Turbulence model reduction by deep learning

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Supported by the Department of Energy under Award Number DE-FG02-04ER54738

Introduction

Preview

- Turbulent transport impacts confinement. How to predict?
- In this talk: use deep supervised learning to find simple model
- As test of concept: apply to well-trodden ground (Hasegawa-Wakatani), and compare to analytics
- Recover existing theory, while finding some new features

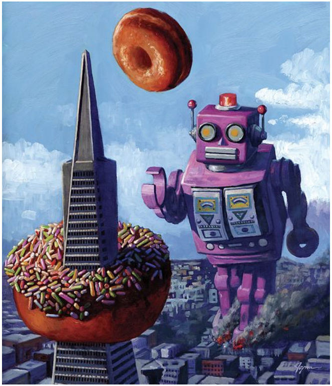


Figure Artist's conception of machine learning applied to the tokamak

Hasegawa-Wakatani

- Simplest realistic framework for understanding collisional drift wave turbulence

$$\frac{dn}{dt} = \alpha(\tilde{\phi} - \tilde{n}) + D\nabla^2 n$$
$$\frac{d\nabla^2\phi}{dt} = \alpha(\tilde{\phi} - \tilde{n}) + \mu\nabla^4\phi$$
$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + (\hat{z} \times \nabla\phi) \cdot \nabla$$

- $\alpha \equiv k_{\parallel}^2 T_e/(n_0 \eta \Omega_i e^2)$ "adiabaticity parameter," measures parallel electron response
- Want theory for *radial* transport (1D reduction)
- Averaging over symmetry directions ($\langle \cdots \rangle$) yields

$$\partial_t \langle n \rangle + \partial_x \Gamma = \text{dissipation}$$
$$\partial_t \langle \nabla^2 \phi \rangle - \partial_x^2 \Pi = \text{dissipation}$$

where $\Gamma = \langle \tilde{n} \tilde{v}_x \rangle$ (particle flux) and $\Pi = \langle \tilde{v}_x \tilde{v}_y \rangle$ (poloidal momentum flux or Reynolds stress)

Methods

Feature selection

- Seek a model that predicts local Γ, Π as function of local zonal averages. How to choose parameters?
- Exact symmetries useful: invariant under uniform shifts $n \to n + n_0$ and $\phi \to \phi + \phi_0$, Galilean boosts in y

$$\begin{cases} \phi & \to \phi + v_0 x \\ y & \to y - v_0 t \end{cases}$$

  Thus cannot depend on $\langle n \rangle, \langle \phi \rangle, \partial_x \langle \phi \rangle$
- Choose minimal set of parameters $N' = \partial_x \langle n \rangle, U = -\partial_x^2 \langle \phi \rangle, U', U'', \varepsilon = \langle (\tilde{n} - \nabla \tilde{\phi})^2 \rangle$
- Close model by coupling with intensity evolution

$$\partial_t \varepsilon + 2\varepsilon(\Gamma - \partial_x \Pi)(N' + U') = -\gamma \varepsilon - \gamma_{NL} \varepsilon^2$$

Deep learning

- Now use deep supervised learning to fit fluxes to choice of params
- Locality → good scaling. Each point in space and time treated on equal footing!
- Exploit 3 reflection symmetries $x \to -x, y \to -y$ and $\phi \to -\phi, n \to -n, x \to -x$ and $\phi \to -\phi, n \to -n, y \to -y$ for data augmentation. Symmetries enforce, e.g. $\Gamma \to -\Gamma$ under $N' \to -N'$ in absence of flow
- Each simulation thus yields $4 N_t N_x$ data points

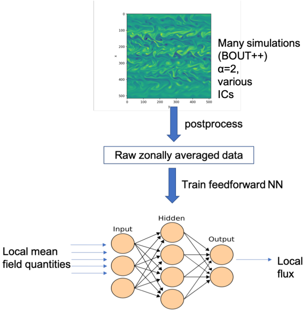


Figure Schematic of deep learning method

Deep neural networks 101

- Method of approximating arbitrary nonlinear functions. We use simplest form: "multi-layer perceptron."
- Inputs $\mathbf{x}$ repeatedly transformed in each layer: $x_j^{(n+1)} = \sigma(W_{ij}^{(n)} x_i^{(n)} + b_j^{(n)})$ where σ is a nonlinear function ("activation")
- Weights $\mathbf{W}^{(n)}$, biases $\mathbf{b}$ are "trained" using SGD
- Bottom line: simply a proven choice of multivariate, fully nonlinear, nonparametric regression

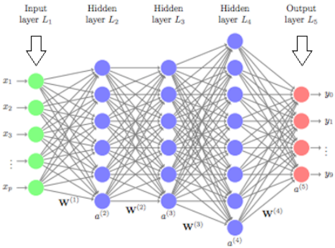


Figure Diagram of MLP, shamelessly stolen from the internet

Results

Particle flux

DNN learns a model roughly of the form (for small gradients)

$$\Gamma \simeq -D_n \varepsilon N' + D_U \varepsilon U'.$$

Large gradients: fluxes saturate. Diffusive term $\propto N'$ well-known, tends relax driving gradient. Second (non-diffusive) term is not so well-known, driven by vorticity gradient!

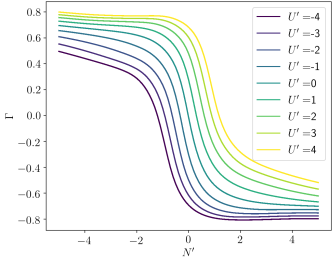


Figure Particle flux at constant ε as function of density and vorticity gradients

Derivation of nondiffusive term

Analytic treatment in $\alpha \to \infty$ limit reproduces nondiffusive term. Need include frequency shift due to convection of mean vorticity. In QLT:

$$\omega_{\mathbf{k}} = \frac{k_y}{1+k^2}(N' + U') + O(\alpha^{-2})$$

$$\gamma_{\mathbf{k}} = \frac{k_y^2}{\alpha(1+k^2)^3}(N' + U')(k^2 N' - U') + O(\alpha^{-2})$$

$$\Gamma = \mathrm{Re} \sum_{\mathbf{k}} -ik_y \tilde{n}_{\mathbf{k}} \tilde{\phi}_{\mathbf{k}}^*$$

$$= \sum_{\mathbf{k}} \frac{-k_y^2 \partial_x n(\gamma_{\mathbf{k}} + \alpha) + \alpha k_y \omega_{r,\mathbf{k}}}{\omega_{r,\mathbf{k}}^2 + (\gamma_{\mathbf{k}} + \alpha)^2} |\tilde{\phi}_{\mathbf{k}}|^2$$

$$= \frac{1}{\alpha} \sum_{\mathbf{k}} -\frac{k_y^2}{1+k^2} \left(k^2 N' - U'\right) |\tilde{\phi}_{\mathbf{k}}|^2 + O(\alpha^{-2})$$

Using ansatz spectrum in this expression yields good agreement with DNN

Implications of nondiffusive term

- Neglected in literature, but coupling same order of magnitude (~ 0.5) that of usual N' term. Γ dependence on shear U comparatively weak!
- Consequence: ZF can induce "staircase" pattern on profile. If $V_y = V_0 \sin(qx)$, U' term will contribute

$$\partial_t \langle n \rangle \sim -\frac{k_y^2 q^3 V_0 \langle \varepsilon \rangle}{\alpha(1+k^2)^3} \cos(qx)$$

- Previous explanation for staircase is some form of bistability. This mechanism is distinct

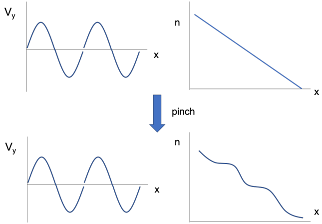


Figure Cartoon indicating how ZF may induce profile staircase via nondiffusive flux/pinch

Reynolds stress

- Learns model of Cahn-Hilliard form

$$\Pi \sim \varepsilon(-\chi_1 U + \chi_3 U^3 - \chi_4 U'')$$

  with $\chi_1, \chi_3, \chi_4 > 0$
- $\partial_t U = \partial_x^2 \Pi \sim \chi_1 \varepsilon k^2 U$. Zonal flow generation by *negative viscosity* $\varepsilon\chi_1$
- Large U stabilized by nonlinearity $\propto U^3$, small scales by hyperviscosity χ_4 (not shown)

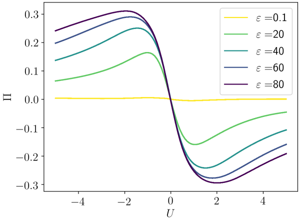


Figure Reynolds stress as function of U, at fixed U', U''

Reynolds stress: gradient corrections

- How does Reynolds stress depend on N', U'? Not easy to calculate
- Learned dependence well-described by overall suppression factor $f \simeq 1/(1+0.04(N'+4U')^2)$, i.e. gradients generally reduce Reynolds stress
- Found to be crucial for stability of learned model. Kinks tend to form in flow in its absence

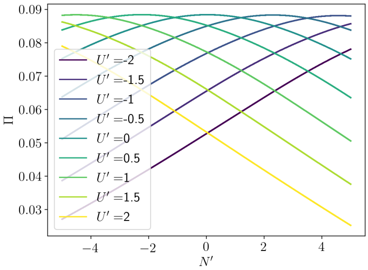
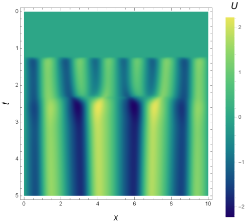


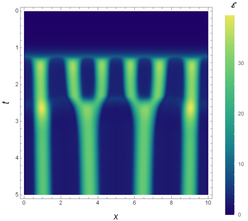
Figure Reynolds stress dependence on gradients at fixed ε, U, U''

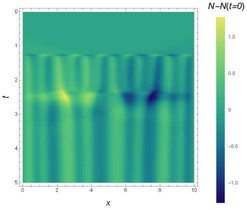
Discussion

Reduced 1-D model

Now have 3 coupled, **one-dimensional** mean field equations describing nonlinear turbulent dynamics. Construct expressions for Γ, Π capturing NN behavior, and numerically solve





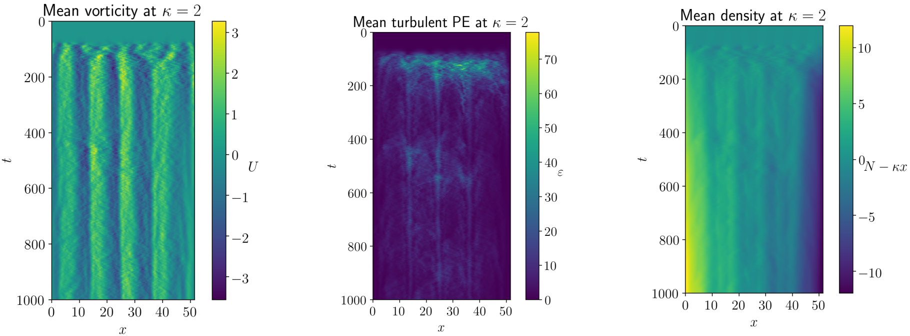


Conclusions

- Have verified, directly from simulation, analytic models for spontaneous ZF. CH is "best" local 1D model
- Identified significant vorticity-gradient-driven particle flux which may induce layering. Shearing effects weak
- Also find higher-order corrections which are harder to anticipate analytically (e.g. effect of shear on Γ, gradients on Π)
- Note: 1D reduction breaks down for strong turbulence due to vortex interactions, $\alpha \lesssim 1$ due to breakdown of ZF
- In future: generalize, apply to more complicated systems? *But:* need sufficient data. Potentially serious issue for application to GK, experiment

Extra slides

Compare to zonally averaged 2D DNS



1D resembles simplified version of DNS. One key difference: 3-field model equivalent to taking stationary "best-fit" spectrum. Some system memory lost

Particle flux dependence on shear

- Shear U is usually invoked as directly involved in suppression of turbulent transport
- We find that direct dependence on U is comparatively weak
- Suppression is $\lesssim 10\%$ for typical values of U
- Conclusion: for HW particle transport in 2D model, shear *gradient* more important than shear itself!

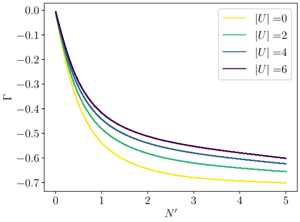


Figure U level curves of particle flux as function of N', at fixed $U'U''$, ε

Reynolds stress: intensity scaling

- Whereas learned Γ is essentially $\propto \varepsilon$, Π scaling with ε is nontrivial
- Learned exponent is 1 for small intensity, close to zero for large intensity
- Jibes with intuition from strong turbulence theory

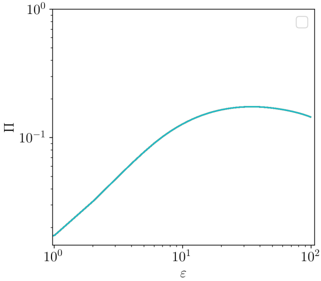


Figure Reynolds stress dependence on gradients at fixed ε, U, U''

Comparison to theory (diffusive term)

Compare DNN result to theory result using spectrum centered at most unstable $\mathbf{k}$ for $U' = 0$

$$\varepsilon_{\mathbf{k}} = \frac{\langle \varepsilon \rangle}{2\pi^2 \Delta k_x \Delta k_y} \frac{1}{1 + k_x^2/\Delta k_x^2} \left(\frac{1}{1 + (k_y - \sqrt{2})^2/\Delta k_y^2} + \frac{1}{1 + (k_y + \sqrt{2})^2/\Delta k_y^2} \right)$$

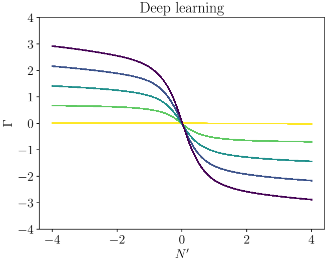


Figure Curves (at fixed $U = U' = U'' = 0$, and various ε) of Γ vs density gradient from DNN

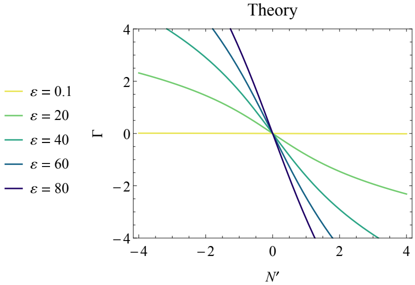


Figure Corresponding curves from QLT+ansatz with $\Delta k_x = \Delta k_y = 0.8$

Comparison to theory (nondiffusive term)

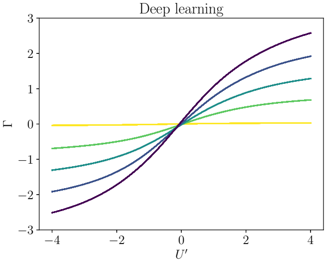


Figure Curves (at fixed $N' = U = U'' = 0$, and various ε) of Γ vs U' from DNN

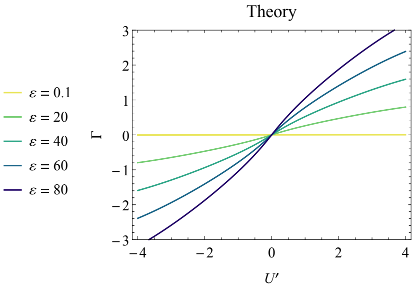


Figure Corresponding curves from QLT+ansatz with $\Delta k_x = \Delta k_y = 0.8$

Good agreement when N', U' are small!

Reynolds stress: hyperviscosity

Hyperviscous term, crucial for stability, has small coefficient.
Sensitive test of method

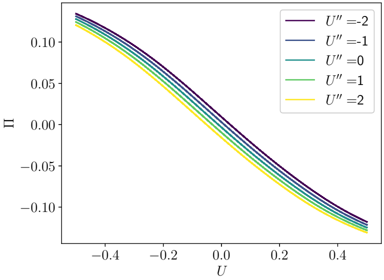


Figure U'' level curves of Reynolds stress as function of U, at fixed ε, U', N'

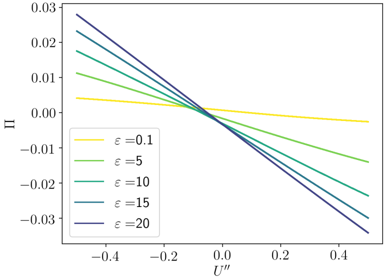


Figure ε level curves of Reynolds stress as function of U'', at fixed U, U', N''

Ideas for future

- No 1D reduction. Replace zonal average with 2D window average
- Relax locality assumption. Can include time derivatives as inputs, or extend to fully non-Markovian and/or spatially nonlocal models. Tradeoff is more predictive power at the expense of simplicity/interpretability
- This work essentially a second-order moment closure. Higher-order moments? Turbulence spreading? (interesting to note: applying this method to PE flux didn't work!)