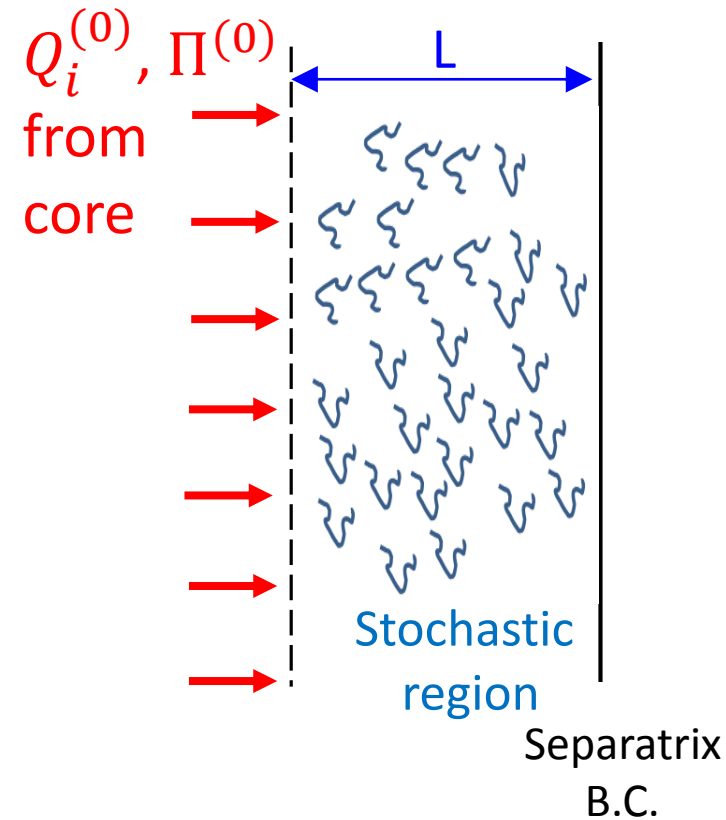


Lessons learned

- Novel $\langle J_r \rangle$ enters due to Stochastic field
 - ✓ Generates Maxwell stress/force, which competes with Reynolds stress/force
 - ✓ Effects on
 - flow (poloidal and toroidal velocity)
 - phase
 - transport (particle and thermal)
 - ✓ Reynolds and Maxwell phases linked by shearing

$$\langle \tilde{b}_r \tilde{b}_\theta \rangle = k_y^2 V_E' \tau_c' |\tilde{b}_r|^2$$

Ongoing: toward to 1D solution



Reduced model as before, the model with:

- $|\tilde{b}_r|^2$

Amplitude; scale

- Reduced RS in $\langle V_\theta \rangle$

Quenching the spin-up of V_θ

- Intrinsic torque in $\langle V_\phi \rangle$

- Effect of $|\tilde{b}_r|^2$ on Q_i, Γ_e

Goals:

➤ Output the time evolution of I, V_ϕ, V_E', n, T_i for $L \rightarrow H$ physics modified by stochastic B-field.

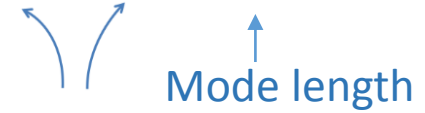
➤ $Q_{i,crit}(|\tilde{b}_r|^2, \frac{\partial}{\partial r} |\tilde{b}_r|^2 \dots \dots)$

Conclusions

- Ambipolarity breaking $\Rightarrow \langle \tilde{b}_r \tilde{b}_\theta \rangle$, contribute to $\langle \mathbf{J}_r \rangle$
- Both **amplitude and profile** of $|b_r|^2$ matter.
- V'_E phasing \Rightarrow stochastic $\langle \tilde{b}_r \tilde{b}_\theta \rangle$ **opposes** turbulence $\langle \tilde{V}_r \tilde{V}_\theta \rangle$
phase linked
- **Intrinsic toroidal torque**, such that reversal or spin-up of edge $\langle V_\phi \rangle$ occur with RMP, $\langle \tilde{b}_r \tilde{b}_\theta \rangle$ enters edge $\langle V_\phi \rangle$
- $|b_r|^2$ can modify T_i and n_e profiles
-

Open issues

- Direct effect of stochasticity on turbulence (l_K v.s. l_{\parallel} v.s. l_{mfpl})
- Collisionality scaling
- Cost of P_{th} to obtain H-mode with $E_r > 0$ (E_r hill) \Rightarrow Trade-off for particle control?
- RMP \rightarrow island + stochastic region
-

 Mode length

Thank you very much !



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