Physics of Turbulence Spreading and Explicit Nonlocality

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"Standard Model" of DW - ZF turbulence:

Disparate profile scale L_T, L_n, L_P and correlation scale Δr_c
$\Rightarrow$ local mixing, *local gradient*: $Q = -\nabla T$
$\Rightarrow D = \rho_* D_B$. $D_B = C_s \rho_*, \rho_* = \rho_i / a$.

- Breaking of gyro-Bohm $D \sim \rho_*^\sigma D_B, \sigma < 1$
- "Nonlocal phenomena"

How do turbulence and transport front propagate? Local but fast propagate? (Explicitly) non-local?

Theory Extension

- Turbulence Spreading
- Avalanching

Core idea is replacing the local Fick's law $Q = -\nabla T$ with a delocalize flux-gradient relation [1, 2, 3]

$$Q = -\int dr' K(r - r') \nabla T(r') \quad (1)$$

where $K(r - r')$ is the nonlocal kernel.

We show that, $\langle \tilde{\phi}^2 \rangle$ evolution is *explicitly* non-local. And such non-locality can affect turbulence spreading.

Explicitly Nonlocal	vs.	Heuristic Model
$\partial_t \langle \tilde{\phi}^2 \rangle = \int \gamma(r - r') \langle \tilde{\phi}^2 \rangle (r') dr' + \cdots$	vs.	$\partial_t \mathcal{E} = \partial_x [(D_0 \mathcal{E}) \partial_x \mathcal{E}] + \gamma(x) \mathcal{E} - \sigma \mathcal{E}^2$

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Roadmap

KE: $\partial_t \bar{f} + \Omega_D E \partial_\alpha \bar{f} - [J_0 \phi, \bar{f}] = 0$, QuasiNeutral: $n_i = n_e$ $\longrightarrow$ Darmet Model: h_i

$\Downarrow$

Goal: Evolution of $\langle \tilde{\phi}^2 \rangle$

Spreading Model From KE to PV

For low frequency turbulence in Tokamak ($\omega < \omega_b$, bounce frequency):

$f(\vec{r}, \vec{p}, t) \xrightarrow[Bounce-average]{Gyro-average} \bar{f}(\psi, \alpha, E, t)$. ψ radial, α angle, and E is the energy[4].

$$\begin{cases} \partial_t \bar{f} + \Omega_D E \partial_\alpha \bar{f} - [J_0\phi, \bar{f}] = 0 \\ n_i = n_e \end{cases} \quad (2)$$

where $[F, G] = \partial_\alpha F \partial_\psi G - \partial_\psi F \partial_\alpha G$.

+

- Mean, adiabatic and non-adiabatic: $\bar{f} = \langle f \rangle - \frac{q_{i,e}\phi}{T_{i,e}} \langle f \rangle + h_{i,e}$.
- Fluctuation not response to zonal potential: $\widetilde{n}_{i,e}/n_0 = -q_{i,e}(\phi - \langle\phi\rangle_\alpha)/T_{i,e}$

The non-adiabatic distribution function h_i and quasi-neutrality equation (Darmet Model [4, 5, 6]):

$$\partial_t h_i + \Omega_D E \partial_\alpha h_i - \left[\bar{\phi}, -\frac{q}{T_i}(\phi - \langle\phi\rangle_\alpha)\langle f_i\rangle + h_i\right] = \partial_t \left(\frac{q}{T_i}(\phi - \langle\phi\rangle_\alpha)\langle f_i\rangle\right) + \partial_\alpha \left(\overline{\phi - \langle\phi\rangle_\alpha}\right)\partial_\psi \langle f_i\rangle \quad (3)$$

$$C_{ad}\left(\phi - \langle\phi\rangle_\alpha\right) - C_i \overline{\Delta}_{i+e}\phi = \frac{2}{n_0\sqrt{\pi}} \int_0^\infty J h_i \sqrt{E} dE - \frac{2}{n_0\sqrt{\pi}} \int_0^\infty J h_e \sqrt{E} dE \quad (4)$$

where $C_i = q/T_i$, $C_{ad} = C_i(1+\tau)/\sqrt{2\varepsilon_0}$, $\tau = T_i/T_e$. $\overline{\Delta}_s = \rho_{0s}^2 \partial_\alpha^2 + \delta_{bs}^2 \partial_\psi^2$. A minimal K.S. for DW turbulence.

Spreading Model From KE to PV

Roadmap

KE: $\partial_t \bar{f} + \Omega_D E \partial_\alpha \bar{f} - [J_0\phi, \bar{f}] = 0$, QuasiNeutral: $n_i = n_e$ → Darmet Model: h_i

Moment

Separate by symmetry

Evolution of Fluctuation $\tilde{T}$、$\tilde{U} \equiv \tilde{\phi} - \overline{\Delta}\tilde{\phi}$

Goal: Evolution of $\langle \tilde{\phi}^2 \rangle$

$h_e = 0$ and neglect $\overline{\Delta}_e$. Taking the derivative of equation (4) w.r.t. time. Separate the results according to symmetry in angle direction. $\phi = \tilde{\phi} + \phi_Z$[7, 8].

$$\left(\frac{\partial}{\partial t} + \widetilde{\mathbf{V}} \cdot \nabla + \mathbf{V}_Z \cdot \nabla\right)(C_i \overline{\Delta}\widetilde{\phi}) = \frac{3}{2}\Omega_D \partial_\alpha \widetilde{T}_i - iC_e(\omega - \omega_E + \frac{\omega_{*n}^i}{\tau})\widetilde{\phi} - C_i \widetilde{V}(\psi)\partial_\psi(\overline{\Delta}\phi_Z) \quad (5)$$

$$\frac{\partial}{\partial t}\left[C_i \overline{\Delta}\phi_Z\right] = C_i \langle \nabla\widetilde{\phi} \times \hat{z} \cdot (\nabla\overline{\Delta}\widetilde{\phi})\rangle_\alpha \equiv C_i \delta_{b0}^2 \partial_\psi^2 \left\langle \tilde{v}_\psi \tilde{v}_\alpha \right\rangle_\alpha \quad (6)$$

Defined *potential-vorticity* quantity: $\tilde{U} \equiv C_e\tilde{\phi} - C_i\overline{\Delta}\tilde{\phi}$. Then:

$$\text{Eq.}(5) \Longrightarrow \left(\frac{\partial}{\partial t} + \widetilde{\mathbf{V}} \cdot \nabla + \mathbf{V}_Z \cdot \nabla\right)\tilde{U} = -\frac{3}{2}\Omega_D \partial_y \widetilde{T}_i + C_i \widetilde{V}(r)\partial_r(\overline{\Delta}\phi_Z) \quad (7)$$

where $(\psi, \alpha) \rightarrow \vec{x} \equiv (r, y)$, Ω_D is a typical (constant) ion precession velocity. Equation above is similar to the H-M eq. Potential vorticity $\tilde{U}$ is a *conserved macro-quantity*, here broken by the linear terms.

Potential vorticity conservation equation:

$$\left(\frac{\partial}{\partial t} + \tilde{\mathbf{V}}\cdot\nabla + \mathbf{V}_Z\cdot\nabla\right)\tilde{U} = -\frac{3}{2}\Omega_D\partial_y\tilde{T}_i + C_i\tilde{V}(r)\partial_r(\overline{\Delta}\phi_Z) \quad (7)$$

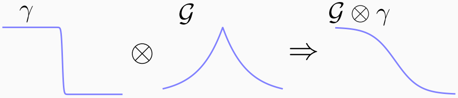


Fig 1: Effect of convolution with $\mathcal{G}$

$\tilde{U} \Rightarrow \tilde{\phi}$?

According to the definition, there is

$$\tilde{U}_{\bar{k}} = (C_e + C_i\bar{k}^2)\tilde{\phi}_{\bar{k}} \longrightarrow \tilde{\phi}_{\bar{k}} = \frac{\tilde{U}_{\bar{k}}}{C_e + C_i\bar{k}^2}$$

$$\xrightarrow{yields} \tilde{\phi} = \int \mathcal{G}(x,x')\tilde{U}(x')\mathrm{d}x' \equiv \mathcal{G}\otimes\tilde{U} \quad (8)$$

where Green's function:

$$\mathcal{G}(x,x') = \frac{\sqrt{A}}{2}e^{-\sqrt{A}|x-x'|}, \quad A \sim \delta_b^{-2} \quad (9)$$

Naturally, the intensity of $\langle\tilde{\phi}^2\rangle$ is:

$$\left\langle\tilde{\phi}^2\right\rangle = \lim_{1\to 2}\iint G(x_1,x_1')G(x_2,x_2')\left\langle\tilde{U}(x_1')\tilde{U}(x_2')\right\rangle \mathrm{d}x_1'\mathrm{d}x_2'$$

Terms like $\left\langle\tilde{v}_{r_1}\widetilde{U_1U_2}\right\rangle$ can be closed by *two-point quasilinear approximation*,

$$\left(\widetilde{U_1U_2}\right)_{\substack{k_y\\ \omega}} = -\Big[R^{(1)}_{\substack{k_y\\ \omega}}\tilde{v}_{\substack{k_y\\ r}}(x_1)e^{ik_yy_1}\partial_{r_1} + R^{(1)}_{\substack{k_y\\ \omega}}\tilde{v}_{\substack{k_y\\ y}}(x_1)e^{ik_yy_1}\partial_{y_1}$$
$$+R^{(2)}_{\substack{k_y\\ \omega}}\tilde{v}_{\substack{k_y\\ r}}(x_2)e^{ik_yy_2}\partial_{r_2} + R^{(2)}_{\substack{k_y\\ \omega}}\tilde{v}_{\substack{k_y\\ y}}(x_2)e^{ik_yy_2}\partial_{y_2}\Big]\left\langle\tilde{U}_1\tilde{U}_2\right\rangle$$

Roadmap

KE: $\partial_t \bar{f} + \Omega_D E \partial_\alpha \bar{f} - [J_0 \phi, \bar{f}] = 0$, QuasiNeutral: $n_i = n_e$ → Darmet Model

→ Moment / Separate by symmetry →

Evolution of Fluctuation $\tilde{T}$、$\tilde{U} \equiv \tilde{\phi} - \overline{\Delta\tilde{\phi}}$

→ 2-p correlation [9] / $\langle \tilde{U}(1)\partial_t \tilde{U}(2) + \tilde{U}(2)\partial_t \tilde{U}(1) \rangle$ →

Evolution of PE: $\langle \tilde{U}(1)\tilde{U}(2) \rangle$

⇢ Goal: Evolution of $\langle \tilde{\phi}^2 \rangle$

PV to $\langle \tilde{\phi}^2 \rangle$

Roadmap

KE: $\partial_t \bar{f} + \Omega_D E \partial_\alpha \bar{f} - [J_0 \phi, \bar{f}] = 0$, QuasiNeutral: $n_i = n_e$ → Darmet Model

→ (Moment; Separate by symmetry) → Evolution of Fluctuation $\tilde{T}$、$\tilde{U} \equiv \tilde{\phi} - \overline{\Delta} \tilde{\phi}$

→ (2-p correlation: $\langle \tilde{U}(1) \partial_t \tilde{U}(2) + \tilde{U}(2) \partial_t \tilde{U}(1) \rangle$) → Evolution of PE: $\langle \tilde{U}(1) \tilde{U}(2) \rangle$

→ (2-p quasilinear method; Green's function) → Goal: Evolution of $\langle \tilde{\phi}^2 \rangle$

The evolution equation of potential intensity:

$$\partial_t \left\langle \tilde{\phi}^2 \right\rangle = \mathcal{G} \otimes \frac{\partial}{\partial r} \left[2D_0 \left\langle \tilde{\phi}^2 \right\rangle \frac{\partial}{\partial r} \left(\left\langle \tilde{\phi}^2 \right\rangle - \frac{\delta_b^2}{2} \frac{\partial^2}{\partial r^2} \left\langle \tilde{\phi}^2 \right\rangle \right) \right] + \mathcal{G} \otimes \left(\gamma_L(r) \left\langle \tilde{\phi}^2 \right\rangle \right) - \frac{D_0}{l_r^2} \left\langle \tilde{\phi}^2 \right\rangle^2 \quad (10)$$

Heat flux drive approximated: $\langle \tilde{v}_r \tilde{T} \rangle \sim -\langle \tilde{\phi}^2 \rangle \partial_r \langle T \rangle \sim -\gamma_L \langle \tilde{\phi}^2 \rangle$ (assumed $\partial_r \langle T \rangle \sim \langle T \rangle / L_T > 0$).
Neglected the ϕ_Z for simplicity.

- Nonlocal nonlinear diffusion: Nonlocality is weak as shown latter, simplified as $\partial_r (2D_0 \langle \tilde{\phi}^2 \rangle \partial_r \langle \tilde{\phi}^2 \rangle)$
- Nonlocal growth: *Distributed pumping of $\langle \tilde{\phi}^2 \rangle$ from the heat flux $\langle \tilde{v}_r \tilde{T} \rangle$.*
  Kernel width of $\mathcal{G}(x, x') \propto \exp(-|x - x'|/\delta_b)$ is several δ_b, thus the growth of $\langle \tilde{\phi}^2 \rangle$ at r is affected by a region of several δ_b in width. Preconditions:
  1. The curvature of the field $\Rightarrow$ trapped ion orbit and ion-precessional motion.
  2. The polarization charge due to trapped ions $\Rightarrow$ redistribution of fluctuating temperature.
- Nonlinear local damping: $D_{y,y} \approx 2 \sum_{k_y, \omega} R_{k_y} \left| \tilde{\phi}_k \right|^2 \frac{k_y^2}{k_y^2 l_r^2} (1 - \cos(k_y y_-)) \xrightarrow{\langle y_-^2 \rangle > 1} \approx 2D_0 \left\langle \tilde{\phi}^2 \right\rangle \frac{1}{\bar{k}_y^2 l_r^2}$

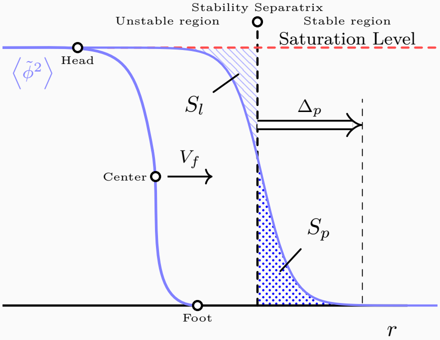
Spreading Model

Heuristic Model[10, 11] vs. Explicitly Nonlocal Model

$$\partial_t \mathcal{E} = \partial_x[(D_0\mathcal{E})\partial_x\mathcal{E}] + \gamma(x)\mathcal{E} - \sigma\mathcal{E}^2 \quad \text{vs.} \quad \partial_t \left\langle \tilde{\phi}^2 \right\rangle = \mathcal{G} \otimes \text{N-lin. Diff.} + \mathcal{G} \otimes \left(\gamma_L(r) \left\langle \tilde{\phi}^2 \right\rangle\right) - \frac{D_0}{l_r^2} \left\langle \tilde{\phi}^2 \right\rangle^2$$

Illustration of quantities:

- V_f, the leading edge propagating speed
- Shape of front characterized with distance between "Foot", "Center" and "Head"
- Penetration of leading edge into the stable region:
  - Depth, Δ_p
  - Area, S_p



How do those nonlocal terms affect spreading front generation and propagation?

Wider, Faster and Deeper

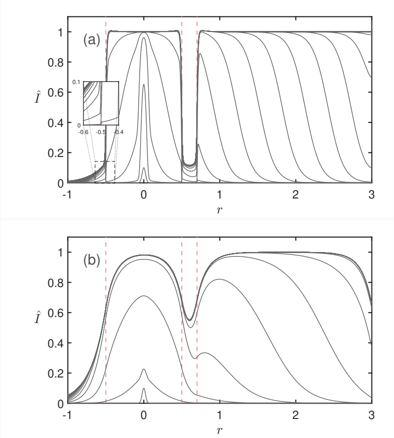


Fig 2: Evolution of (a) with nonlocal diffusion, (b) with nonlocal growth.

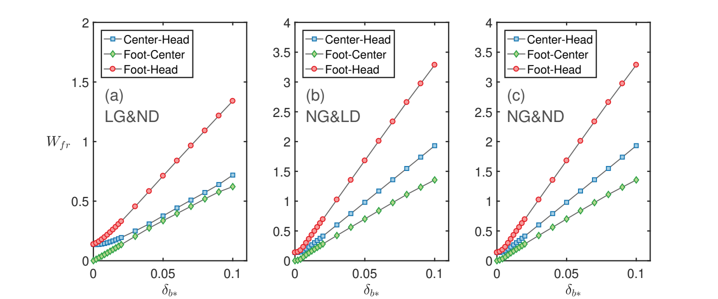


Fig 3: Width of the propagating front in different equations with a fixed ρ_i when varying δ_b.

- $W_f \propto \delta_b$
- $\mathcal{G} \otimes (\gamma_L(r)\langle\tilde{\phi}^2\rangle)$ is much more effective.

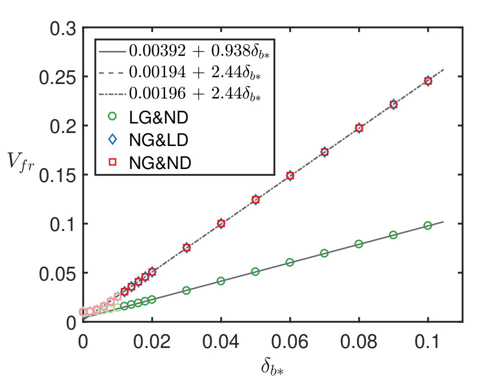


Fig 4: Leading edge propagation speed for different models when varying δ_b with $\rho_i = 0.01$. Data points with lighter colors indicate where $\delta_b < \rho_i$ and are excluded from the fit lines.

- $\delta_b \to 0$, the speed converges to classic Fisher-KPP front speed $\sqrt{2\gamma D} = 0.01$ [11].
- $\delta_b > \rho_i$, $V_f \propto \sqrt{2\gamma D}(1 + \delta_b)$
- Data form NG&ND and NG&LD overlapping indicates that the nonlocal growth effect dominates.

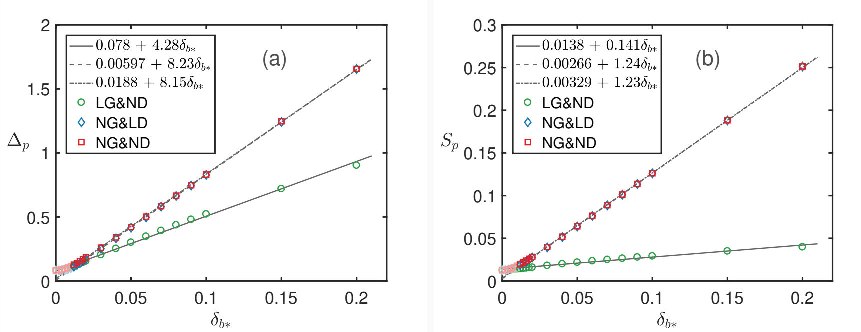


Fig 5: Front penetration Δ_p (a) and effective penetration S_p (b) against δ_{b*} for different equations. Simple linear relation can fit both Δ_p and S_p, when $\delta_{b*} > \rho_*$. Data points in lighter colors are excluded from the fits.

$$\Delta_p, S_p \propto \delta_{b*} \xrightarrow[\text{Domain}]{\text{Symmetric}} \bar{D}(\langle\tilde{\phi}^2\rangle) \propto 1 - S_I = 1 - \delta_{b*} \tag{11}$$

where $\delta_{b*} = \delta_b / L_T$.

Conclusions and Discussions

Roadmap

KE & QuasiNeutrality $\longrightarrow$ Darmet Model $\longrightarrow$ $\tilde{T}$、$\tilde{U} \equiv \tilde{\phi} - \overline{\Delta}\tilde{\phi}$ $\longrightarrow$ $\langle \tilde{U}(1)\tilde{U}(2)\rangle$

$$\partial_t \left\langle \tilde{\phi}^2 \right\rangle = \partial_r \left[D_0 \left\langle \tilde{\phi}^2 \right\rangle \partial_r \left\langle \tilde{\phi}^2 \right\rangle \right] + \mathcal{G} \otimes \left(\gamma_L(r) \left\langle \tilde{\phi}^2 \right\rangle \right) - \frac{D_0}{l_r^2} \left\langle \tilde{\phi}^2 \right\rangle^2 \longleftarrow \text{Green's function}$$

Conclusions

1. $\partial_t \langle \tilde{\phi}^2 \rangle$ is *explicitly nonlocal*.
2. *Explicit non-local growth* is the principal new effect.
3. Potential vorticity $\tilde{U} = A\tilde{\phi} - \overline{\Delta}\tilde{\phi}$ conservation.
4. Inverting PV to $\tilde{\phi} \Rightarrow$ Green's Function:
   $\mathcal{G}(x, x') \propto \sqrt{A} e^{-\sqrt{A}|x - x'|}$
   $\Rightarrow \delta_b$ sets range of nonlocality, which is modest.
5. $V_f \simeq (\gamma D)^{1/2}(1 + \delta_b)$, $\Delta_p \propto \delta_{b*}$

Discussions and Future Plans

- The utility of PV (potential vorticity).
- Near macro-marginality $\Rightarrow$ Explicit nonlocality $\uparrow$.
- Pedestal $\Rightarrow \delta_b / L_T \uparrow$.
- Energetic particle-driven turbulence$\Rightarrow \delta_b \uparrow$.
- Including zonal flows.
- Jamming...

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Questions and Answers

Thanks!