What Limits Zonal Flow Shears in Collisionless Drift-Wave Turbulence?

T. Zhang[1], R.A. Heinonen[1,2], and P.H. Diamond[1]
UCSD Department of Physics[1] and University of Rome, Italy[2]
Supported by US D.O.E under Grant No. DE-FG02-04ER54738

Introduction

- Drift wave - zonal flow turbulence is self-regulating and frequently modeled as a predator-prey interaction [5].
- Turbulent friction or viscosity could be damping the zonal flow.
- There must then be a feedback on the prey (drift wave) to conserve energy.
- Thus, zonal flow stability is an important factor.
- We test the viability of the gradient of potential vorticity profile, PV $= n - \nabla^2\phi$ as a measure of zonal flow stability by comparing R $= \frac{E_{ZF}}{E_{ZF}+E_{DW}}$ with $\nabla(PV)$.
- Results: For $\nabla(PV)$ large, R $\rightarrow 1$ and for $\nabla(PV) \rightarrow 0$, R $\rightarrow 0$, indicating a higher turbulence level co-existing with a generalized inflection point.
- **Ultimately, we want to quantify zonal flow stability in the predator-prey model and derive a turbulent viscosity for collisionless saturation.**

Motivation

- The predator-prey model between zonal flows and drift waves can be shown by the following equations where N is the turbulence energy, E_V is the energy associated with zonal flows [3, 4, 9]:

$$\partial_t N = \gamma N - \alpha E_V N - \Delta\omega N^2$$

$$\partial_t E_V = \alpha N E_V - \nu_F E_V - \gamma_{nl} E_V^2$$

- α is the shearing efficiency, γ is the drift wave's linear growth rate, and ν_F represents zonal flow damping.
- $\Delta\omega$ and γ_{nl} are the non-linear damping rates of drift waves and zonal flows.
- We want to test the effect zonal flow stability has on regulating the zonal flow and extend the predator-prey model to include the effect of zonal flow instability

Critical Questions

- What regulates zonal flow instability?
- Can the gradient of the mean potential vorticity be used to indicate zonal flow instability?
- How does the value of the potential vorticity correlate with saturated turbulence levels?
- How does zonal flow marginality correlate to fluctuation levels?
- Does $\mathrm{R} = \frac{E_{ZF}}{E_{ZF}+E_{DW}}$ show a correlation with $\nabla(PV)$?

Hasegawa-Wakatani Model

$$\partial_t \nabla_\perp^2 \phi + \{\phi, \nabla_\perp^2 \phi\} = \alpha(\phi - n) - \mu \nabla_\perp^2 \phi + \nu \nabla_\perp^6 \phi \rightarrow \partial_t \nabla_\perp^2 \phi - \partial_x(\nabla_\perp^2 \phi \partial_y \phi) = \nu \nabla_\perp^6 \phi \text{ [1,8]}$$

$$\partial_t n + \{\phi, n\} = \alpha(\phi - n) - \kappa \partial_y \phi + D \nabla_\perp^4 n \rightarrow \partial_t n - \partial_x(n \partial_y \phi) = -D \nabla_\perp^4 n$$

- α is the adiabatic operator, $\alpha = \frac{K_{||}^2 V_{Th}^2}{\omega \nu}$
- κ is a ∇n gradient drive, $\rho^* = \frac{\rho_s}{L_n} = 0.01$
- μ is a flow-damping parameter, $\mu << \omega$
- ν is a hyperviscosity constant and D is a hyperdiffusive constant
- Usual Normalizations: $ln(\frac{n}{n_0}) \rightarrow n, \phi \rightarrow \frac{e\phi}{T_e}, x \rightarrow \rho_s x, t \rightarrow \frac{t}{\omega_{ci}}$
- Zonal Flow Energy $= E_{ZF} = \int \int |\nabla_\perp \phi|^2 dxdy$ for $\alpha > 1$
- Turbulence Energy $= E_{DW} = \int \int |n|^2 + |\nabla_\perp \phi|^2 dxdy \simeq \int \int |\phi|^2 + |\nabla_\perp \phi|^2 dxdy$ for $\alpha > 1$
- We defined R $= \frac{E_{ZF}}{E_{ZF} + E_{DW}}$

Rayleigh-Kuo Criterion

- For an invisicid, incompressible 2D fluid, the Rayleigh inflection point theorem has classically been used as a necessary condition for instability within the shear flow [6].
- Derived from Euler's fluid equations, the Rayleigh inflection point theorem states that the $\nabla(vorticity) = 0$ for shear flow instability to occur.
- Rayleigh's Equation: $(U - c)(\phi'' - k^2\phi) - U''\phi = 0$ with $U(y)$ as the shear flow velocity, $\phi(y)$ is a complex valued amplitude, k as the wavenumber and c as the velocity of the infinitesimal disturbances [2].
- It can be derived from this equation that $U_{yy} \rightarrow 0$, so an inflection point must be located in the shear flow for instability to occur
- However, in the case of drift wave turbulence with $\alpha > 1$, the Hasegawa-Mima equation is the more relevant equation for shear flow stability, which is due to finite $K_{||}$, $\frac{\omega}{K_{||}V_{Th}} < 1$.
- This produces a similar theorem, known as the Rayleigh-Kuo criterion, which has $\nabla(PV) = 0$ [7].

Rayleigh-Kuo Criterion Continued

$$\partial_t(\rho_s^2\nabla^2 - 1)\delta\phi + \mathbf{u}_e\cdot\nabla[(\rho_s^2\nabla^2 - 1)\delta\phi] + V_*(\partial_x\delta\phi) = 0$$

- Using the HM equation with proper normalization and letting $\zeta = \nabla_\perp^2\phi - \phi$, and $V_* \to 0$,

$$\partial_t\zeta + \{\phi, \zeta\} = 0$$

Letting $\phi = \varphi(y)e^{(ik_x x - i\omega t)}$, with k_x real and ω complex, we get

$$(\partial_y^2 - k_x^2 - 1 - (\frac{\partial_y\zeta}{\zeta - \frac{w}{k_x}}))\varphi = 0$$

Multiplying by φ^* and integrating the imaginary part of our equation,

$$\int_0^L \frac{\partial_y\zeta}{|\zeta - \frac{w}{k_x}|^2}|\varphi|^2 = 0$$

- For equality to hold, $\partial_y\zeta = 0 => \partial_y(\nabla_\perp^2\phi - \phi)$, which ends up being the following equation after removing our assumption $\phi = n$:

$$\partial_r(n - \nabla_r^2\phi) = 0$$

- This is known as the Rayleigh-Kuo criterion; similar to the Rayleigh inflection point theorem, we get a necessary condition stating that the gradient of the potential vorticity must go to 0 for instability to occur.
- For fixed ∇n, R-K criterion sets a condition on the zonal vorticity profile relative to the density profile.

Results I

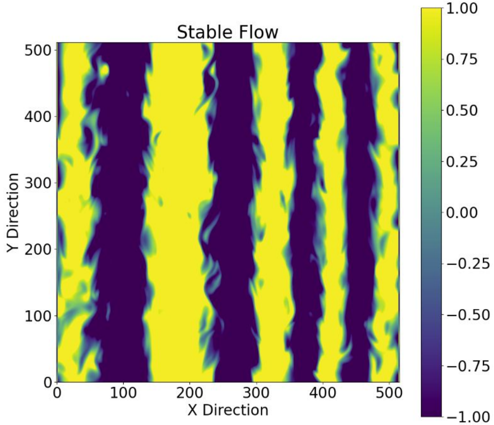

Figure 1: $\nabla(PV) \neq 0$

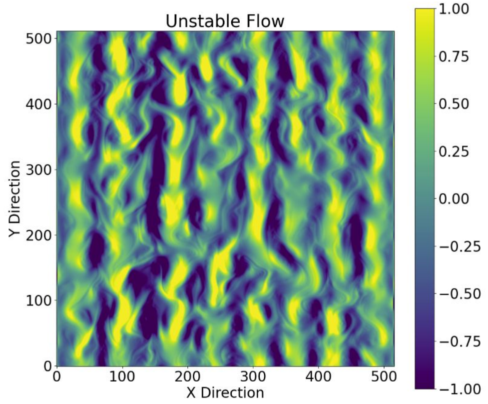

Figure 2: $\nabla(PV) \to 0$

- For $\nabla(PV) \neq 0$, we get more robust zonal flows as seen in Figure 1.
- $\nabla(PV) \to 0$ has very distorted zonal flows, as shown in Figure 2.
- Zonal flow appears to be near marginal as zonal jets can still be made out, as seen in both figures.

Results II

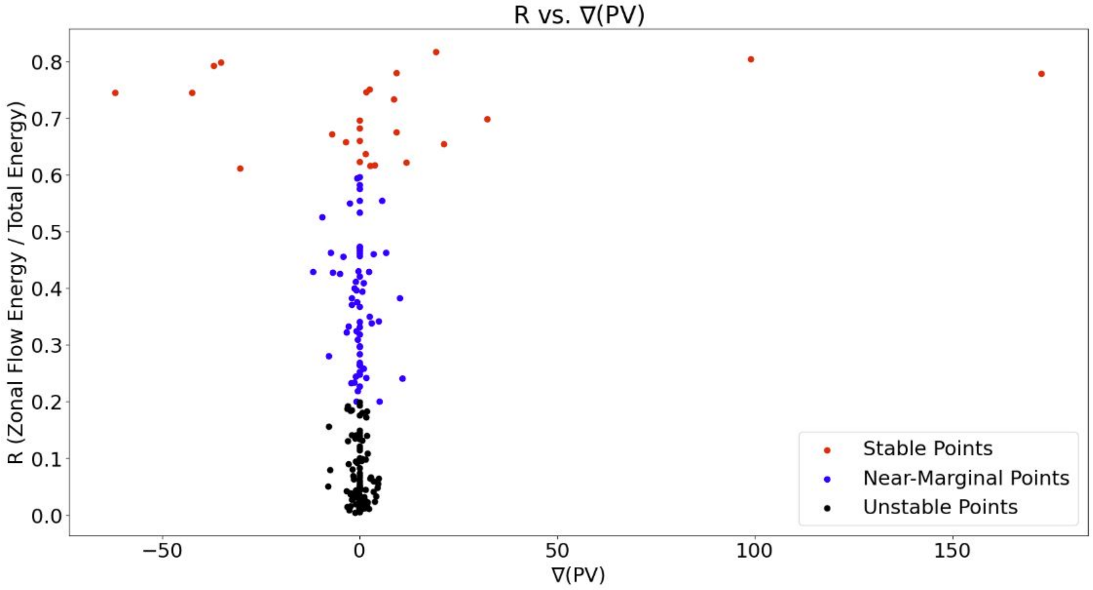

Figure 3: Distribution of R vs. $\nabla(PV)$

- Red points have high $\nabla(PV)$, having R $\rightarrow 1$
- Black points have low $\nabla(PV)$, having R $\rightarrow 0$
- There doesn't seem to be a clear division between stable and unstable points

Analysis

- We have 3 groups of points here, two of which are easily identifiable as stable (red) and unstable (blue) regions.
- The group of points in the middle may be hovering near marginality, making zonal flow damping more of a factor.
- There is a clear trend of unstable points localizing around $\nabla(PV) \to 0$, which is what is expected from the R-K criterion.
- Changing the intergration limits for evaluating the energies didn't change the overall trend of the diagram.

Conclusion + Next Steps

- Our results appear to be consistent with what is expected, stable zonal flows contain more zonal flow energy than drift wave energy while zonal flow instabilities have more drift wave interaction, hence comparatively higher turbulent energies.
- We want to extend our analysis to the variation of zonal friction and density drive, ∇n so that we can quantify the effect of zonal flow stability in the predator-prey model.
- **One may think that increasing ∇n would drive the turbulence, increasing the drift wave energy, however, R-K dictates that it may make it more difficult to have $\nabla(PV) \to 0$.**

References I

[1] Akira Hasegawa and Masahiro Wakatani, “Self-Organization of Electrostatic Turbulence in a Cylindrical Plasma”, Physical Review Letters, 59 (14), 1987.

[2] Balmforth, N. J., and P. J. Morrison. ”A necessary and sufficient instability condition for inviscid shear flow.” Studies in Applied Mathematics 102.3 (1999): 309-344.

[3] Diamond, P., Liang, Y.M., Carreras, B., & Terry, P. (1994). Self-Regulating Shear Flow Turbulence: A Paradigm for the L to H Transition. Phys. Rev. Lett., 72, 2565–2568.

[4] Fujisawa, A. (2008). A review of zonal flow experiments. Nuclear Fusion, 49, 013001.

[5] Gurcan, O., & Diamond, P. (2015). Zonal flows and pattern formation. Journal of Physics A, 48(29), 293001.

References II

[6] J. W. S. Rayleigh. On the stability or instability of certain fluid motions, Proc. Lond. Math. Soc. 9: 57–70 (1880).

[7] H.-L. Kuo, J. Meteor. 6, 105 (1949).

[8] Numata, R., Ball, R., & Dewar, R. (2007). Bifurcation in electrostatic resistive drift wave turbulence. Physics of Plasmas, 14(10), 102312.

[9] Schmitz, L., Zeng, L., Rhodes, T., Hillesheim, J., Peebles, W., Groebner, R., Burrell, K., McKee, G., Yan, Z., Tynan, G., Diamond, P., Boedo, J., Doyle, E., Grierson, B., Chrystal, C., Austin, M., Solomon, W., & Wang, G. (2014). The role of zonal flows and predator–prey oscillations in triggering the formation of edge and core transport barriers. Nuclear Fusion, 54(7).

[10] Zhu, H., Zhou, Y., & Dodin, I. (2018). On the Rayleigh–Kuo criterion for the tertiary instability of zonal flows. Physics of Plasmas, 25(8), 082121.