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Physics of Micro-Turbulence in a Stochastic Magnetic Field

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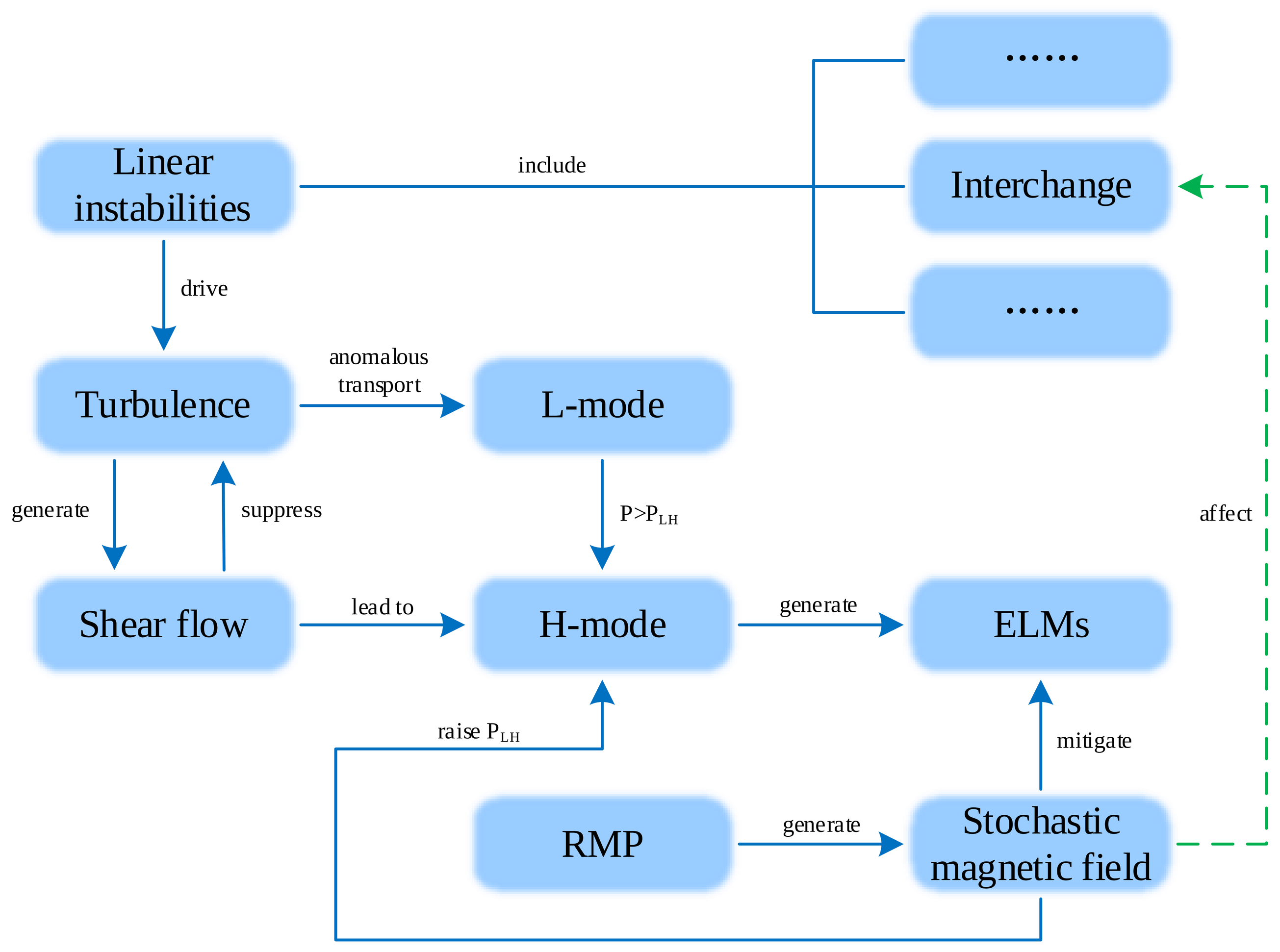
Background

Syntheses of **good confinement** & **optimal power handing**

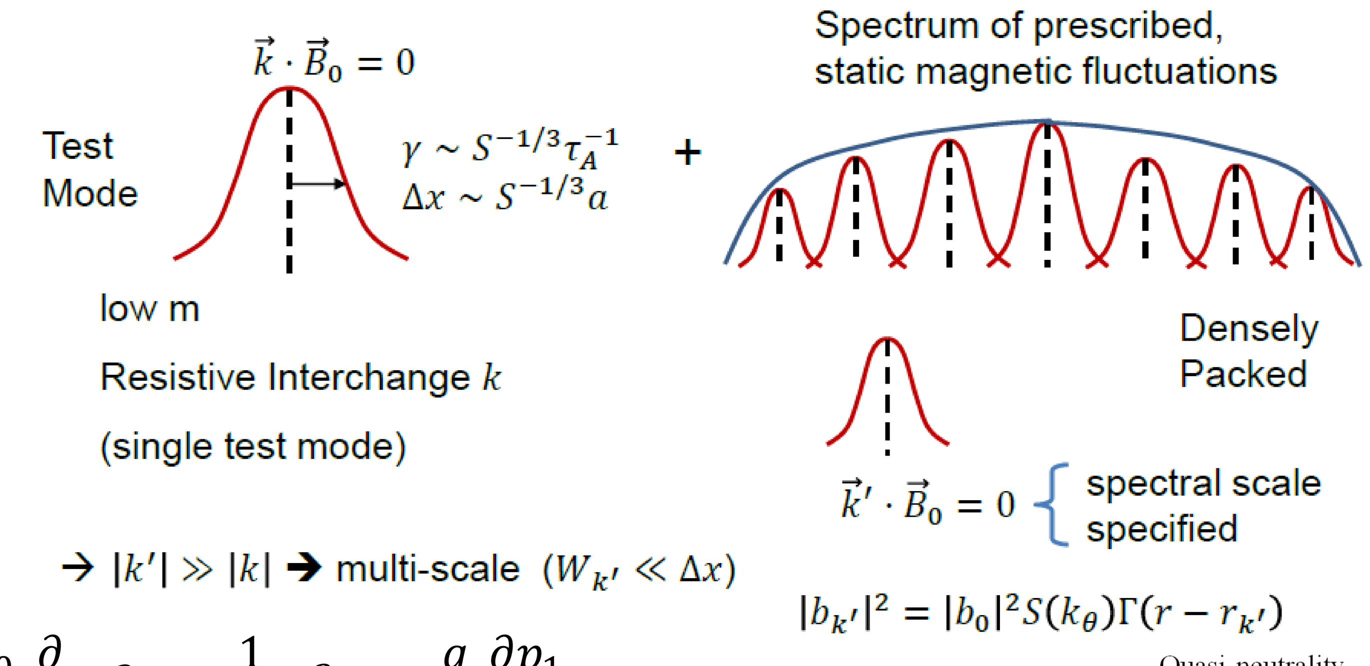


Key question: how does a stochastic field modify the instability process?

Resistive interchange mode: a good focus

Model

A **low-*k*** resistive interchange mode under a high-***k*** static stochastic magnetic field

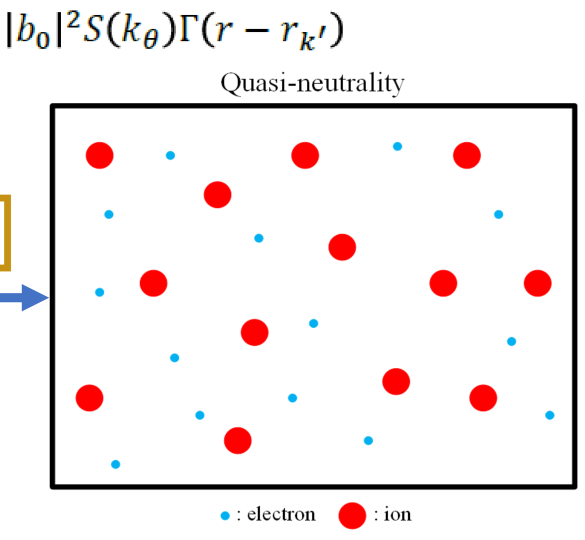


→ $|k'|\gg|k|$ → multi-scale $(W_{k'}\ll\Delta x)$

$|b_{k'}|^2=|b_0|^2S(k_\theta)\Gamma(r-r_{k'})$

$$\begin{cases} \underbrace{-\dfrac{\rho_0}{B_0^2}\dfrac{\partial}{\partial t}\nabla_\perp^2\phi}_{J_{pol}} \underbrace{-\dfrac{1}{\eta_\parallel}\nabla_\parallel^2\phi}_{J_\parallel} \underbrace{-\dfrac{g}{B_0}\dfrac{\partial p_1}{\partial y}}_{J_{PS}}=0 \\ \dfrac{\partial p_1}{\partial t}-\dfrac{\nabla\phi\times\hat{\mathbf{z}}}{B_0}\cdot\nabla p_0=0 \end{cases}$$

$\boxed{\nabla\cdot\boldsymbol{J}=0}$



Clarification: $\nabla_\parallel=\nabla_\parallel^{(0)}+\tilde{\boldsymbol{b}}\cdot\nabla_\perp$, $\tilde{\boldsymbol{b}}=\dfrac{\tilde{\boldsymbol{B}}_\perp}{B_0}$

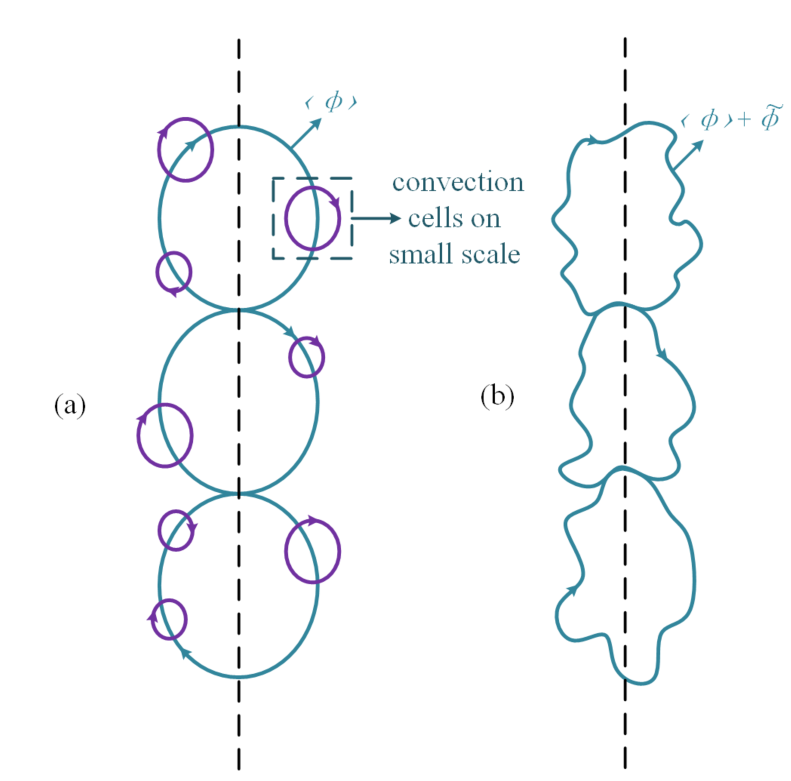
Development

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi=-\frac{S}{\tau_A}\left(\nabla_\parallel^{(0)}+\tilde{\boldsymbol{b}}\cdot\nabla_\perp\right)^2\phi-\frac{gB_0}{\rho_0}\frac{\partial p_1}{\partial y}\qquad \frac{\partial}{\partial t}p_1-\frac{\nabla\phi\times\hat{\mathbf{z}}}{B_0}\cdot\nabla p_0=0$$

In the beginning, we thought the main physics of this problem came from "random-bending", i.e.,$\left(\nabla_\parallel^{(0)}+\boldsymbol{b}\cdot\nabla_\perp\right)^2$. But soon we found that things are more than that. To keep the quasi-neutrality of plasma, a potential fluctuation $\tilde{\phi}$, which indicates the existence of small-scale convective cells, is generated by the stochastic magnetic field and low-***k*** perturbation. This small-scale convective cells imply the existence of a turbulent viscosity ν and a turbulent diffusivity χ. We got this insight from Kadomtsev and Pogutse's study of electron conductivity of the plasma in a stochastic magnetic field. Then the equations become:

$$\begin{cases} \left(\dfrac{\partial}{\partial t}-\nu\nabla_\perp^2\right)\nabla_\perp^2(\bar{\phi}+\tilde{\phi})=-\dfrac{S}{\tau_A}\left(\nabla_\parallel^{(0)}+\tilde{\boldsymbol{b}}\cdot\nabla_\perp\right)^2(\bar{\phi}+\tilde{\phi})-\dfrac{gB_0}{\rho_0}\dfrac{\partial(\bar{p}_1+\tilde{p}_1)}{\partial y} \\ \left(\dfrac{\partial}{\partial t}-\chi\nabla_\perp^2\right)(\bar{p}_1+\tilde{p}_1)-\dfrac{\nabla(\bar{\phi}+\tilde{\phi})\times\hat{\mathbf{z}}}{B_0}\cdot\nabla p_0=0 \end{cases}$$

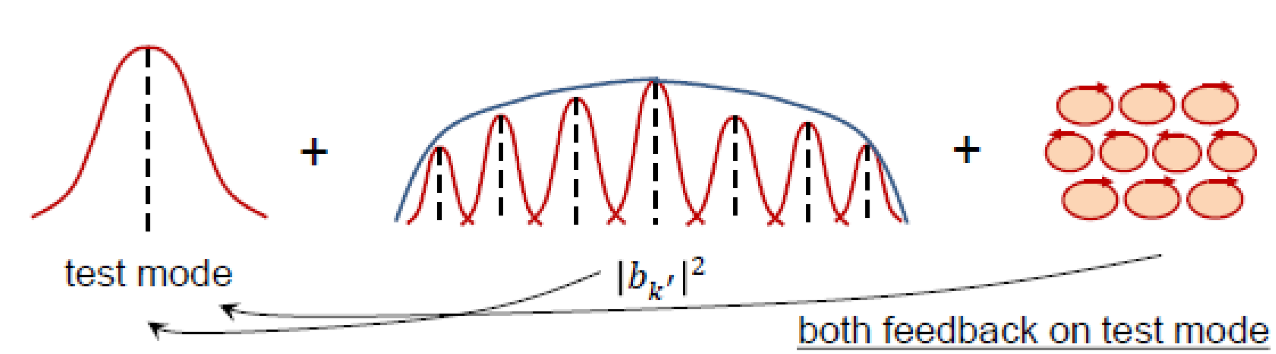
$\phi=\bar{\phi}+\tilde{\phi}$	ν: viscosity
$p_1=\bar{p}_1+\tilde{p}_1$	χ: diffusivity



(a) convection cells on small scale
(b) total electrostatic potential in a stochastic magnetic field

By using the method of averaging, we can separate the evolution of the system into those of macro and micro scales, for which slow-interchange and fast-interchange approximations apply, respectively. The final equations are:

$$\begin{cases} \left(\dfrac{\partial}{\partial t}-\nu\nabla_\perp^2\right)\nabla_\perp^2\bar{\phi}=-\dfrac{S}{\tau_A}\left[\nabla_\parallel^{(0)^2}\bar{\phi}+\left(\nabla_\perp\cdot\langle\tilde{b}\tilde{b}\rangle\right)\cdot\nabla_\perp\bar{\phi}+\left\langle\nabla_\parallel^{(0)}\nabla_\perp\cdot\left(\tilde{\boldsymbol{b}}\tilde{\phi}\right)\right\rangle+\left\langle\nabla_\perp\cdot\left(\tilde{\boldsymbol{b}}\nabla_\parallel^{(0)}\tilde{\phi}\right)\right\rangle\right]-\dfrac{gB_0}{\rho_0}\dfrac{\partial\bar{p}_1}{\partial y} \\ \left(\dfrac{\partial}{\partial t}-\nu\nabla_\perp^2\right)\nabla_\perp^2\tilde{\phi}+\dfrac{S}{\tau_A}\nabla_\parallel^{(0)^2}\tilde{\phi}+\dfrac{gB_0}{\rho_0}\dfrac{\partial\tilde{p}_1}{\partial y}=-\dfrac{S}{\tau_A}\left[\left(\tilde{\boldsymbol{b}}\cdot\nabla_\perp\right)\nabla_\parallel^{(0)}\bar{\phi}+\nabla_\parallel^{(0)}\left(\tilde{\boldsymbol{b}}\cdot\nabla_\perp\right)\bar{\phi}\right] \\ \left(\dfrac{\partial}{\partial t}-\chi\nabla_\perp^2\right)\bar{p}_1-\dfrac{\nabla\bar{\phi}\times\hat{\mathbf{z}}}{B_0}\cdot\nabla p_0=0\qquad\left(\dfrac{\partial}{\partial t}-\chi\nabla_\perp^2\right)\tilde{p}_1-\dfrac{\nabla\tilde{\phi}\times\hat{\mathbf{z}}}{B_0}\cdot\nabla p_0=0 \end{cases}$$



Actual system: a single mode + a stochastic magnetic field background + small scale convective cells

Results

Three new terms, which represent 3rd order magnetic torque (see Rutherford '73) appear. They are:

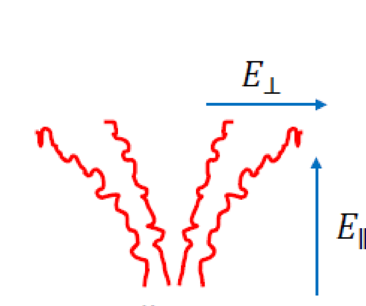
① $(\nabla_\parallel J_\parallel)^{(3)}\sim\dfrac{S}{\tau_A}\left(\nabla_\perp\cdot\langle\tilde{b}\tilde{b}\rangle\right)\cdot\nabla_\perp\bar{\phi}\sim\dfrac{v_A^2}{\eta}\dfrac{k_y^2}{L_s^2}\dfrac{w_I'^4}{(\Delta x)^2}\bar{\phi}$ ← Magnetic vorticity damping (**enhancing the inertia**)

Since $(\nabla_\parallel J_\parallel)^{(1)}\sim\frac{S}{\tau_A}\nabla_\parallel^{(0)^2}\bar{\phi}\sim\frac{v_A^2}{\eta}\frac{k_y^2}{L_s^2}(\Delta x)^2$, when $w_I'\sim\left[\frac{k_y^2}{k_y'^2}(\Delta x)^4\right]^{\frac{1}{4}}$, 3rd order magnetic torque balance 1st order.

For ② $\dfrac{S}{\tau_A}\nabla_\perp\cdot\langle\tilde{\boldsymbol{b}}_\perp\tilde{E}_{\parallel^0}\rangle$ and ③ $\dfrac{S}{\tau_A}\left\langle\nabla_\parallel^{(0)}\left(\tilde{\boldsymbol{b}}_\perp\cdot\tilde{\boldsymbol{E}}_\perp\right)\right\rangle$, we need to get $\tilde{\phi}$. But how? **Mean Field Theory!!!**

From the microscopic vorticity equation, we can find the linear response of $\tilde{\phi}$ to $\tilde{\boldsymbol{b}}$:

$$\tilde{\phi}_{k'}=\int dx''G(x,x'')\left\{-\frac{S}{\tau_A\nu k_y'^2}\left[\nabla_\perp\cdot\left(\tilde{\boldsymbol{b}}_{k'}\nabla_\parallel^{(0)}\bar{\phi}\right)+\nabla_\parallel^{(0)}\left(\tilde{\boldsymbol{b}}_{k'}\cdot\nabla_\perp\right)\bar{\phi}\right]\right\}$$

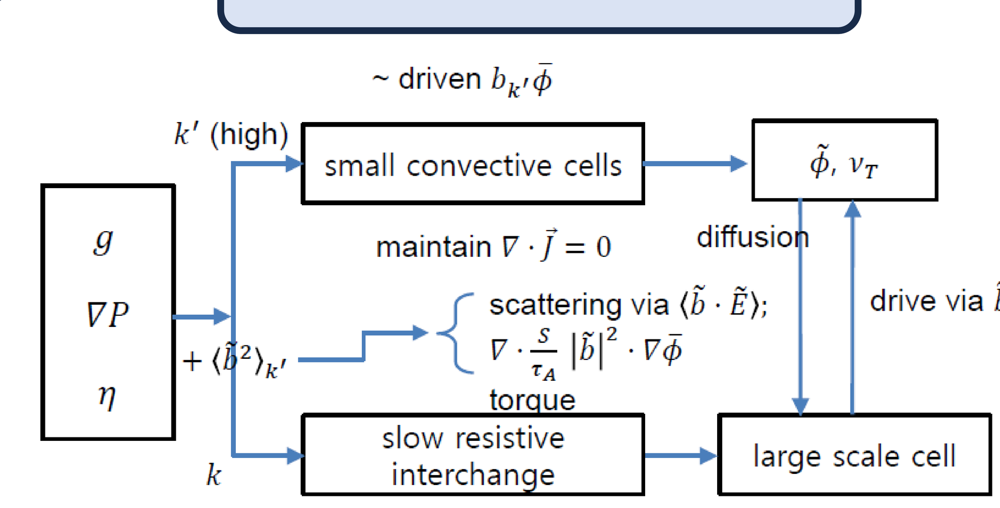


And by using the simplest non-linear closure, the turbulent viscosity is approximated by

$$\nu_T\sim\left(g/L_p\right)^{1/2}1/k_y'^2+\delta\nu$$

The effects of ② and ③ are to be determined.

Conclusions



- Basically this is a multi-scale problem: $\bar{\phi}$, $\tilde{\phi}$ and $\tilde{\boldsymbol{b}}$.
- To main quasi-neutrality, we have to introduce $\tilde{\phi}$ and we have a non-trivial $\langle\tilde{\boldsymbol{b}}\tilde{\phi}\rangle$.
- There is a magnetic vorticity damping effect, which can enhance the inertia of plasma.
- A criterion when the effect of stochastic magnetic field is nonnegligible is given.