How Decoherence of Reynolds Force by Stochastic Magnetic Fields Raises the L-H Transition Power Threshold

Chang-Chun Chen[1], Patrick H. Diamond[1], Rameswar Singh, and Steven M. Tobias[2]

[1]University of California, San Diego, US
[2]University of Leeds, Leeds LS2 9JT, UK

This work is supported by the U.S. Department of Energy under Award No. DE-FG02-04ER54738

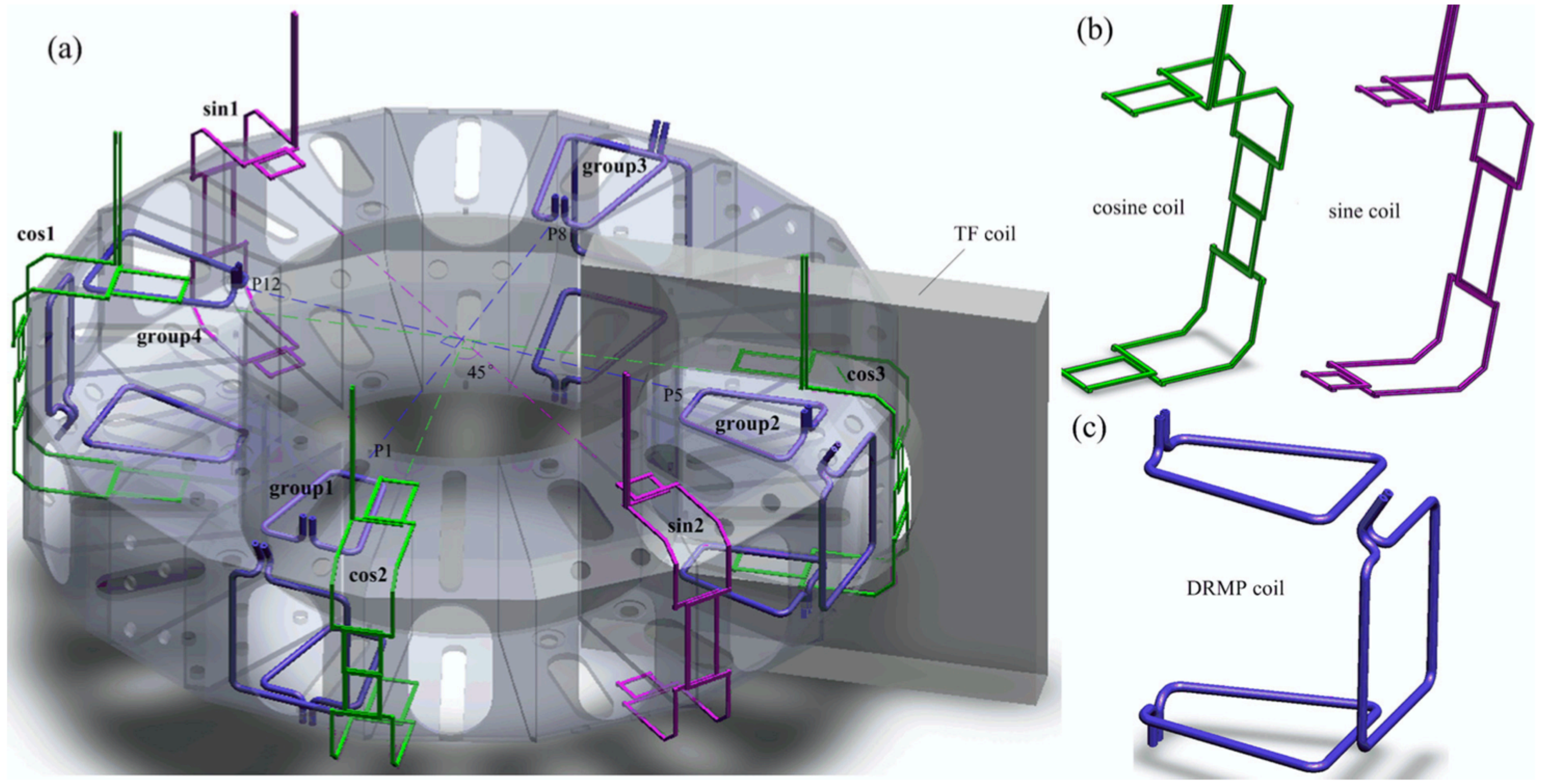
APS-DPP, Virtual Meeting, November 09th 2020

Introduction— Why

- The resonant magnetic perturbation (RMP) raises L-H transition power thresholds.
- Studies have shown that Reynolds stress bursts at the edge are suppressed and hence so is the zonal flow.

The tokamak

3D with $\underline{k} \cdot \underline{B} = 0$ resonance



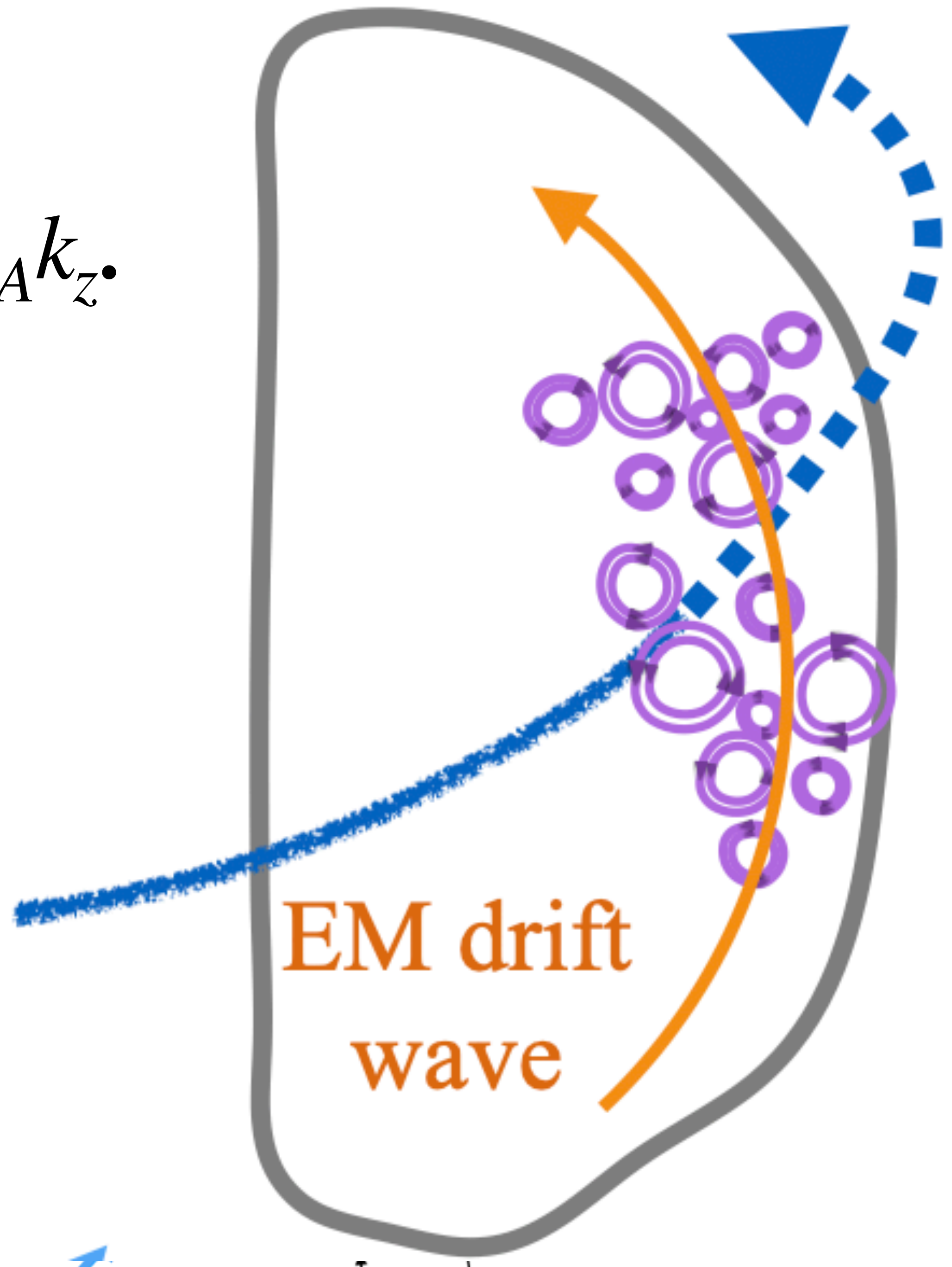
(J-TEXT)

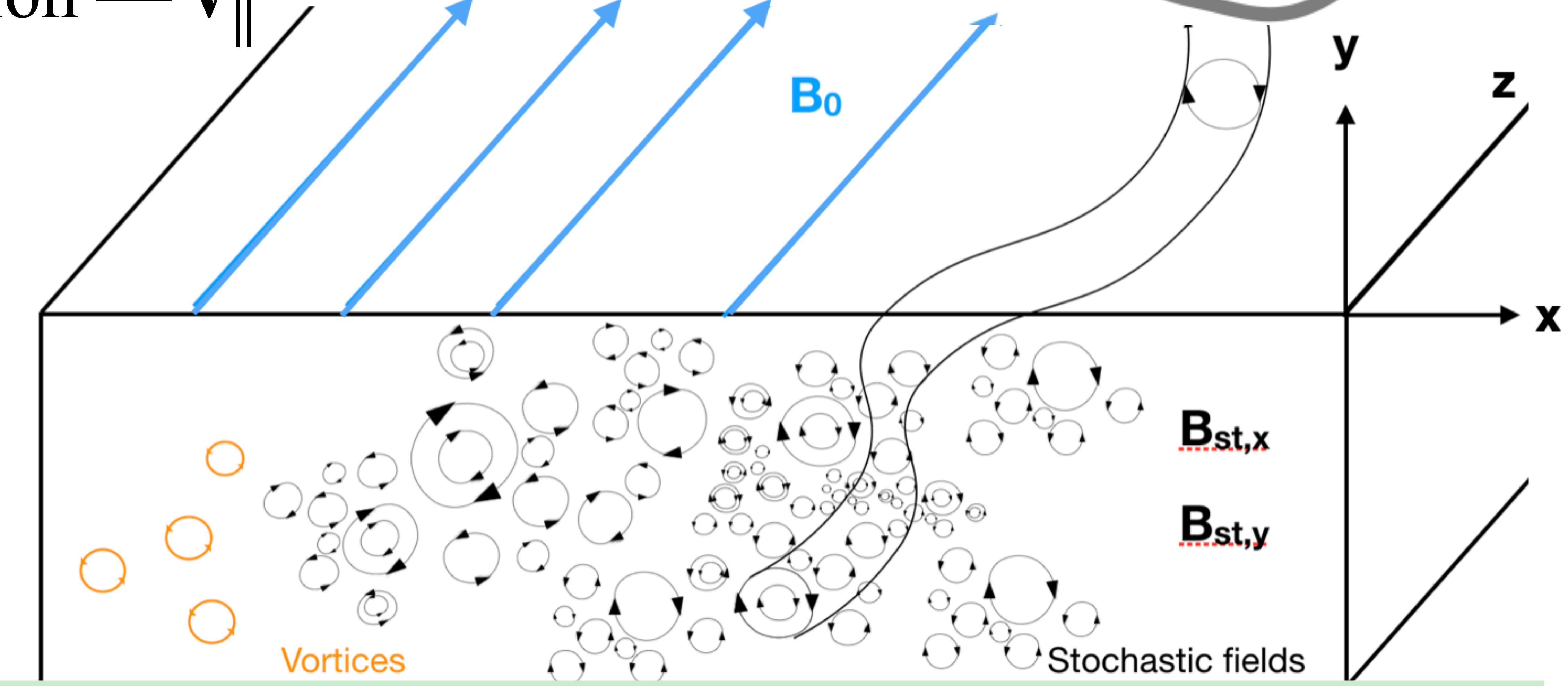
We examines the physics of stochastic fields interaction with zonal flow near the edge.

Model

The model (Cartesian Coordinate):

1. Strong mean field (3D).
2. $\underline{k} \cdot \underline{B} = 0$ (or $k_{\parallel} = 0$) **resonant at rational surface has third direction** — $\omega \rightarrow \omega \pm v_A k_z$.
3. Kubo number: $Ku_{mag} = \dfrac{l_{ac} |\widetilde{\mathbf{B}}|}{\Delta_{\perp} B_0} < 1$).
4. Four-field equations —
   - (a) Vorticity equation — vorticity $-\nabla^2\psi \equiv \zeta$
   - (b) Induction equation — **A, J**
   - (c) Pressure equation — **P**
   - (d) Parallel flow equation — $\mathbf{v}_{\parallel}$





Mean-field Approximation:

$$\zeta = \langle \zeta \rangle + \boxed{\widetilde{\zeta}}$$
$$\psi = \langle \psi \rangle + \boxed{\widetilde{\psi}}$$
$$A = \langle A \rangle + \boxed{\widetilde{A}}$$

Perturbations produced by turbulences

$$, where \ \langle\ \rangle = \frac{1}{L}\int dx \frac{1}{T}\int dt$$

ensemble average over the zonal scales

$$\boxed{b^2 \equiv \overline{B_{st}^2}/B_0^2}$$

Decoherence — L-H Transotion

When does stochastic Fields dephasing become effective?

Basic scales:

Shear flow rate	Alfvénic Dispersion	Natural linewidth	Stochastic broadening	
$k_\theta \Delta x \frac{\partial}{\partial x} u_y$	$v_A \| \Delta k_\parallel \|$	$\boxed{\Delta\omega}$	$\boxed{D k_\perp^2}$	ω

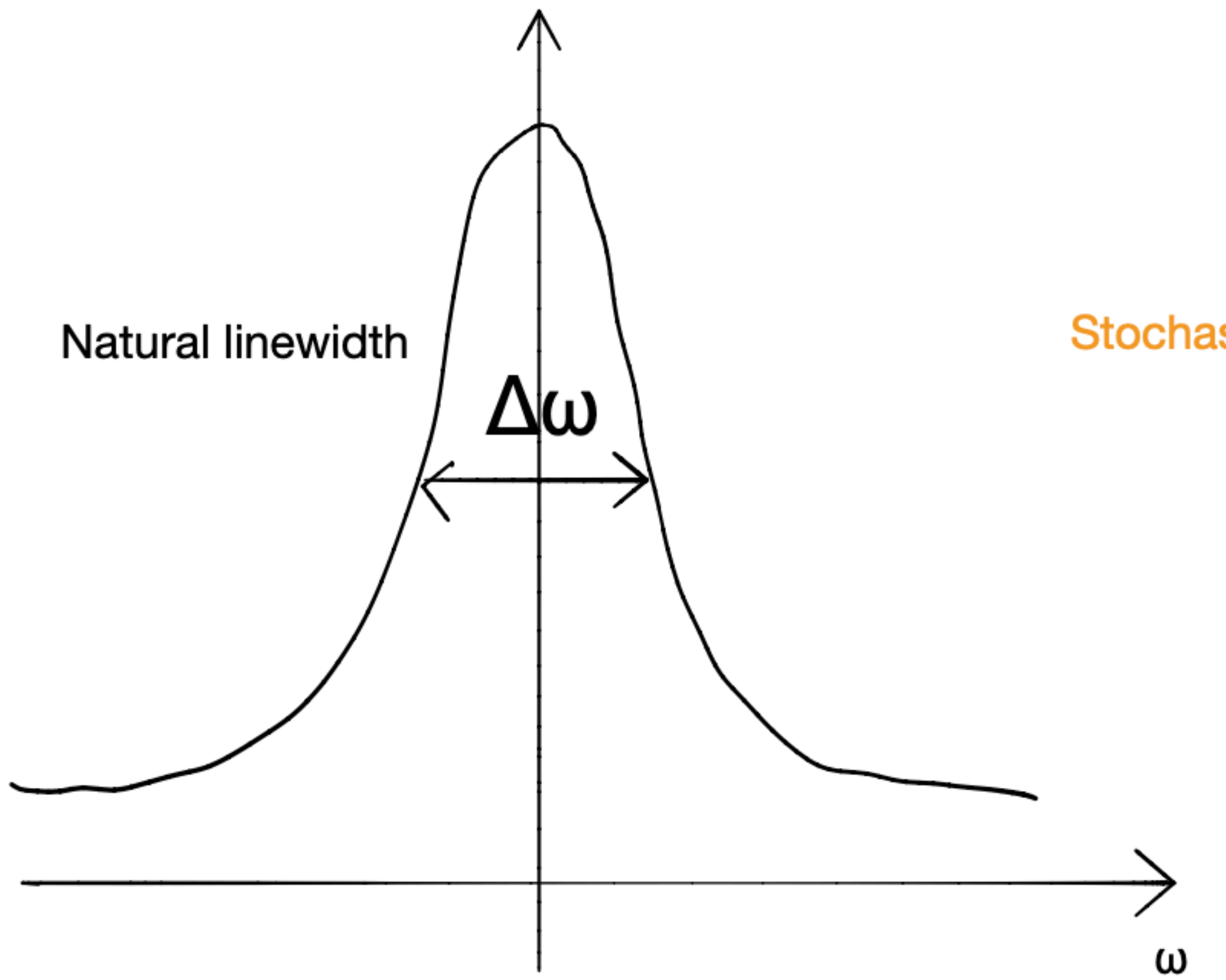
(non-linear micro-instability process)

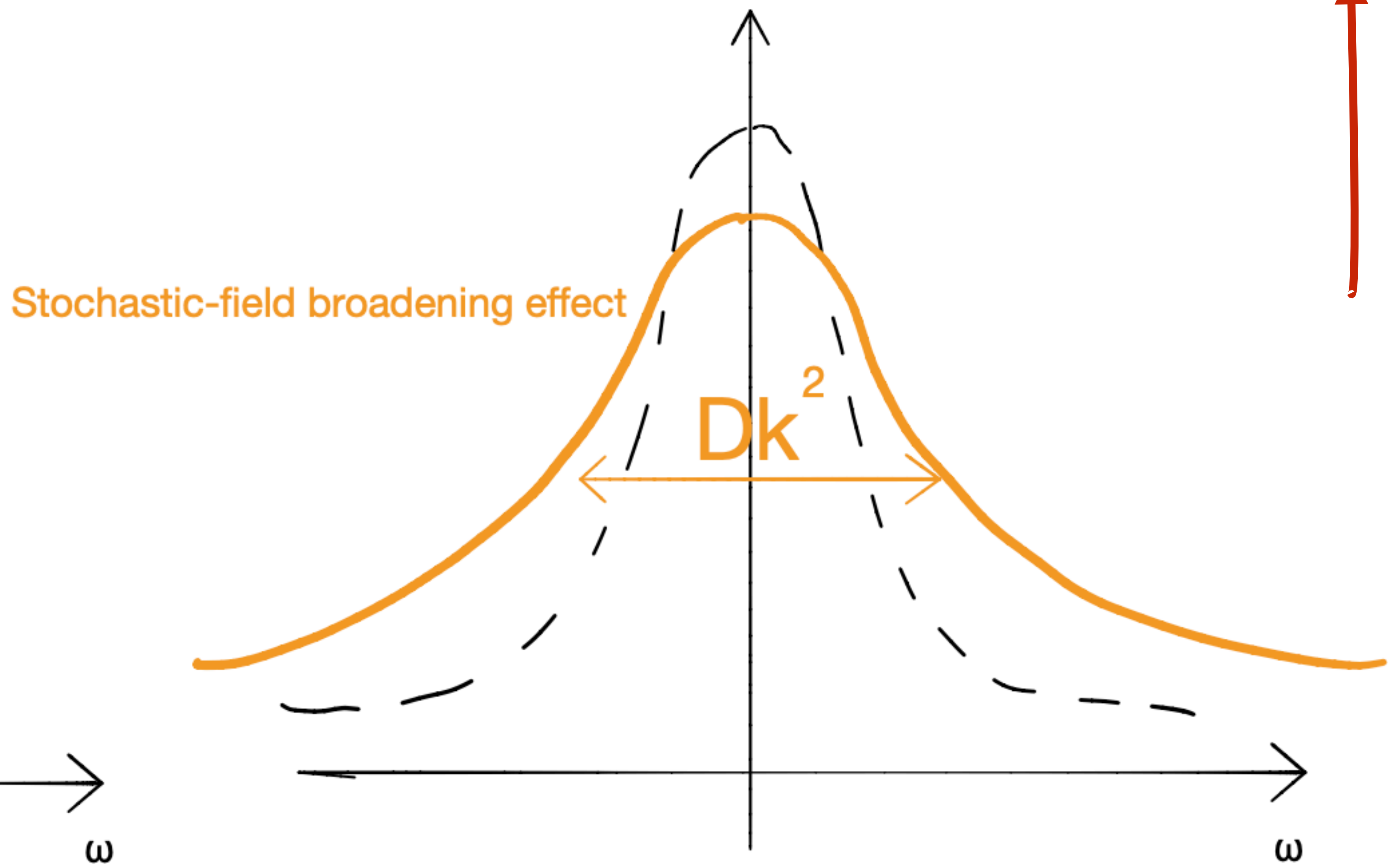
Stochastic field decoherence beats self-decoherence.

$$D \equiv v_A D_M = v_A \sum_k \pi \delta(k_\parallel) b_k^2$$

Magnetic diffusivity

Auto-correlation length l_{ac}





Alfvén wave propagate via stochastic fields
→ characteristic velocity from $\underline{\nabla} \cdot \underline{J} = 0$

Decoherence — L-H Transotion

- $Dk_\perp^2 > \Delta\omega$ **gives a dimensionless parameter (α):**

1. $$\begin{cases} l_{ac} \simeq Rq \\ \epsilon \equiv L_n/R \sim 10^{-2} \\ \beta \simeq 10^{-2\sim-3} \\ \rho_* \equiv \frac{\rho_s}{L_n} \simeq 10^{-2\sim-3} \end{cases}$$

$$b^2 \equiv \left(\frac{\delta B_r}{B_0}\right)^2 > \sqrt{\beta}\rho_*^2\frac{\epsilon}{q} \sim 10^{-7}$$

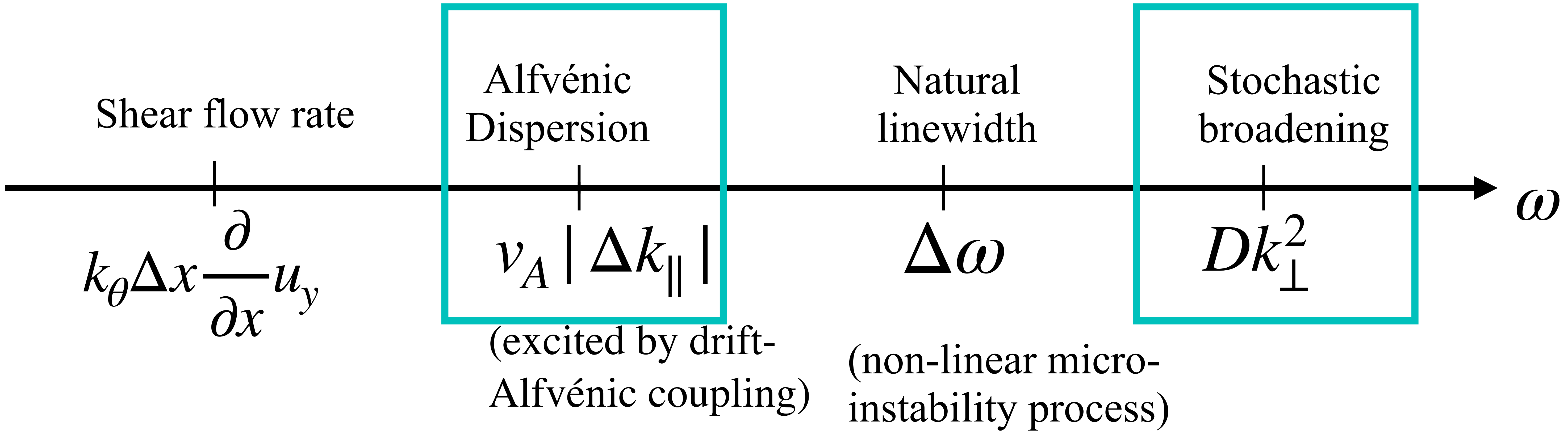
2. $$\alpha \equiv \frac{b^2}{\rho_*^2\sqrt{\beta}}\frac{q}{\epsilon} > 1$$

Extended Kim-Diamond Model

Criterion for stochastic fields effect important to L-H transition.

- **How 'stochastic' is this? Magnetic Kubo number?**

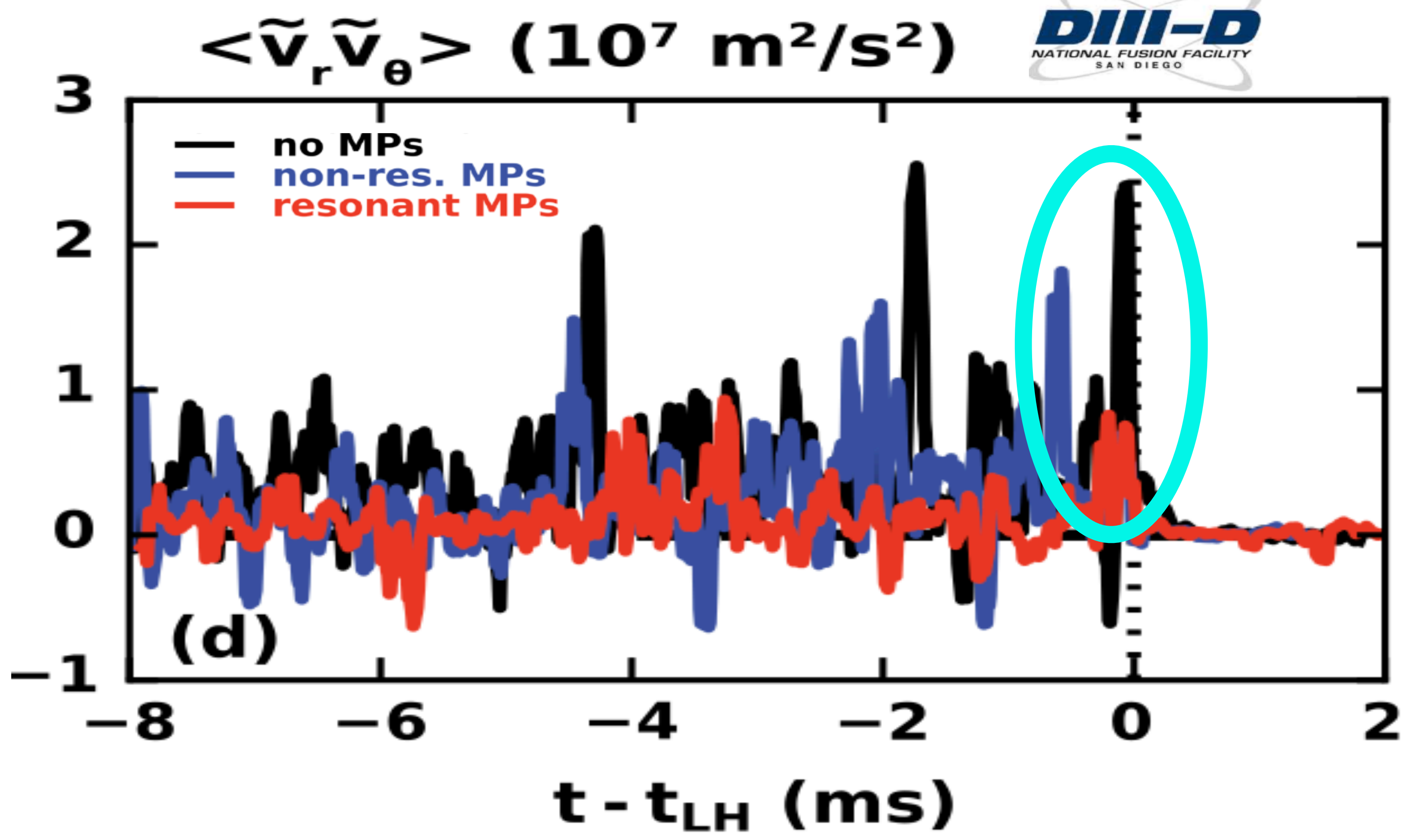
Basic scales:



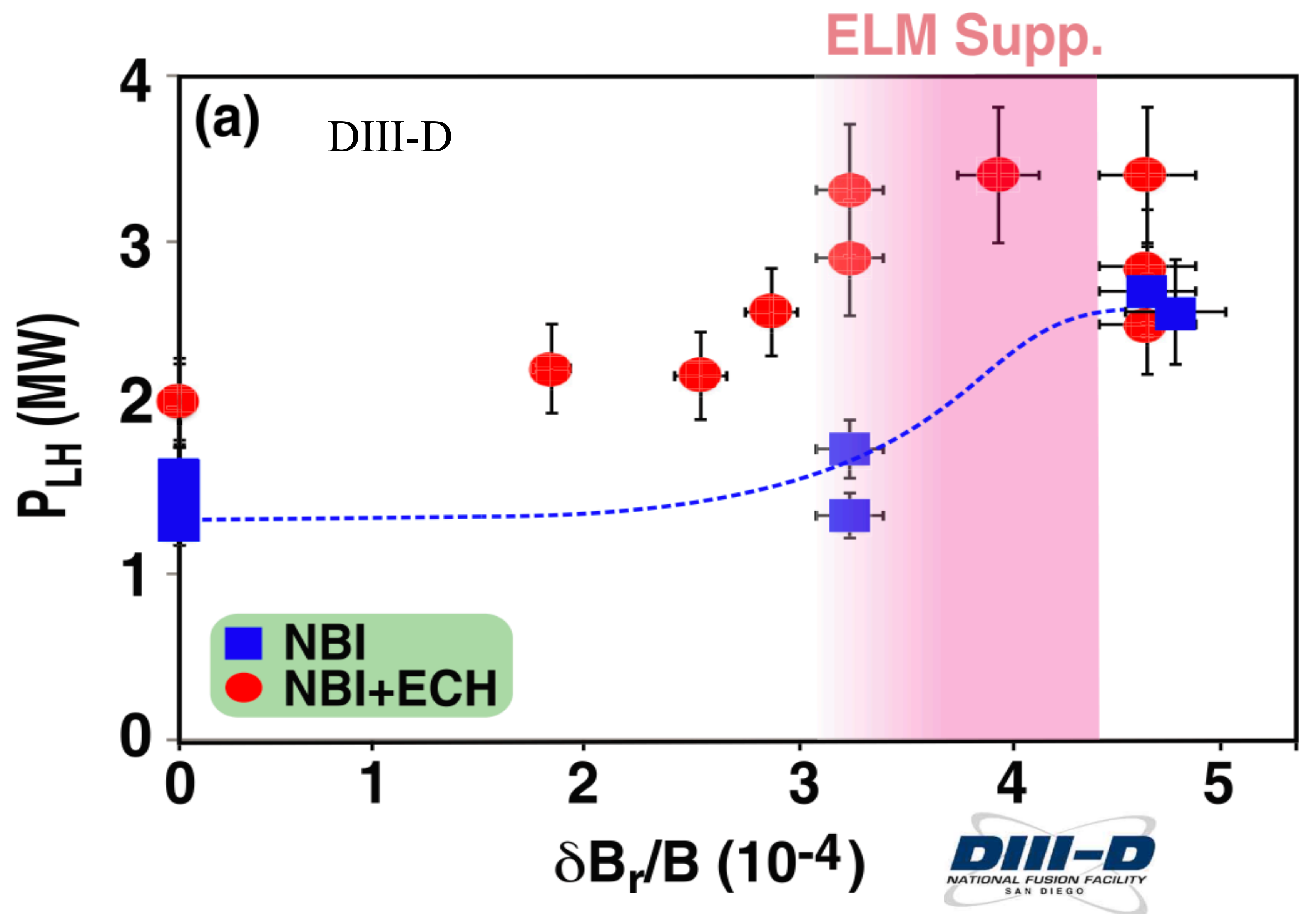
$$Ku_{mag} \equiv \frac{\text{stochastic field scattering length}}{\text{perpendicular magnetic fluctuation size}} \simeq 1$$

Experimental Results in L-H Transition

- **Experimental results in L-H transition (DIII-D):**



(D. Kriete et al, PoP **27** 062507 (2020))



(L. Schmitz et al, NF **59** 126010 (2019))

- **Suppression of poloidal Reynolds stress:**

$$\langle \tilde{u}_x \tilde{u}_y \rangle = -\boxed{D_{PV}} \frac{\partial}{\partial x} \langle u_y \rangle + \boxed{F_{res}} \kappa \langle p \rangle$$

Residual Stress

Curvature

Suppressed by stochastic fields

$$\bar{\omega} \equiv \omega - \langle u_y \rangle k_y$$

$$D_{PV} = \sum_{k\omega} |\tilde{u}_{x,k\omega}|^2 \frac{v_A b^2 l_{ac} k^2}{\bar{\omega}^2 + \left(v_A b^2 l_{ac} k^2\right)^2}$$

Reynolds stress will be suppressed as stochastic fields via PV diffusivity and residual stress.

This stochastic dephasing is insensitive to turbulent mode (e.g. ITG, TEM,…etc.).

Decoherence of eddy tilting feedback — the physics

- **Snell's law:**

$$\frac{d}{dt}k_x = -\frac{\partial \omega_k}{\partial x} = -k_y\frac{\partial u_y}{\partial x}$$ shear flow

Leads to non-zero $\langle k_x k_y \rangle$
$\rightarrow \langle \tilde{u}_x \tilde{u}_y \rangle \propto \langle k_x k_y \rangle$

- **Self-feedback of Reynolds stress:**

$$\langle \tilde{u}_x \tilde{u}_y \rangle \simeq -\sum_k \frac{|\tilde{\phi}_k|^2}{B_0^2}\left(k_y^2 \frac{\partial u_y}{\partial x}\tau_c\right)$$

The $E \times B$ shear generates the $\langle k_x k_y \rangle$ correlation and hence support the non-zero Reynolds stress.

The Reynold stress modifies the shear via momentum transport.

➤ **The shear flow reenforce the self-tilting.**

- **Now, the dispersion relation with drift-Alfvén coupling is:**

$$\boxed{\omega}^2 - \omega_D\omega - \boxed{k_\parallel}^2 v_A^2 = 0$$

$$\boxed{k_\parallel = k_\parallel^{(0)} + \underline{b}\cdot\underline{k}_\perp}$$

Drift-wave frequency

$$\boxed{\omega = \omega_D + \delta\omega}$$

Frequency shift induced by b^2

$$(\omega_D + \delta\omega)^2 - \omega_D(\omega_D + \delta\omega) - (k_\parallel + \underline{b}\cdot\underline{k}_\perp)^2 v_A^2 = 0$$

Decoherence of eddy tilting feedback — the physics

Stochastic fields dephase the self-feedback loop of Reynolds stress:

Expectation of frequency in stochastic fields: $\langle\omega\rangle = \langle\omega_0\rangle + \langle\delta\omega\rangle$.

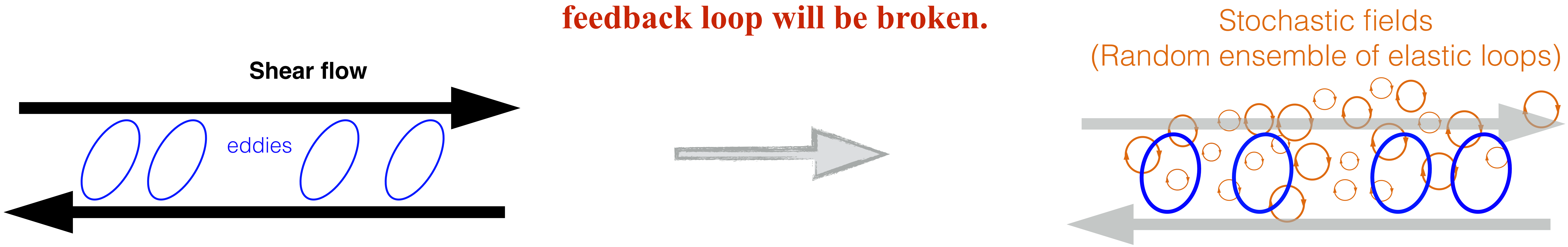
$$\langle\omega\rangle \simeq \omega_D + \frac{1}{2}\frac{v_A^2}{\omega_0}b^2k_\perp^2$$

Ensemble average frequency shift

$$\langle\tilde{u}_x\tilde{u}_y\rangle \simeq -\sum_k \frac{|\tilde{\phi}_k|^2}{B_0^2}\left(k_y^2\frac{\partial u_y}{\partial x}\tau_c - \frac{1}{2}\frac{v_A^2k_\perp^2}{\omega_0}\frac{\partial b^2}{\partial x}\tau_c\right)$$

Stochastic dephasing

When these two are comparable, the feedback loop will be broken.



Stochastic fields act as elastic loops and resist the tilting of eddies.

➤ Stochastic fields interfere with shear-tilting feedback loop.

Results — Increment of P_{LH}

Macroscopic Impact

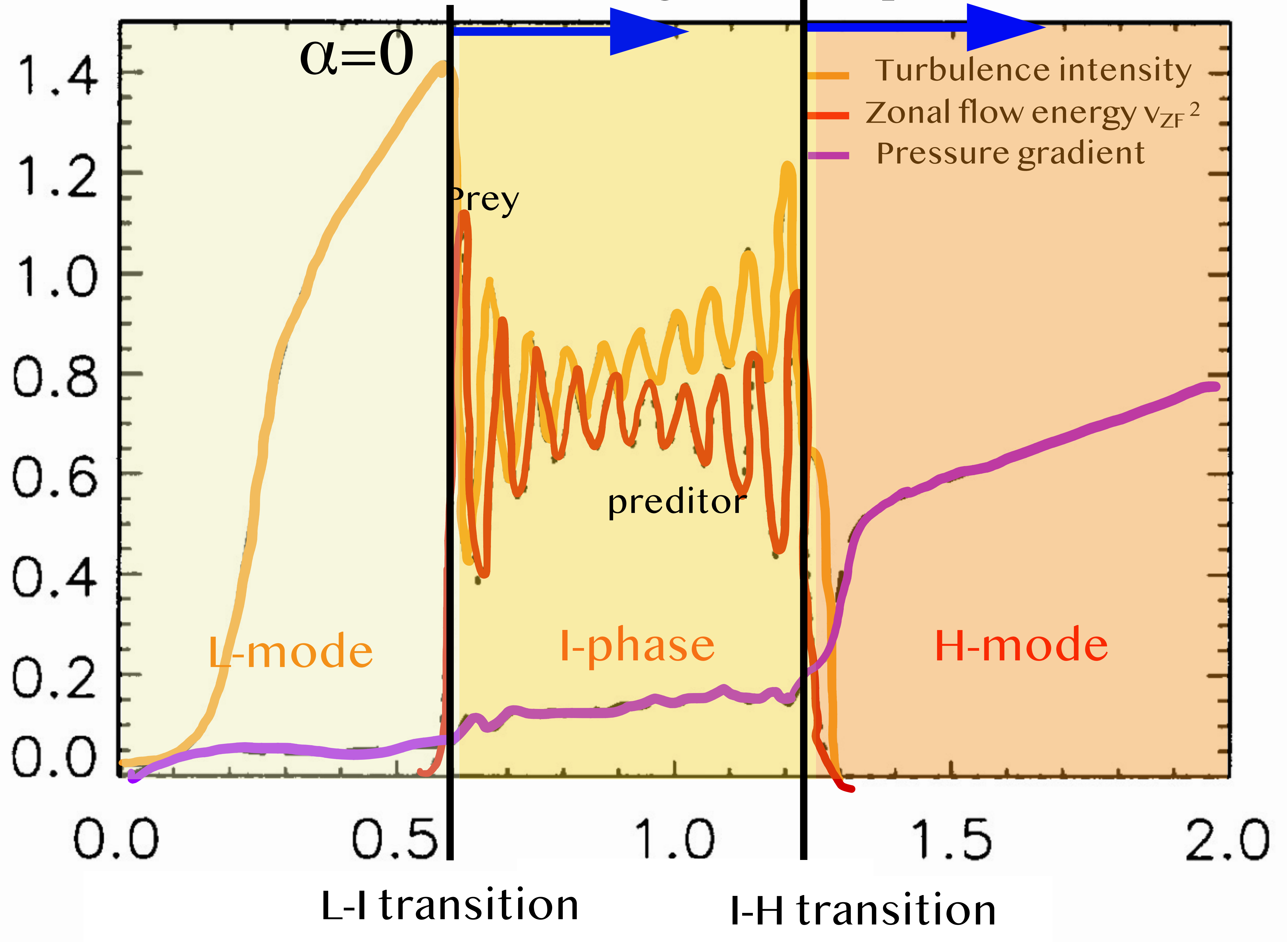
- **Extended Kim-Diamond Model (Simple reduced model):**

Stochastic fields broadening effect requires: $\Delta\omega \leq k_\perp^2 D$. This gives dimensionless parameter (α):

$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} > 1$$

- **1D Theory of power threshold:** M. A. Malkov et al. (PoP **22**, 032506 (2015)).

Kim-Diamond model is useful for testing trends in power threshold increment induced by stochastic fields.



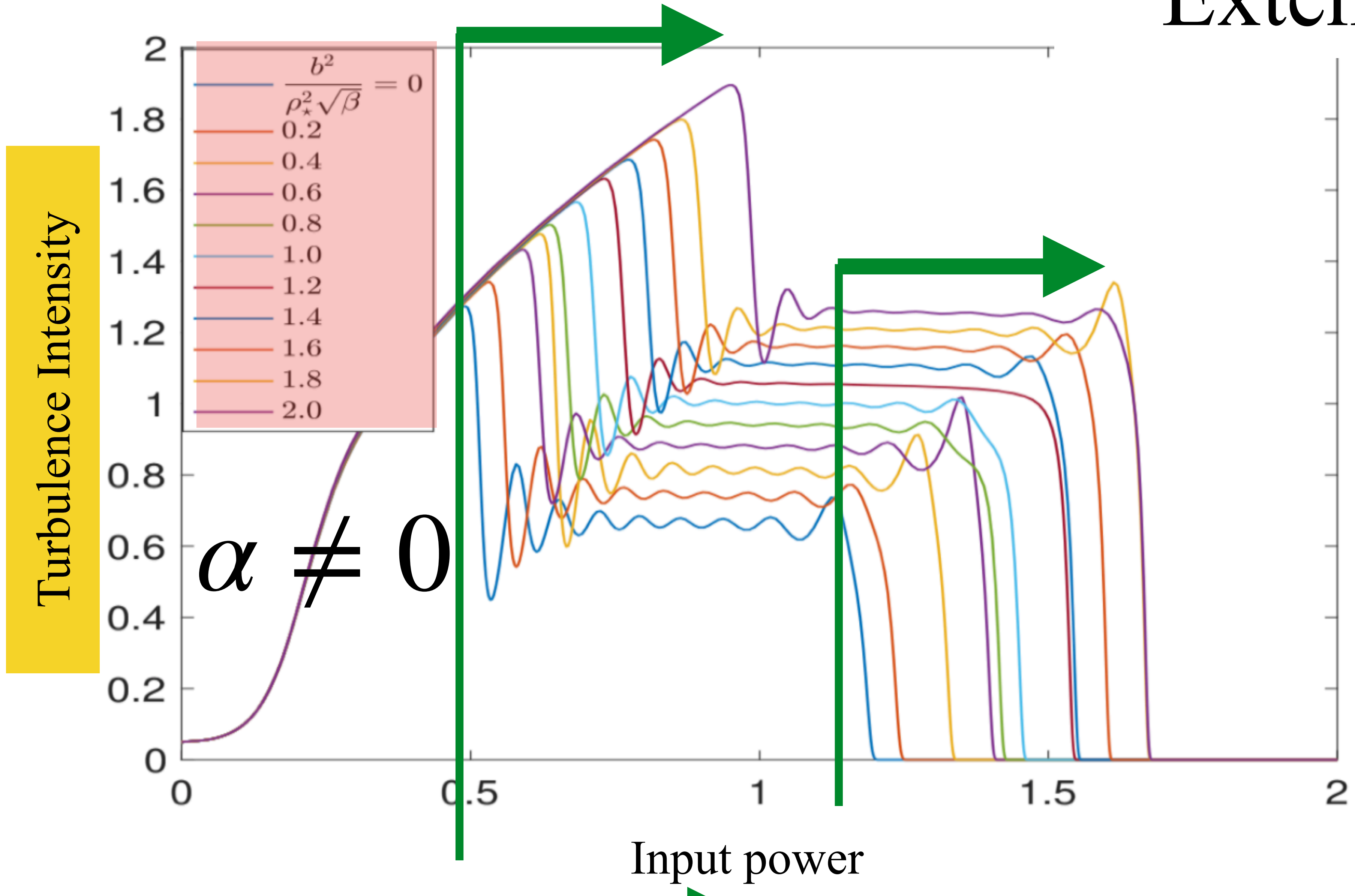
➤ **We expect stochastic fields to raise L-I and I-H transition thresholds.**

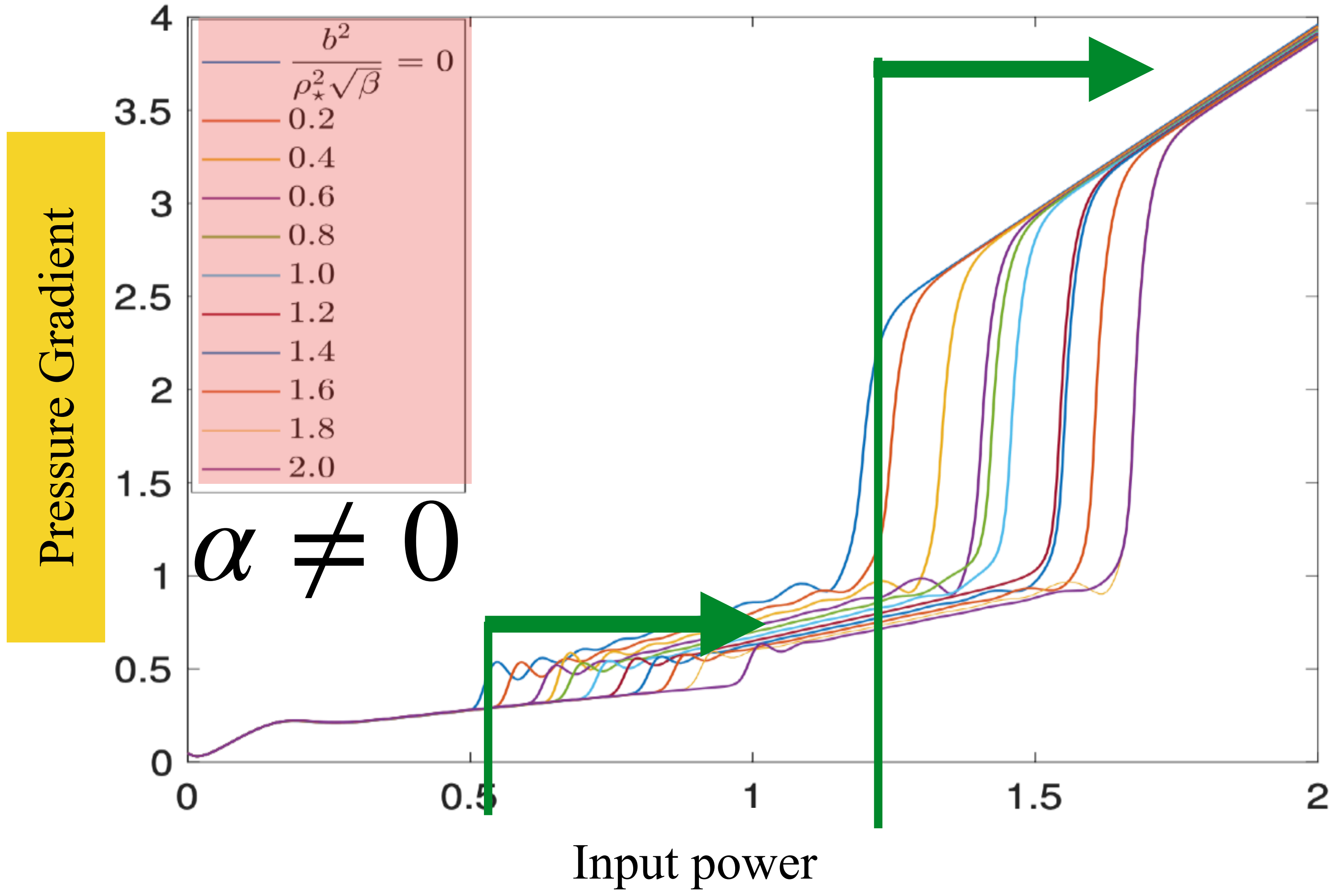
$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$$

quantifies the strength of stochastic dephasing.

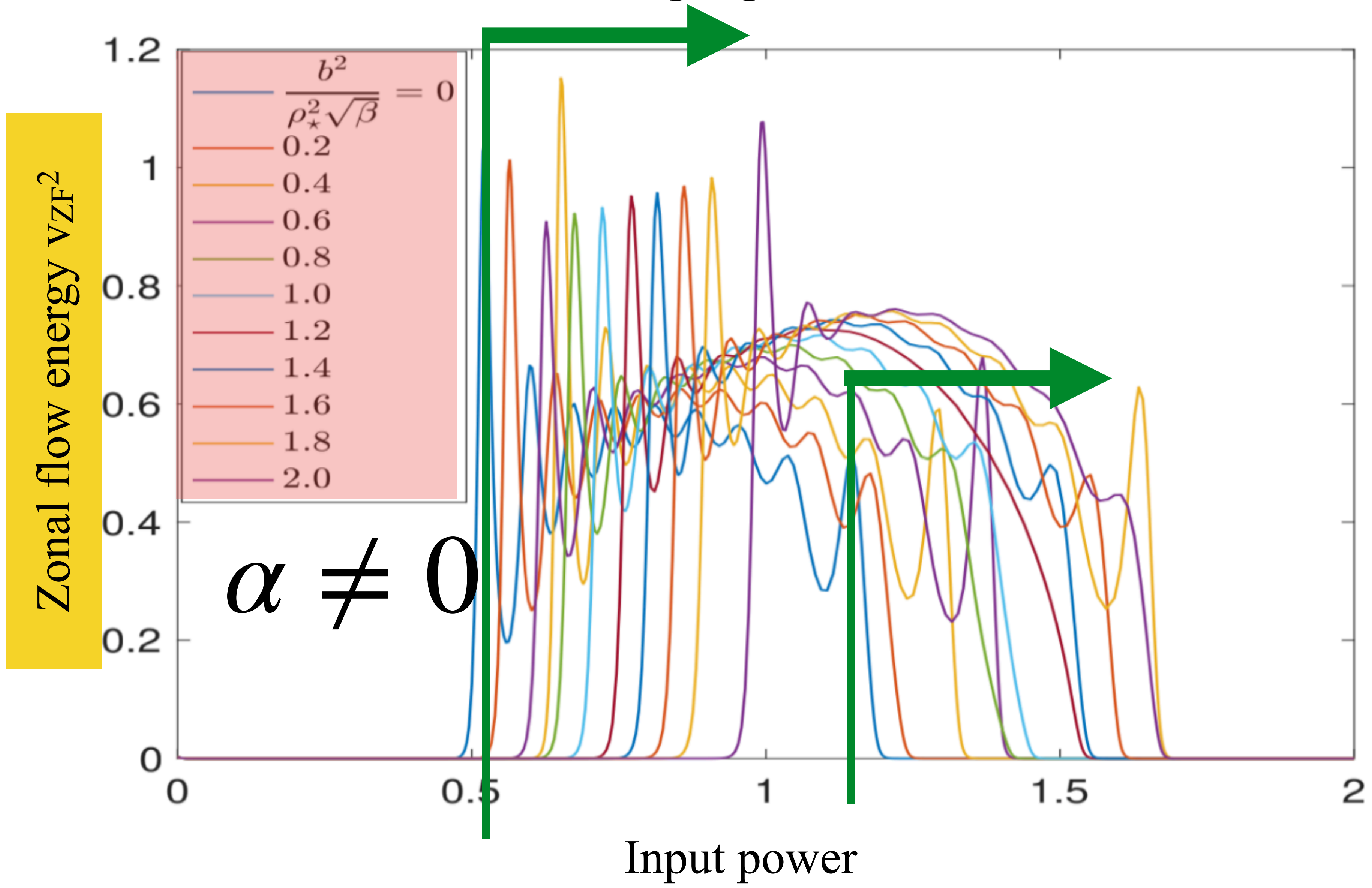
Results

Extended Kim-Diamond Model





$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$$

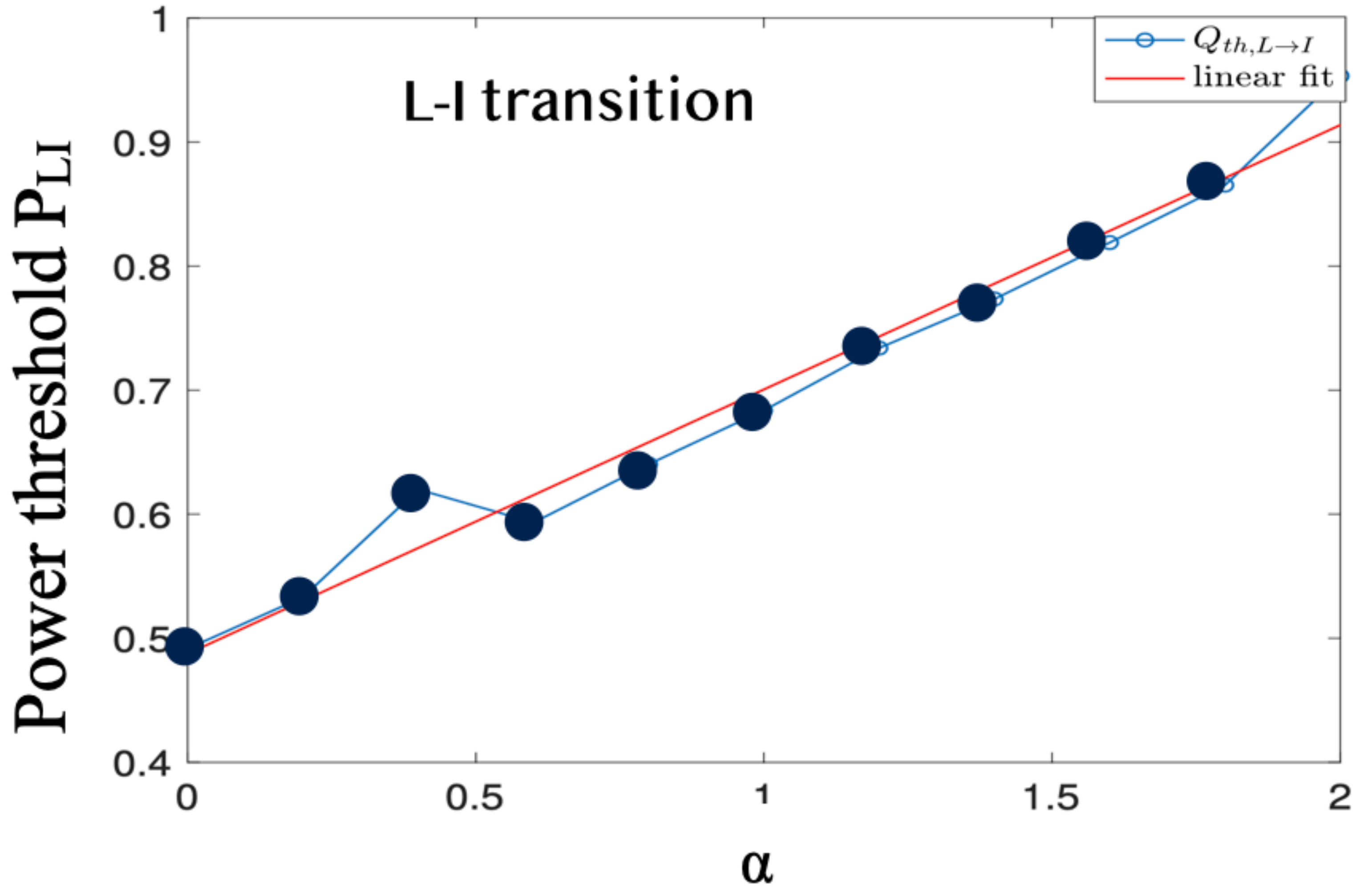


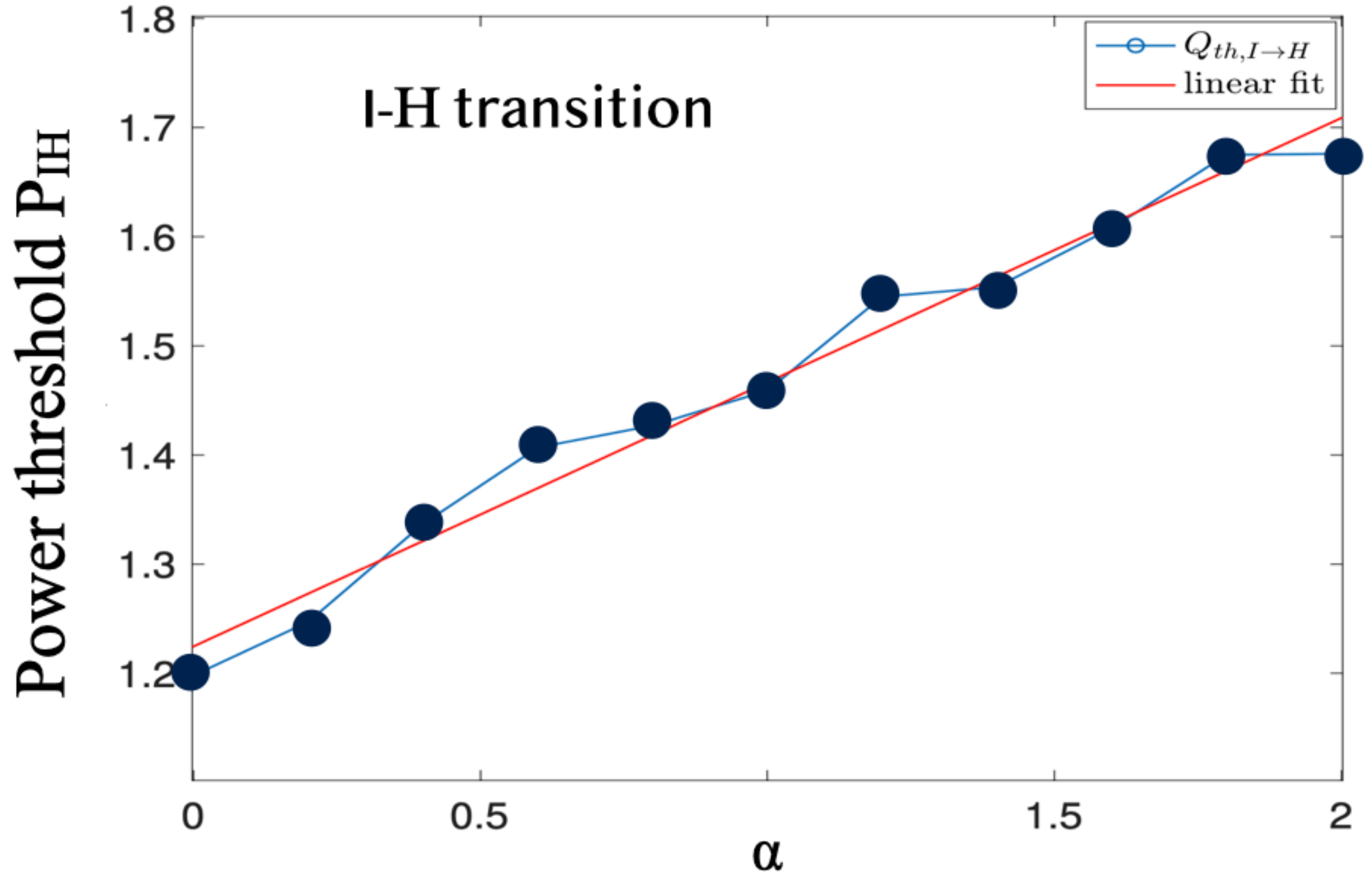
The threshold increase due to stochastic dephasing effect is seen in turbulence intensity, zonal flow, and pressure gradient.

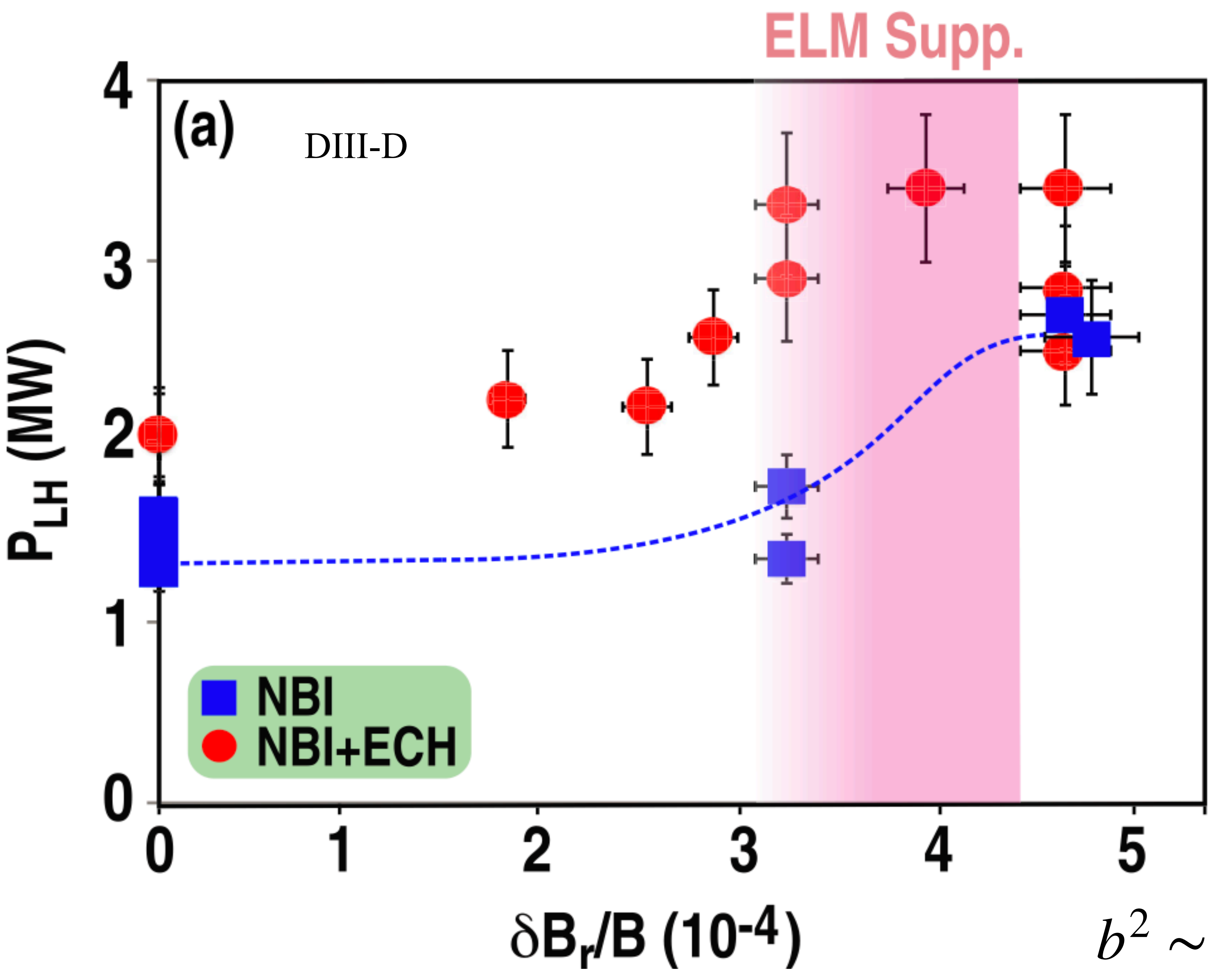
Results— Transitions in DIII-D

Increment of Power threshold:

The power threshold increases linearly with the increment of stochastic fields intensity $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$.







$b^2 \sim 10^{-7}$

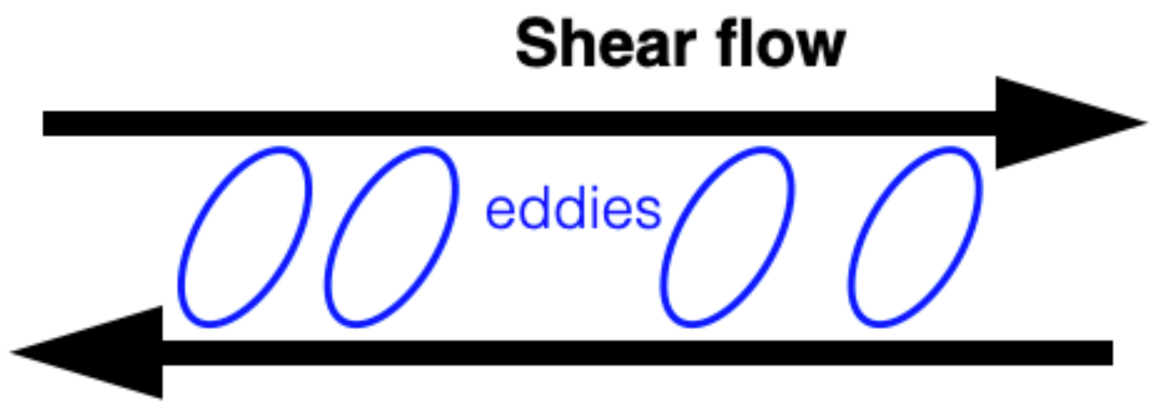
(L. Schmitz et al, NF **59** 126010 (2019))

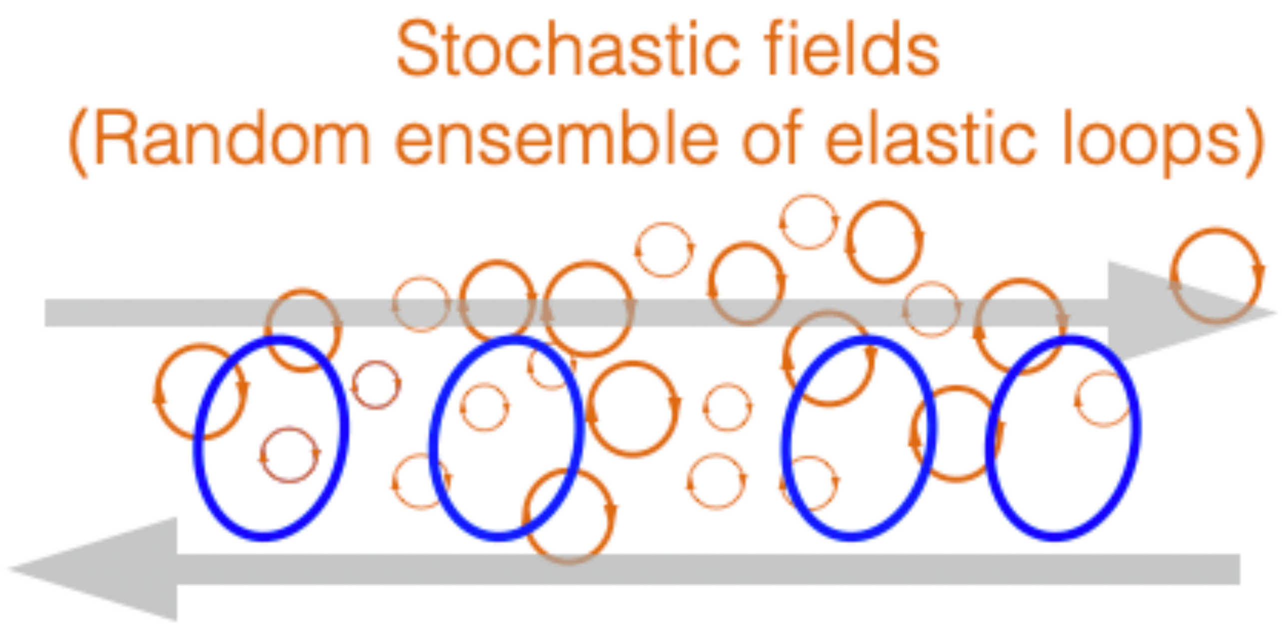
$\overline{b^2}$ shift L-H, I-H thresholds to higher power, in proportional to α.

Conclusion and Discussion

What we have learned:

Dephasing effect caused by stochastic fields quenches Reynolds stress.





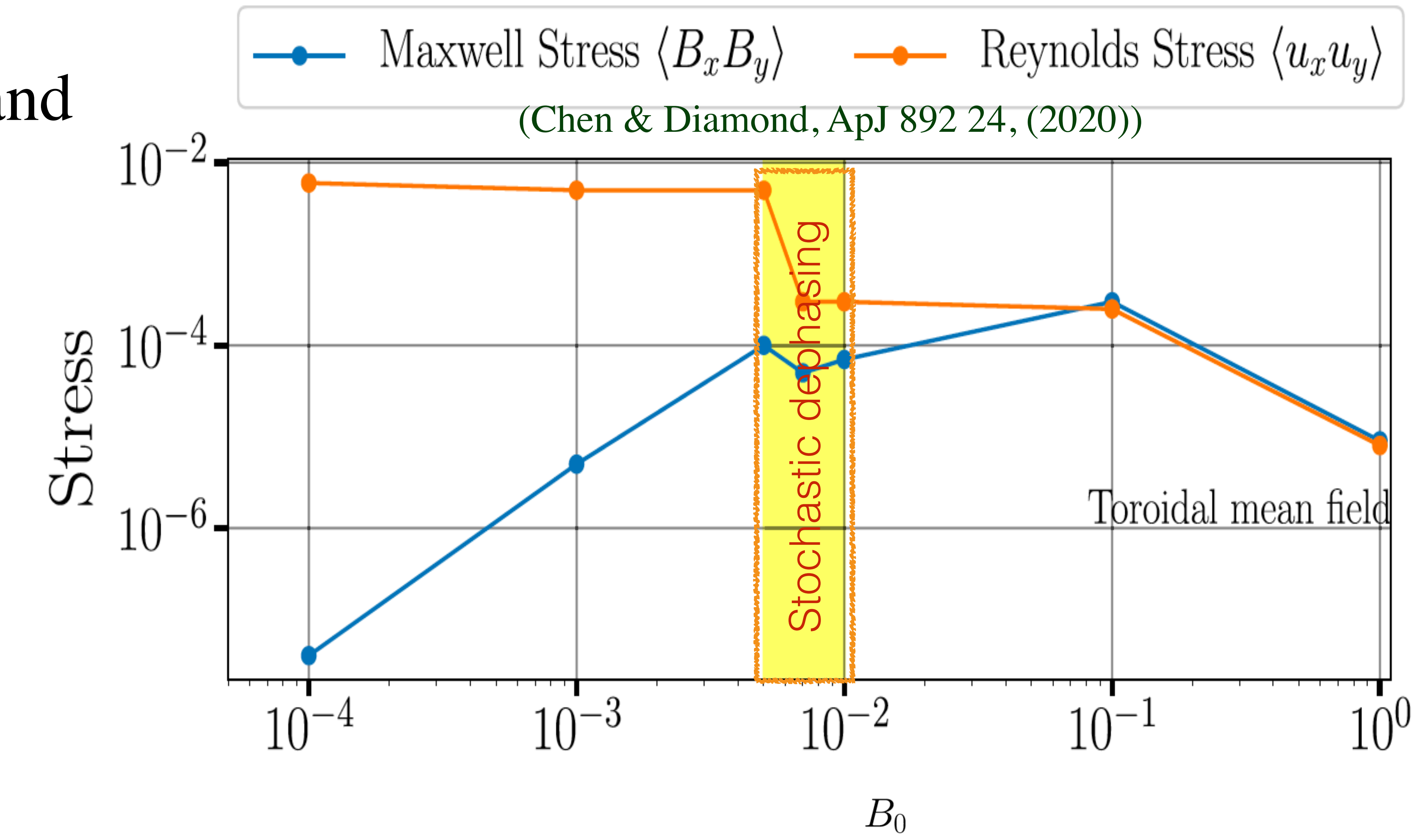
Message for experimentalists:

1. Reynolds stress is **dephased by stochastic fields due to RMP**, and **power thresholds increases**.

2. **Critical parameter** is $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$.

Related Work :

Reynolds stress will undergo decoherence at levels of field intensities **well below that of Alfvénization (**where Maxwell stress balances the Reynolds stress).



- C. Chen, P. Diamond, R. Singh, and S. Tobias (GI-02. Nov. 10, 09:30 am). Potential Vorticity Mixing in a Tangled Magnetic Field.
- Maya Katz, Robin Heinonen, and Patrick Diamond (TO-16. Nov. 12, 10:30 am). Cross-Helicity Generation and Structure Formation in β-plane MHD Turbulence.

Thank you!