Physics of the SOL Heat Load Scale: Stability and Turbulence Spreading

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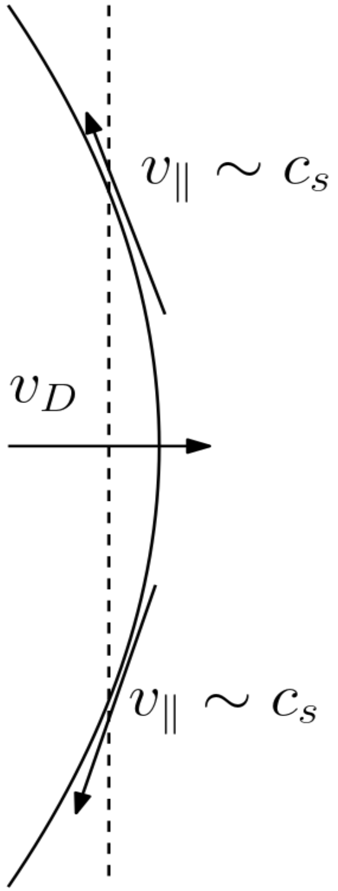
The Problem: Small SOL Width for Current Day H-mode discharges

- Goldston's Scaling: 2012, Goldston, *Nucl. Fusion*
  - $\lambda = v_D \tau_{\|} = \rho_\theta \epsilon \sim B_\theta^{-1}$



Note:

- Based on drifts, no turbulent transport
- Size independent
- Produce very narrow λ
- Small layer for large device with high current

Outline – Overview of the Talk

- SOL stable due to $E \times B$ shear and sheath resistivity:
  - Point to Turbulent Spreading from Pedestal
- Influence of Spreading: 2Box model: Pedestal → SOL
  - SOL as a flux (Q, Γ_n, Γ_e) driven Boundary Layer Γ_e: Turbulence Intensity Flux
  - Γ_e from pedestal: Spreading with shearing
- New Trade Off: Layer Broadening $v.s.$ Confinement (Can turbulent pedestal broaden λ while maintaining adequate confinement?)
  - SOL width λ as a function of Γ_e
  - Estimate Minimal level of Pedestal Fluctuation Needed to Broaden the SOL (Scaling)
- Implication for HDL and H→L back transition

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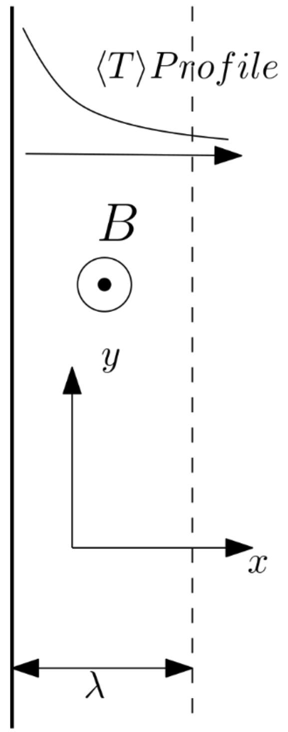
Why does Goldston's Scaling Work?

SOL $E \times B$ shearing $\sim \lambda^{-2}$

Linear Analysis: SOL Stability

Model: (Linear Perturbation of the model by Myra. et. al. 2002):

$$\begin{cases} \partial_t \Delta\delta\phi + \lambda_T \omega_s e^{-x/\lambda_T} \partial_y \Delta\delta\phi - e^{-x/\lambda_T} \beta \partial_y \delta n \\ \qquad = \alpha e^{x/2\lambda_T} \delta\phi + \nu \Delta^2 \delta\phi \\ \partial_t \delta n + \lambda_T \omega_s e^{-x/\lambda_T} \partial_y \delta n + \partial_y \delta\phi \partial_x ln n_0 \\ \qquad = D\Delta\delta n/n + 2D\partial_x ln n_0 \partial_x \delta n \end{cases}$$



Small Heat Load Width

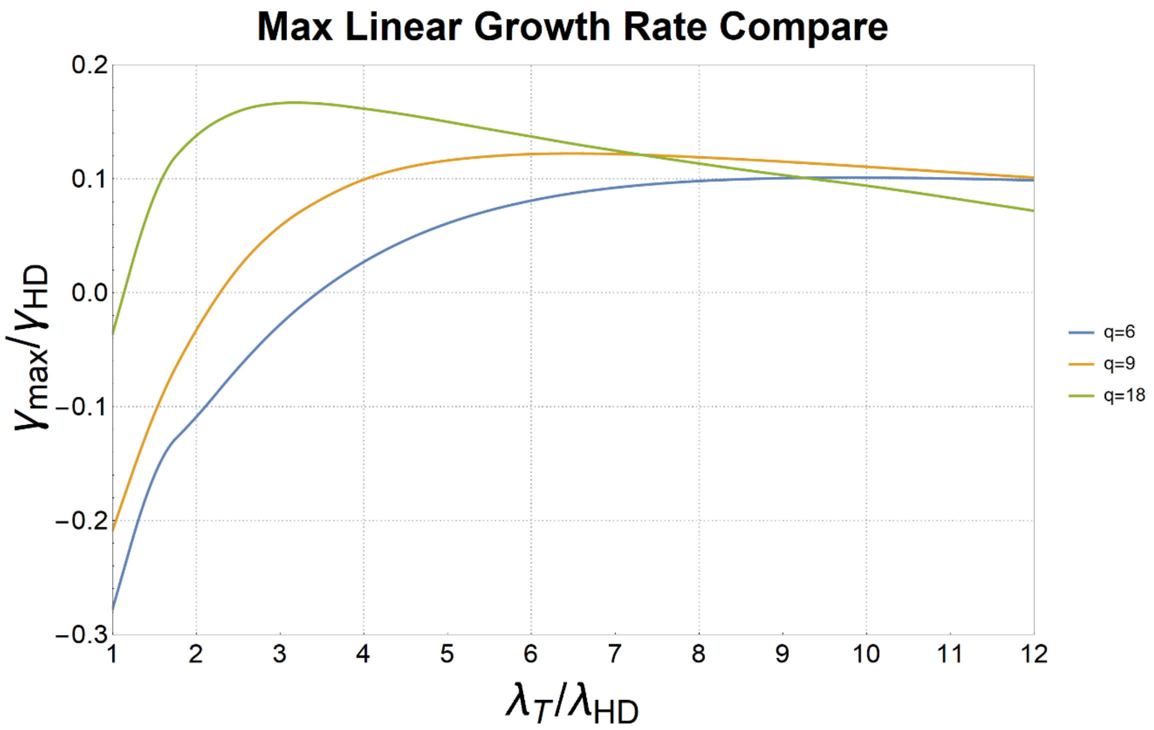
Note:

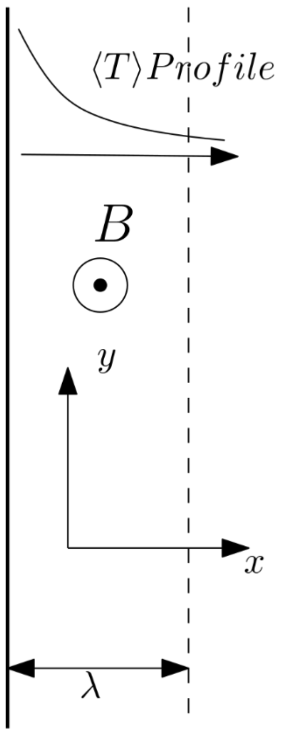
- $\beta = 2\rho/R$. Usual: denoted by β_t
- $E \times B$ Shearing Rate: $\omega_s = \frac{3\langle T\rangle}{eB\lambda_T^2}$, determined by the width

$$\gamma \approx c_s/\sqrt{\lambda R} - \omega_s$$

Linear Analysis: SOL Stability

Linear growth rate with neoclassical diffusion.





Interchange mode is stabilized by the combination of shear, sheath resistivity and plateau region diffusion:
Goldston Scaling Works

Origin of SOL fluctuations:

- Local Instability ×
- Spreading √

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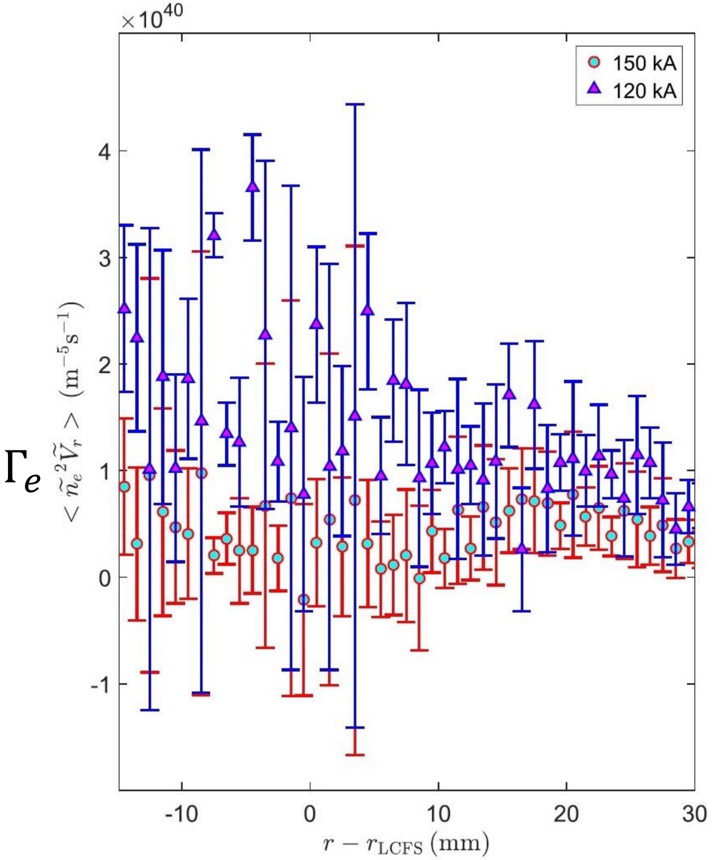
Edge-SOL Connection Already Indicated

L mode: HL-2A   Ting Wu, et. al., 2021, *Plasma Sci. Technol.*



- Flux of turbulence energy into the SOL indicated
- Turbulence spreading reduced at larger current

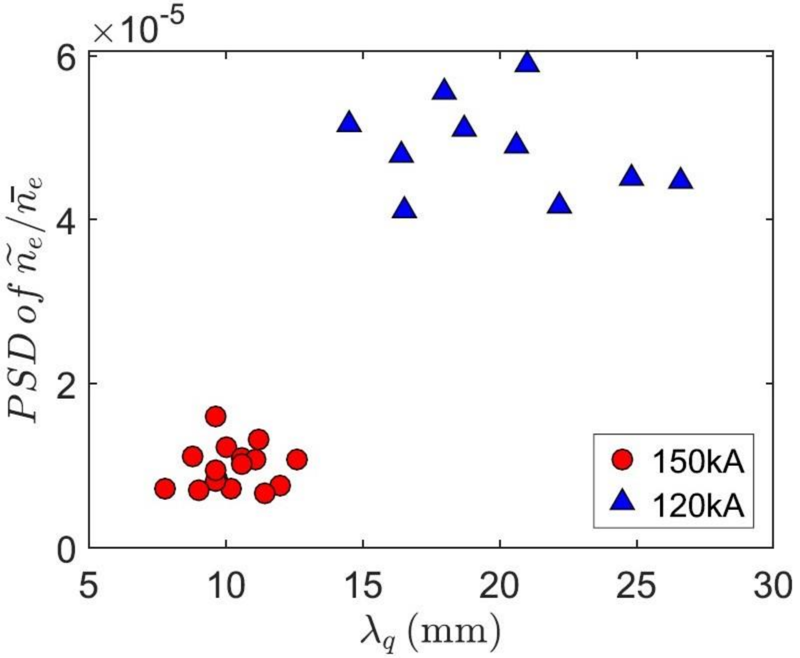


Figure 7. The power of relative density fluctuation level at $r - r_{\mathrm{LCFS}} \approx -15$ to -10 mm versus SOL width with different plasma currents.

PSD $\tilde{n}/n$ inside separatrix vs. SOL width

SOL width larger for stronger edge turbulence levels at lower current.

Suggests Inside turbulence→SOL width influence due to spreading.

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Turbulence in Pedestal

- Reduced but Finite Fluctuation
  - Wide Pedestal QH Regime
  - Grassy ELM Regime
  - …

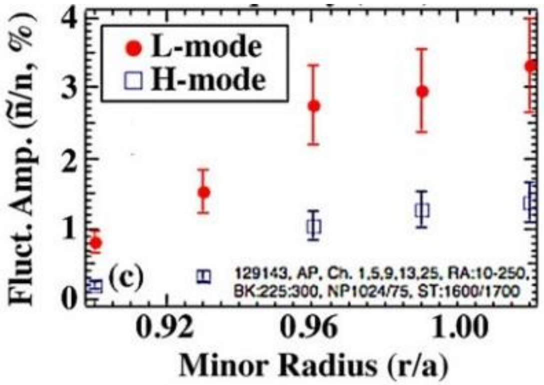

Figure 4. Density fluctuation characteristics measured near the edge ($0.9 < r/a \leqslant 1.0$) of an upper-single-null discharge: (*a*) frequency and time resolved spectrogram of density fluctuations at $r/a = 0.95$ showing rapid fluctuation suppression at the L–H transition at $t = 1696$ ms, (*b*) spectra of fluctuations before and after the L–H transition, (*c*) the integrated fluctuation amplitude edge profile in L and H-mode.

McKee, et. al., 2009, *Nucl. Fusion*

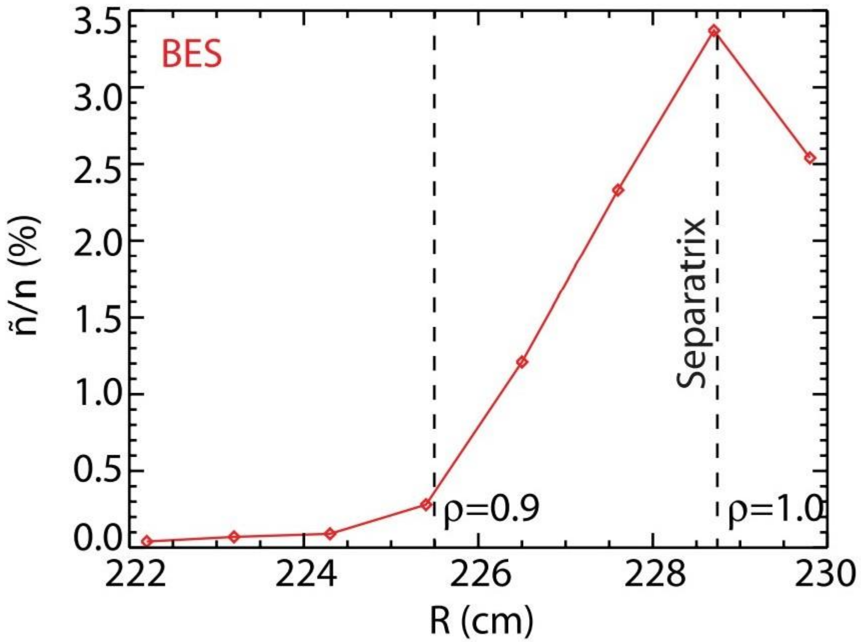

Figure 9. Radial profile of relative density fluctuations integrated over 10 kHz through 80 kHz from BES measurements around 4300 ms in discharge 163518.

Xi Chen, et. al., 2018, *Nucl. Fusion*

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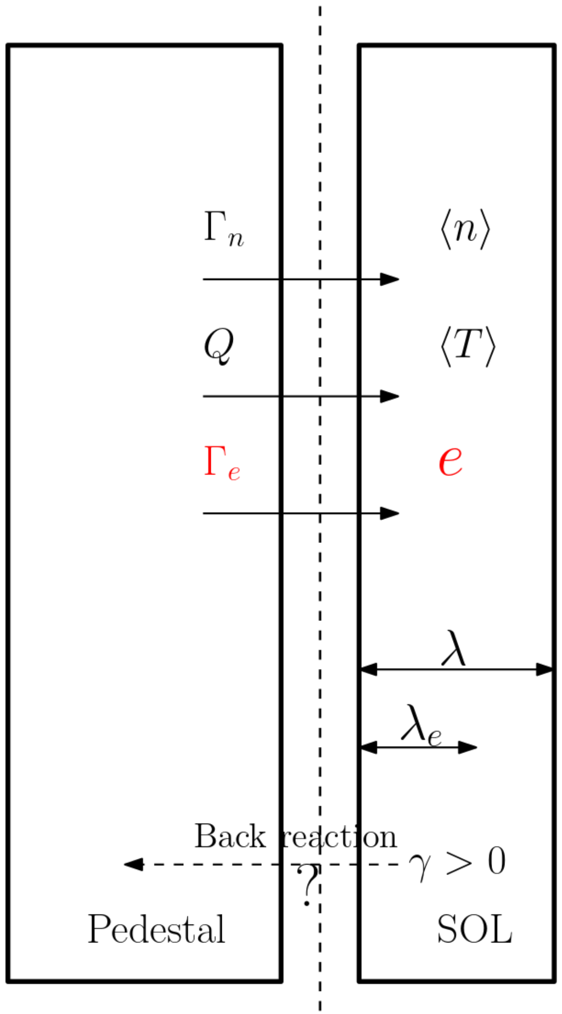
How Spreading Modifies SOL Width

Spreading Model Overview

- Pedestal Model
  - Relating pedestal parameters to intensity flux
  - Intensity gradient combined with pedestal $E \times B$ shear
  - Consider microturbulence and MHD (ballooning)



- SOL Model
  - SOL as a BL with multiple drives: Q, Γ_e
  - Calculate λ from Q and Γ_e
  - Γ_e balances linear and nonlinear damping, determining λ
  - Q determines $\langle T \rangle_{sep}$

Γ_e: Turbulent Intensity Flux from Pedestal

New Trade off:
Layer Broadening v.s. Confinement

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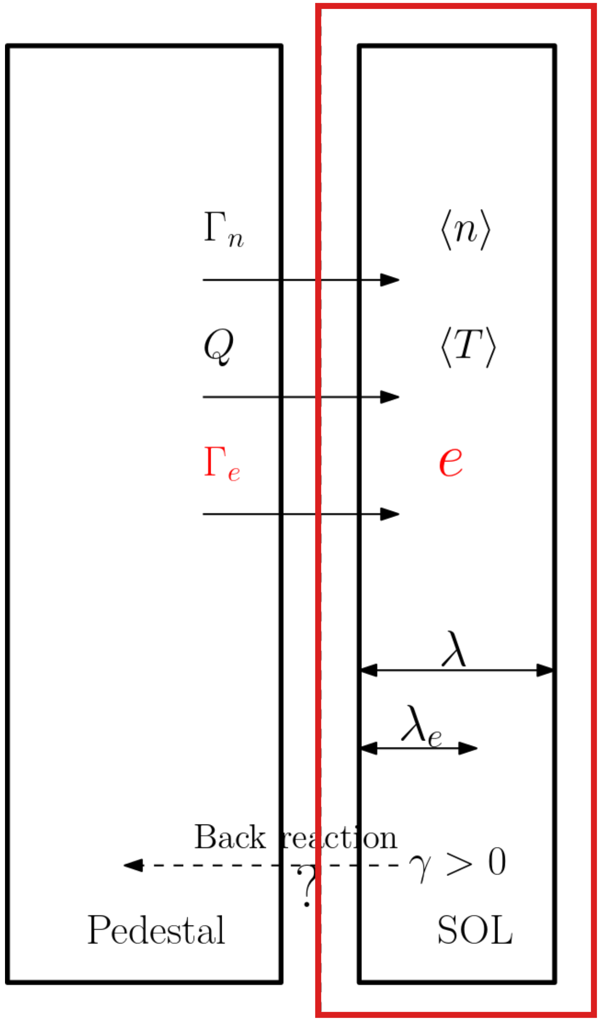
SOL Model: Theory

- Turbulent Intensity Equation: Hahm, Diamond, et. al.
  - $\partial_t e = \gamma e - \sigma e^{1+\kappa} - \partial_x \Gamma_e$, Spatial 1D: $e(x,t)$
- Integrate radially over the SOL
- Turbulent Energy Balance:
  - $\Gamma_{e0} = \lambda_e|\gamma|e + \sigma e^{1+\kappa}\lambda_e$
  - Γ_{e0}: Intensity flux from pedestal (at the separatrix)



Note:

- No assumptions on the form of intensity flux
- Heat flux Q determines $\langle T \rangle_{sep}$ and enters the model from c_s in γ_0
- SOL as a BL with 2 drives (Q, Γ_e)

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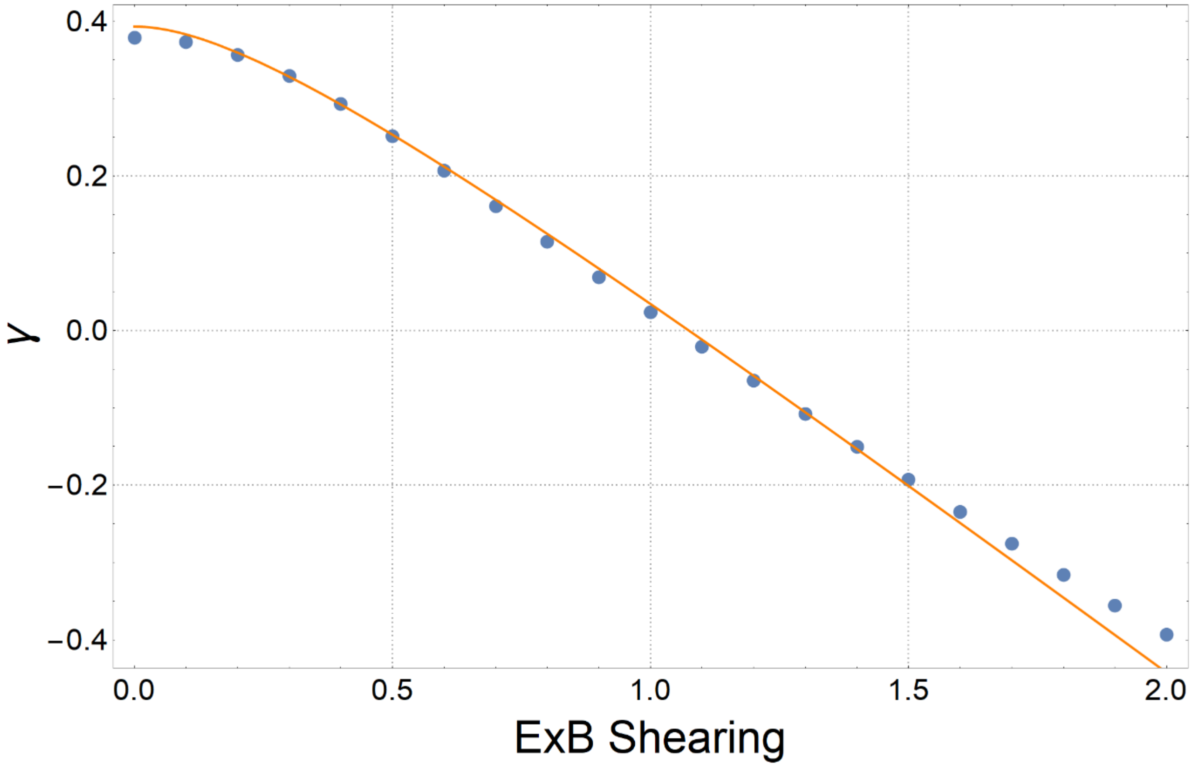
SOL Model: Linear and Nonlinear Damping

- Linear Damping:
  - $\gamma = \gamma_0 - 3T_e/eB\lambda_T^2 \approx -3T_e/eB\lambda_T^2$
  - $\gamma_0 = c_s/\sqrt{R\lambda}$
- Nonlinear Damping:
  - General Form: $\sigma e^{1+\kappa}$
  - Cascade: $\kappa = 1/3,\ \sigma = \alpha^{1/3}$ (One possibility)



Numerical Result of $\gamma\ v.s.\ \omega_s$

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SOL $\lambda - \Gamma_e$ Relation: Unified Relation

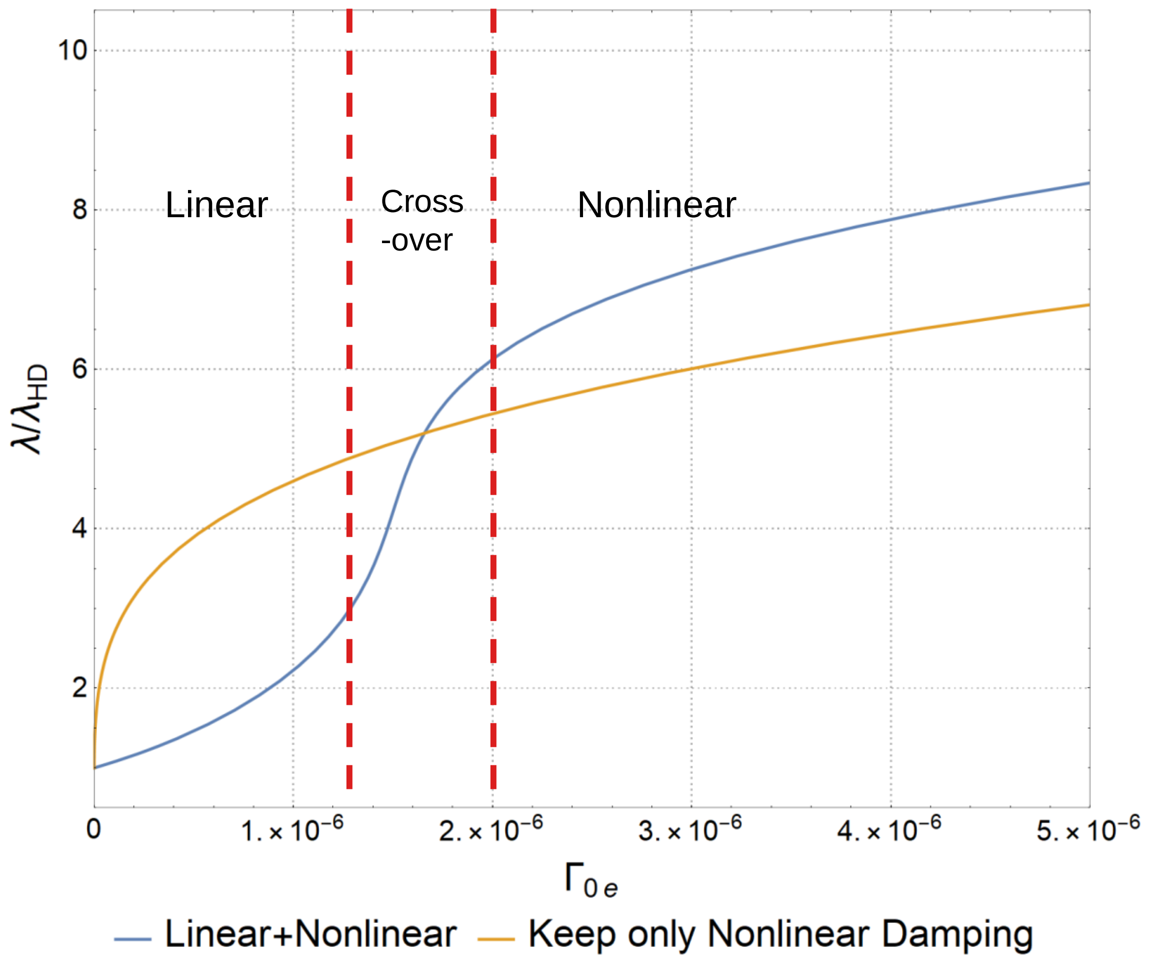
- Account for both drift and $\tilde{v}$
  - $\lambda_e = \lambda = \sqrt{\lambda_{HD}^2 + \tau_{\parallel}^2 e}$
- $\Gamma_{0e} = -\left(\sqrt{\beta/\lambda} - 3/\lambda^2\right) e\lambda + \sigma e^{1+\kappa}\lambda$

Linear Damping

Nonlinear Damping

Note:

- Shape of the curve is sensitive to linear and nonlinear damping



The Cost –

How Strong Pedestal Fluctuation is needed to Broaden the Layer

(Calculate Γ_{0e} from Pedestal)

Pedestal Model: Spreading with $E \times B$ Shear

- $\Gamma_e = -\tau_c K \partial_x K$ (interpreted coarse grained)
- $\Gamma_{e0} \sim \tau_c K^2 / w_{ped}$
- Shearing(ω_s) modifies τ_c
  - $\tau_c^{-1} = Dk^2\left(1 + \omega_s^2 \tau_c^2\right)$
    - Strong Shear Limit: $\tau_c = \left(Dk^2\right)^{-1/3} \omega_s^{-2/3}$
  - $D = \int_0^\infty \langle v(0) v(\tau) \rangle d\tau = \int_0^\infty d\tau \sum_k |v_k|^2 \, e^{-k_y^2 \omega_s^2 D \tau^3 - k^2 D \tau}$ (Kubo formular)
    - Strong Shear Limit: $D \sim |v|^{1.5} k^{-0.5} \omega_s^{-0.5}$
  - In strong shear limit: $\tau_c = \tau_t^{0.5} \omega_s^{-0.5}$, τ_t is eddy turnover rate, or the τ_c when $\omega_s = 0$, denoted by τ_{c0}

τ_c: 3 processes

- Wave interaction
- Nonlinear Decorrelation
- Shearing: ω_s

Note: Strong shear reduces τ_c, reflects transport barrier

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Estimation of Intensity Flux from Pedestal

- Drift Wave: (microturbulence)
  - $\Gamma \sim \tau_c K \partial_x K$
    - K: turbulent kinetic energy
  - $\tau_{c0} v_* = \rho$
  - Weak Shear Limit:
    - $\Gamma_e \sim a \frac{L_n}{\Omega\rho} K^2 / w_{ped}$, a of $O(1)$
  - In strong shear limit:
    - $\Gamma_e \sim a \left(\frac{L_n}{\Omega\rho}\right)^{0.5} \omega_s^{-0.5} K^2 / w_{ped}$

- Ballooning Mode: (MHD)
  - $\tilde{v} \sim \gamma \Delta_r = L_p \omega_A \left(\frac{L_{pc}}{L_p} - 1\right)^{0.5} \frac{\Delta_r}{L_p}$
    - L_p is estimated by pedestal width w_{ped}
  - $\Gamma_e \sim \tau_c K^2 / w_{ped}$, $\tau_{c0} \sim 1/\gamma$
  - Weak Shear Limit:
    - $\Gamma_e \sim \frac{1}{\gamma} (\gamma \Delta_r)^4 / w_{ped} \sim \omega_A^3 \left(\frac{L_{pc}}{L_p} - 1\right)^{1.5} \Delta_r^4 / w_{ped}$
  - In strong shear limit:
    - $\Gamma_e \sim \omega_s^{-0.5} \omega_A^{3.5} \left(\frac{L_{pc}}{L_p} - 1\right)^{1.75} \Delta_r^4 / w_{ped}$

Analogue: Spreading of a turbulent spot

$$E_r \approx \frac{\nabla p}{ne}, \omega_s \sim \frac{\rho^2 \Omega}{w_{ped}^2}$$

Spreading: Γ_{min}, Microturbulence (DW)

- What's the minimal pedestal fluctuation needed to broaden the SOL?
- The criterion: $\lambda_e = \lambda_{HD}$, or equivalently $\tilde{v} = v_D$ when $\tau = \tau_{\parallel}$.($\lambda_e = \tau e^{0.5}$)

For small scale fluctuations:

$$\tau_c \omega_s = \frac{\rho}{w_{ped}}$$

Effect of shear is not strong

1. Balance Linear Damping:
   - $\Gamma_0 = |\gamma| \lambda_{HD}^3 \tau_{\parallel}^{-2}$

   Balancing with the estimation in the pedestal

   - $\frac{e\delta\phi}{T_e} \sim \frac{|\delta v|}{c_s} \sim \left(\frac{3L_e}{aL_n}\right)^{0.25} q^{-0.25} (\rho/R)^{0.5}$
2. Balance Nonlinear Damping:

   $\Gamma_0 = \alpha^3 \lambda_{HD}^9$

   Balancing with the estimation in the pedestal

   - $\frac{e\delta\phi}{T_e} \sim \frac{|\delta v|}{c_s} \sim \left(\frac{L_e}{aL_n}\right)^{0.25} q^{1.5} (\rho/R)^{0.75}$

THE Question:
Is the turbulence level in pedestal to broaden the layer compatible with good confinement? → $e\delta\phi/T_e$ small

Spreading: Γ_{min}, MHD (Ballooning)

- For Ballooning: $\omega_s \tau_c = \sqrt{\beta_t} q\rho R / \left(w_{ped}^2 \sqrt{\frac{L_{pc}}{L_p} - 1} \right)$

Weak Shear Limit:

1. Balance Linear Damping:
   - $\Gamma_0 \sim |\gamma| \lambda_{HD} v_D^2$
   - $\frac{L_{pc}}{L_p} - 1 \sim (q\rho)^{4/3} \frac{R^{2/3}}{L_p^2} \frac{L_p^{8/3}}{\Delta_r^{8/3}} \beta_t$
2. Balance Nonlinear Damping:
   - $\Gamma_0 \sim \alpha^3 \lambda_{HD}^9$
   - $\frac{L_{pc}}{L_p} - 1 \sim \frac{\rho^2 q^6}{L_p^2} \frac{L_p^{8/3}}{\Delta_r^{8/3}} \beta_t$

Strong Shear Limit:

1. Balance Linear Damping:
   - $\Gamma_0 \sim |\gamma| \lambda_{HD} v_D^2$
   - $\left(\frac{L_{pc}}{L_p} - 1\right) \sim (q\rho)^{14/9} \frac{R^{10/9}}{L_p^{8/3}} \left(\frac{L_p}{\Delta_r}\right)^{16/9} \beta_t$
2. Balance Nonlinear Damping:
   - $\Gamma_0 \sim \alpha^3 \lambda_{HD}^9$
   - $\left(\frac{L_{pc}}{L_p} - 1\right) \sim \frac{\rho^2 q^{14/3} R^{2/3}}{L_p^{8/3}} \left(\frac{L_p}{\Delta_r}\right)^{16/9} \beta_t$

$L_{pc}/L_p \sim 1 + \epsilon$, weakly unstable → Grassy ELM

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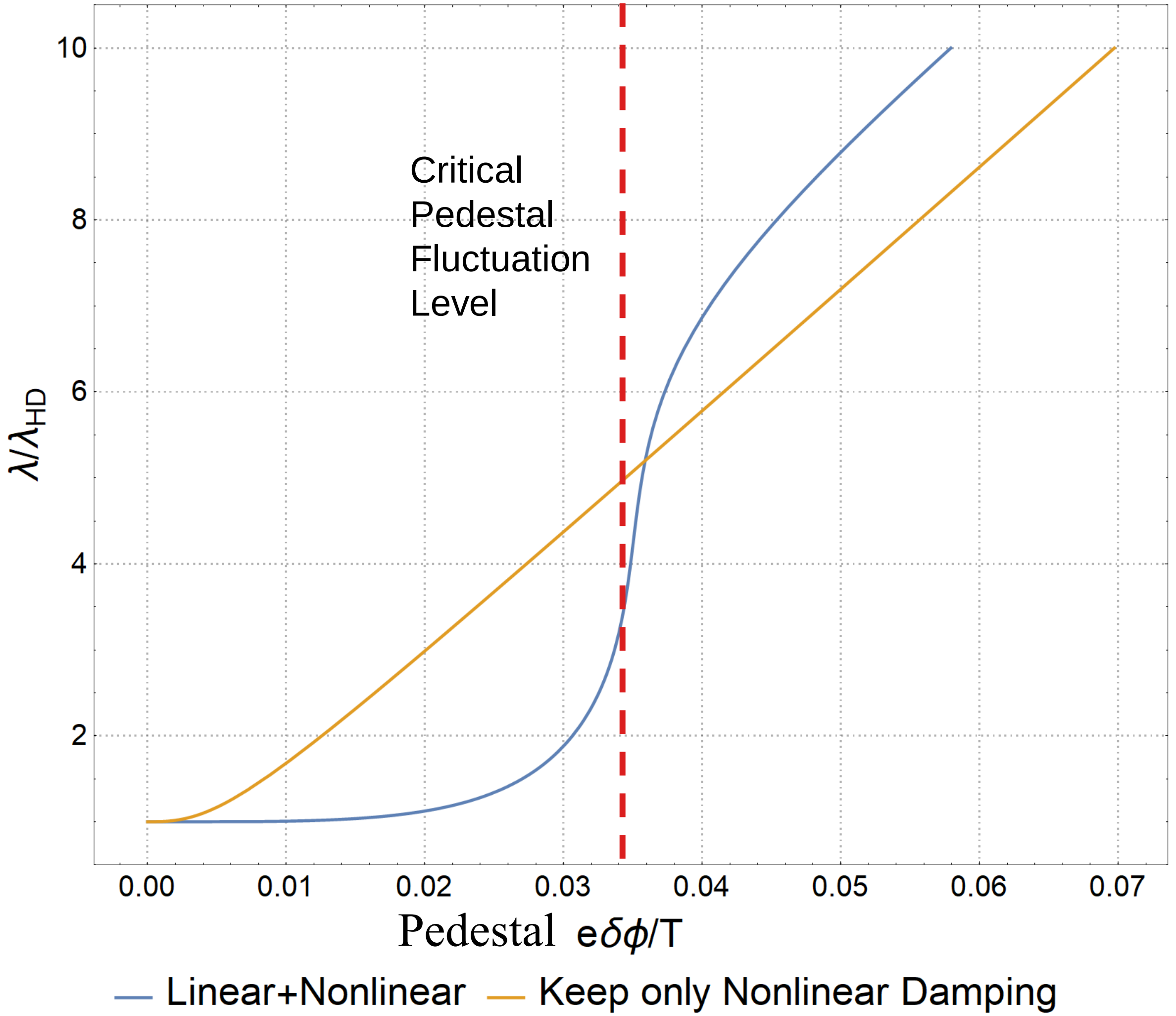
SOL Layer Width – Unified Estimation

- $\lambda_e = \lambda = \sqrt{\lambda_{HD}^2 + \tau_\parallel^2 e}$
- $\Gamma_e = \left(\sqrt{\beta/\lambda} - 3/\lambda^2\right) e\lambda + \sigma e^{1+\kappa}\lambda$
- $\sigma = 0.6, \kappa = 0.5$
- The fluctuation level is converted from intensity flux using DW estimation

- Effective critical fluctuation level is required.



Maybe not so bad but soft pedestal

Conclusions

- SOL is linearly stable in H mode due to large $E \times B$ shear and sheath resistivity
- Turbulent spreading from the pedestal can broaden the layer and should be considered
- SOL width is related to intensity flux across the separatrix which is in turn determined by pedestal parameters
- Intensity flux balances linear and nonlinear damping in the SOL
- There exist a minimal intensity flux for spreading such that $\lambda > \lambda_{HD}$
- Identifies the key question: Can turbulent pedestal broaden λ while maintaining adequate confinement?)
  - Point to turbulent pedestal states

For Experimentalists

- Direct Measurement of Turbulence Spreading at Separatrix (Ting Wu)
- Correlation of Spreading, Edge Intensity Levels to SOL Width
  - Special Attention to Heat Load Width in Turbulent Pedestal States

- Connection between Spreading and Blob Generation
- Is blob simply the end state of turbulent spreading event?

Back transition and HDL

- Future research:
  - HDL: Strong layer broadening weakens shear stabilization and makes SOL interchange unstable.
  - Does SOL turbulence then invade pedestal, cause H→L, defining HDL?
    - From Dog→Tail to Tail→Dog
  - Two levels: onset and invasion, Gap?

- Γ_{max} broadens the SOL width and makes SOL linearly marginally stable: $\gamma(\lambda_m) = 0$
  - $\lambda_m = \beta^{-1/3} 3^{2/3}, \Gamma_{max} = \sigma e^{1+\kappa} \lambda_e$

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