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Intrinsic Multi-Scale Microturbulence in a Stochastic Magnetic Field

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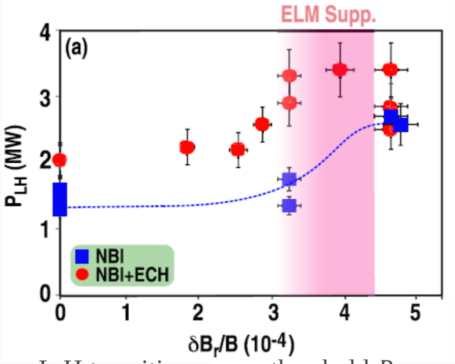
OUTLINE

- Motivation: Instability in a Stochastic Magnetic Field
- Formulation: A Simple Model Maintaining $\nabla \cdot \boldsymbol{J} = 0$
- Analysis: Physical Picture Behind the Calculation
- Conclusion: Lessons learned
- Future: What Next?

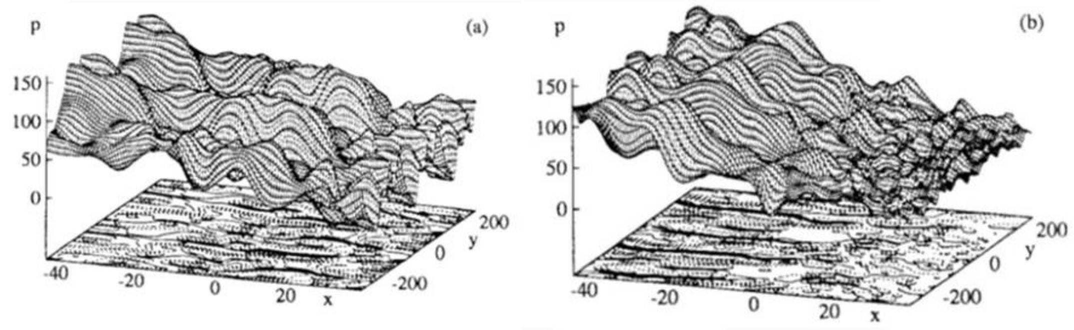
MOTIVATION: INSTABILITY IN A STOCHASTIC MAGNETIC FIELD

Hot topic: a trade-off between good confinement and good power handling



L-H transition power threshold P_{LH} versus RMP perturbation strength[1]

One interesting topic: plasma turbulence in the stochastic layer



plasma pressure in a sector at the low field side with and without RMP[2]

Large-scale pressure fluctuations are suppressed and fluctuation of electric field increases.[2]

How to distinguish the effect of stochastic magnetic field?

—— Complexity-Entropy analysis

Results: the spatial structure of turbulence becomes less predictable[3]

But the physical picture is still unclear…

1. L. Schmitz, D.M. Kriete, et al., 2019.
2. P. Beyer, X. Garbet, and P. Ghendrih, 1998
3. M.J. Choi, et al., 2021.

MOTIVATION: INSTABILITY IN A STOCHASTIC MAGNETIC FIELD

Basic problem: How does stochastic magnetic field modify instability process?

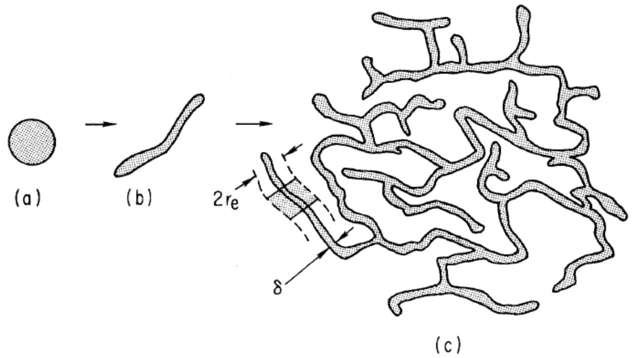
Some history:

- Motivation: Stochastic field transport in the late 1970s[1, 2].
- Early research: Tearing modes in a braided magnetic field[3].
- Defects:
    1. Is quasi-neutrality maintained at all scales?
    2. No micro-macro feedback.

Target: Construct a simple model to get insights and guide simulations

The model is supposed to:

- maintain $\nabla \cdot \boldsymbol{J} = 0$ at all scales
- connect micro and macro scales
- be tractable → resistive interchange

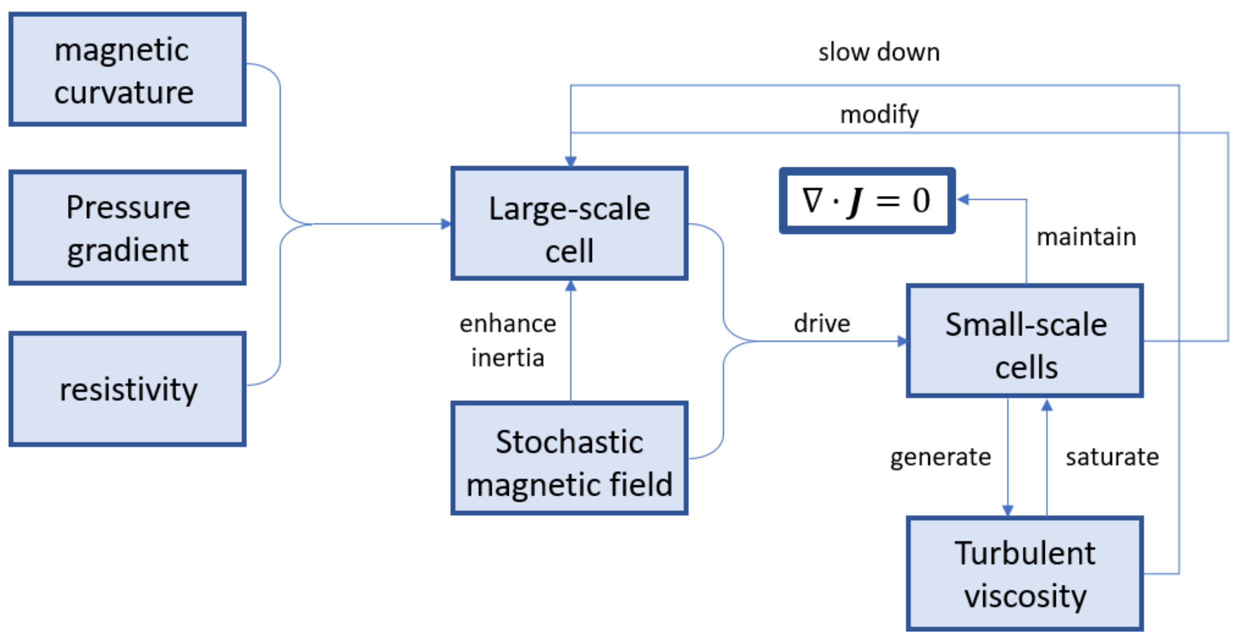


The evolution of area mapping of field lines and guiding-center trajectories (a test particle picture)

1. B.B. Kadomtsev, O.P. Pogutse, 1979.
2. A.B. Rechester, and M.N. Rosenbluth, 1977
3. P.K. Kaw, E.J. Valeo, and P.H. Rutherford, 1979.

FORMULATION: A SIMPLE MODEL MAINTAINING $\nabla \cdot J = 0$

The basic logic:



Key point: potential fluctuations are generated due to stochastic magnetic field

FORMULATION: A SIMPLE MODEL MAINTAINING $\nabla \cdot \boldsymbol{J} = 0$

Where we start:

1. Classical resistive interchange:

- Linearized vorticity equation

$$\underbrace{-(\rho_0/B_0^2)\partial_t \nabla_\perp^2 \varphi}_{\nabla_\perp \cdot \boldsymbol{J_{pol}}} \underbrace{-(g/B_0)\partial_y p}_{\nabla_\perp \cdot \boldsymbol{J_{PS}}} + \underbrace{\boldsymbol{b_0} \cdot \nabla J_\parallel}_{\nabla_\parallel J_\parallel} = 0 \longrightarrow \boxed{\nabla \cdot \boldsymbol{J} = 0}$$

- Electrostatic Ohm's law of resistive MHD

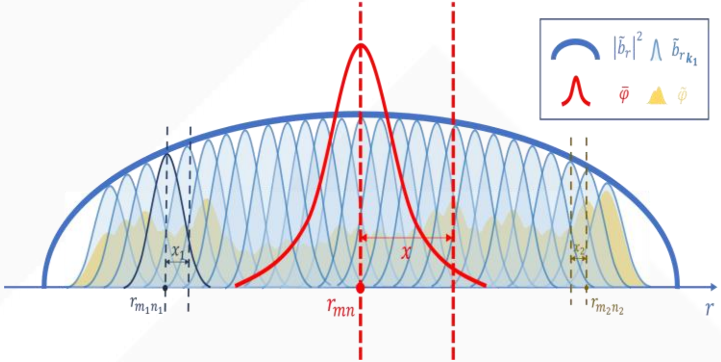
$$E_\parallel = -\nabla_\parallel \varphi = \eta_\parallel J_\parallel$$

- Linearized pressure equation

$$\partial_t p - (\nabla\varphi \times \hat{\boldsymbol{z}})/B_0 \cdot \nabla p_0 = 0$$

2. Magnetic perturbation:

$$\tilde{\boldsymbol{b}} = \tilde{\boldsymbol{B}}_\perp / B_0 = \sum_{m,n} \tilde{\boldsymbol{b}}_{m,n}(x') e^{i(m\theta - n\phi)}$$



Sketch of the mode and stochastic magnetic field

Since $\boldsymbol{B_{tot}} = \boldsymbol{B_0} + \tilde{\boldsymbol{B}}_\perp$, now the parallel gradient is $\nabla_\parallel = \nabla_\parallel^{(0)} + \tilde{\boldsymbol{b}} \cdot \nabla_\perp$.

Compared to mode, the profile of stochastic field evolves much slowly in space.

FORMULATION: A SIMPLE MODEL MAINTAINING $\nabla \cdot \boldsymbol{J} = 0$

Remember: we want to keep $\nabla \cdot \boldsymbol{J} = 0$ at all scales.

If there are only $\tilde{\boldsymbol{b}}$ and $\bar{\varphi}$, $\nabla \cdot \boldsymbol{J} = 0$ is not guaranteed!

At micro scale:

$$\tilde{\boldsymbol{J}} = \tilde{\boldsymbol{J}}_{\parallel} = \tilde{\boldsymbol{J}}_{\parallel^0} + \tilde{\boldsymbol{J}}_{\perp} = -\frac{1}{\eta_{\parallel}}(\tilde{\boldsymbol{b}} \cdot \nabla_{\perp})\bar{\phi}\boldsymbol{b_0} - \frac{1}{\eta_{\parallel}}\nabla_{\parallel}^{(0)}\bar{\phi}\tilde{\boldsymbol{b}}$$

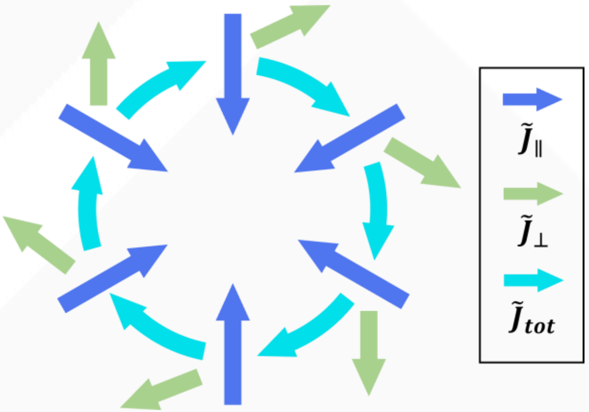
$$\nabla \cdot \tilde{\boldsymbol{J}} = \nabla_{\parallel}^{(0)}\tilde{J}_{\parallel^0} + \nabla_{\perp} \cdot \tilde{\boldsymbol{J}}_{\perp} = -\frac{1}{\eta_{\parallel}}\left\{\nabla_{\parallel}^{(0)}\left[(\tilde{\boldsymbol{b}} \cdot \nabla_{\perp})\bar{\phi}\right] + (\tilde{\boldsymbol{b}} \cdot \nabla_{\perp})\nabla_{\parallel}^{(0)}\bar{\phi}\right\} \neq 0$$

Insights from the classic: Kadomtsev and Pogutse'78[1]:

Electron heat flux is divergence free at all scales → $\nabla \cdot \boldsymbol{q} = 0$

Analogy	K&P	C&D
Goal	$\langle q_r \rangle_{NL}$	$\gamma_{\boldsymbol{k}}^{(1)}$
Base State	$\bar{T}$	$\bar{\varphi}$
Stochastic quantity	$\tilde{\boldsymbol{b}}$	$\tilde{\boldsymbol{b}}$
Constraint	$\nabla \cdot \boldsymbol{q} = 0$	$\nabla \cdot \boldsymbol{J} = 0$
Resulting Fluctuations	$\tilde{T}$	$\tilde{\varphi}$

→ **Intrinsic Multi-Scale Microturbulence**



Small-scale current

1. B. B. Kadomtsev, and O. P. Pogutse, 1979.

FORMULATION: A SIMPLE MODEL MAINTAINING $\nabla\cdot J = 0$

The full set of equations is

① $$\left[\frac{\partial}{\partial t}+\tilde{\boldsymbol{v}}\cdot\nabla\right]\nabla_\perp^2\bar\varphi = -\frac{S}{\tau_A}\Big[\nabla_\parallel^{(0)^2}\bar\varphi + \underbrace{\left(\nabla_\perp\cdot\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}\rangle\right)\cdot\nabla_\perp\bar\varphi}_{(a)} + \underbrace{\left\langle\nabla_\parallel^{(0)}\tilde b\cdot\nabla_\perp\tilde\varphi\right\rangle}_{(b)} + \underbrace{\left\langle\left(\tilde b\cdot\nabla_\perp\right)\nabla_{\parallel'}^{(0)}\tilde\varphi\right\rangle}_{(c)}\Big] - \frac{gB_0}{\rho_0}\frac{\partial\bar p_1}{\partial y},$$

② $$\left[\frac{\partial}{\partial t}+\tilde{\boldsymbol{v}}\cdot\nabla\right]\nabla_\perp^2\tilde\varphi = -\frac{S}{\tau_A}\Big[\nabla_\parallel^{(0)^2}\tilde\varphi + \underbrace{\left(\tilde b\cdot\nabla_\perp\right)\nabla_\parallel^{(0)}\bar\varphi}_{(\alpha)} + \underbrace{\nabla_\parallel^{(0)}\left(\tilde b\cdot\nabla_\perp\right)\bar\varphi}_{(\beta)}\Big] - \frac{gB_0}{\rho_0}\frac{\partial\tilde p_1}{\partial y},$$

→ Relate $\tilde\varphi$ to $\tilde{\boldsymbol{b}}$

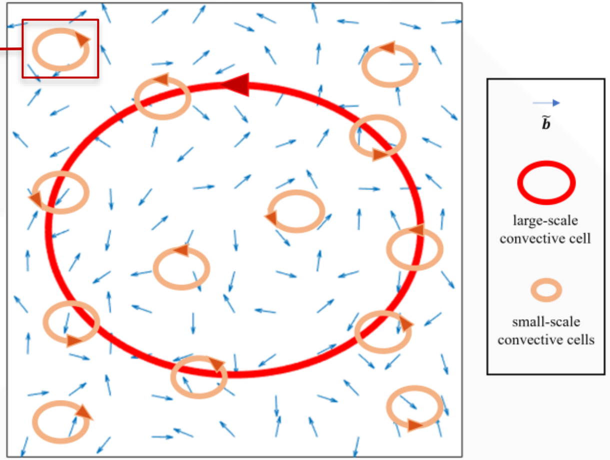
③ $$\left[\frac{\partial}{\partial t}+\tilde{\boldsymbol{v}}\cdot\nabla\right]\bar p_1 - \frac{\nabla\bar\varphi\times\hat{\mathbf{z}}}{B_0}\cdot\nabla p_0 = 0,$$

④ $$\left[\frac{\partial}{\partial t}+\tilde{\boldsymbol{v}}\cdot\nabla\right]\tilde p_1 - \frac{\nabla\tilde\varphi\times\mathbf{z}}{B_0}\cdot\nabla p_0 = 0,$$

$\left[\tilde{\boldsymbol{v}}\cdot\nabla\right]$ replaced by $-\nu_T\nabla_\perp^2$ or $\chi_T\nabla_\perp^2$

$$\langle A\rangle = \bar A = \frac{1}{(2\pi)^2}\iint d\theta d\phi e^{-i(m\theta-n\phi)}A$$

Turbulent viscosity or Turbulent diffusivity



Three players in the model: $\tilde{\boldsymbol{b}}$, $\bar\varphi$, and $\tilde\varphi$

Some assumptions/observations:

- $\bar\varphi$: low $\boldsymbol{k}$, slow interchange approximation $\left(1/w_{\boldsymbol{k}}^2 \gg k_y^2\right)$
- $\tilde\varphi$: high $\boldsymbol{k}'$, fast interchange approximation $\left(1/w_{\boldsymbol{k}'}^2 \ll {k_y'}^2\right)$
- The beat of $\tilde{\boldsymbol{b}}$ and $\tilde\varphi$ serves as the drive of $\bar\varphi$ while $\bar\varphi$ modifies $\tilde\varphi$, thus small scale and large scale are now connected.

ANALYSIS: PHYSICAL PICTURE BEHIND THE CALCULATION

Main equation:

$$① \left(\frac{\partial}{\partial t} - \nu_T \nabla_\perp^2\right)\nabla_\perp^2 \bar{\varphi} + \frac{S}{\tau_A}\underbrace{\left(\nabla_\perp \cdot \langle \tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}\rangle\right) \cdot \nabla_\perp \bar{\varphi}}_{(a)} = -\frac{S}{\tau_A}\left[\nabla_\parallel^{(0)^2} \bar{\varphi} + \underbrace{\left\langle \nabla_\parallel^{(0)} \tilde{\boldsymbol{b}} \cdot \nabla_\perp \tilde{\varphi}\right\rangle}_{(b)} + \underbrace{\left\langle (\tilde{\boldsymbol{b}} \cdot \nabla_\perp)\nabla_\parallel^{(0)} \tilde{\varphi}\right\rangle}_{(c)}\right] - \frac{gB_0}{\rho_0}\frac{\partial \bar{p}_1}{\partial y}$$

(a): $\frac{S}{\tau_A}\left(\nabla_\perp \cdot \langle \tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}\rangle\right) \cdot \nabla_\perp \bar{\varphi} = \frac{S}{\tau_A}\partial_x |b_r|^2 \partial_x \bar{\varphi}_{\boldsymbol{k}}(x)$ → magnetic vorticity damping

→ 3^{rd} order $\nabla_\| J_\|$

$\gamma_{\boldsymbol{k}} \partial_x^2 \bar{\varphi}_{\boldsymbol{k}} + \frac{S}{\tau_A}\partial_x |b_r|^2 \partial_x \bar{\varphi}_{\boldsymbol{k}}(x)$ → Enhance inertia

As $\frac{S}{\tau_A}\left|\frac{\tilde{B}_{rk'}}{B_0}\right|^2 \sim \frac{V_A^2}{\eta}\frac{k_\theta'^2}{L_S^2} W_I'^4$ → $\frac{S}{\tau_A}\partial_x\left(\left|\tilde{b}_r\right|^2 \partial_x \bar{\varphi}\right) \sim \frac{v_A^2}{\eta}\frac{k_y^2}{L_s^2}\frac{o_I'^4}{(\Delta x)^2}\bar{\varphi}$ (3rd order magnetic torque)

$(\nabla_\| J_\|)^{(1)} \sim \frac{S}{\tau_A}\nabla_\|^{(0)^2}\bar{\varphi} \sim \frac{v_A^2}{\eta}\frac{k_y^2}{L_S^2}(\Delta x)^2$

$o_I' \equiv$ island width for stochastic field
$\Delta x \equiv \bar{\varphi}$ layer width

When $w_I' \sim \left[\frac{k_y^2}{k_y'^2}(\Delta x)^4\right]^{\frac{1}{4}}$, 3rd order magnetic torque balances 1st order. This is a reminiscent of Rutherford '73[1] while the difference is **significant**: the ratio $(k_y^2/k_y'^2)$ is due to the multi-scale characteristic.

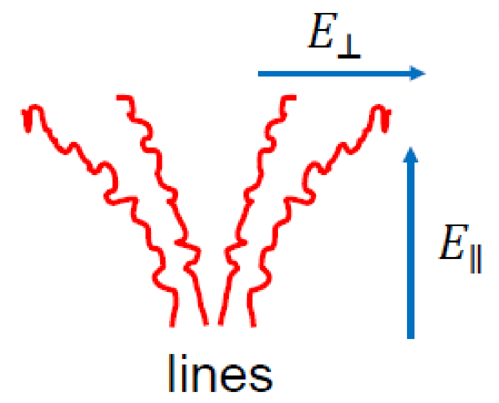
1. P. H. Rutherford, 1973.

ANALYSIS: PHYSICAL PICTURE BEHIND THE CALCULATION

$$(b) = \left\langle \nabla_{\parallel}^{(0)} \nabla_{\perp} \cdot (\tilde{\boldsymbol{b}}\tilde{\varphi}) \right\rangle = \left\langle -\nabla_{\parallel}^{(0)} (\underline{\tilde{\boldsymbol{b}} \cdot \tilde{\boldsymbol{E}}_{\perp}}) \right\rangle$$

$$(c) = \left\langle \nabla_{\perp} \cdot \left(\tilde{\boldsymbol{b}} \nabla_{\parallel}^{(0)} \tilde{\varphi}\right) \right\rangle = -\nabla_{\perp} \cdot \langle \underline{\tilde{\boldsymbol{b}}_{\perp} \tilde{E}_{\parallel 0}} \rangle$$

E field projections along wandering tilting lines



- Perpendicular electric field $\tilde{\boldsymbol{E}}_{\perp}$ generates a parallel current.
- Parallel electric field $\tilde{E}_{\parallel 0}$ generates a perpendicular current

Why do we have ν_T? What's the value of ν_T?

$\tilde{\varphi}$ is the so-called intrinsic multi-scale microturbulence, which is the origin of ν_T.

Separation of time scales:

The time scale of $\bar{\varphi}$ is much larger than that of $\tilde{\varphi}$. Therefore, ν_T should be large enough so that $\tilde{\varphi}$ is over-saturated during the evolution of $\bar{\varphi}$. The scaling of ν_T is given by the nonlinear closure theory:

$$\nu = \sum_{\boldsymbol{k}_2} \left|\tilde{v}_{\boldsymbol{k}_2}\right|^2 \tau_{\boldsymbol{k}_2}$$

CONCLUSION : LESSONS LEARNED

- A full set of equations for $\bar{\varphi}$'s evolution in presence of $|b_{\boldsymbol{k}'}|^2$.
- Calculate the correction to the growth rate of resistive interchange in a stochastic magnetic field:

$$\gamma_k^{(1)} = -\frac{1}{3}\left(\frac{S}{\tau_A}\right)|\tilde{b}_r|^2 - \frac{5}{6}\nu\left(\frac{S}{\tau_A}\right)^{\frac{2}{3}}\left(\frac{k_\theta}{L_s^2}\right)^{\frac{2}{3}}\left(\frac{\rho_0 L_p}{g p_0}\right)^{\frac{1}{3}} - \frac{2\sqrt{2}}{3} Rr_{mn}\left(\frac{S}{\tau_A}\right)^{\frac{4}{3}}\left(\frac{k_\theta^4}{L_s^5}\right)^{\frac{1}{3}}\left(\frac{\rho_0 L_p}{g p_0}\right)^{-\frac{1}{3}} I .$$

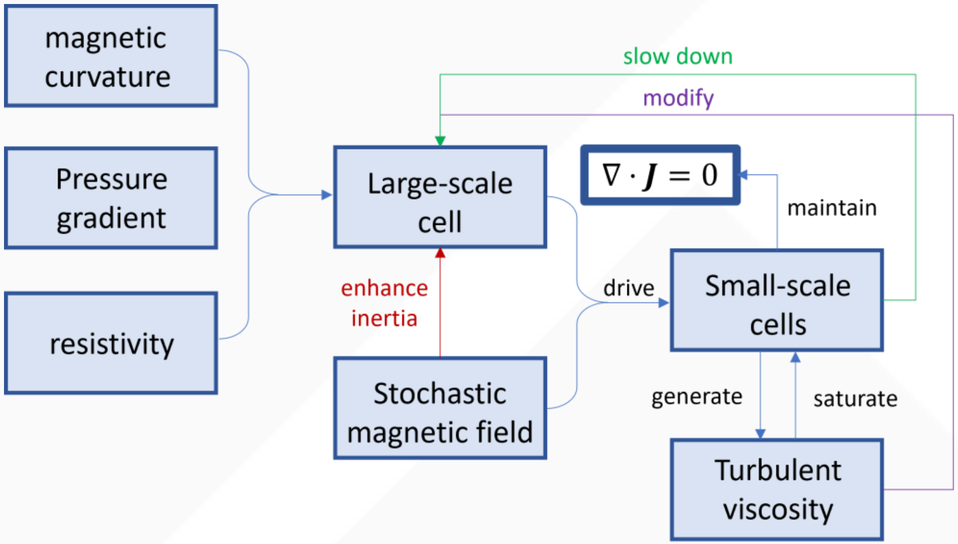
- Calculate the scaling of turbulent viscosity in the weak-mean-pressure-gradient limit:

$$\nu = \left[\pi^{\frac{1}{2}}\frac{Rr_{mn}}{B_0^2}\frac{k_\theta^2}{L_s^3}\left(\frac{S}{\tau_A}\right)^2 \bar{\varphi}_{\boldsymbol{k}}^2(0)\int dk_{2\theta}\,\frac{c^2 Z^2 w_{\boldsymbol{k}_2} o_{\boldsymbol{k}_2}^2}{|k_{2\theta}|^5 \gamma_{\boldsymbol{k}_2}^{(0)}}\right]^{\frac{1}{3}}$$

- Obtain the non-trivial $\langle \tilde{b}_r \tilde{\varphi} \rangle$

$$\langle \tilde{b}_r \tilde{v}_r \rangle = \pi^{\frac{1}{2}}\frac{k_\theta Rr_{mn}}{L_s^2 B_0}\frac{S}{\tau_A}\bar{\varphi}_{\boldsymbol{k}}(0)\int d\,k_{2\theta}|k_{2\theta}|k_{2\theta}\,\frac{c^2 Z^2 (k_\theta - k_{2\theta}) w_{\boldsymbol{k}_2} o_{\boldsymbol{k}_2}^2}{\Lambda_{\boldsymbol{k}_2}^0 - \Lambda_{\boldsymbol{k}_2}}$$

The Feedback Loop



CONCLUSION : LESSONS LEARNED

- Intrinsically a multi-scale problem: $\bar{\varphi}$; $\tilde{\varphi}$ and $\tilde{\boldsymbol{b}}$

  To maintain $\nabla \cdot \boldsymbol{J} = 0$ at all scales for prescribed $\tilde{\boldsymbol{b}}$ and instability$\bar{\varphi}$, $\tilde{\varphi}$(microscopic convective cells) is generated[1]. (The fluctuation amplitude increases significantly with the RMP ELM suppression, and the fluctuations exhibit the less predictable characteristics → imply the existence of $\tilde{\varphi}$)

- This yields a non-trivial $\langle \tilde{b}_r \tilde{\varphi} \rangle$, i.e., electrostatic turbulence ‘locks on’ to magnetic perturbation.
- In the weak-mean-pressure-gradient limit, the scaling of turbulent viscosity is obtained by using nonlinear closure theory.

1. M.J. Choi, et al., 2021.

CONCLUSION : LESSONS LEARNED

$$\gamma_k^{(1)} = -\frac{1}{3}\left(\frac{S}{\tau_A}\right)|\tilde{b}_r|^2 - \frac{5}{6}\nu\left(\frac{S}{\tau_A}\right)^{\frac{2}{3}}\left(\frac{k_\theta}{L_s^2}\right)^{\frac{2}{3}}\left(\frac{\rho_0 L_p}{g p_0}\right)^{\frac{1}{3}} - \frac{2\sqrt{2}}{3} R r_{mn}\left(\frac{S}{\tau_A}\right)^{\frac{4}{3}}\left(\frac{k_\theta^4}{L_s^5}\right)^{\frac{1}{3}}\left(\frac{\rho_0 L_p}{g p_0}\right)^{-\frac{1}{3}} I$$

- The effect of stochastic magnetic field is to slow down the mode growth through three channels:
  - Magnetic vorticity damping effect (enhanced inertia)

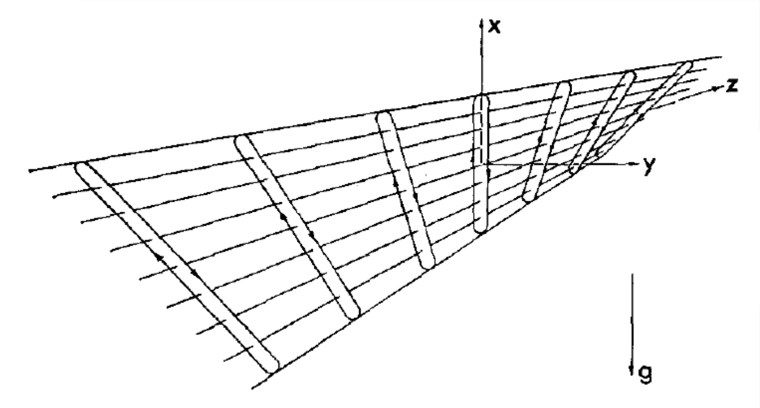
    $$inertia \rightarrow inertia + \frac{S}{\tau_A}\partial_x |b_r|^2 \partial_x \bar{\varphi}$$

    $w_I' \sim \left[(k_y^2/k_y'^2)(\Delta x)^4\right]^{1/4}$, when $(\nabla_\parallel J_\parallel)^{(1)} \sim (\nabla_\parallel J_\parallel)^{(3)}$.

    Magnetic vorticity damping is stronger than Rutherford's problem, for $k_y \ll k_y'$.
  - Turbulent viscosity ν_T tends to stabilize the mode.
  - Large-scale mode is electrostatically scattered by small-scale convective cells. In the weak-mean-pressure-gradient limit, the existence of small-scale convective cells can reduce the growth rate of large-scale mode.

FUTURE: WHAT NEXT?

- Another way to solve it? Schrodinger equation with 1-D random potential.
- Can the turbulent diffusivity χ_T we find explain “RMP pump-out” effect?
- Look at effects of stochastic magnetic field $\tilde{\boldsymbol{b}}$ on twisted slicing modes, i.e., include toroidicity.



Twisted slicing modes

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Thank you