Shear Layer and Staircase Formation in a Stochastic Magnetic Field

Chang-Chun Chen[1], Patrick Diamond[1],
Rameswar Singh[1], and Steven Tobias[2]

[1]University of California San Diego, USA
[2]University of Leeds, UK

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Outline

- Introduction

  Critical question: How resilient is barriers in stochastic magnetic field?

  What is FOM for resilience?

- Model & Calculation

- Results

  a. Suppression of PV diffusivity and the shear-eddy tilting feedback loop.

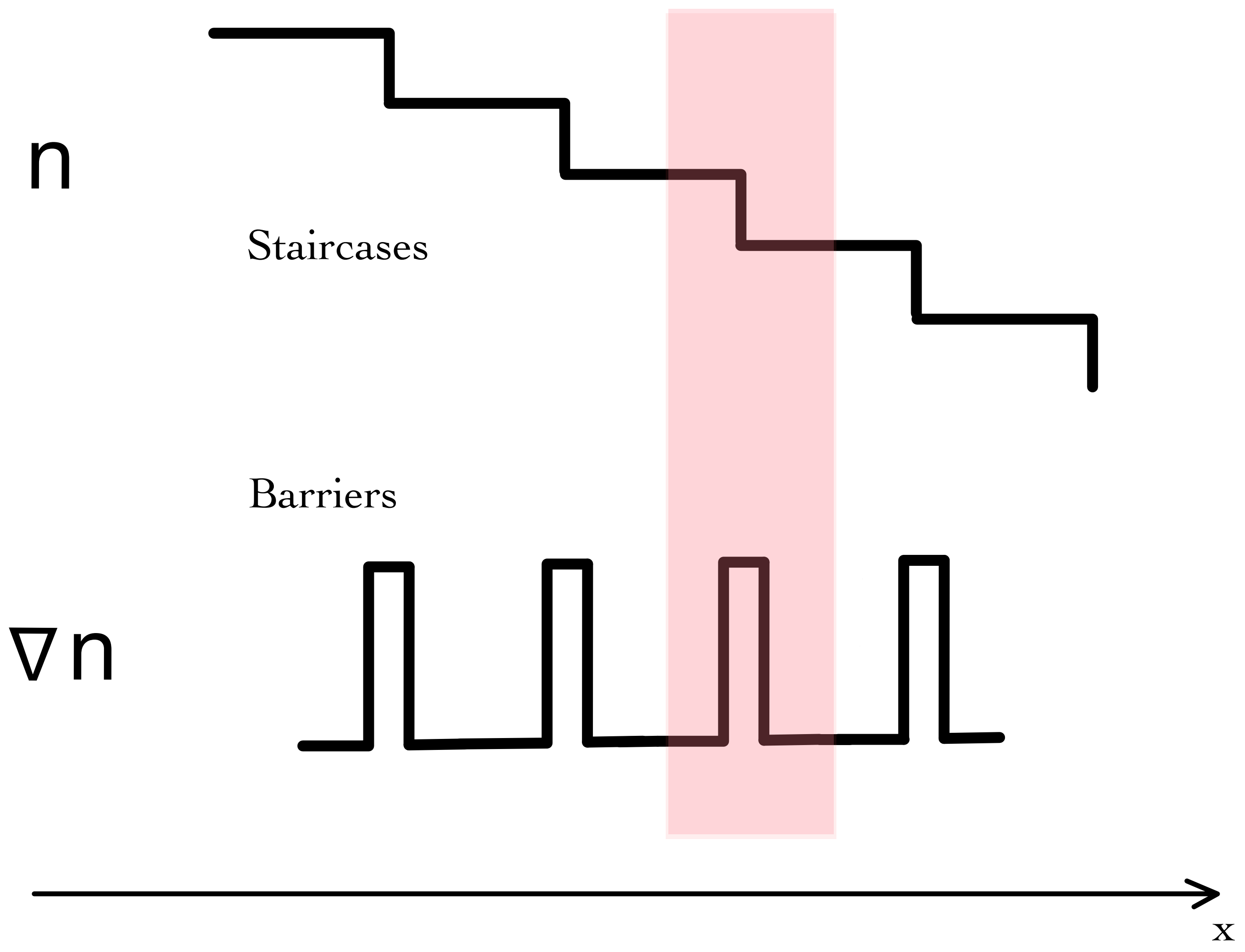
  b. Power threshold increment for L-H transition.

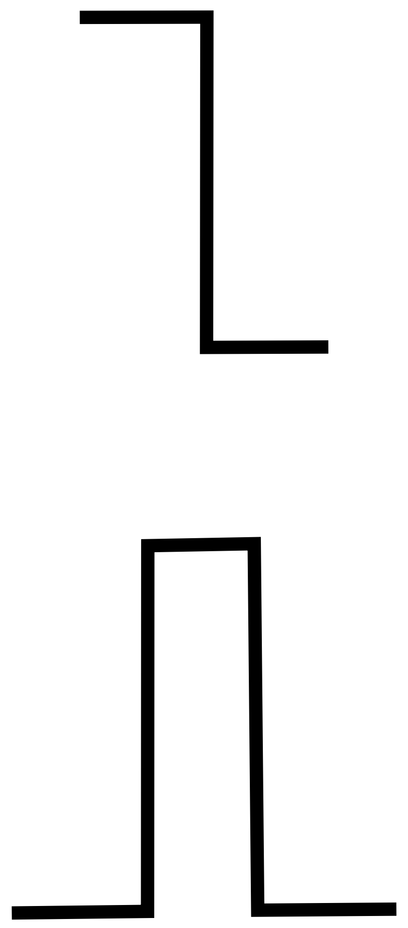
  c. From single barrier to BLY (BALMFORTH, LLEWELLYN SMITH, and YOUNG 1998): timescale, induced by stochastic fields, that might modify the mixing length for the barrier.

- Conclusions

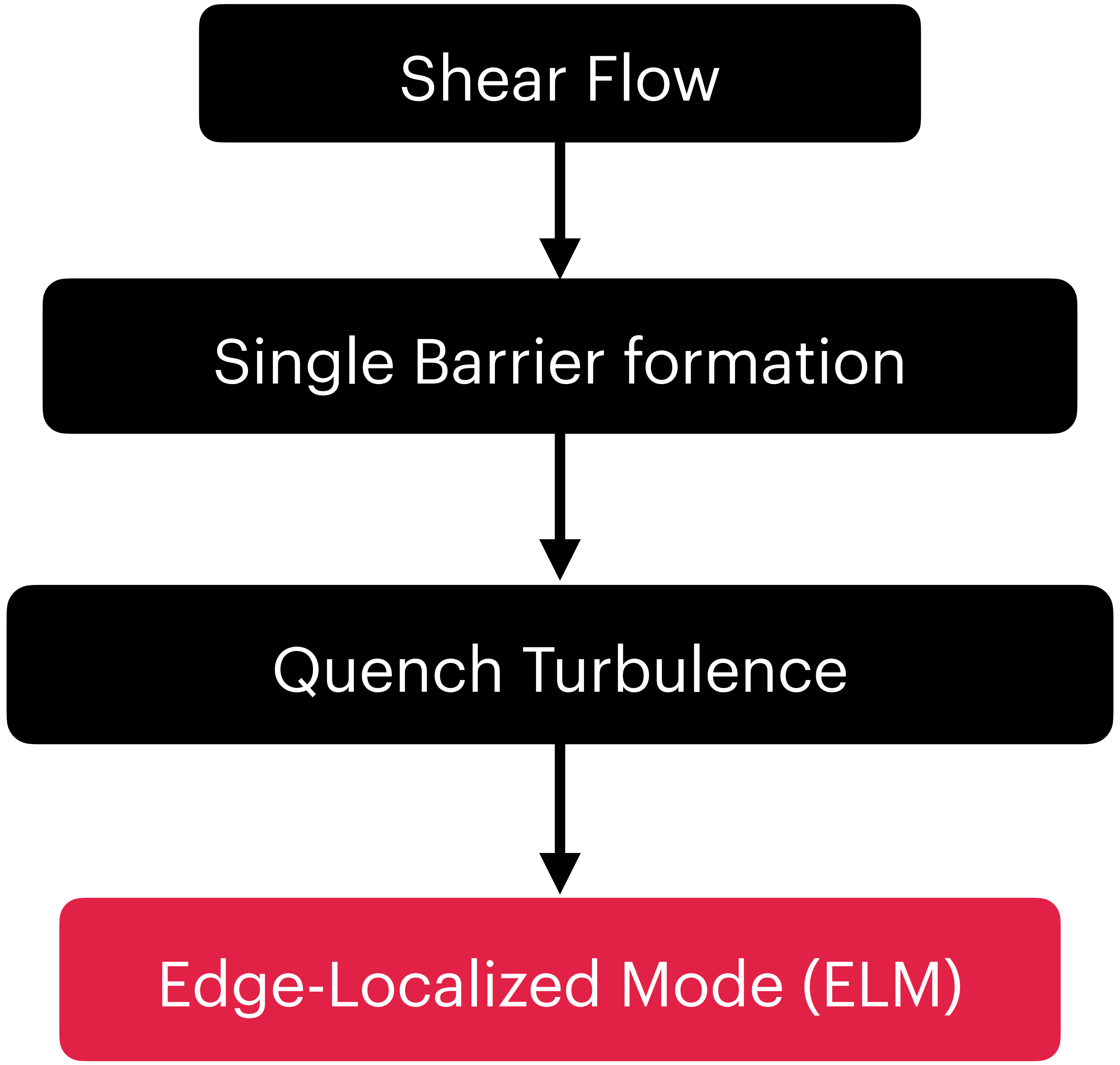
Why we study staircase in fusion device?

Staircase-like structure in Fusion Plasma:





Single barrier



- ELMs are quasi-periodic relaxation events occurring at edge pedestal in H-mode plasma.
- ELMs can damage wall components of a fusion device.

Stochastic field effect is important for boundary control

Shear Flow

Edge-Localized Mode (ELM)

↓

Edge gradient ↑

↓

Peeling-ballooning mode

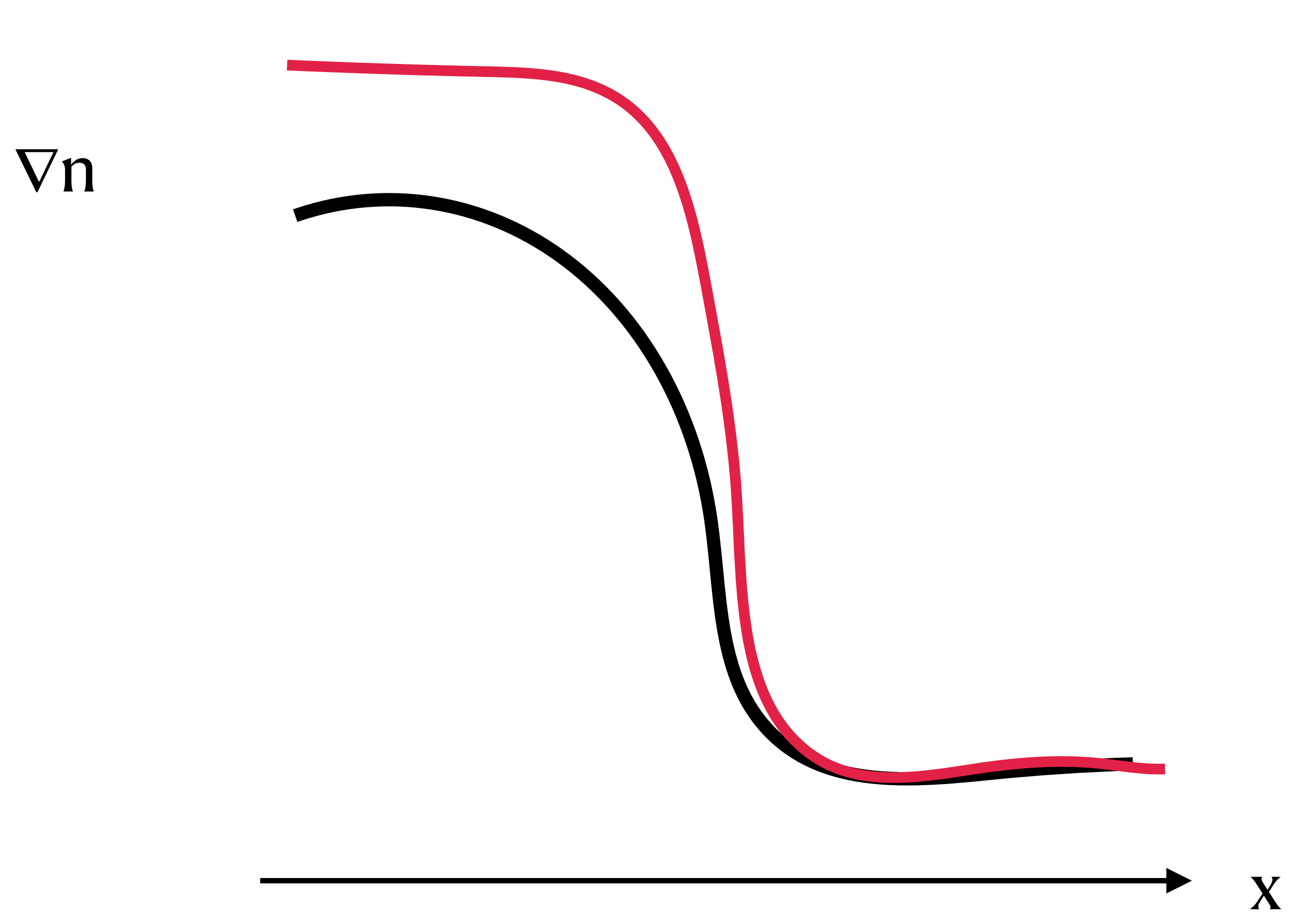
↓

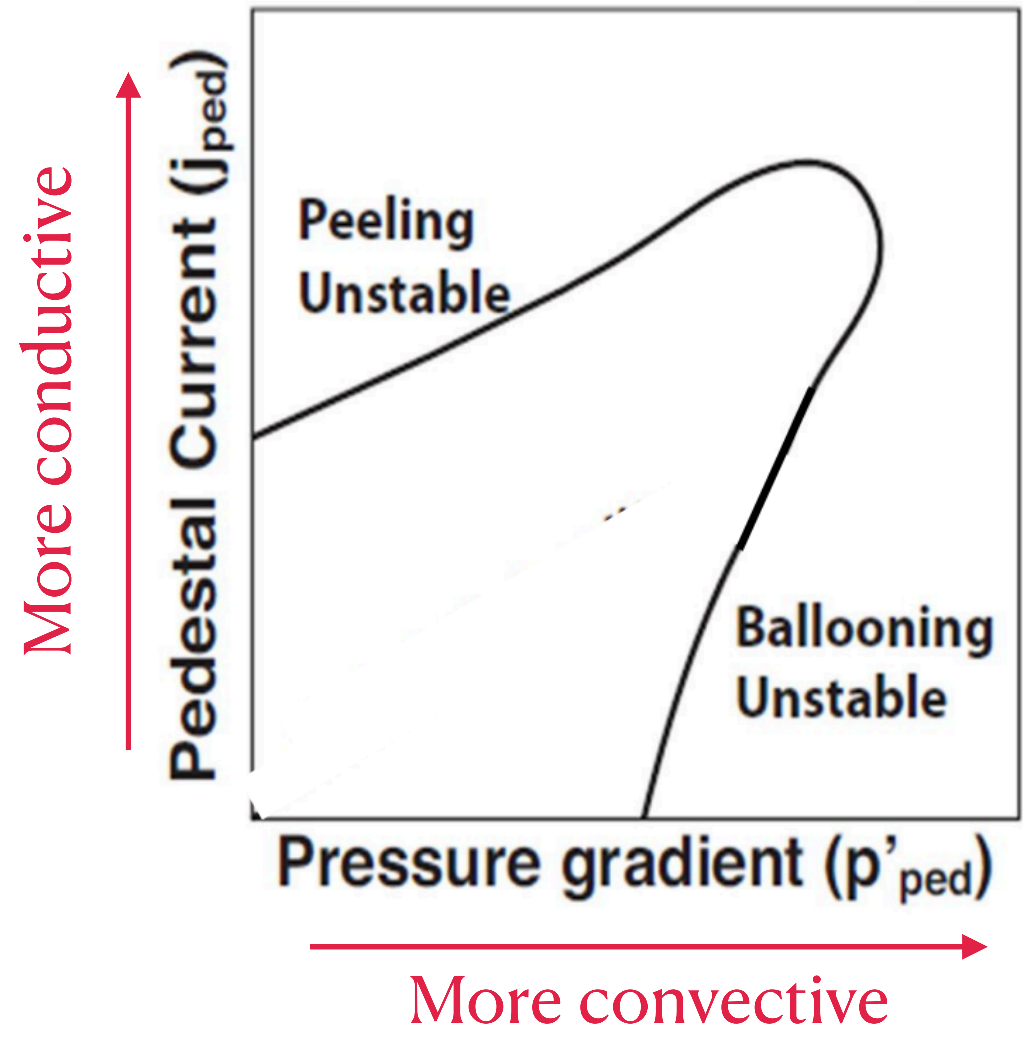
ELM Burst

Boundary Control: Resonant Magnetic Perturbation (RMP)

Suppress (by inducing magnetic perturbation)

Trade off: RMP controls gradients and mitigates ELM, but raise the power threshold.

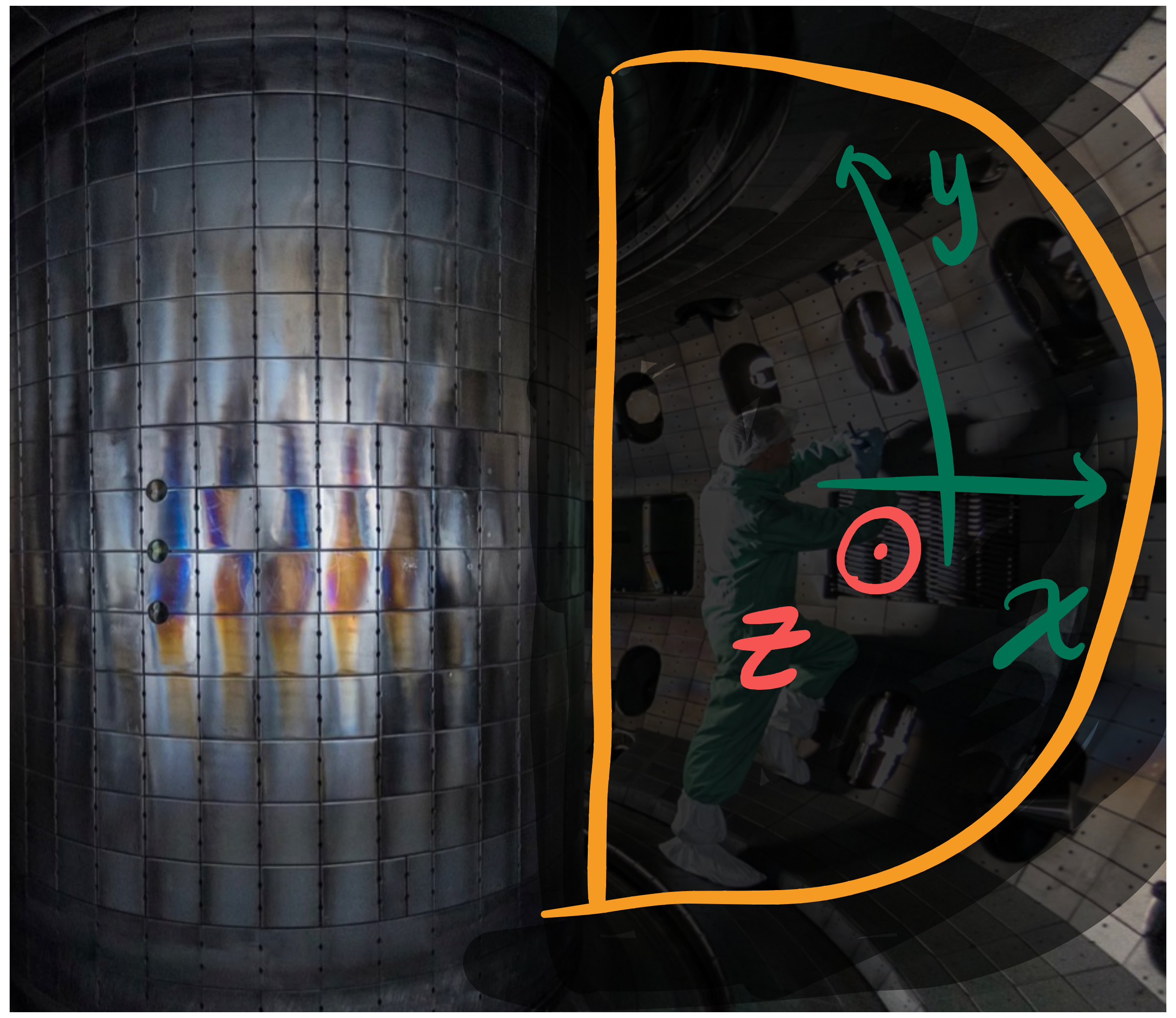




How resilient is the barrier under the influence disordered field?

How stochastic fields influence the shear flow, and the barrier formation?

Model



1. Cartesian coordinate: strong mean field B_0 is in z direction (3D).
2. Rechester & Rosenbluth (1978): waves, instabilities, and transport are studied in the presence of **an ensemble of prescribed, static, stochastic fields.**
3. $\underline{k} \cdot \underline{B} = 0$ (or $k_{\parallel} = 0$) **resonant at rational surface in third direction —** $\omega \rightarrow \omega \pm v_A k_z$, **and** Kubo number: $Ku_{mag} = \dfrac{l_{ac} |\widetilde{\mathbf{B}}|}{\Delta_{\perp} B_0}$).
4. Four-field equations —
   - (a) Potential vorticity equation—vorticity $- \nabla^2 \psi \equiv \zeta$
   - (b) Induction equation — $\mathbf{A}, \mathbf{J}$
   - (c) Pressure equation — $\mathbf{p}$
   - (d) Parallel flow equation — $\mathbf{u_z}$

Well beyond HM model

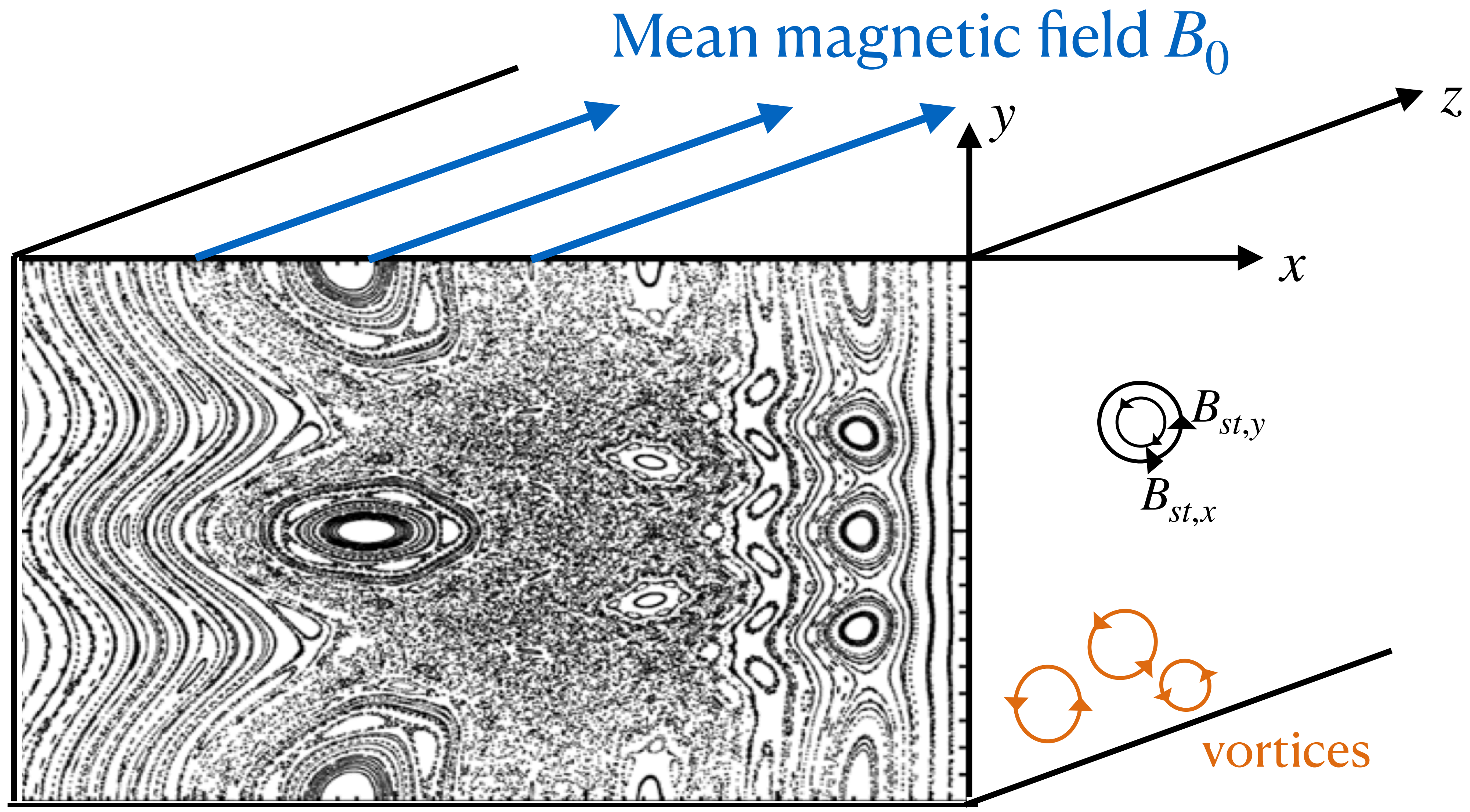
We use mean field approximation:

$$\zeta = \langle \zeta \rangle + \tilde{\zeta},$$ Perturbations produced by turbulences

$$where \ \langle \ \rangle = \frac{1}{L} \int dx \frac{1}{T} \int dt$$

ensemble average over the zonal scales

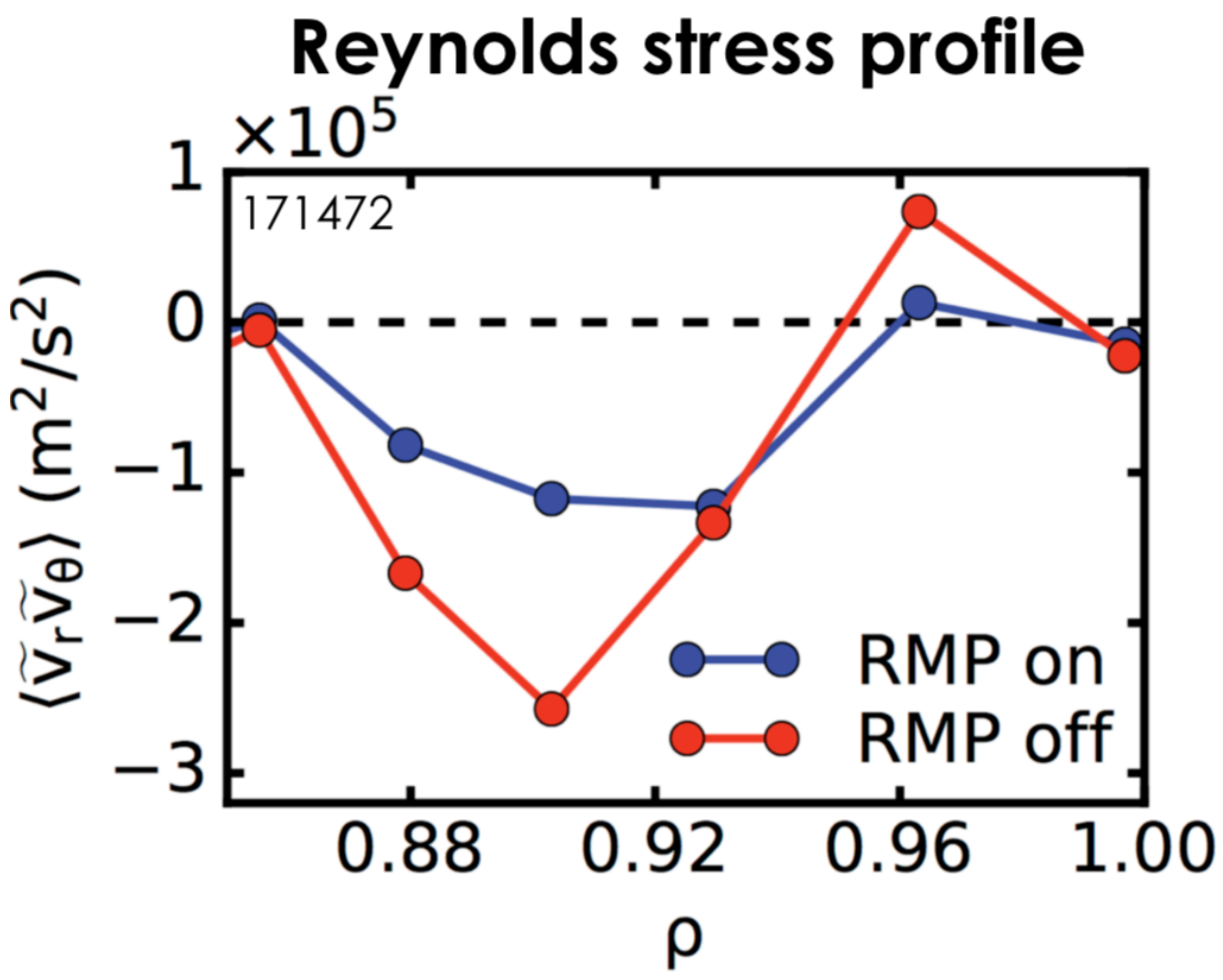
We define rms of normalized stochastic field $b \equiv \sqrt{\overline{(B_{st}/B_0)^2}}$



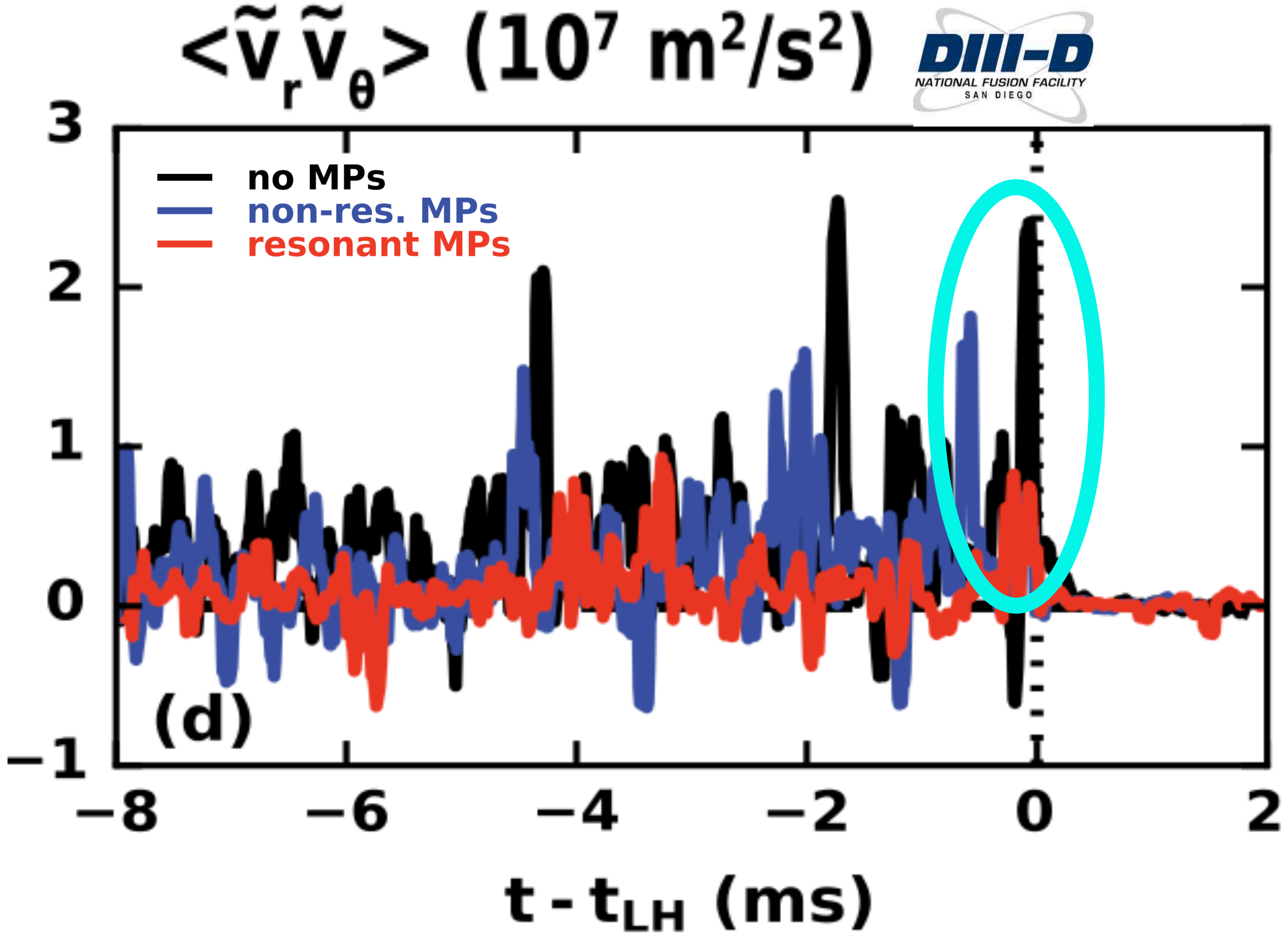
Magnetic islands overlapping forms stochastic

Key Physics

Experimental Results with RMP for L–H Transition — fluctuations



(D. Kriete et al, PoP **27** 062507 (2020))



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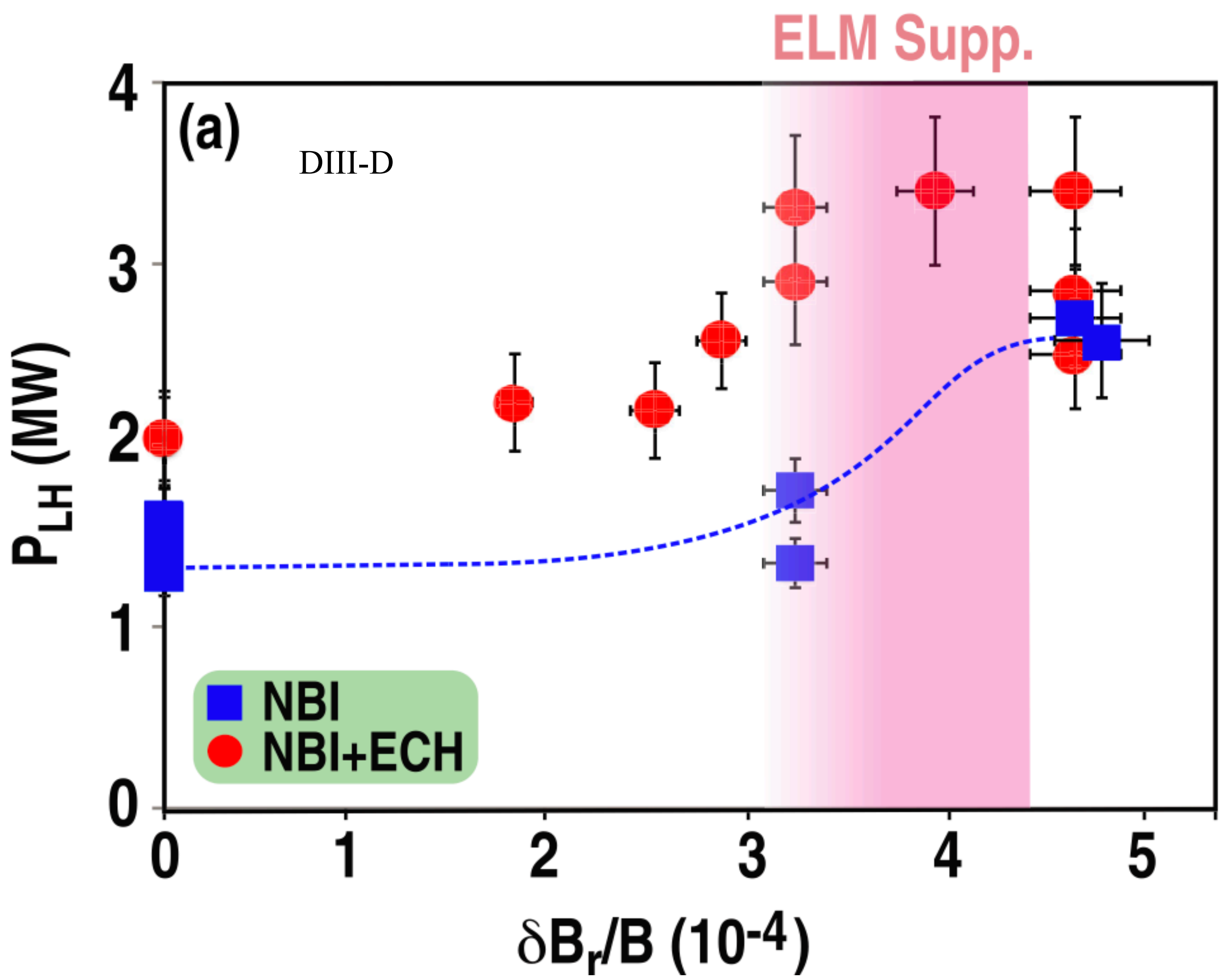
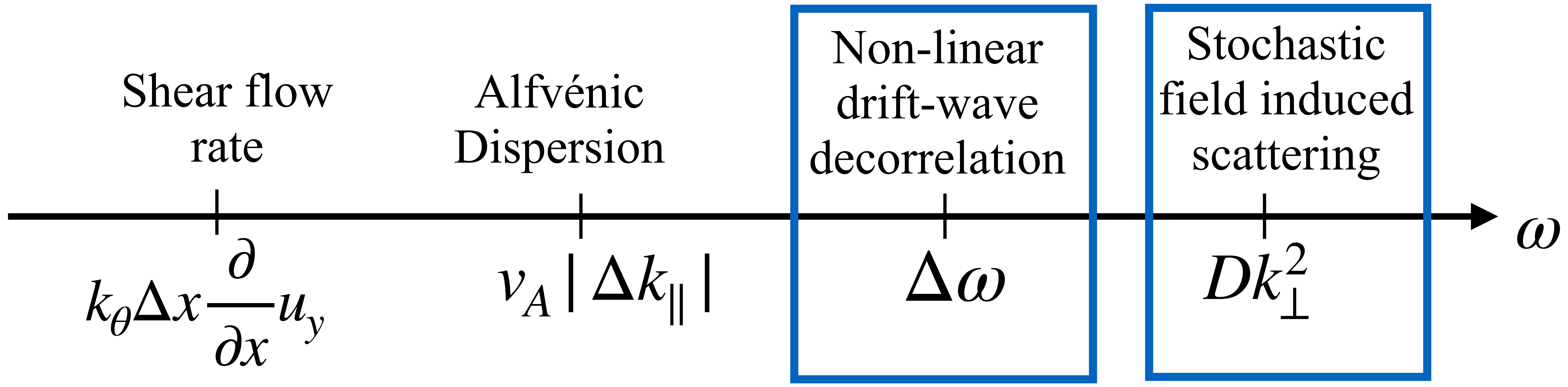


(L. Schmitz et al, NF **59** 126010 (2019))

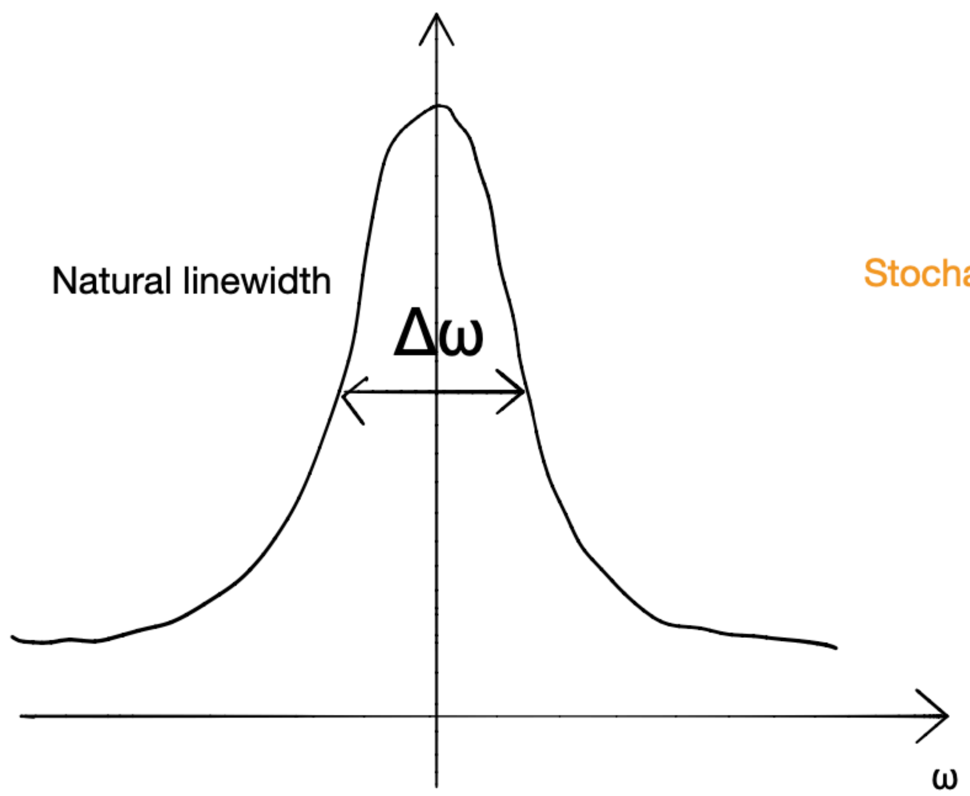
Key Questions: What is the effect of stochastic fields (b^2) on the Reynolds stress and the power threshold for the L-H transition?

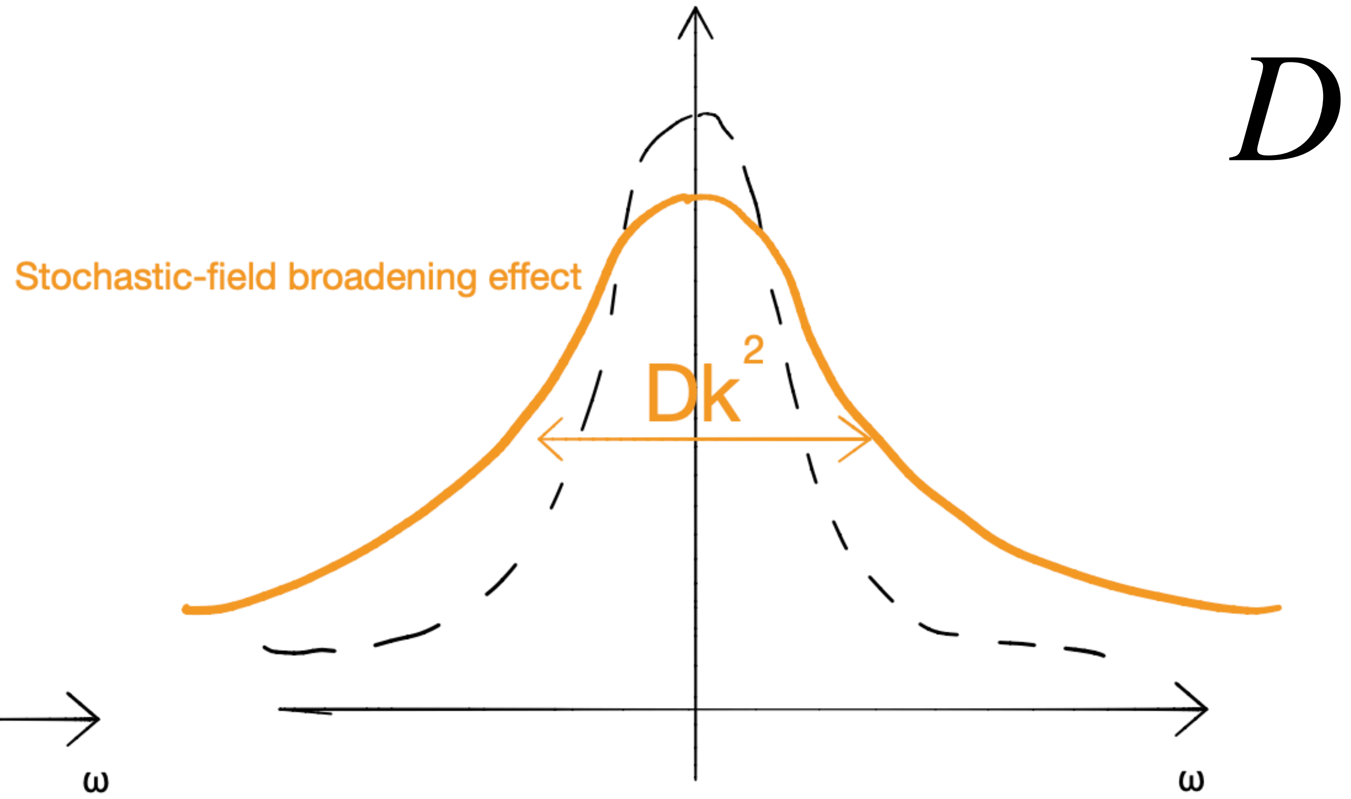
When does stochastic field effect becomes significant?

We consider timescales:



Stochastic field decoherence beats self-decoherence.





$$D \equiv v_A D_M = v_A \sum_k \pi \delta(k_z) b_k^2 \propto B_{st}^2$$

(Independent of B_0)

Magnetic diffusivity

Auto-correlation length l_{ac}

Alfvén wave propagate along stochastic fields
→ characteristic velocity emerges from the calculation of $\underline{\nabla} \cdot \underline{J} = 0$

Decoherence of eddy tilting feedback

Snell's law:

$$\frac{d}{dt}k_x = -\frac{\partial(\omega_0 + u_y k_y)}{\partial x} = -k_y \frac{\partial u_y}{\partial x}$$

Gives an non-zero $\langle k_x k_y \rangle$

$\rightarrow \langle \tilde{u}_x \tilde{u}_y \rangle \propto \langle k_x k_y \rangle$

shear flow

Self-feedback loop:

The $E \times B$ shear generates the $\langle k_x k_y \rangle$ correlation and hence support the non-zero Reynolds stress.

$$\langle \tilde{u}_x \tilde{u}_y \rangle \simeq \sum_k \frac{|\tilde{\phi}_k|^2}{B_0^2}\left(k_y^2 \frac{\partial u_y}{\partial x}\tau_c\right)$$

The Reynold stress modifies the shear via momentum transport.

Shear flow reinforce the self-tilting.

Stochastic Fields Effect

$$k_\parallel = k_\parallel^{(0)} + \underline{b} \cdot \underline{k}_\perp$$

$$\omega = \omega_D + \delta\omega$$

Dispersion relation with drift-Alfvén coupling

$$\omega^2 - \omega_D \omega - k_\parallel^2 v_A^2 = 0$$

$$(\omega_D + \delta\omega)^2 - \omega_D(\omega_D + \delta\omega) - (k_\parallel + \underline{b} \cdot \underline{k}_\perp)^2 v_A^2 = 0$$

$$\delta\omega \simeq \frac{v_A^2}{\omega_D}\left(2k_\parallel \underline{b} \cdot \underline{k}_\perp + (\underline{b} \cdot \underline{k}_\perp)^2\right)$$

$$\omega_D \text{ (drift wave turbulence frequency)} \equiv \frac{k_y \rho_s C_s}{L_n}$$

Decoherence of eddy tilting feedback

Expectation frequency:

$$\left\langle \delta\omega \simeq \frac{v_A^2}{\omega_0}(2k_\parallel \underline{b}\cdot\underline{k}_\perp + (\underline{b}\cdot\underline{k}_\perp)^2)\right\rangle$$

$$\langle\delta\omega\rangle \simeq \frac{v_A^2}{\omega_0}\langle(\underline{b}\cdot\underline{k}_\perp)^2\rangle = \frac{1}{2}\frac{v_A^2}{\omega_0}b^2k_\perp^2$$

$$\omega = \omega_D + \delta\omega$$

$$\langle\omega\rangle \simeq \omega_D + \frac{1}{2}\frac{v_A^2}{\omega_D}b^2k_\perp^2$$

Snell's law:

$$\frac{d}{dt}k_x = -\frac{\partial\omega_k}{\partial x}$$

$$= -k_y\frac{\partial u_y}{\partial x} - \frac{1}{2}\frac{v_A^2k_\perp^2}{\omega_D}\frac{\partial b^2}{\partial x}$$

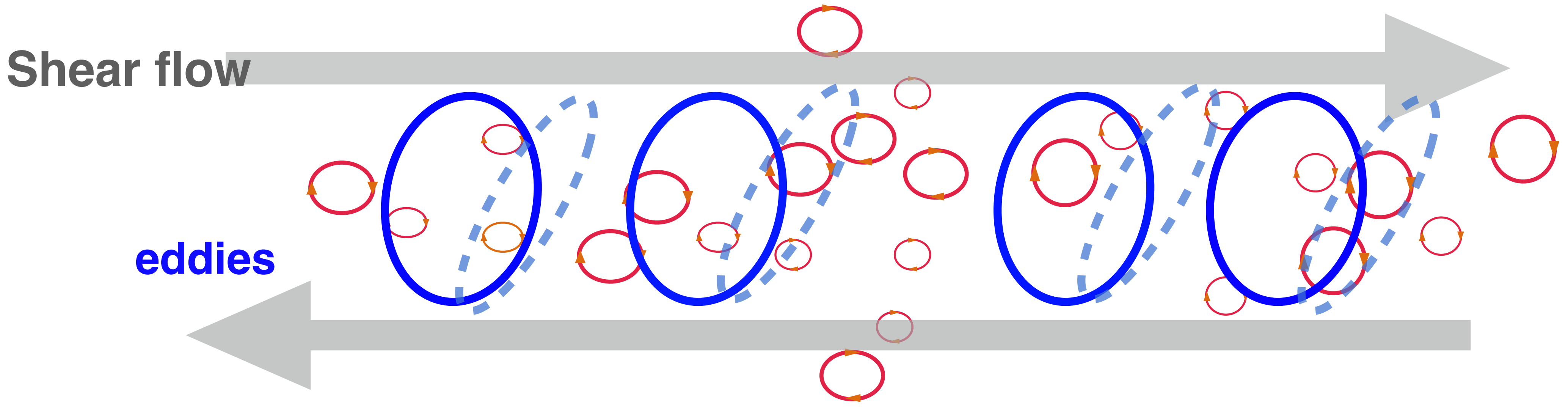
Ensemble average frequency shift

Self-feedback loop is broken by b^2:

$$\langle\tilde{u}_x\tilde{u}_y\rangle \simeq \sum_k \frac{|\tilde{\phi}_k|^2}{B_0^2}\left(k_y^2\frac{\partial u_y}{\partial x}\tau_c + \frac{1}{2}k_y\frac{v_A^2k_\perp^2}{\omega_D}\frac{\partial b^2}{\partial x}\tau_c\right)$$

Stochastic dephasing

Stochastic fields (random ensemble of elastic loops) act as elastic loops and resist the tilting of eddies.
→change the cross-phase btw $\tilde{u}_x$ and $\tilde{u}_y$.



Stochastic fields interfere with shear-tilting feedback loop.

Results—Suppression of PV diffusivity

The ensemble average Reynolds force $\frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle$:

$$\langle \tilde{u}_x \tilde{\zeta} \rangle = \frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle = -D_{PV}\frac{\partial}{\partial x}\langle \zeta \rangle + F_{res}\kappa\frac{\partial}{\partial x}\langle p \rangle$$

PV diffusivity (D_{PV}), Residual Stress (F_{res}), Curvature (κ)

Suppressed by stochastic fields

Taylor Identity: $\underbrace{\langle \tilde{u}_x \tilde{\zeta} \rangle}_{PV\ flux} = \frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle$ (Reynolds force)

Mean vorticity $\langle \tilde{\zeta} \rangle = \frac{\partial v_{E\times B}}{\partial x}$ ($E \times B$ shear)

$$D_{PV} = \sum_{k\omega} |\tilde{u}_{x,k\omega}|^2 \frac{v_A b^2 l_{ac} k^2}{\bar{\omega}^2 + \left(v_A b^2 l_{ac} k^2\right)^2}$$

v_A: Low $\beta \equiv P_{thermal}/P_{mag}$, so it is v_A instead of sound speed C_s (small).

$$F_{res} \simeq \sum_{k\omega} \frac{-2k_y}{\bar{\omega}\rho} D_{PV,k\omega}$$

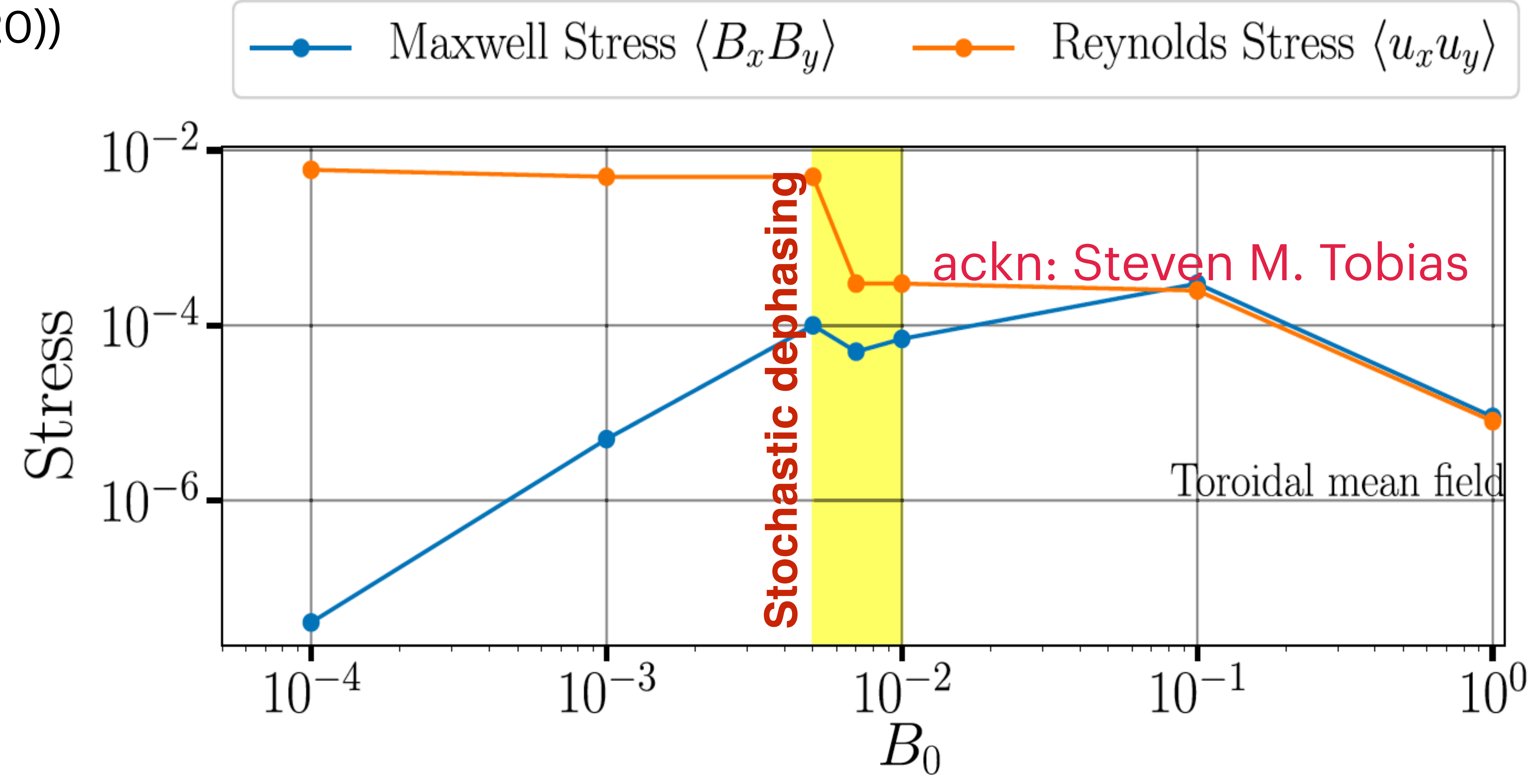
$$\bar{\omega} \equiv \omega - \langle u_y \rangle k_y$$

This **stochastic dephasing** is insensitive to turbulent modes (e.g. ITG, TEM,...etc.).

PV transport will be suppressed by stochastic fields via decoherence.

Similar Simulation Results for β–plane MHD

2D MHD (Chen & Diamond, ApJ **892**, 24 (2020))



More details:

Reynolds stress will undergo decoherence at levels of field intensities **well below that of Alfvénization (**where Maxwell stress balances the Reynolds stress).

PV flux ↓ Magnetic drag ↓

$$\frac{\partial}{\partial t}\langle u_x \rangle = \langle \Gamma \rangle - \frac{1}{\eta \mu_0 \rho} \langle \overline{B^2_{st,y}} \rangle \langle u_x \rangle + \nu \nabla^2 \langle u_x \rangle$$

1. Coupling to resisto-elastic waves, which is $\overline{B^2_{st}}$ dependent.
2. Increase of the magnetic drag.

Stochastic fields reduce the Reynolds stress at a B_0 smaller than that for Alfvénization.

Macroscopic Impact

Results — Increment of P_{LH}

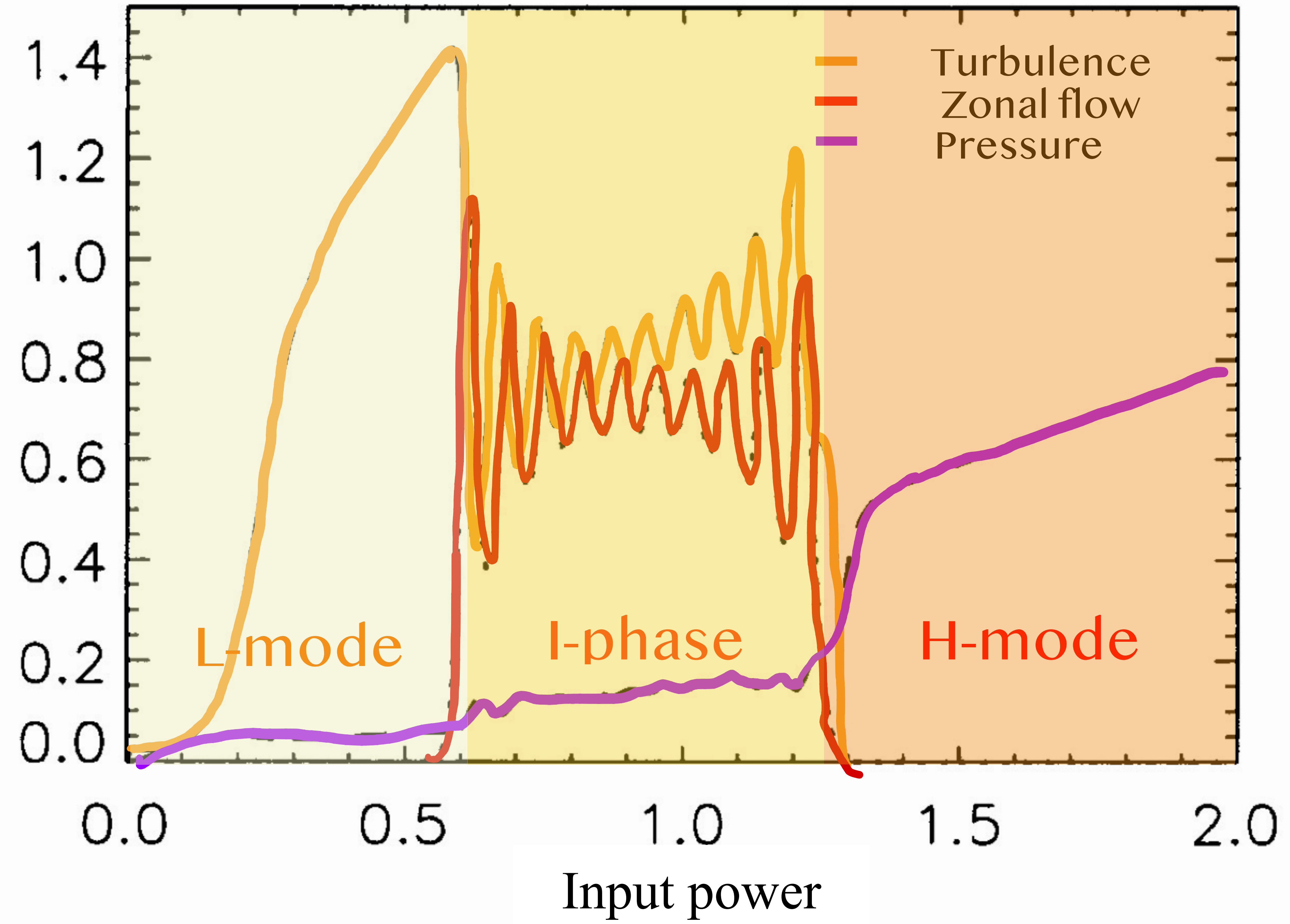
Stochastic field stress dephasing effect requires: $\Delta\omega \leq k_\perp^2 D$ (where $D = D_M v_A$).

This gives **dimensionless parameter** (α): $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} > 1$

ρ_* is small → $\alpha \uparrow$ (pessimistic)

$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$ **quantifies the strength of stochastic dephasing.**

$$\begin{cases} l_{ac} \simeq Rq \\ \epsilon \equiv L_n/R \sim 10^{-2} \\ \beta \equiv \frac{P_{thermal}}{P_{mag}} \simeq 10^{-2\sim-3} \\ \rho_* \equiv \frac{\text{gyro-radius}}{\text{density scale length}} \\ \quad \equiv \frac{\rho_s}{L_n} \simeq 10^{-2\sim-3} \\ q(\text{stafety factor}) \equiv \frac{rB_t}{RB_p} \end{cases}$$



Kim-Diamond Model

(Kim & Diamond, PoP **10**, 1698 (2003))

This reduce model for the L-H transition is useful for testing trends in power threshold increment induced by stochastic fields.

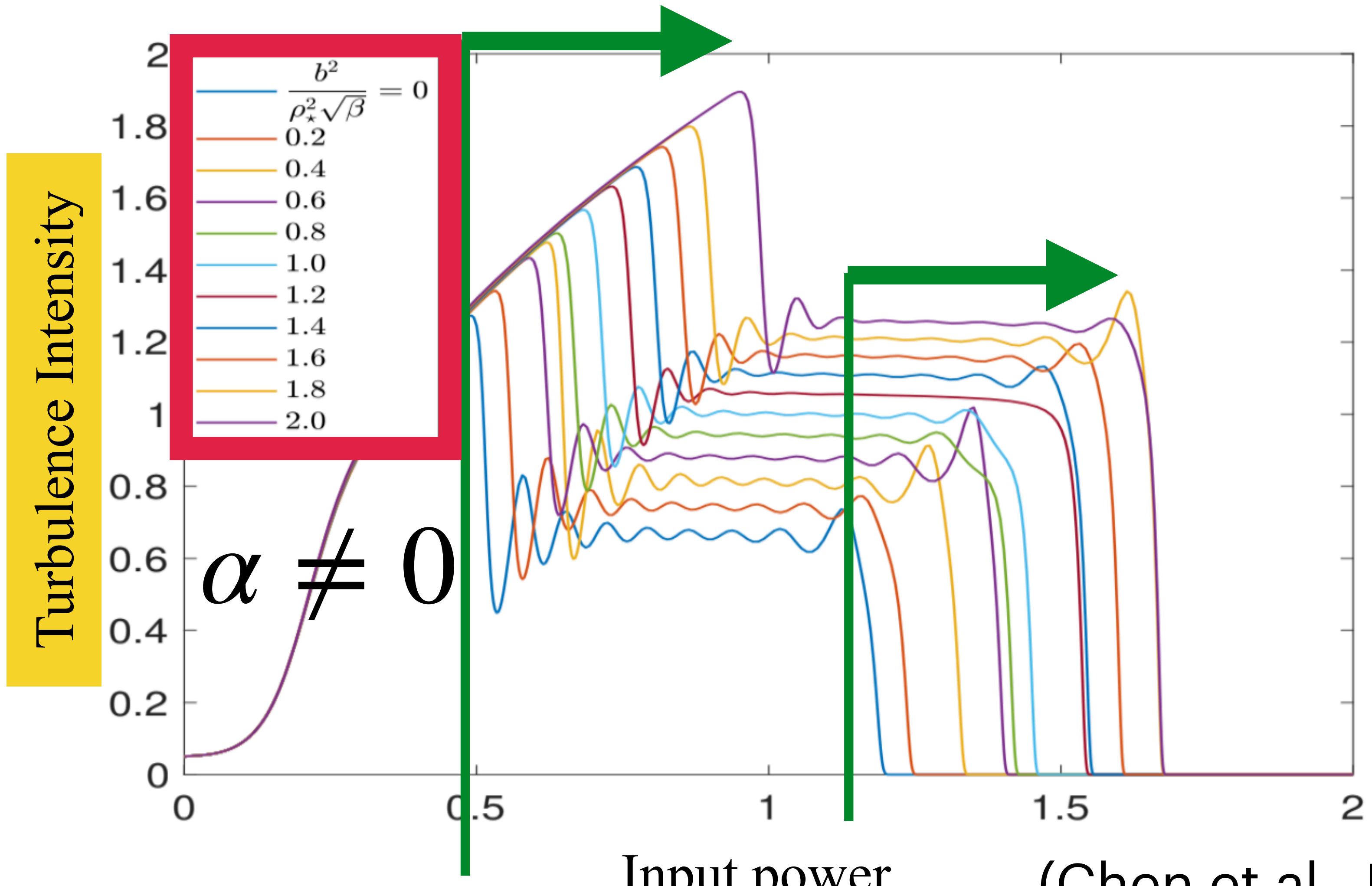
Predator: zonal flow
prey: turbulence

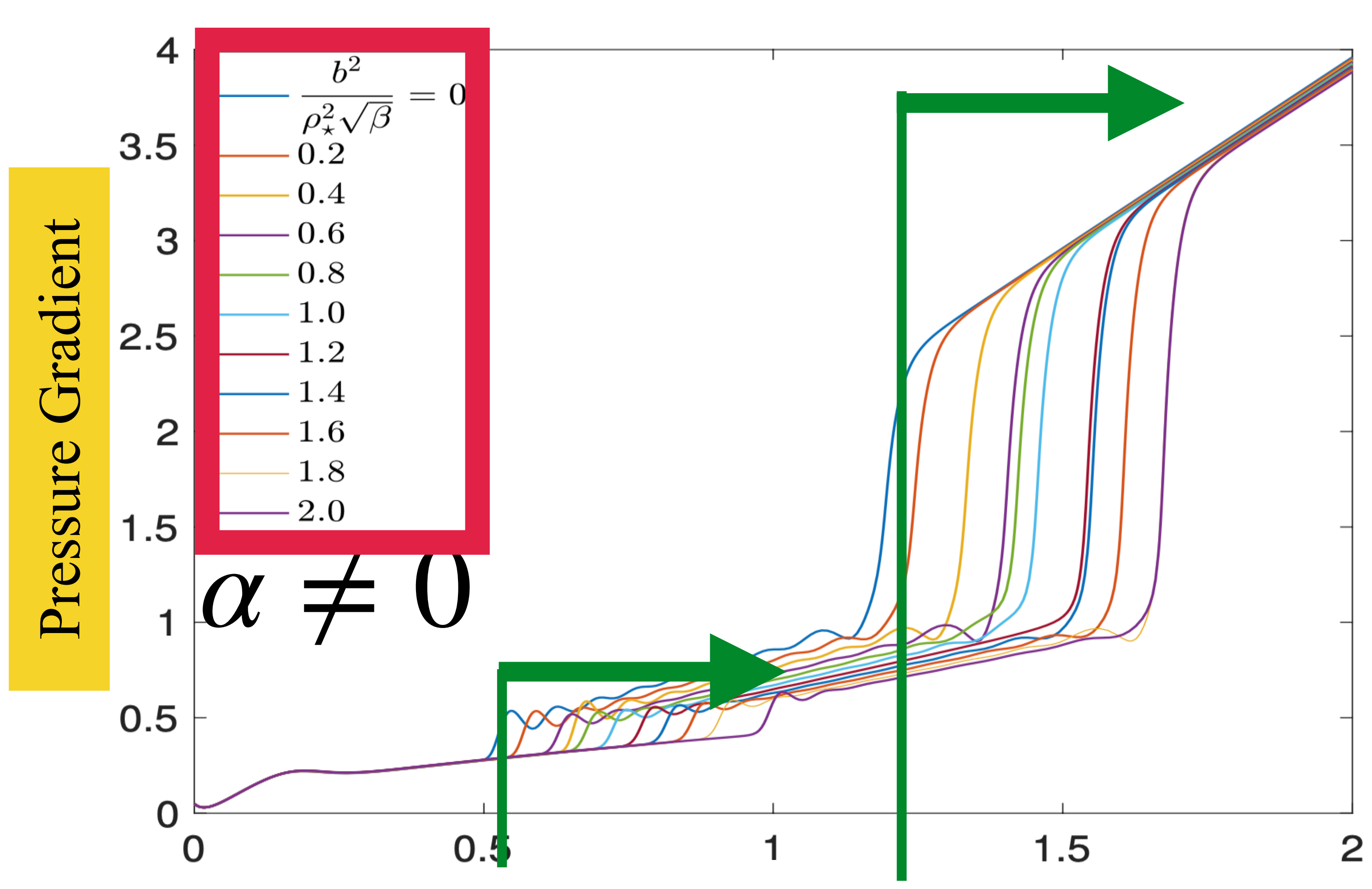
We expect stochastic fields to raise L-I and I-H transition thresholds.

Macroscopic Impact

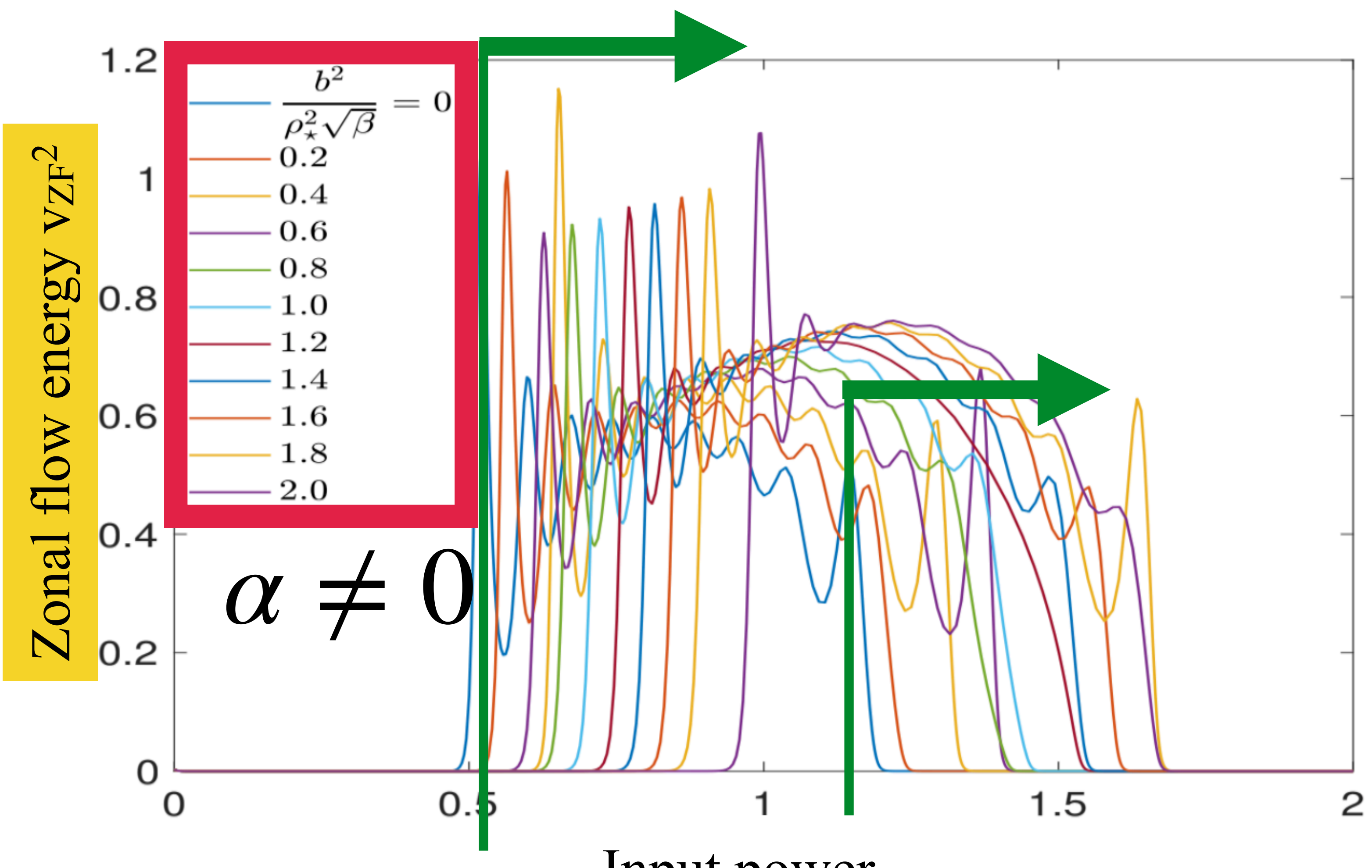
Results — Increment of P_{LH}

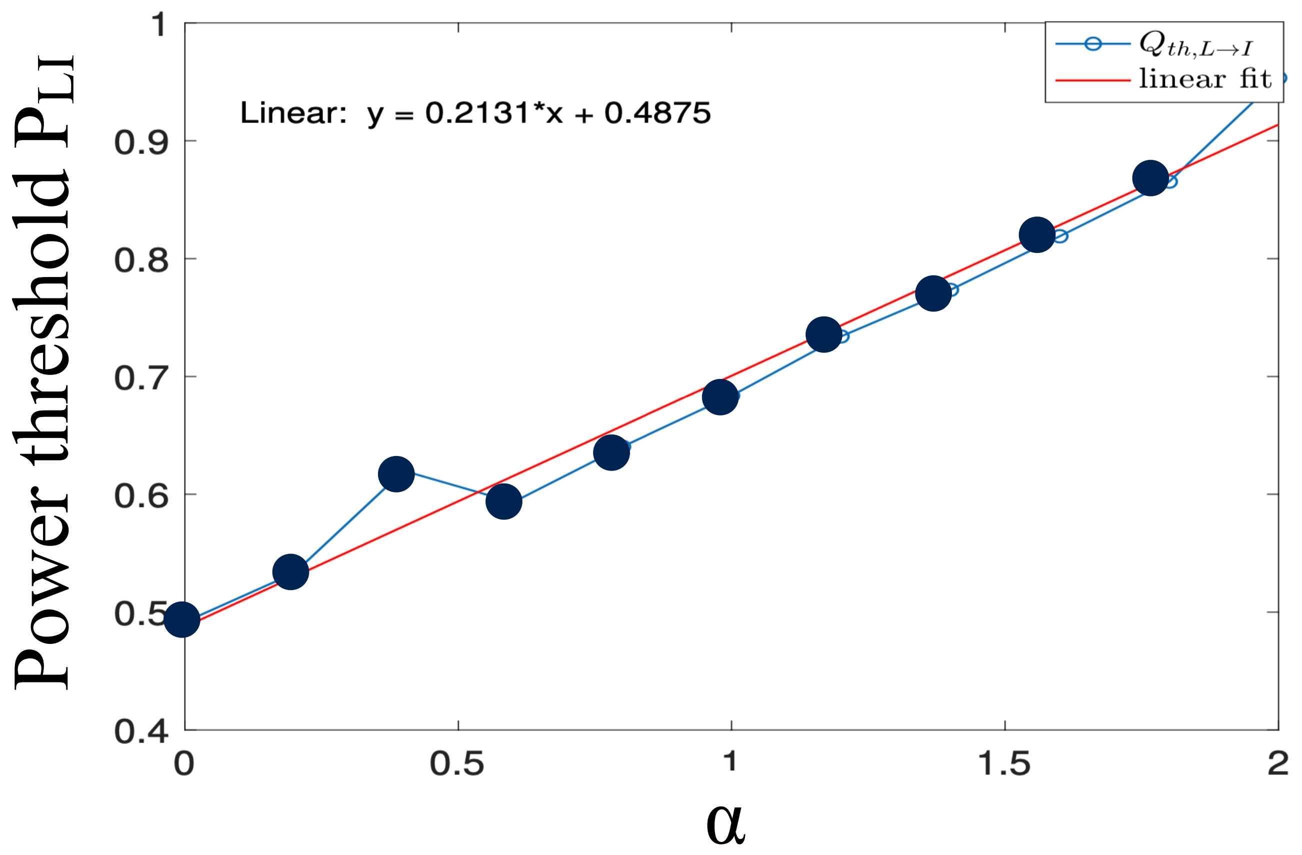
$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$$

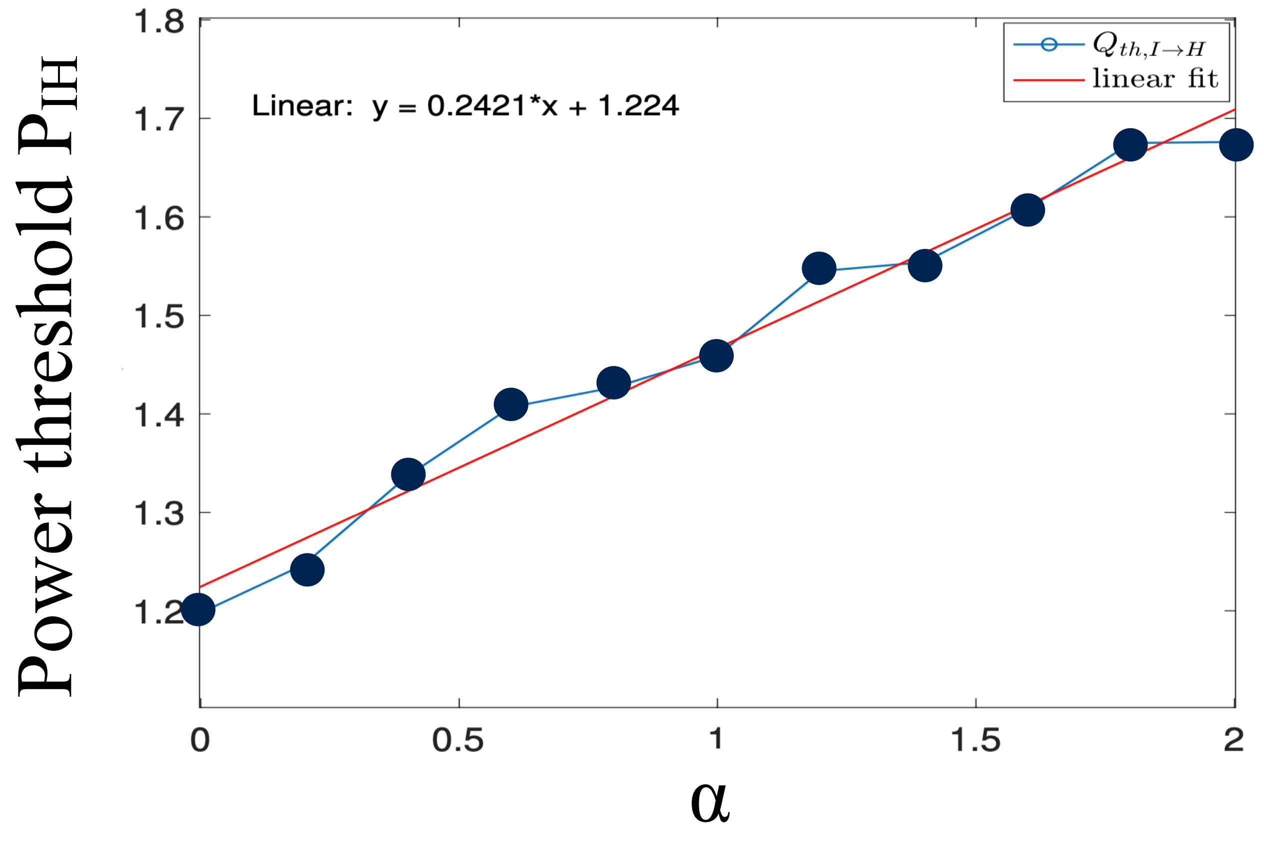


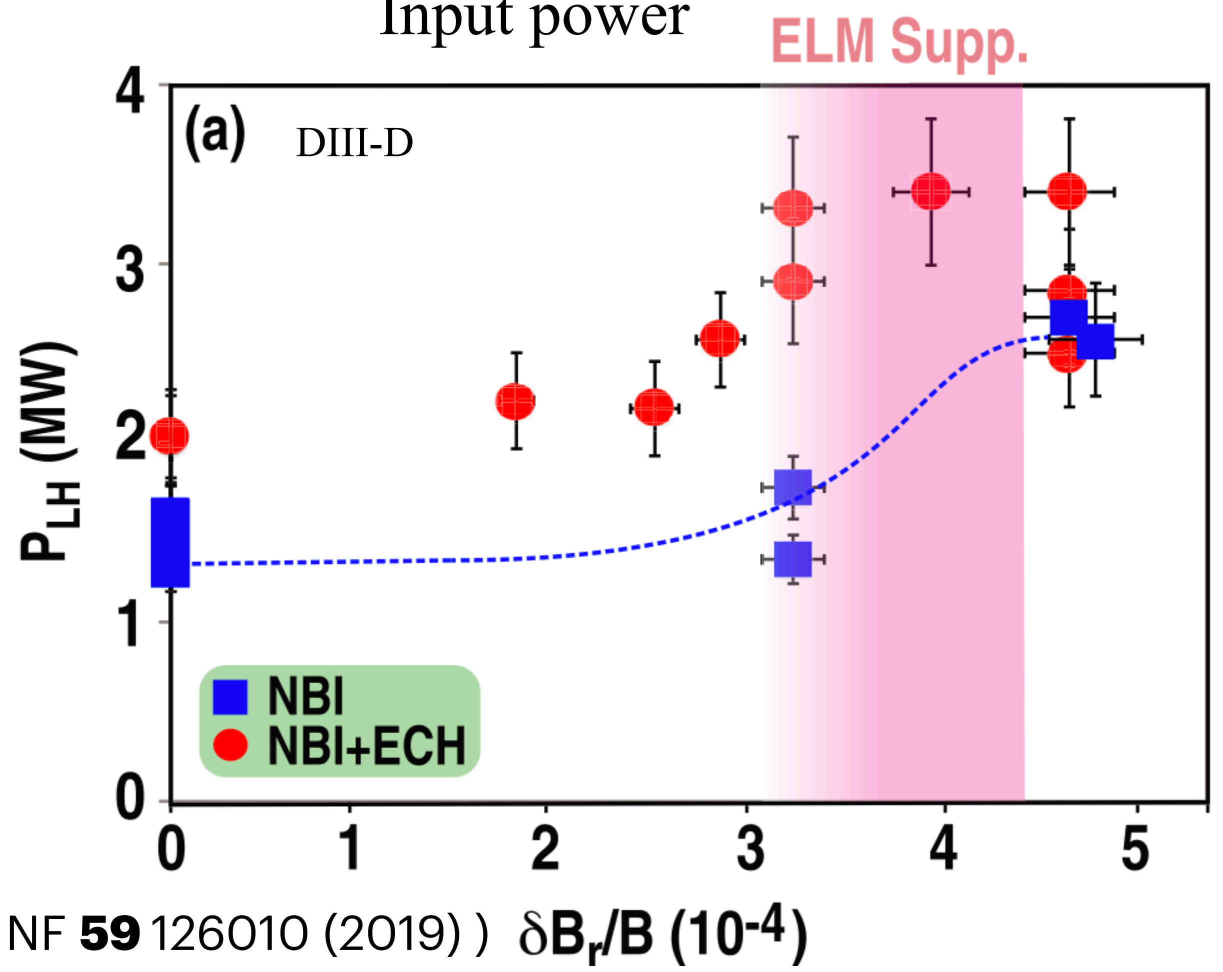


(Chen et al., PoP in press (2021))









(L. Schmitz et al, NF **59** 126010 (2019))

The threshold increase due to stochastic dephasing effect is seen in turbulence intensity, zonal flow, and pressure gradient.

From Single Barrier to Layering?

After BLY's mixing length model (Balmforth et al. JFM **355,** 239 (1998)), Ashourvan & Diamond, PoP **24,** 012305 (2017) proposed a mixing length model for H-W turbulence:

- Reduce evolution equations (based on H-W model).
- Energy and Potential entropy (PE) conserved.

Density: $\frac{\partial}{\partial t}\langle n\rangle = \frac{\partial}{\partial x}\left(D_n \frac{\partial \langle n\rangle}{\partial x}\right) + D_c \frac{\partial^2}{\partial x^2}\langle n\rangle$ (turb. particle diffusion)

Potential Vorticity: $\frac{\partial}{\partial t}\langle \zeta\rangle = \frac{\partial}{\partial x}\left((D_n - \chi) \frac{\partial \langle n\rangle}{\partial x}\right) + \chi \frac{\partial^2}{\partial x^2}\langle \zeta\rangle + \mu_c \frac{\partial^2}{\partial x^2}\langle \zeta\rangle$ (residual stress; turb. Viscous diffusion)

Turbulent potential Enstrophy: $\frac{\partial}{\partial t}\epsilon = \frac{\partial}{\partial x}\left(D_\epsilon \frac{\partial \epsilon}{\partial x}\right) + \chi\left[\frac{\partial (n-\zeta)}{\partial x}\right]^2 - \epsilon_c^{-1/2}\epsilon^{3/2} + P$ (PE diffusion; mean-turb PE Coupling; PE Dissipation)

- n : density
- ζ : potential vorticity
- ϵ : turbulent PE
  - $\epsilon \equiv (\delta n - \delta\zeta)^2/2$
- D_n : turbulent particle diffusivity
- χ : turbulent vorticity
- P : production

The mixing length depends on two scales:

- Forcing scale: l_0
- Rhines scale: $l_{RH} = \frac{\sqrt{\epsilon}}{|\partial_x q|}$

→ mixing scale: $l_{mix} = \frac{l_0}{(1 + l_0^2(\partial_x q)^2/\epsilon)^{\kappa/2}} = \frac{l_0}{(1 + l_0^2/l_{RH}^2)^{\kappa/2}}$

$$\begin{cases} \text{Strong mixing } (l_{RH} > l_0): & l_{mix} \simeq l_0 \text{ (Weak mean PV gradient)} \\ \text{Weak mixing } (l_0 > l_{RH}): & l_{mix} \simeq l_0^{1-\kappa} l_{RH}^{\kappa} \text{ (Strong PV gradient)} \end{cases}$$

→ **Transport bifurcation**

Very Preliminary

From Single Barrier to Layering?

Essential physics of Ashourvan & Diamond model (after BLY's) is contained in l_{mix} and transport coefficient density diffusivity D_n and turbulent viscosity χ (called D_{PV} previously).

These evolve $\langle n \rangle$ and $\langle \partial_x v_{E\times B} \rangle$ fields and govern PE exchange with fluctuations.

For α_{DW} (a measurement of the resistive diffusion rate in the parallel direction) > 1 in H-W regime (where $\alpha_{DW} = \frac{k_{\parallel}^2 v_{the}^2}{\nu}$), we have

$$D_n \simeq \frac{l_{mix}^2 \epsilon}{\alpha_{DW}}$$

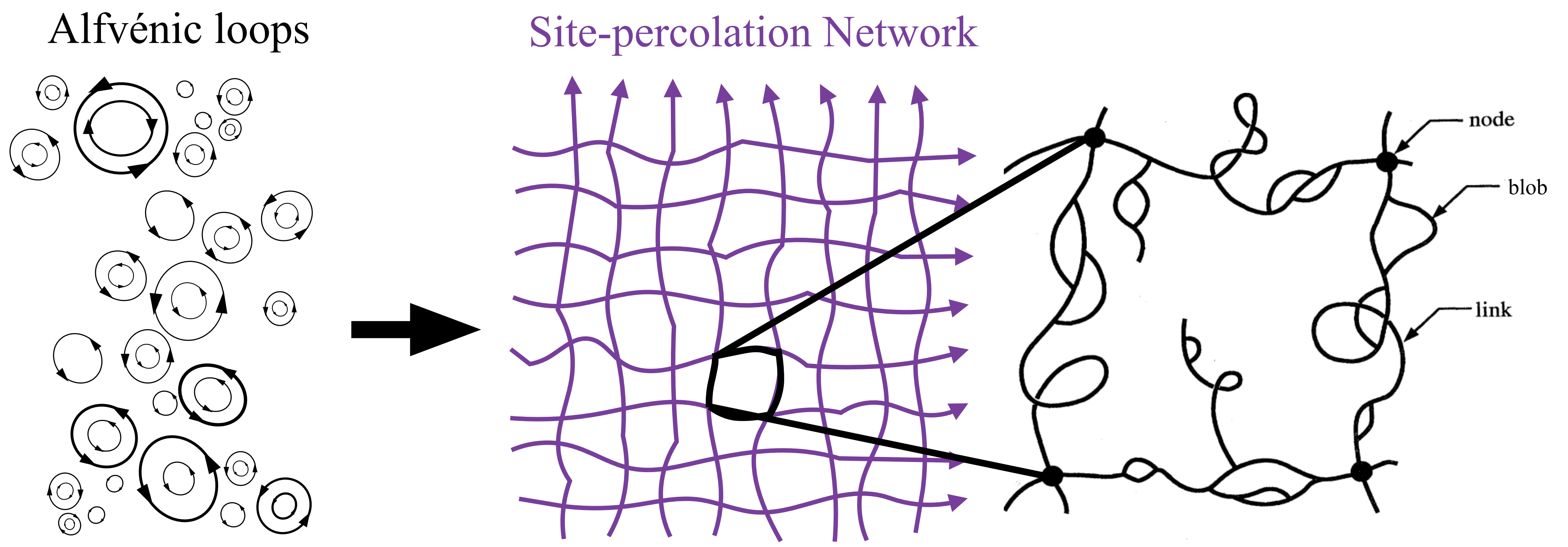
By replacing $k_{\parallel}$ with $k_{\parallel} = \underline{k} \cdot \underline{\hat{b}_0} \simeq \frac{1}{Rq} + \underline{b}_{\perp} \cdot \underline{k}_{\perp} \simeq \frac{1}{Rq} + \frac{b_{\perp}}{l_{mix}}$, we estimate $D_n \simeq \frac{l_{mix}^2 \epsilon \nu / v_{the}^2}{(\frac{1}{Rq})^2 + (\frac{b}{l_{mix}})^2}$.

Notice that $\frac{1}{Rq}$ v.s. $\frac{b_{\perp}}{l_{mix}}$ $\rightarrow$ magnetic Kubo number $Ku_{mag} = bRq/l_{mix}$. ➡ $Ku_{mag} = Ku_{mag}(l_{mix})$

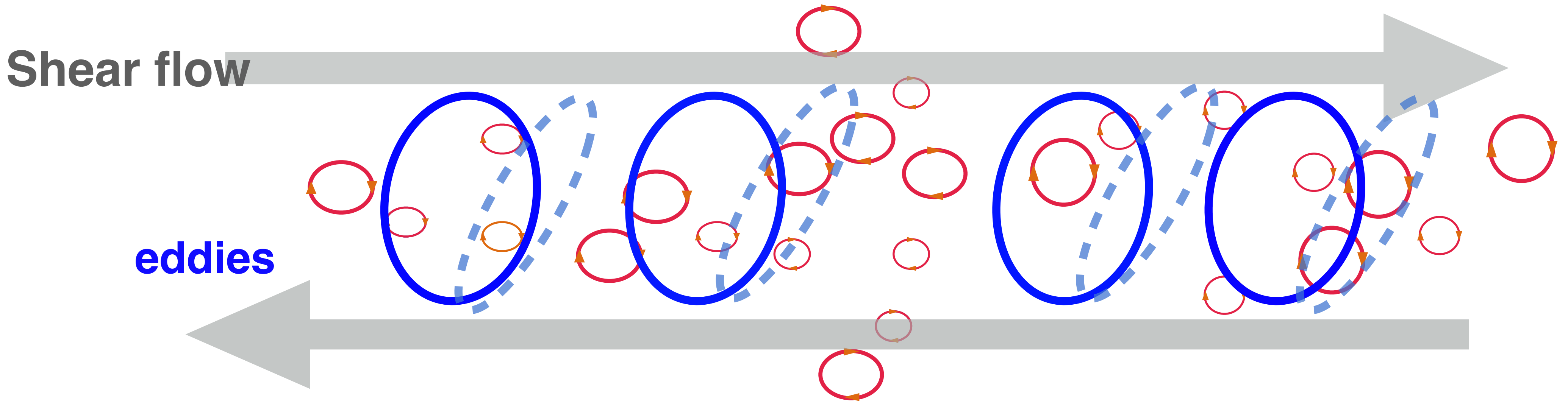
To **reduce** D_n significantly requires $Ku_{mag} \geq 1$. Same for χ as we discussed in previous slides.

Conclusions

- Stochastic fields can form a **fractal, elastic network**. Strong coupling of flow turbulence to the fractal network **prevents** PV mixing and hence zonal flow formation.



- **Dephasing effect** caused by stochastic fields quenches Reynolds stress (e.g. $\Delta\omega < Dk_\perp^2$).



- b^2 shift L-H threshold to higher power, in proportional to $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$.

- Preliminary estimation suggests $Ku_{mag}(l_{mix}) \geq 1$ required for significant change in mean-turbulence coupling.
  Hence, a staircase appears **resilient** in H-W model.