On How Decoherence of Vorticity Flux by Stochastic Magnetic Fields Quenches Zonal Flow Generation

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Outline

- Introduction

  Resonant Magnetic Perturbation plays an important role in momentum transport in edge plasma evolution.

- Model & Calculation

- Results

  a. Suppression of PV diffusivity and the shear-eddy tilting feedback loop.

  b. Power threshold increment for L-H transition.

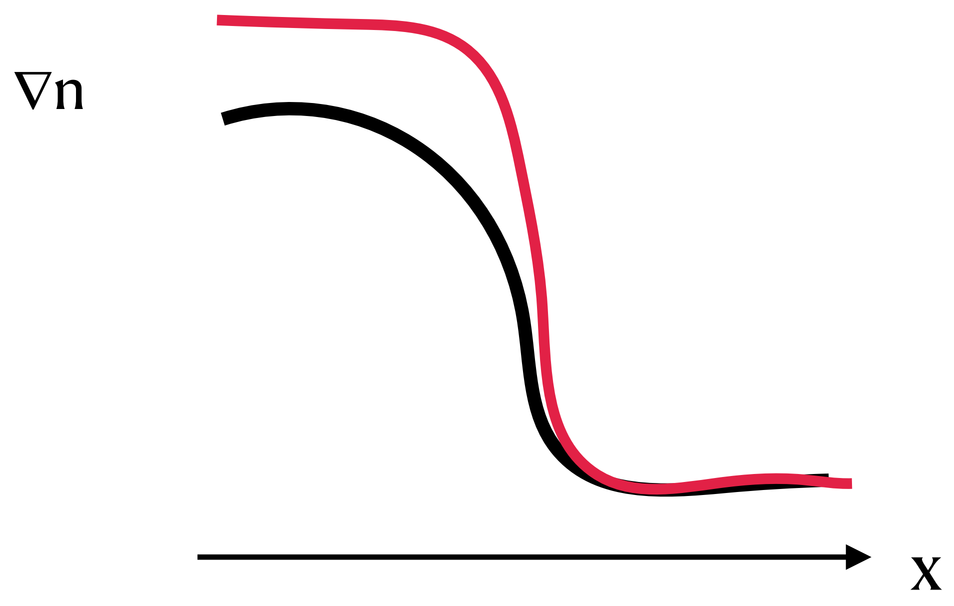
  c. Intrinsic Rotation in presence of stochastic fields.

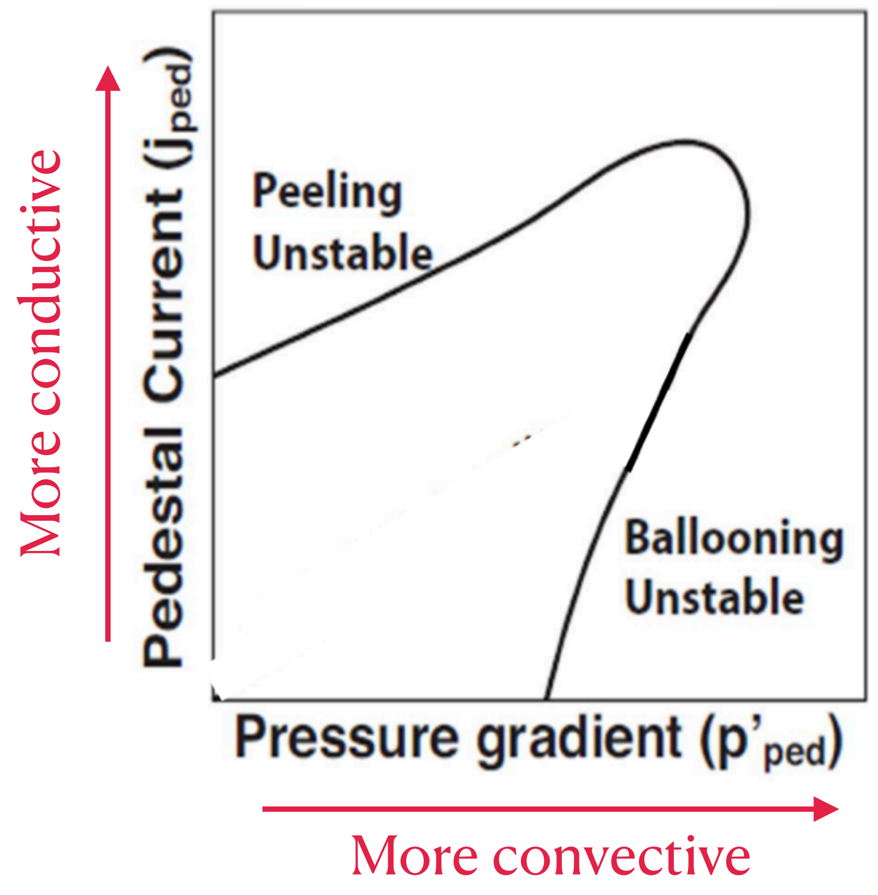
  d. Mixing length in presence of stochastic fields.

- Conclusions

Why we study stochastic fields in fusion device?

Shear Flow
↓
Quench Turbulence
↓
Pedestal Formation
↓
Edge-Localized Mode (ELM)





Edge-Localized Mode (ELM)
↓
Edge gradient ↑
↓
Peeling-ballooning mode
↓
ELM Burst

Suppress (by inducing magnetic perturbation)

Boundary Control: Resonant Magnetic Perturbation (RMP)

- ELMs are quasi-periodic relaxation events occurring at edge pedestal in H-mode plasma.
- ELMs can damage wall components of a fusion device.

Stochastic field effect is important for boundary control

Shear Flow

Edge-Localized Mode (ELM)

ELM Burst

Boundary Control: Resonant Magnetic Perturbation (RMP)

Suppress (by inducing magnetic perturbation)

Trade off: RMPs controls gradients and mitigates ELM, but raise the **power threshold**.

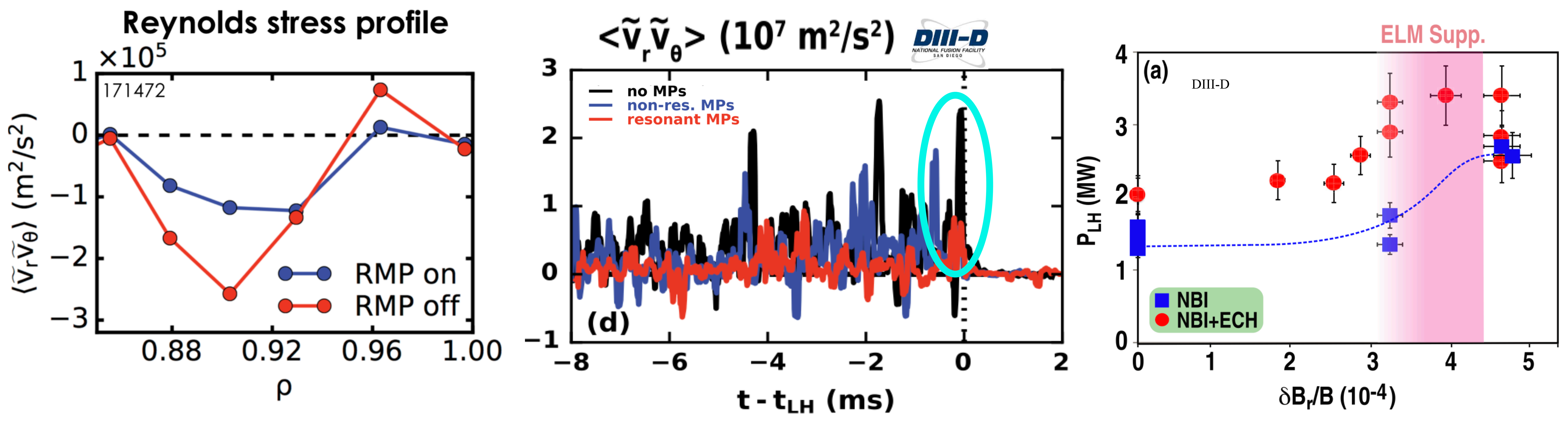
Key Questions:

How RMPs influence the Reynolds stress and hence suppress the zonal flow?

How stochastic fields increase the power threshold of L-H transition?

We examine the physics of stochastic fields interaction with zonal flow near the edge.

Experimental Results with RMP for L–H Transition — fluctuations



(D. Kriete et al, PoP **27** 062507 (2020))

(D. Kriete et al, PoP **27** 062507 (2020))

(L. Schmitz et al, NF **59** 126010 (2019))

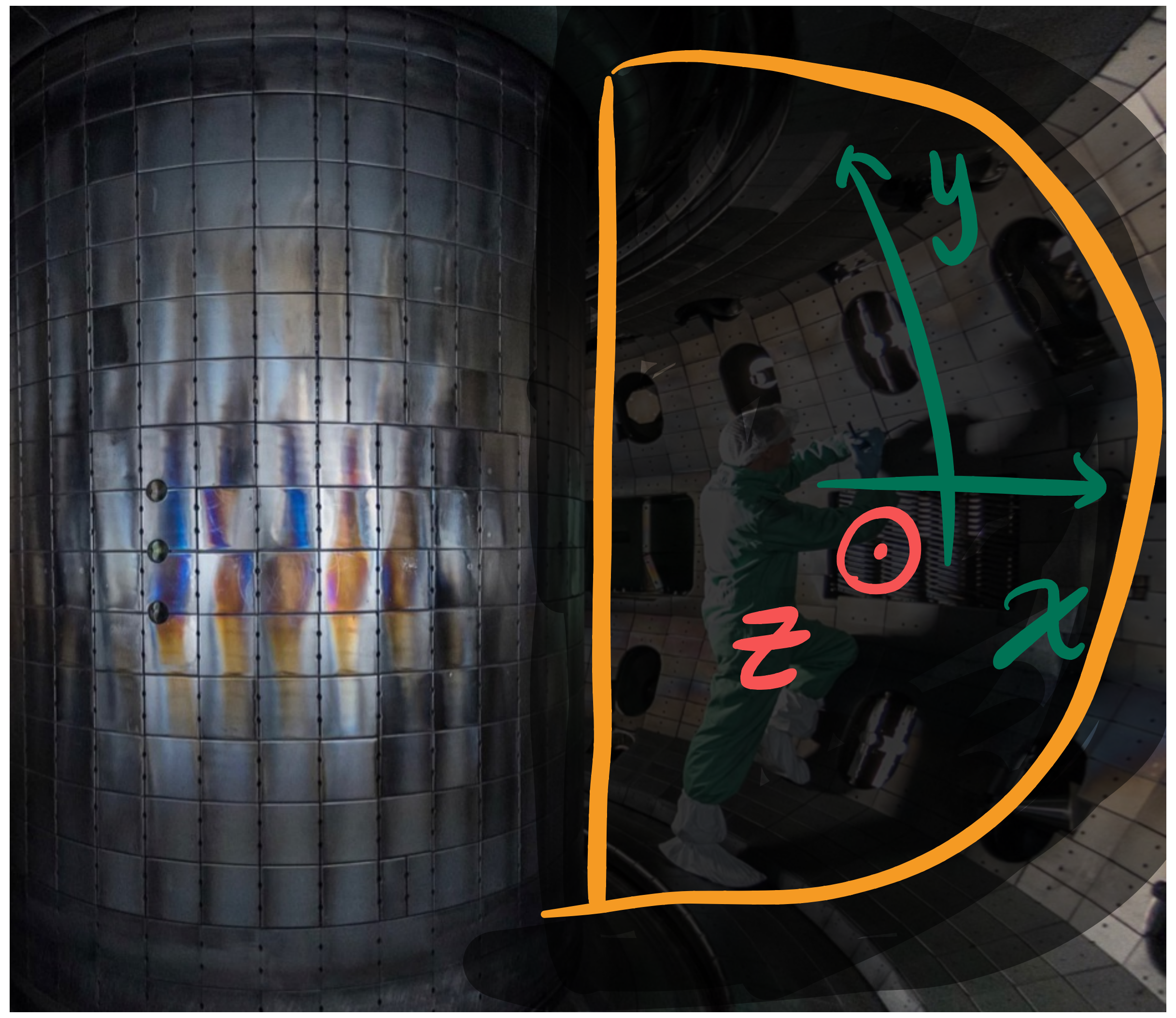
DIII-D Experimental results: RMPs lower the Reynolds stress and increase the power threshold of L-H transition.

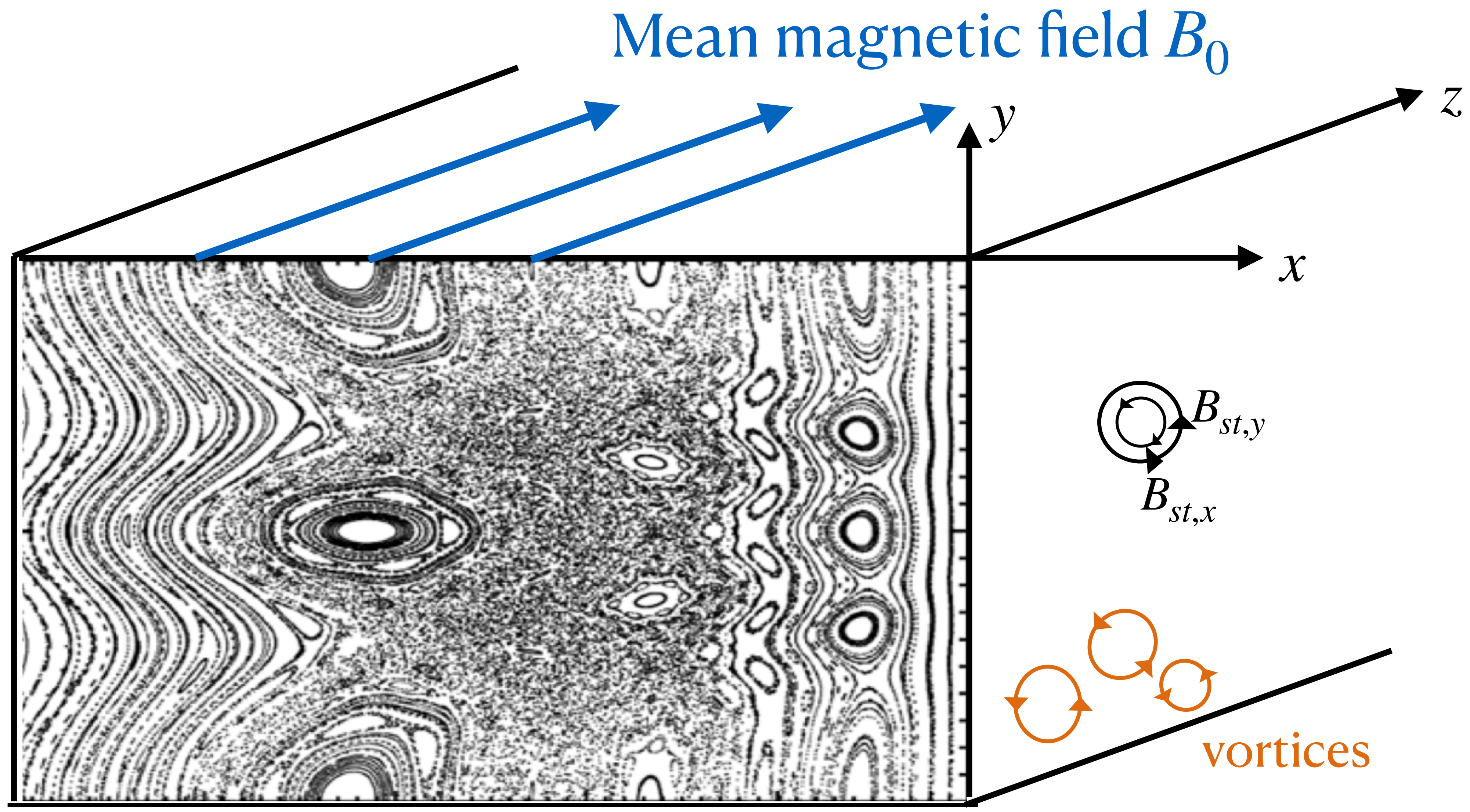
Model

1. Cartesian coordinate: strong mean field B_0 is in z direction (3D).
2. Rechester & Rosenbluth (1978): waves, instabilities, and transport are studied in the presence of **external excited, static, stochastic fields.**
3. $\underline{k} \cdot \underline{B} = 0$ (or $k_{\|} = 0$) **resonant at rational surface in third direction —**

   $\omega \to \omega \pm v_A k_z$, **and** Kubo number: $Ku_{mag} = \frac{l_{ac} |\widetilde{\mathbf{B}}|}{\Delta_\perp B_0}$).
4. Four-field equations —
   - (a) Potential vorticity equation—vorticity $-\nabla^2 \psi \equiv \zeta$
   - (b) Induction equation — $\mathbf{A}, \mathbf{J}$
   - (c) Pressure equation — $\mathbf{p}$
   - (d) Parallel flow equation — $\mathbf{u_z}$

Well beyond HM model





Magnetic islands overlapping forms stochastic

We use mean field approximation:

$$\zeta = \langle \zeta \rangle + \tilde{\zeta},$$

Perturbations produced by turbulences

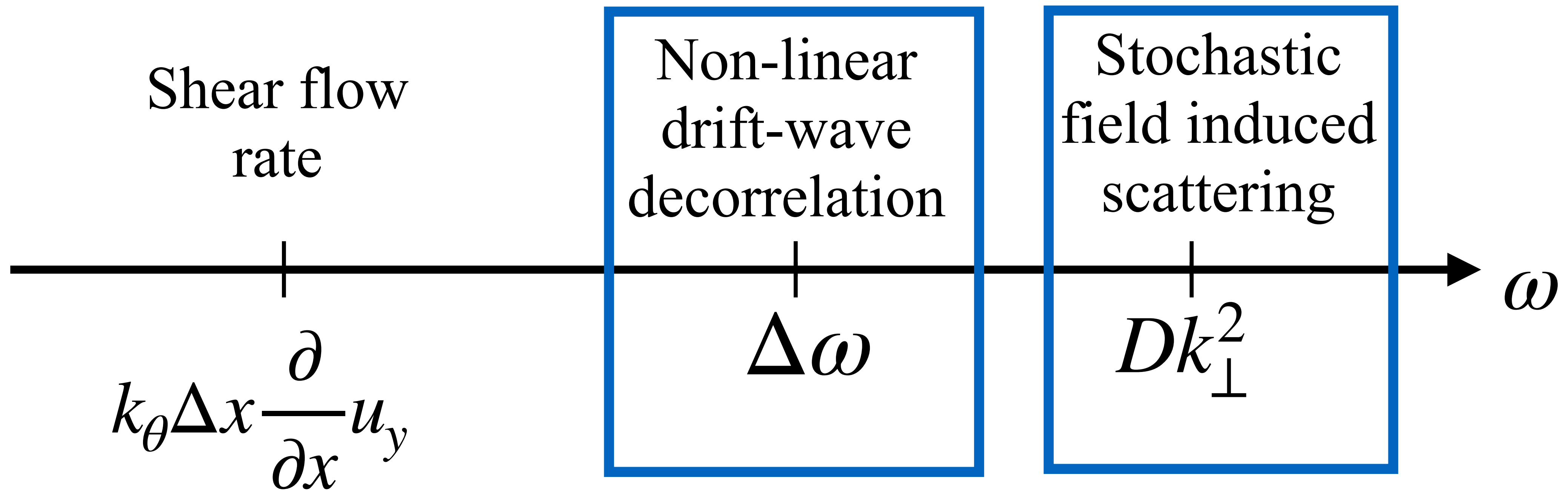
$$where \ \langle \ \rangle = \frac{1}{L} \int dx \frac{1}{T} \int dt$$

ensemble average over the zonal scales

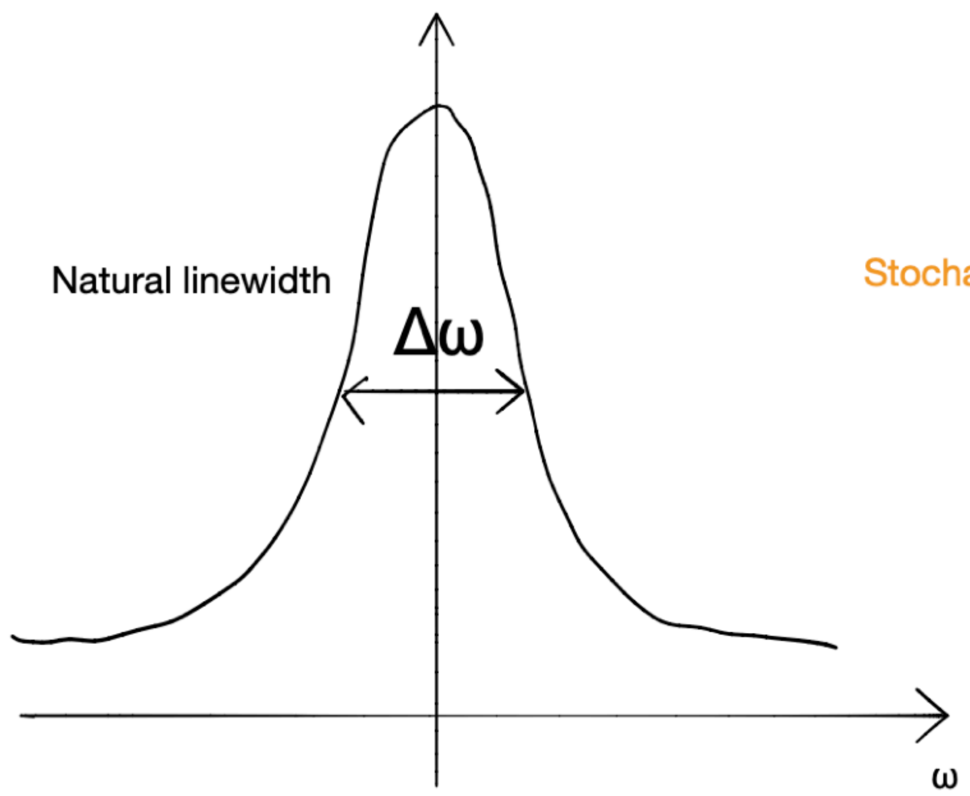
We define rms of normalized stochastic field $b \equiv \sqrt{\overline{(B_{st}/B_0)^2}}$

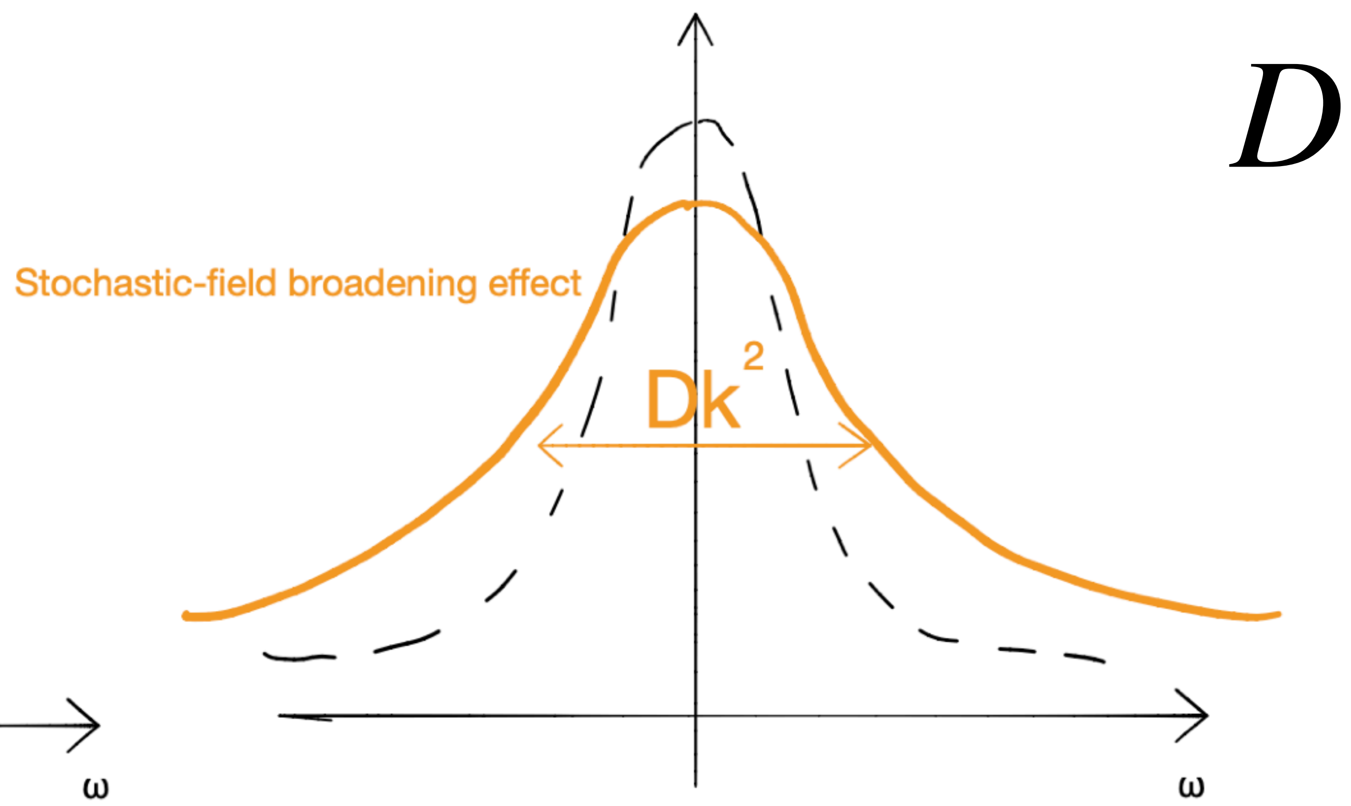
When does stochastic field effect becomes significant?

We consider timescales:



Stochastic field decoherence beats the self-decoherence.





$$D \equiv v_A D_M = v_A \sum_k \pi\delta(k_z) b_k^2 \propto B_{st}^2$$

(Independent of B_0)

Magnetic diffusivity

Auto-correlation length l_{ac}

Alfvén wave propagate along stochastic fields
→ characteristic velocity emerges from the calculation of $\underline{\nabla} \cdot \underline{J} = 0$

Dimensionless Parameters

Two dimensionless Parameters:

$$\begin{cases} l_{ac} \simeq Rq \\ \epsilon \equiv L_n/R \sim 10^{-2} \\ \beta \simeq 10^{-2\sim-3} \\ \rho_* \equiv \frac{\rho_s}{L_n} \simeq 10^{-2\sim-3} \end{cases}$$

1. $\Delta\omega < Dk_\perp^2$

$$b^2 \equiv (\frac{\delta B_r}{B_0})^2 > \sqrt{\beta}\rho_*^2 \frac{\epsilon}{q} \sim 10^{-8}$$

Criterion for stochastic fields effect important to L-H transition.

2. $$\alpha \equiv \frac{b^2}{\rho_*^2\sqrt{\beta}}\frac{q}{\epsilon} > 1$$

α quantifies the strength of stochastic dephasing.

How `stochastic' is magnetic field?

Alfvénic Dispersion		Stochastic broadening
$v_A/L_\parallel$	v.s	$Dk_\perp^2$

(excited by drift-Alfvénic coupling)

Ku_{mag} **(Magnetic Kubo number)**

$$\equiv \frac{\text{stochastic field scattering length}}{\text{perpendicular magnetic fluctuation size}} \lesssim 1,$$

(for a b^2 given)

Decoherence of eddy tilting feedback

Snell's law:

$$\frac{d}{dt}k_x = -\frac{\partial(\omega_0 + u_y k_y)}{\partial x} = -k_y \frac{\partial u_y}{\partial x}$$

Gives an non-zero $\langle k_x k_y \rangle$

$\rightarrow \langle \tilde{u}_x \tilde{u}_y \rangle \propto \langle k_x k_y \rangle$ shear flow

Self-feedback loop:

The $E \times B$ shear generates the $\langle k_x k_y \rangle$ correlation and hence support the non-zero Reynolds stress.

$$\langle \tilde{u}_x \tilde{u}_y \rangle \simeq \sum_k \frac{|\tilde{\phi}_k|^2}{B_0^2}(k_y^2 \frac{\partial u_y}{\partial x} \tau_c)$$

The Reynold stress modifies the shear via momentum transport.

Shear flow reinforce the self-tilting.

Dispersion relation with drift-Alfvén coupling

$$\omega^2 - \omega_D \omega - k_\parallel^2 v_A^2 = 0$$

+

Stochastic Fields Effect

$$k_\parallel = k_\parallel^{(0)} + \underline{b} \cdot \underline{k}_\perp$$

$$\omega = \omega_D + \delta\omega$$

→

$$(\omega_D + \delta\omega)^2 - \omega_D(\omega_D + \delta\omega) - (k_\parallel + \underline{b} \cdot \underline{k}_\perp)^2 v_A^2 = 0$$

$$\delta\omega \simeq \frac{v_A^2}{\omega_D}(2k_\parallel \underline{b} \cdot \underline{k}_\perp + (\underline{b} \cdot \underline{k}_\perp)^2)$$

$$\omega_D \text{ (drift wave turbulence frequency)} \equiv \frac{k_y \rho_s C_s}{L_n}$$

Decoherence of eddy tilting feedback

Expectation frequency:

$$\left\langle \delta\omega \simeq \frac{v_A^2}{\omega_0}\left(2k_{\|}\underline{b}\cdot\underline{k}_{\perp} + (\underline{b}\cdot\underline{k}_{\perp})^2\right)\right\rangle \;\rightarrow\; \langle\delta\omega\rangle \simeq \frac{v_A^2}{\omega_0}\langle(\underline{b}\cdot\underline{k}_{\perp})^2\rangle = \frac{1}{2}\frac{v_A^2}{\omega_0}b^2k_{\perp}^2$$

$$\omega = \omega_D + \delta\omega$$

$$\langle\omega\rangle \simeq \omega_D + \frac{1}{2}\frac{v_A^2}{\omega_D}b^2k_{\perp}^2$$

Snell's law:

$$\frac{d}{dt}k_x = -\frac{\partial\omega_k}{\partial x}$$

$$= -k_y\frac{\partial u_y}{\partial x} - \frac{1}{2}\frac{v_A^2k_{\perp}^2}{\omega_D}\frac{\partial b^2}{\partial x}$$

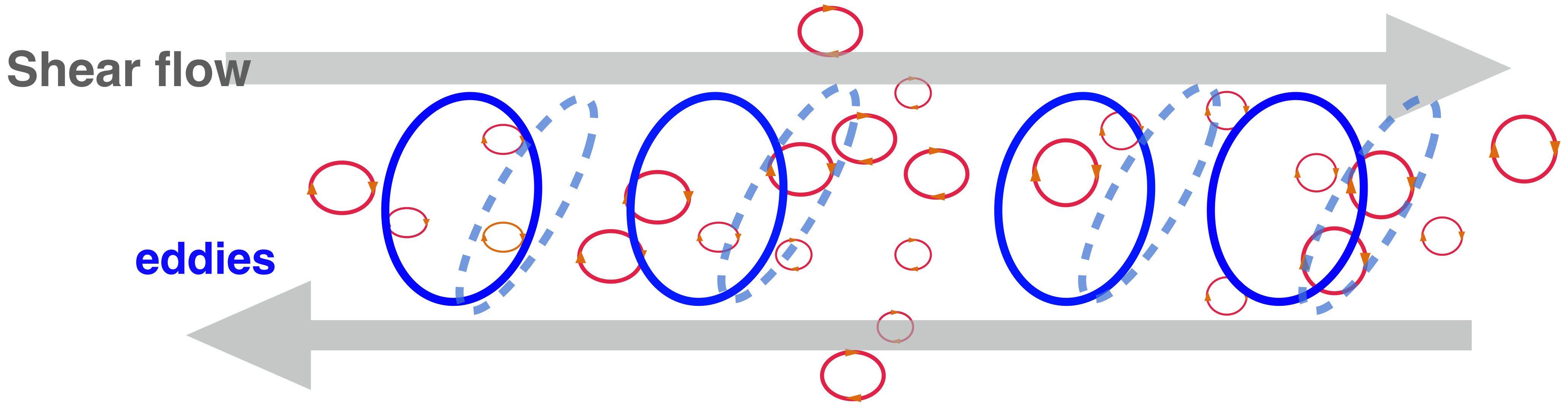
Ensemble average frequency shift

Self-feedback loop is broken by b^2:

$$\langle\tilde{u}_x\tilde{u}_y\rangle \simeq \sum_k \frac{|\widetilde{\phi}_k|^2}{B_0^2}\left(k_y^2\frac{\partial u_y}{\partial x}\tau_c + \frac{1}{2}k_y\frac{v_A^2k_{\perp}^2}{\omega_D}\frac{\partial b^2}{\partial x}\tau_c\right)$$

Stochastic dephasing

Stochastic fields (random ensemble of elastic loops) act as elastic loops and resist the tilting of eddies.
→change the cross-phase btw $\tilde{u}_x$ and $\tilde{u}_y$.



Stochastic fields interfere with shear-tilting feedback loop.

Results—Suppression of PV diffusivity

The ensemble average Reynolds force $\frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle$:

$$\text{PV flux} = \langle \tilde{u}_x \tilde{\zeta} \rangle = \frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle = -D_{PV}\frac{\partial}{\partial x}\langle \zeta \rangle + F_{res}\kappa \frac{\partial}{\partial x}\langle p \rangle$$

PV diffusivity — Residual Stress — Curvature

Suppressed by stochastic fields

Taylor Identity: $\langle \tilde{u}_x \tilde{\zeta} \rangle = \frac{\partial}{\partial x}\langle \tilde{u}_x \tilde{u}_y \rangle$

Mean vorticity $\langle \zeta \rangle = \frac{\partial v_{E\times B}}{\partial x}$ ($E \times B$ shear)

$$D_{PV} = \sum_{k\omega} |\tilde{u}_{x,k\omega}|^2 \frac{v_A b^2 l_{ac} k^2}{\bar{\omega}^2 + \left(v_A b^2 l_{ac} k^2\right)^2}$$

v_A: Low $\beta \equiv P_{thermal}/P_{mag}$, so it is v_A instead of sound speed C_s (small).

$$F_{res} \simeq \sum_{k\omega} \frac{-2k_y}{\bar{\omega}\rho} D_{PV,k\omega}$$

$$\bar{\omega} \equiv \omega - \langle u_y \rangle k_y$$

This **stochastic dephasing** is insensitive to turbulent modes (e.g. ITG, TEM,...etc.).

(Chen et al.,PoP **28**, 042301 (2021))

PV transport will be suppressed by stochastic fields via decoherence.

Macroscopic Impact

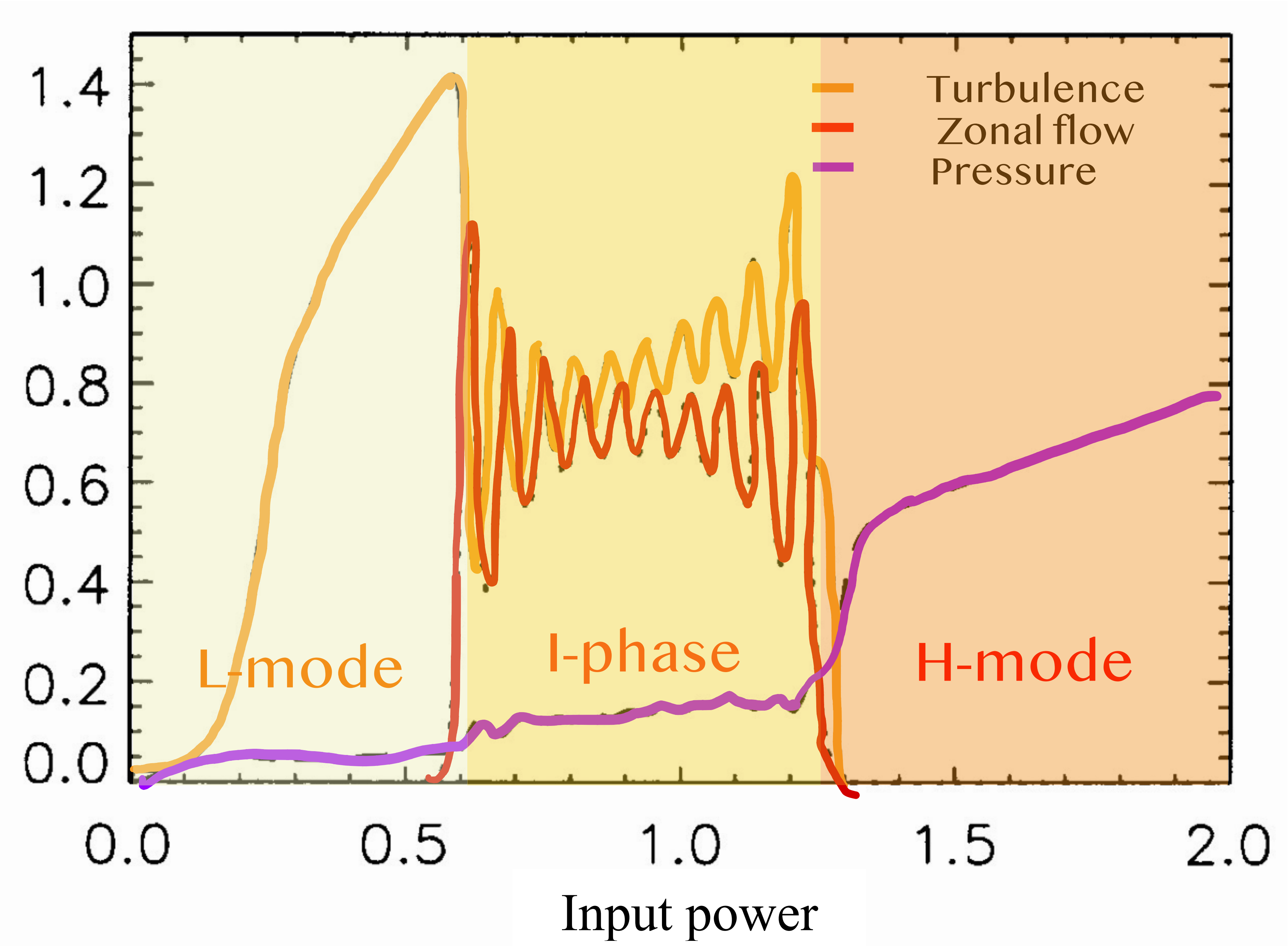
Results — Increment of P_{LH}

Stochastic field stress dephasing effect requires: $\Delta\omega \leq k_\perp^2 D$ (where $D = D_M v_A$).

This gives **dimensionless parameter** (α): $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} > 1$

α quantifies the strength of stochastic dephasing.

$$\begin{cases} l_{ac} \simeq Rq \\ \epsilon \equiv L_n/R \sim 10^{-2} \\ \beta \equiv \frac{P_{thermal}}{P_{mag}} \simeq 10^{-2\sim-3} \\ \rho_* \equiv \frac{\text{gyro-radius}}{\text{density scale length}} \\ \quad \equiv \frac{\rho_s}{L_n} \simeq 10^{-2\sim-3} \\ q(\text{stafety factor}) \equiv \frac{rB_t}{RB_p} \end{cases}$$



Kim-Diamond Model

(Kim & Diamond, PoP **10**, 1698 (2003))

This reduce model for the L-H transition is useful for testing trends in power threshold increment induced by stochastic fields.

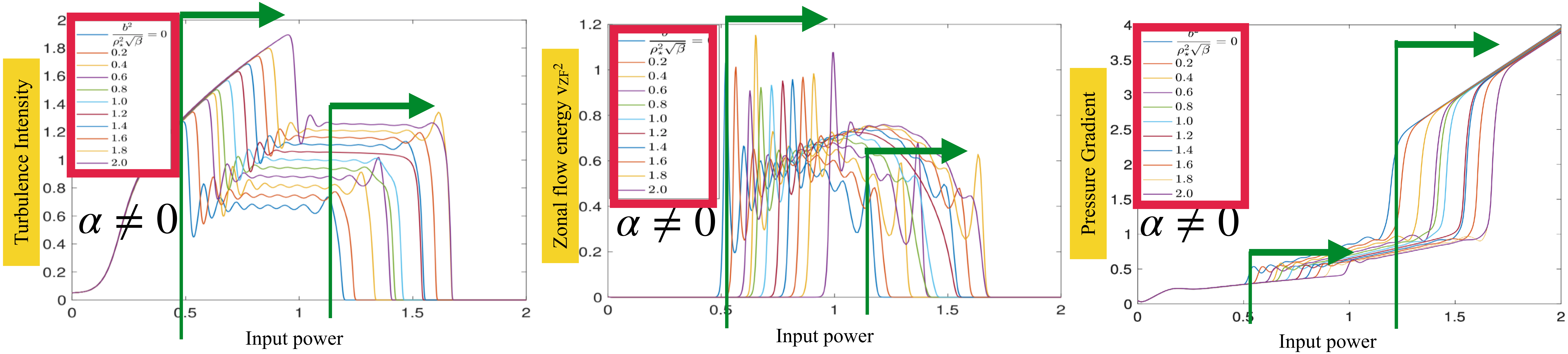
Predator: zonal flow
prey: turbulence

We expect stochastic fields to raise L-H transition thresholds.

Macroscopic Impact

Results — Increment of P_{LH}

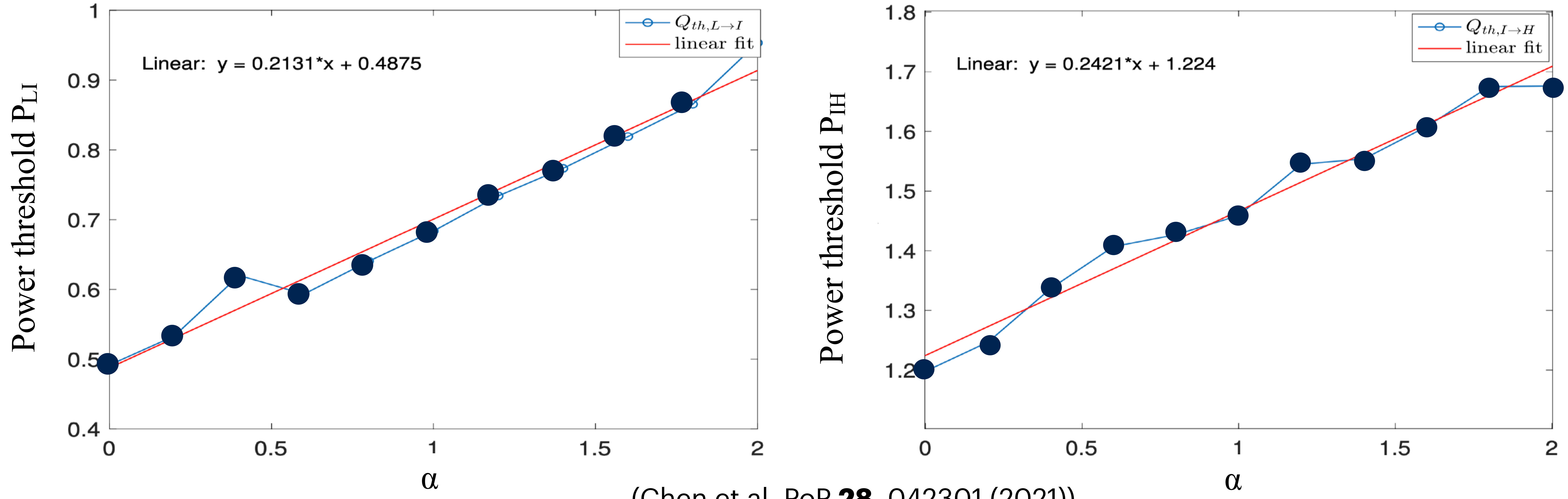
$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} = 0.0, 0.2, 0.4, 0.6, 0.8...., 2.0$$



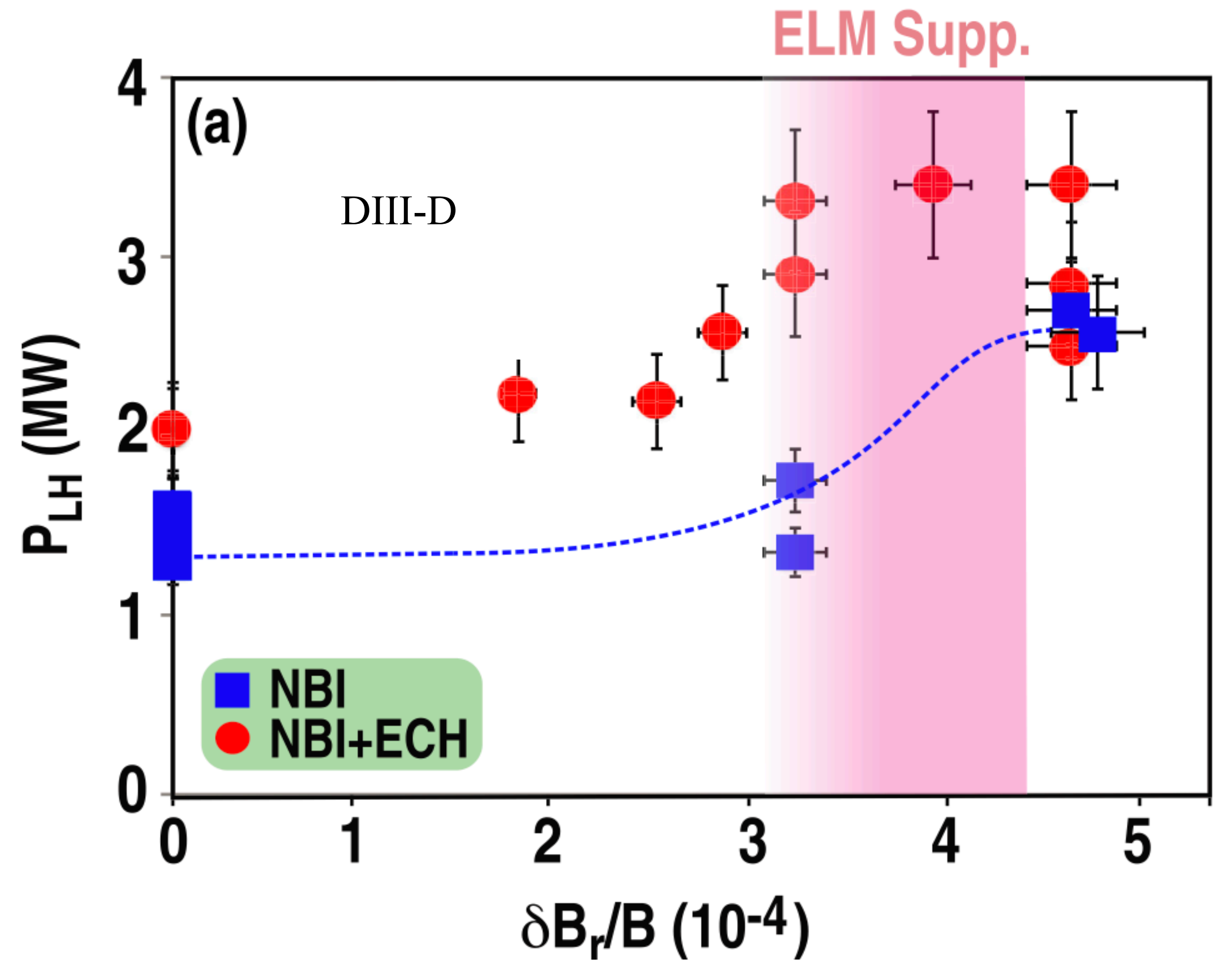
The threshold increase due to stochastic dephasing effect is seen in turbulence intensity, zonal flow, and pressure gradient.

(Chen et al.,PoP **28**, 042301 (2021))

Results — Increment of P_{LH}



(Chen et al.,PoP **28**, 042301 (2021))



(L. Schmitz et al, NF **59** 126010 (2019))

$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$$

$$\alpha \propto \frac{1}{\rho_*^2}$$

ρ_* is small $\rightarrow \alpha \uparrow$ (pessimistic)

The threshold increase, in proportional to α, due to stochastic dephasing effect. This can be seen in turbulence intensity, zonal flow, and pressure gradient.

Intrinsic Rotation and Kinetic Stress

From parallel acceleration:

$$\frac{\partial}{\partial t}u_z + (\mathbf{u}\cdot\nabla)u_z = -\frac{1}{\rho}\frac{\partial}{\partial z}p$$

+

Stochastic Fields Effect

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z}^{(0)} + \underline{b}\cdot\underline{\nabla}_\perp$$

→

$$\frac{\partial}{\partial t}\langle u_z\rangle + \frac{\partial}{\partial x}\langle \tilde{u}_x\tilde{u}_z\rangle = -\frac{1}{\rho}\frac{\partial}{\partial x}\langle b\tilde{p}\rangle$$

Toroidal Reynolds Stress

Kinetic Stress

Pat Diamond's talk this afternoon 12:30 pm

$$\langle \tilde{u}_x\tilde{u}_z\rangle = -\nu_{turb}\frac{\partial}{\partial x}\langle u_z\rangle + F_{z,res}\frac{\partial}{\partial x}\langle p\rangle$$

Turbulent viscosity

Toroidal Residual Stress

↓ Influence intrinsic rotation

$$\nu_{turb} = \sum_{k\omega} |\tilde{u}_{x,k\omega}|^2 \frac{2C_s b^2 l_{ac} k^2}{\omega_{sh}^2 + (2C_s b^2 l_{ac} k^2)^2}$$

- The sound speed is the relevant speed (acoustic dynamics). Stochastic fields effect is weak ($C_s D_M < v_A D_M$).
- $F_{z,res} \sim \sum_{k\omega} \frac{-k_z}{\omega_{sh}\rho}\nu_{turb,k\omega}\cdot$

$F_{z,res}$ Requires symmetry breaking $\langle k_z k_y\rangle \neq 0$ → Detail calculation is needed.

(Chen et al.,PoP **28**, 042301 (2021))

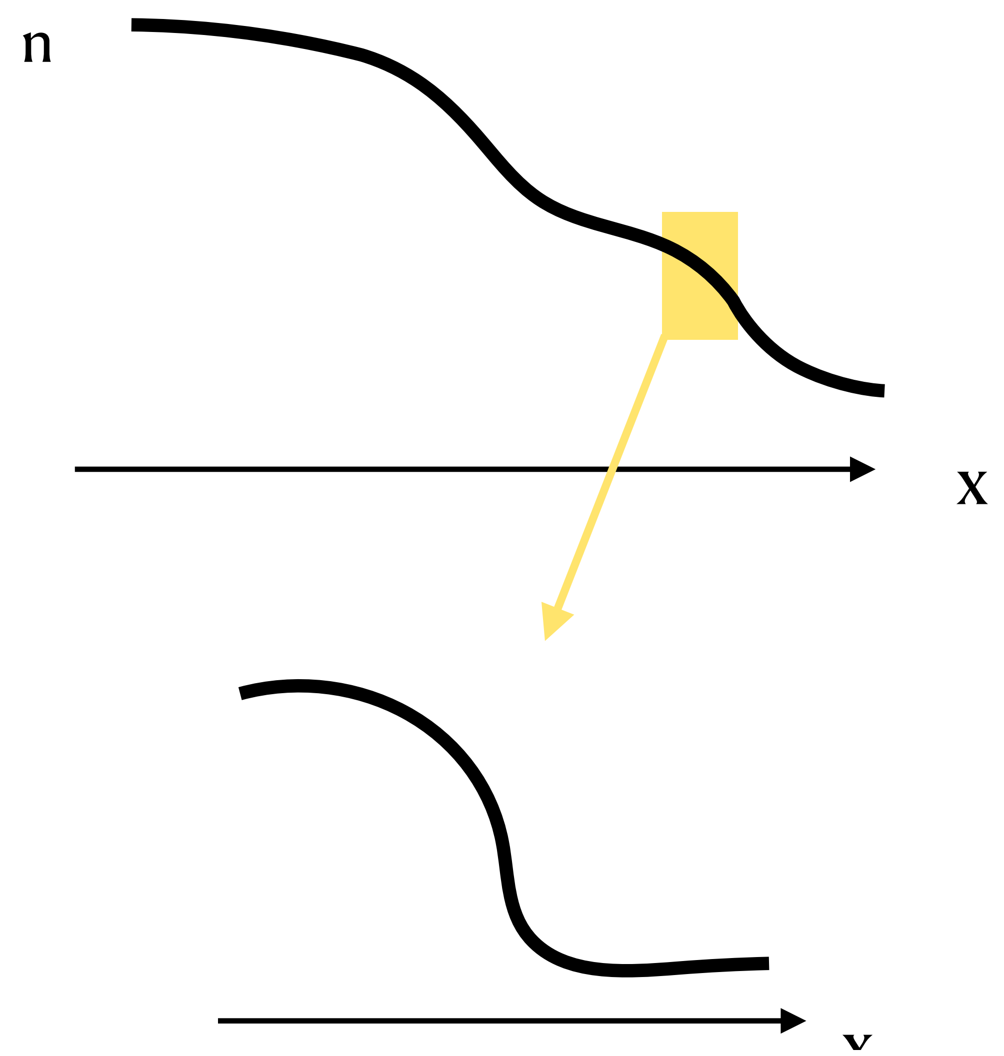
Stochastic fields reduce the toroidal stress and hence slow down the intrinsic rotation.

Fate of Spatial structure of zonal flow?

Shear Flow

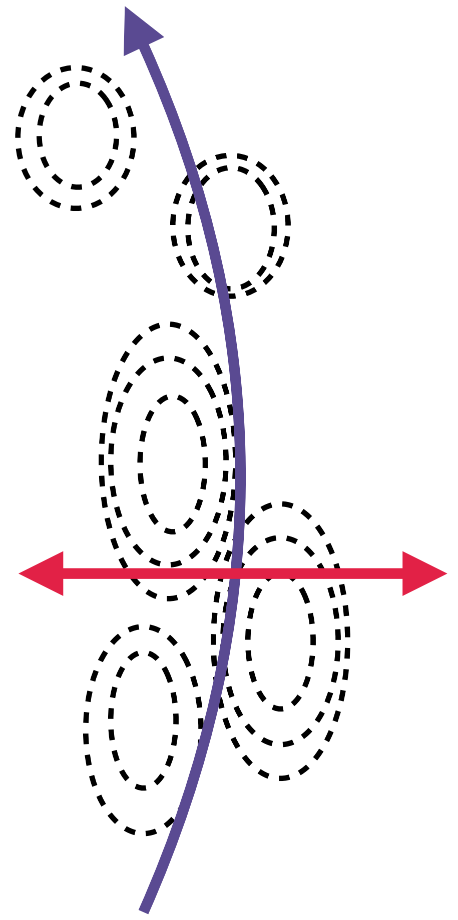
Edge-Localized Mode (ELM)

Edge gradient ↑



Density corrugation

Poloidal zonal



Zonal flow width

Zonal flow width is related to corrugation length.

We are interested in zonal flow width in presence of stochastic fields.

Layering Structure—Mixing Length Model

A mixing length model for layering:

- Reduce evolution equations (based on H-W model).
- Energy and Potential entropy (PE) conserved.

Density: $$\frac{\partial}{\partial t}\langle n\rangle = \frac{\partial}{\partial x}\left(D_n\frac{\partial\langle n\rangle}{\partial x}\right) + D_c\frac{\partial^2}{\partial x^2}\langle n\rangle$$

turb. particle diffusion

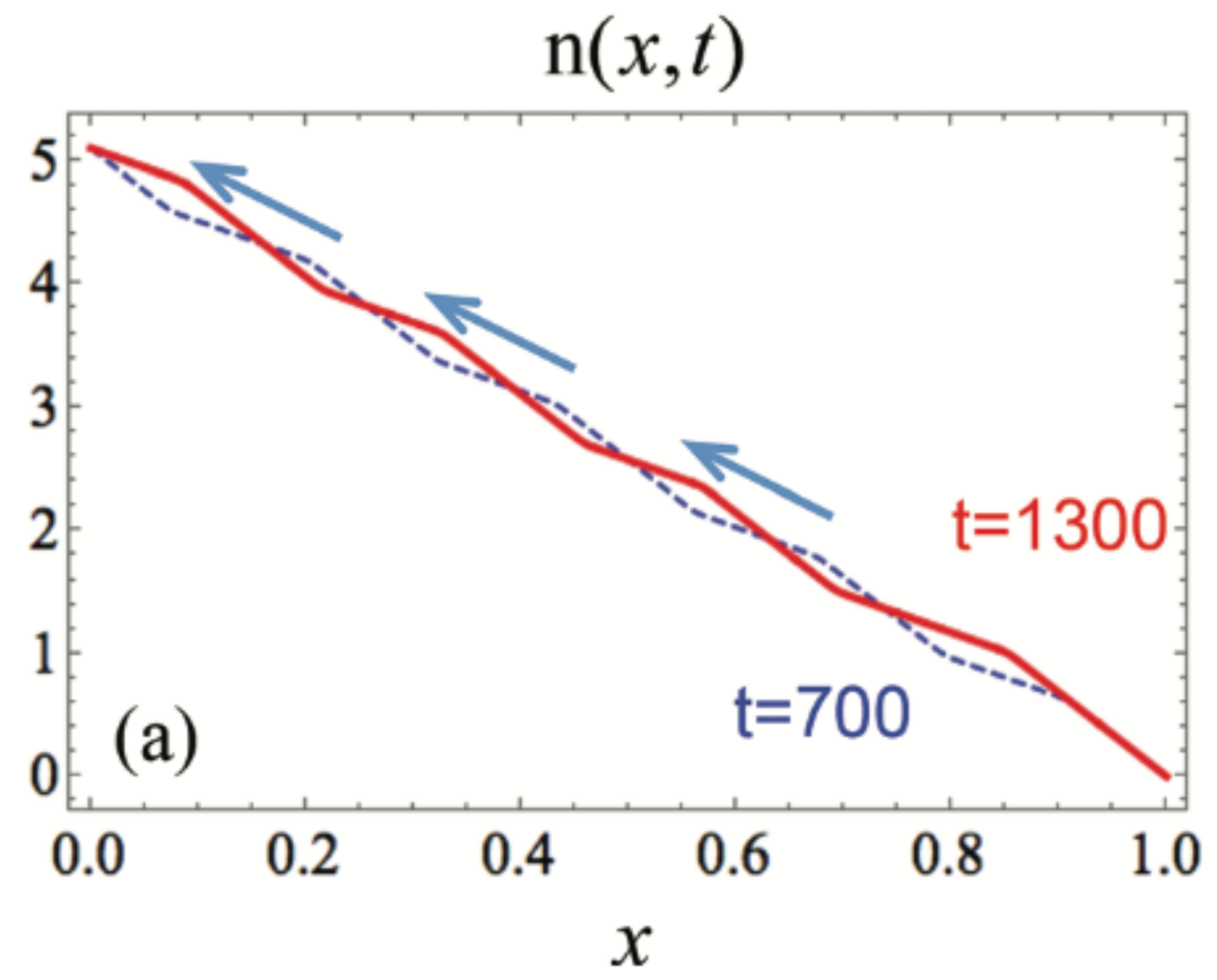
Potential Vorticity: $$\frac{\partial}{\partial t}\langle\zeta\rangle = \frac{\partial}{\partial x}\left((D_n-\chi)\frac{\partial\langle n\rangle}{\partial x}\right) + \chi\frac{\partial^2}{\partial x^2}\langle\zeta\rangle + \mu_c\frac{\partial^2}{\partial x^2}\langle\zeta\rangle$$

residual stress; turb. Viscous diffusion

Turbulent potential Enstrophy: $$\frac{\partial}{\partial t}\epsilon = \frac{\partial}{\partial x}\left(D_\epsilon\frac{\partial\epsilon}{\partial x}\right) + \chi[\frac{\partial(n-\zeta)}{\partial x}]^2 - \epsilon_c^{-1/2}\epsilon^{3/2} + P$$

PE diffusion; mean-turb PE Coupling; PE Dissipation

- n : density
- ζ : potential vorticity
- ϵ : turbulent PE $\quad \epsilon \equiv (\delta n - \delta\zeta)^2/2$
- D_n : turbulent particle diffusivity
- χ : turbulent vorticity
- P : production



Ashourvan & Diamond, PoP **24,** 012305 (2017)

Density corrugation forms staircase-like structure.

Scale Selection

The mixing length (l_{mix}) depends on **two scales:**

- Driving scale: l_0
- Rhines scale: $l_{RH} = \dfrac{\sqrt{\epsilon}}{|\partial_x q|}$

➡ mixing scale: $l_{mix} = \dfrac{l_0}{(1 + l_0^2(\partial_x q)^2/\epsilon)^{\kappa/2}} = \boxed{\dfrac{l_0}{(1 + l_0^2/l_{RH}^2)^{\kappa/2}}}$

$$\begin{cases} \text{Strong mixing } (l_{RH} > l_0): & l_{mix} \simeq l_0 \text{ (Weak mean PV gradient)} \\ \text{Weak mixing } (l_0 > l_{RH}): & l_{mix} \simeq l_0^{1-\kappa} l_{RH}^{\kappa} \text{ (Strong PV gradient)} \end{cases}$$

l_{mix} (hybrid length scale) sets the scale of zonal flow.

What is the effect of stochastic fields on staircases?

Very Preliminary

Main effect of diffusivity D_n and χ

For α_{DW} (a measurement of the resistive diffusion rate in the parallel direction) > 1 in H-W regime:

Density diffusivity:

$$D_n \simeq \frac{l_{mix}^2 \epsilon}{\alpha_{DW}}$$

Resistive diffusion rate:

$$\alpha_{DW} = \frac{k_{\parallel}^2 v_{the}^2}{\nu}$$

+

Stochastic Fields Effect

$$k_{\parallel} = \underline{k} \cdot \underline{\hat{b}_0} \simeq \frac{1}{Rq} + \underline{b_{\perp}} \cdot \underline{k_{\perp}} \simeq \frac{1}{Rq} + \frac{b_{\perp}}{l_{mix}}$$

→

$$D_n \simeq \frac{l_{mix}^2 \epsilon \nu / v_{the}^2}{(\frac{1}{Rq})^2 + (\frac{b}{l_{mix}})^2}$$

Same for χ (or D_{PV} in this case).

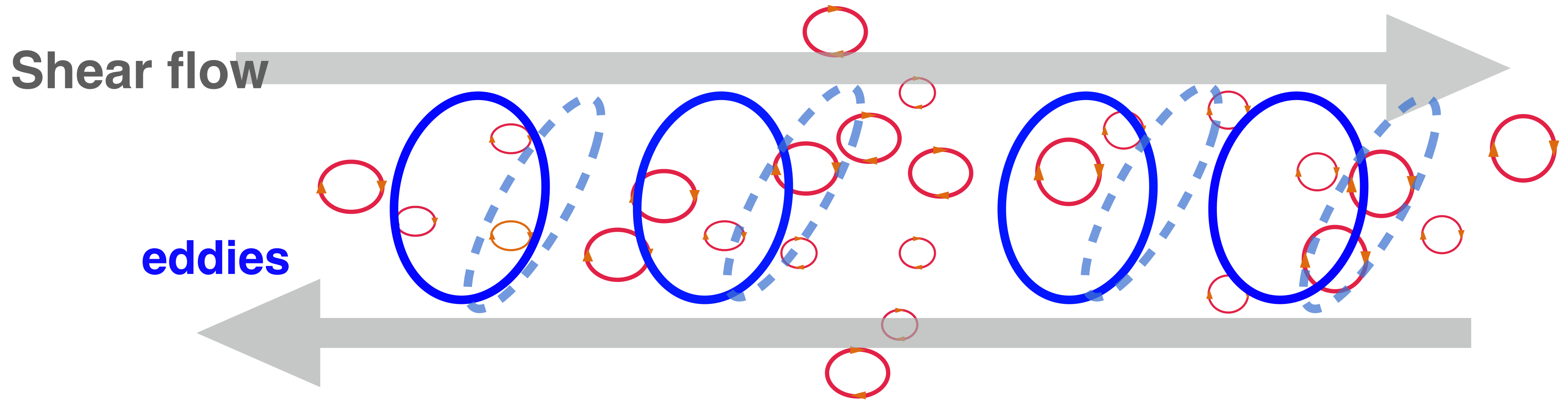
Competition btw $\frac{1}{Rq}$ v.s. $\frac{b_{\perp}}{l_{mix}}$ gives $Ku_{mag} = bRq/l_{mix}$ → $Ku_{mag} = Ku_{mag}(l_{mix})$

The mixing length is not likely affected by b^2.

Scalar selection and staircase corrugation change requires $Ku_{mag} \geq 1$.

Conclusions

- **Dephasing effect** caused by stochastic fields quenches poloidal Reynolds stress (e.g. $\Delta\omega < Dk_\perp^2$). Here, $D = v_A D_M$.



- b^2 shift L-H threshold to higher power, in proportional to $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$.
- Stochastic fields have weaker effect on reducing toroidal Reynolds stress, since $C_s D_M < v_A D_M$. Need to revisit symmetry breaking $\langle k_y k_z \rangle \neq 0$ calculation (for $F_{z,res}$) in stochastic magnetic field.
- The mixing length is not likely affected by b^2. To change mixing length, we need $Ku_{mag} \geq 1$.