Ion Heat and Parallel Momentum Transport by Stochastic Magnetic Fields and Turbulence

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Outline

- Introduction

  Transport of momentum and ion heat in stochastic magnetic field .

- Results

  a. In strong turbulence regime, the mean flow is driven by **stochastic-turbulent scattering**.

  b. Stochastic lines and parallel ion flow gradient drives a **net electron particle flux**, in additional to the Maxwell force contribution.

- Conclusions

Why we study stochastic fields in fusion device?

Shear Flow

↓

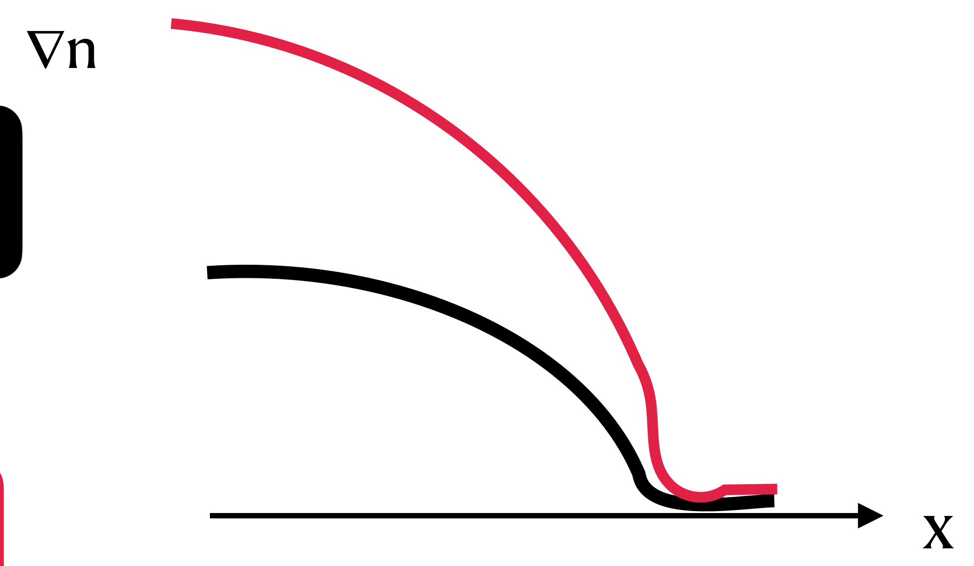
Quench Turbulence

↓

Pedestal Formation

↓

Edge-Localized Mode (ELM)



Boundary Control: Resonant Magnetic Perturbation (RMP)

Suppress (by inducing magnetic perturbation)

↓

Edge-Localized Mode (ELM)

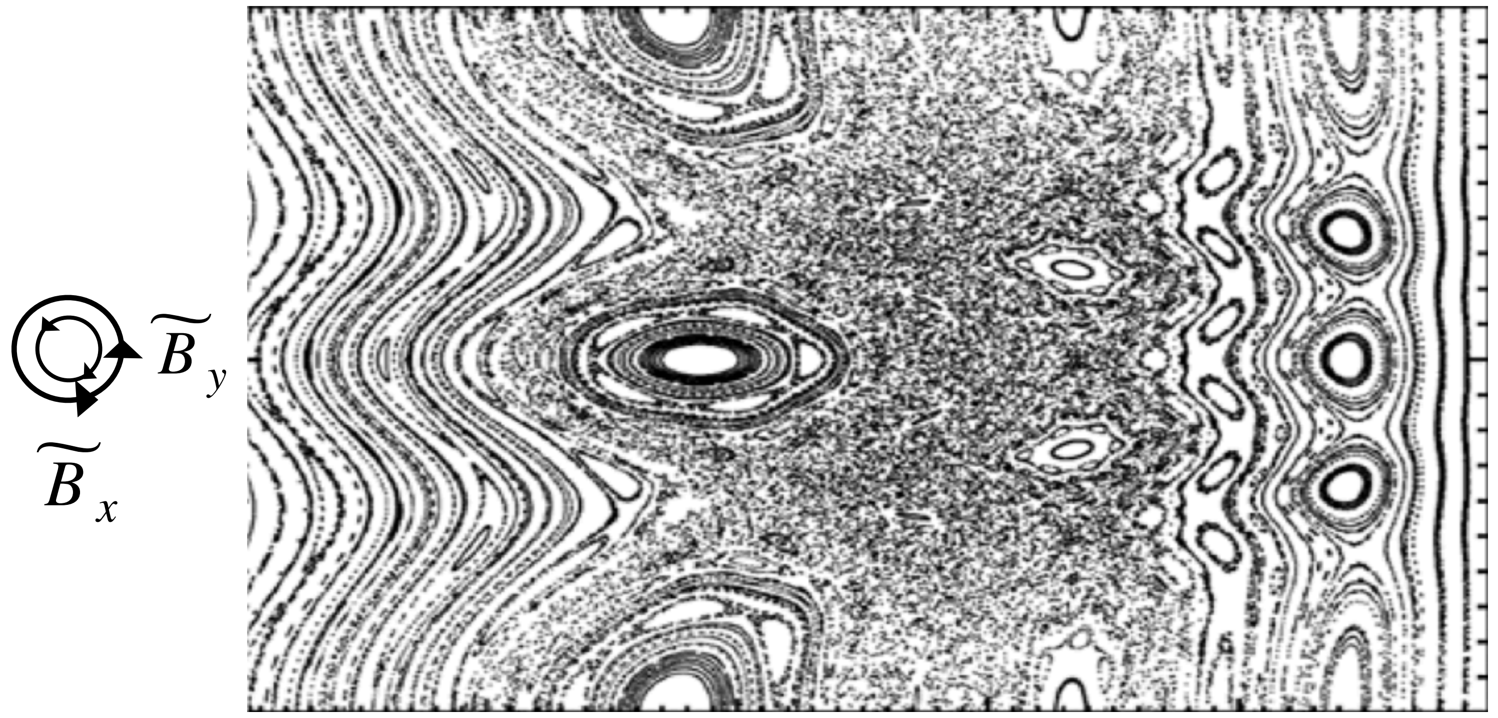
Trade off: RMPs controls gradients and mitigates ELM, but raise the **power threshold**.

- ELMs are quasi-periodic relaxation events occurring at edge pedestal in H-mode plasma.
- ELMs can damage wall components of a fusion device.

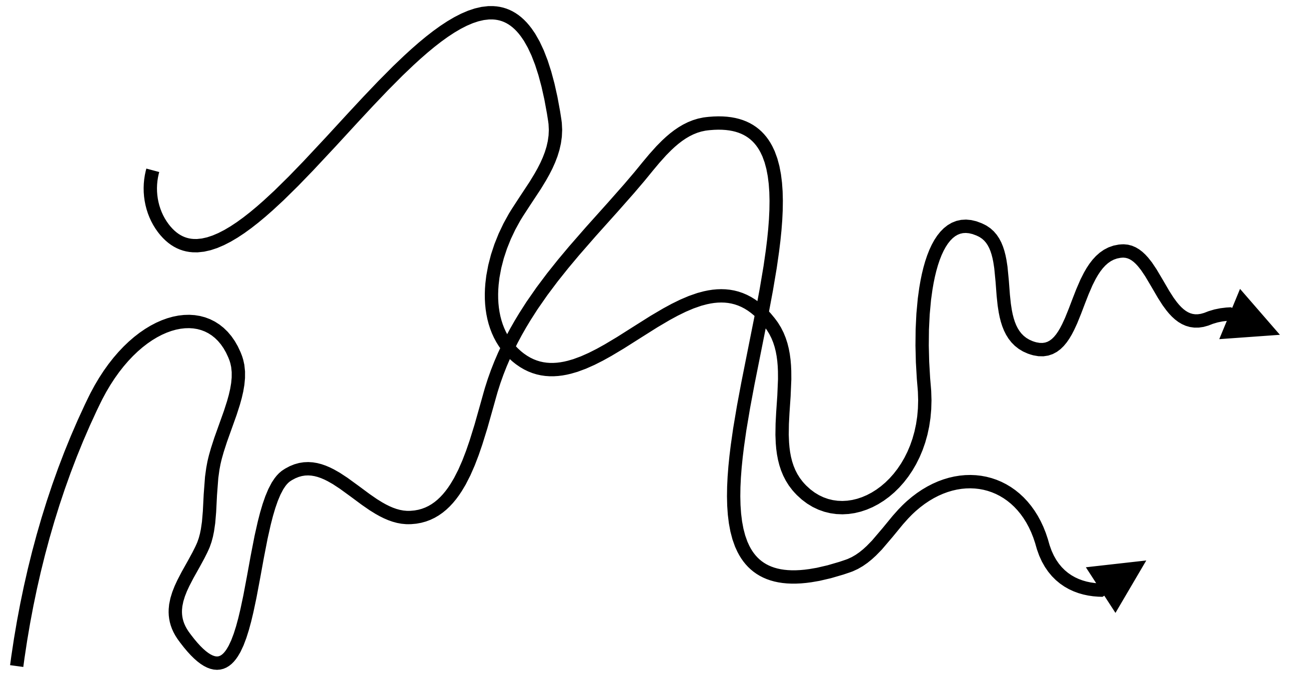
The **transport of parallel momentum** and **pressure** in presence of stochastic field is important in studies of fusion plasma.

Coexistence of Stochastic Field and Turbulence

Before L-H transition, L-mode plasma with RMP:



Magnetic islands overlapping forms stochastic fields



Strong electrostatic turbulence

Both **stochastic field** and **turbulence** enter the cross-phase $\langle \widetilde{b}\widetilde{p} \rangle$, $\langle \widetilde{b}\widetilde{u}_{\parallel} \rangle$, and hence enter the dephasing mechanism.

Key question:

How does stochastic fields influence on the response of parallel flow and pressure in strong/weak turbulence regime?

We analyze the dephasing effect of stochastic field in strong and weak electrostatic turbulence—how they *together* drives transport.

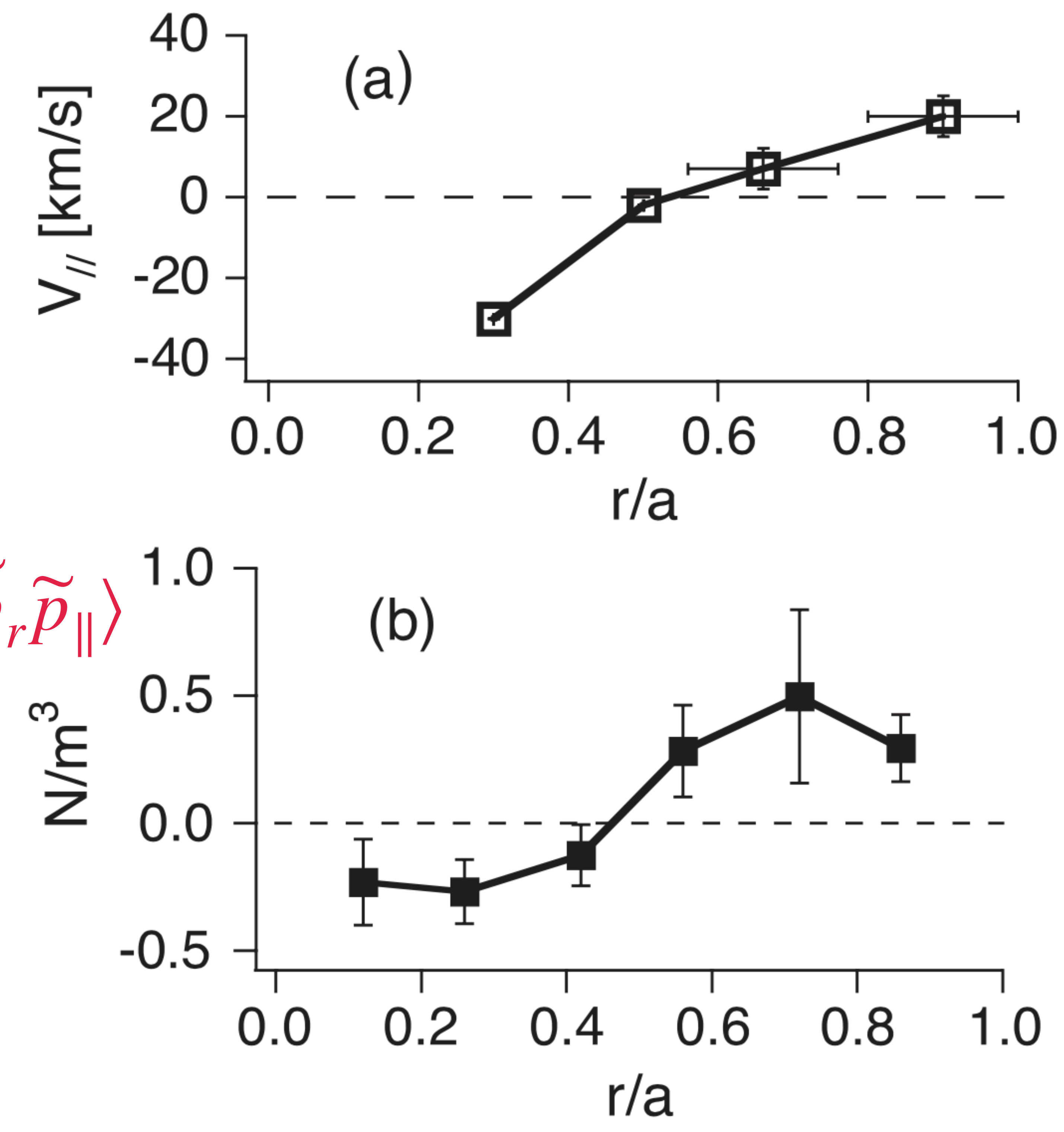
(Chen et al., PPCF, accepted (2021))

Example

Experimental Result of Madison Symmetric Torus (MST)

$\langle u_{\parallel} \rangle$

$-\rho \frac{\partial}{\partial r} \langle \tilde{b}_r \tilde{p}_{\parallel} \rangle$



(Ding et al., PRL **110**, 065008 (2013))

Macroscopic parallel flow dynamics.

Nonlinear momentum transport

Microscopic effect measured from the fluctuations of the pressure and the stochastic field.

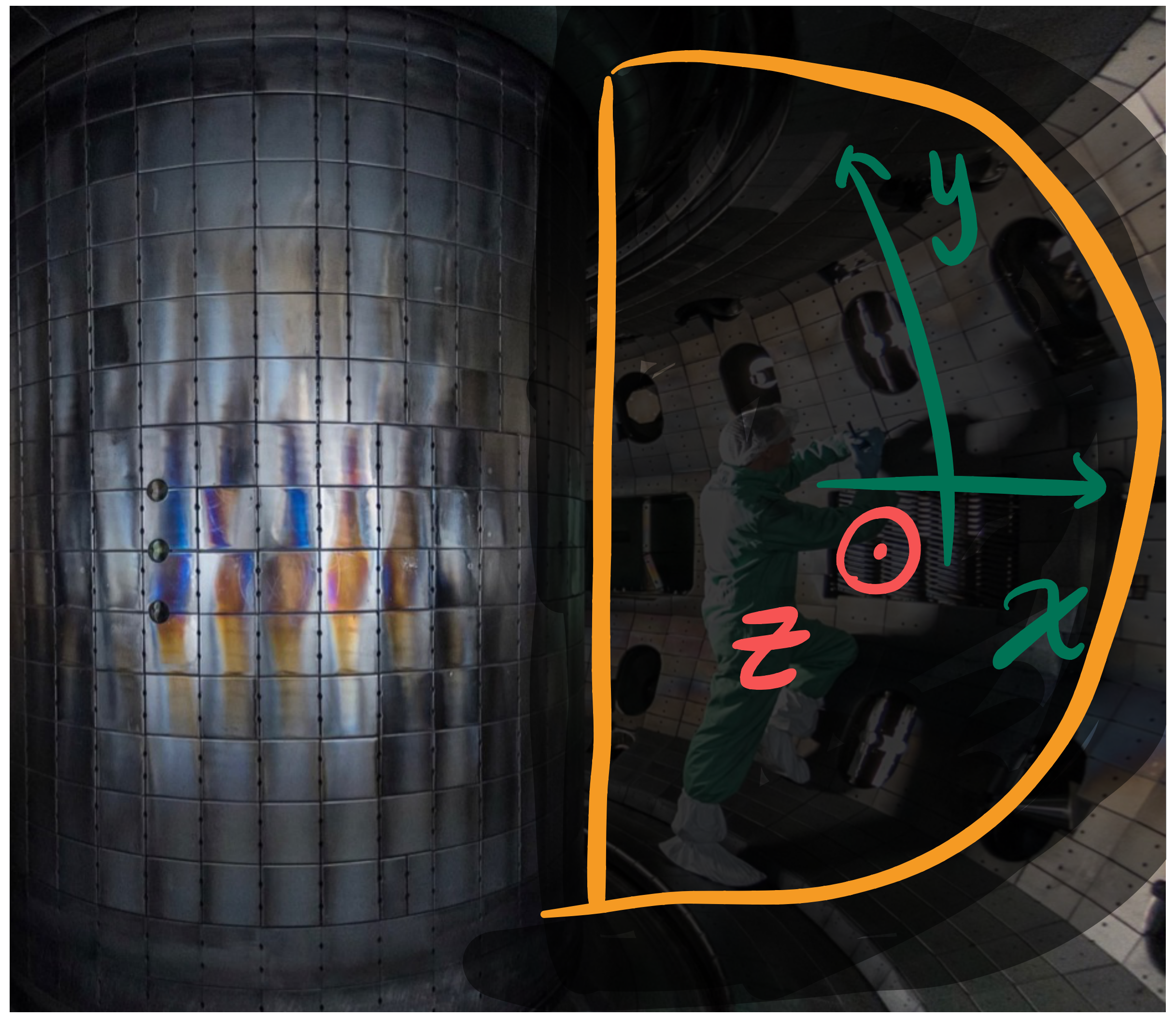
MST Experimental results: demonstrated the similarity of the kinetic stress to the parallel flow.

Model

1. Cartesian coordinate: strong mean field B_0 is in z direction (3D).
2. Rechester & Rosenbluth (1978): waves, instabilities, and transport are studied in the presence of **external excited, static, stochastic fields.**
3. $\underline{k} \cdot \underline{B} = 0$ (or $k_{\parallel} = 0$) **resonant at rational surface in third direction, and** Kubo number: $Ku_{mag} = l_{ac} | \widetilde{\mathbf{B}} | / \Delta_{\perp} B_0$).
4. Equations:

(a) Pressure equation: $\frac{\partial}{\partial t} p + (\mathbf{u} \cdot \nabla) p = - \gamma p (\nabla_z \cdot \mathbf{u}_z)$

(b) Parallel flow equation: $\frac{\partial}{\partial t} u_z + (\mathbf{u} \cdot \nabla) u_z = - \frac{1}{\rho} \nabla_z p$



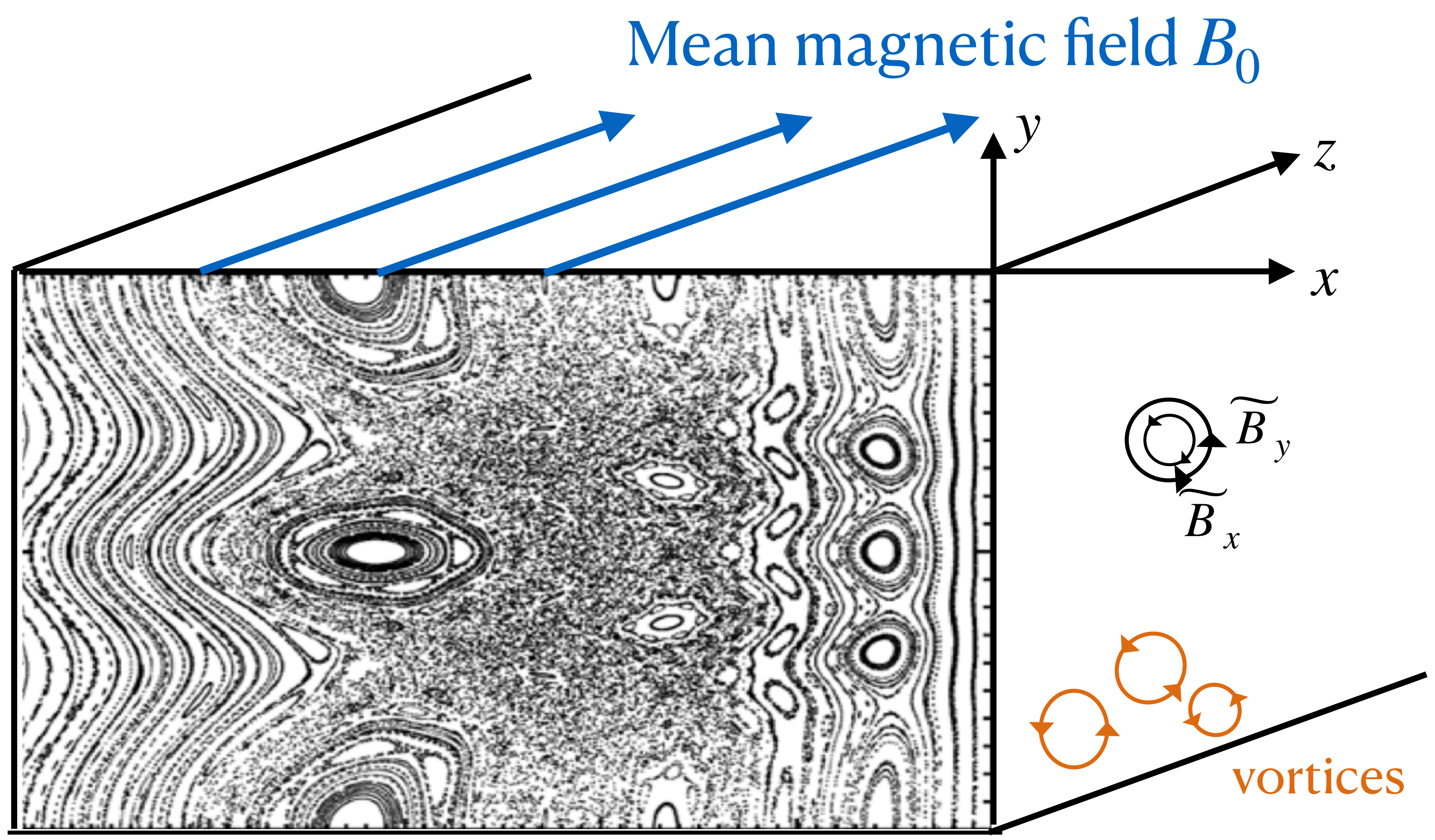
We use mean field approximation: $\nabla_z = \nabla_z^{(0)} + \widetilde{b} \cdot \nabla_{\perp}$

$p = \langle p \rangle + \widetilde{p}$, Perturbations produced by turbulences

$$where \ \langle \ \rangle = \frac{1}{L_{\parallel}} \int dL_{\parallel} \frac{1}{2\pi r} \int r d\theta$$

ensemble average over the symmetry direction

We define rms of normalized stochastic field $\widetilde{b} \equiv \sqrt{\langle \widetilde{B}^2 \rangle / B_0^2}$



Magnetic islands overlapping forms stochastic

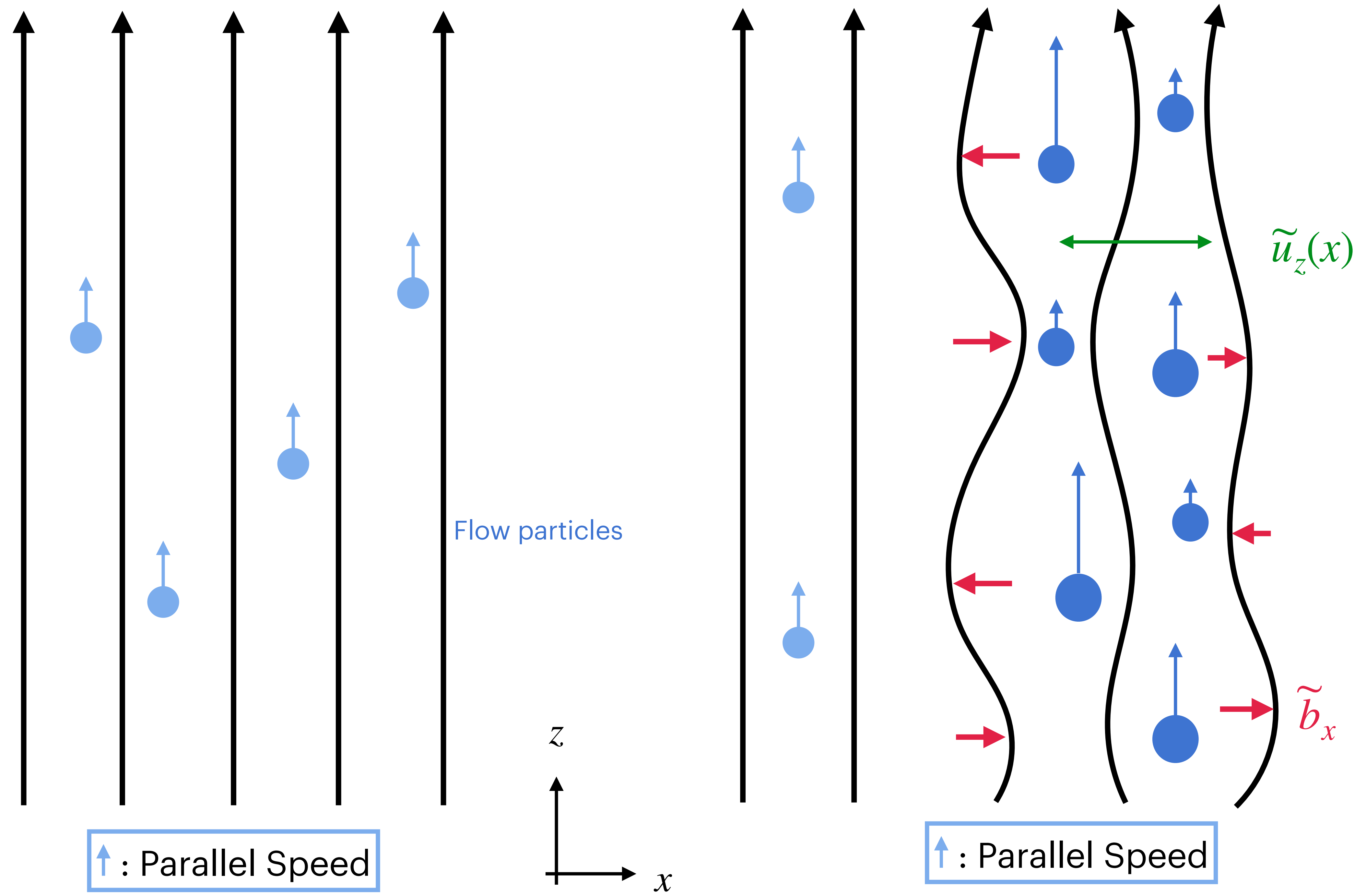
Physical Picture of Pressure Response

Local **pressure excess** ($\tilde{b}_r \partial_r \langle p \rangle$) caused by magnetic perturbation is balanced by: ($u_\parallel$ response in the same way)

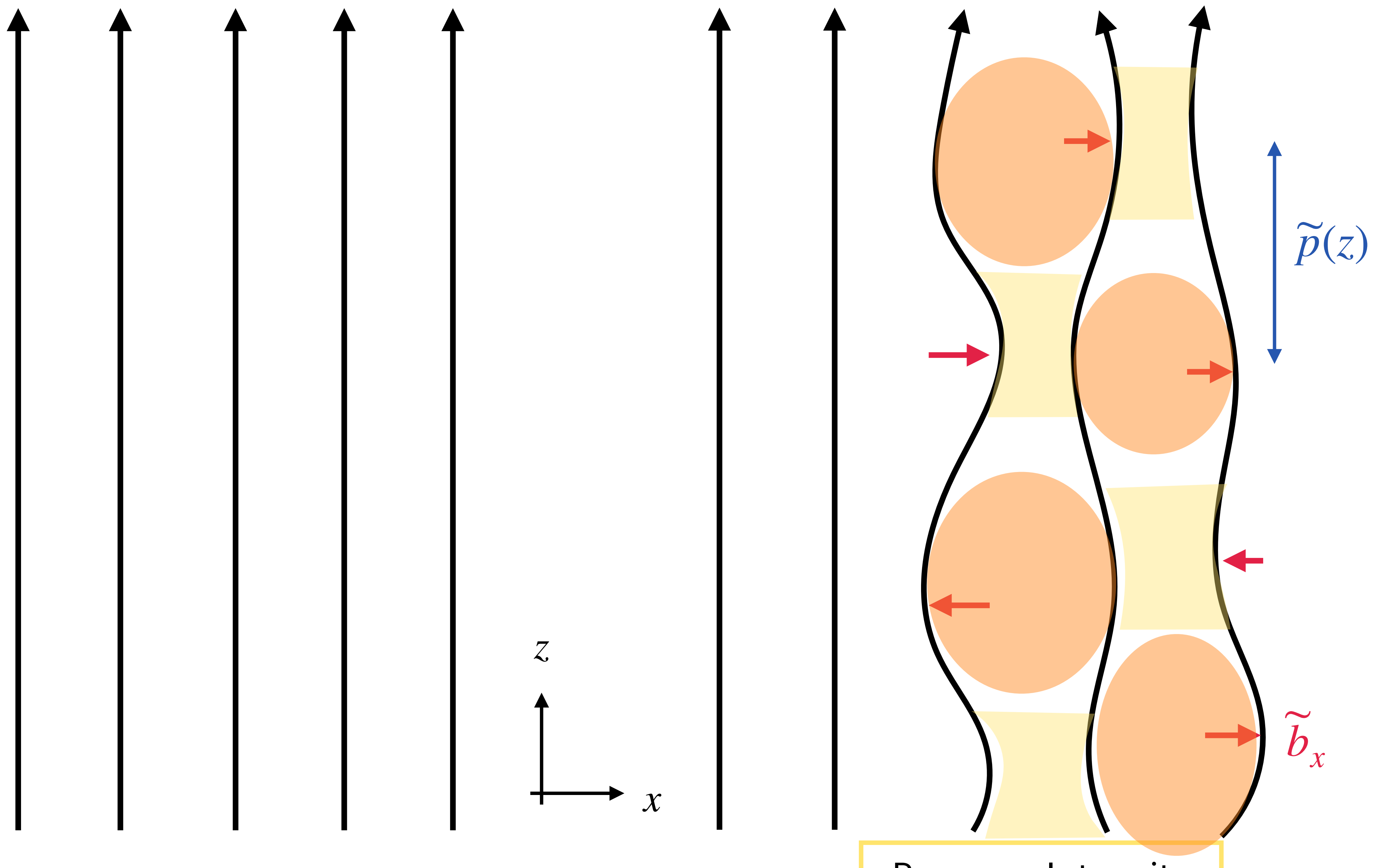
... by parallel flow perturbation, which is damped by turbulent viscosity.

... by parallel pressure gradient. → Finn et al., PoP **4**, 1152 (1992)

1 Strong Turbulence: $\tilde{b}_r \nabla_r \frac{\langle p \rangle}{\rho} \simeq D_T \nabla_\perp^2 \tilde{u}_z$



2 Weak Turbulence: $\tilde{b}_r \nabla_r \langle p \rangle \simeq -\nabla_z \tilde{p}$ $(\mathbf{B} \cdot \nabla p = 0)$.



Rate of turbulent (i.e. viscous) mixing $D_T/l_\perp^2$ > other rate: turbulent viscosity will dissipate the parallel flow.

Only strong turbulent cases are relevant!

Rate of sound propagation $c_s/l_\parallel$ > other rate: pressure gradient builds up parallelly.

Kinetic Stress and Compressible Energy Flux

Mean field equation for parallel flow and the pressure equation:

$$\frac{\partial}{\partial t}\langle u_z\rangle + \frac{\partial}{\partial x}\langle \widetilde{u}_x\widetilde{u}_z\rangle = -\frac{1}{\rho}\frac{\partial}{\partial x}\langle \widetilde{b}_x\widetilde{p}\rangle \equiv -\frac{\partial}{\partial x}K, \text{ where the } \textbf{kinetic stress } K \equiv \frac{1}{\rho}\langle \widetilde{b}_x\widetilde{p}\rangle$$

$$\frac{\partial}{\partial t}\langle p\rangle + \frac{\partial}{\partial x}\langle \widetilde{u}_x\widetilde{p}\rangle = -\rho c_s^2\frac{\partial}{\partial x}\langle \widetilde{b}_x\widetilde{u}_z\rangle \equiv -\frac{\partial}{\partial x}H, \text{ where the } \textbf{compressible heat flux } H \equiv \rho c_s^2\langle \widetilde{b}_x\widetilde{u}_z\rangle$$

(or, ion heat density flux)

Perturbed equation with **Riemann variables** $f_\pm \equiv \widetilde{u}_{z,k\omega} \pm \dfrac{\widetilde{p}_{k\omega}}{\rho c_s}$:

The propagator $1/(k_\perp^4 D_T^2 + k_z^2 c_s^2)$ contains the turbulent mixing ($k_\perp^4 D_T^2$) and the magnetic shear effect ($k_z^2 c_s^2$).

Magnetic shear Effect

$$k_z^2 c_s^2 = \left(\frac{k_y x}{L_s}\right)^2 c_s^2$$

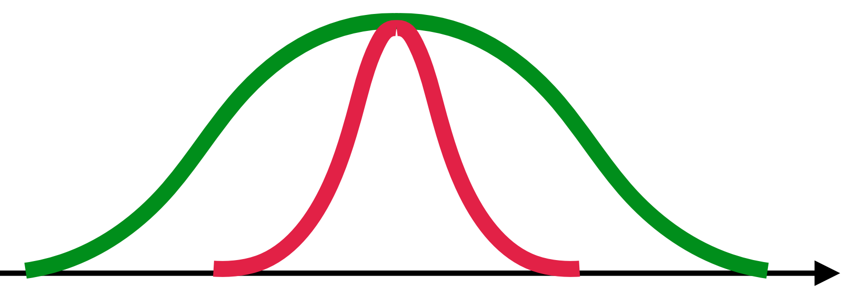
Magnetic shear

$$\hat{s} = \frac{r_0}{q}\frac{dq}{dr}$$

Summation:

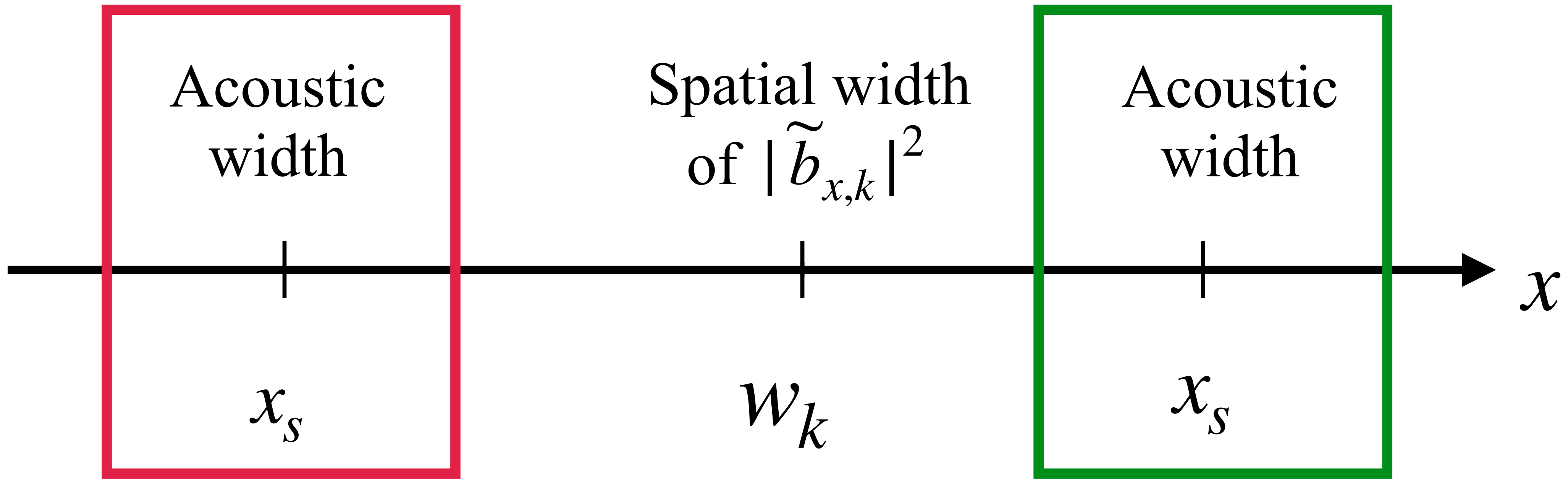
$$\sum_{k_y k_z} = \int dm \int dn = \int dm \int dx \frac{|m|}{q^2} q' = r_0 \int dk_y \int dx \frac{|k_y|}{q}\hat{s}$$

$$\langle \widetilde{b}_x \widetilde{p}\rangle = \overset{①}{\int dk_y \frac{k_y r_0^2 q'}{q^2}\int dx\, |\widetilde{b}_{x,k}|^2 \frac{\tau_{c,k}}{1+(x/x_s)^2}\left(-\rho c_s^2 \frac{\partial}{\partial x}\langle u_z\rangle\right)} \overset{②}{+ \int dk_y \frac{k_y r_0^2 q'}{q^2}\int dx\, |\widetilde{b}_{x,k}|^2 \frac{1}{(k_\perp^2 D_T)^2 + k_z^2 c_s^2}\left(ik_z c_s^2 \frac{\partial}{\partial x}\langle p\rangle\right)}$$



Scales

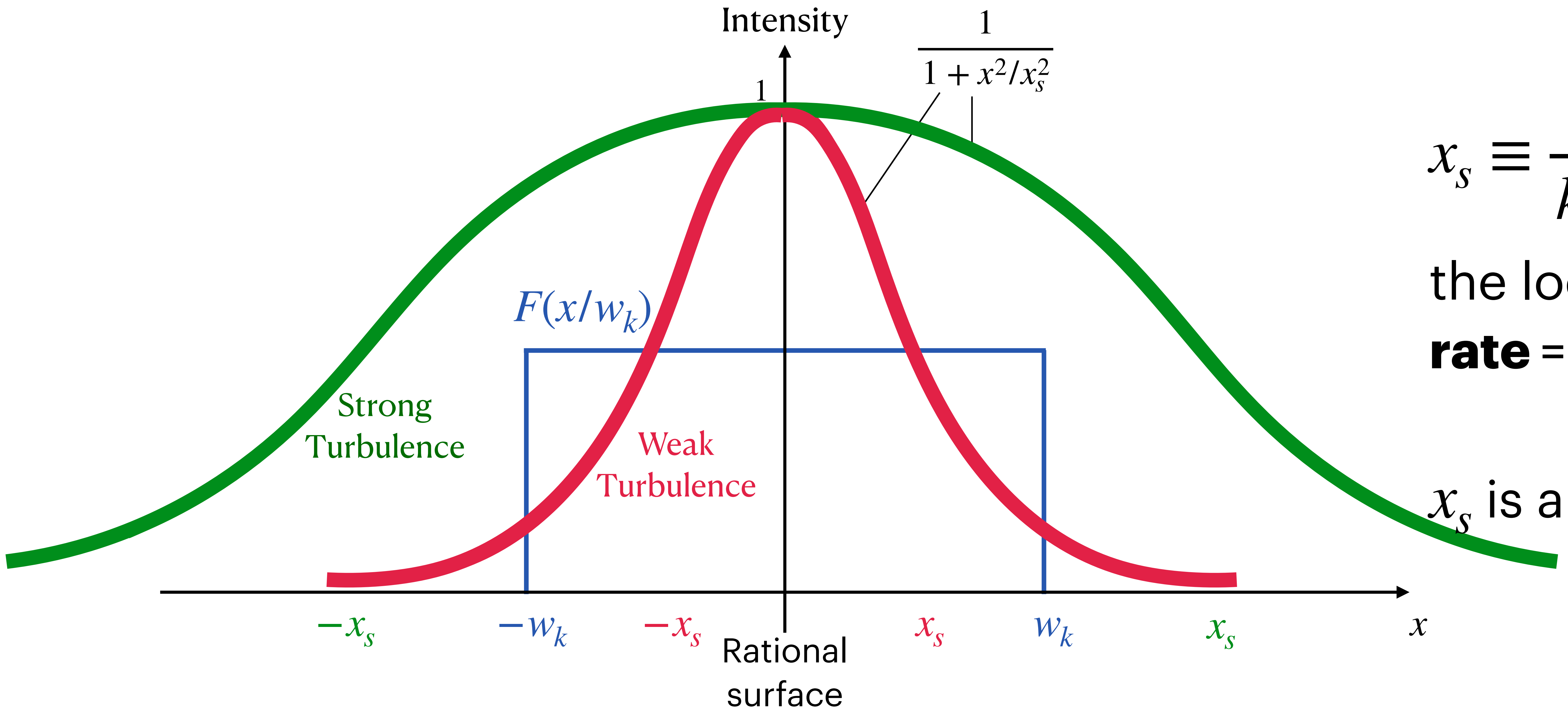
We consider length scales:



Weak turbulence regime

Strong turbulence regime

Dimensionless parameter
$$\lambda \equiv \frac{x_s}{w_k},$$
defines the competition between the stochastic-field and turbulent effect.



$x_s \equiv \dfrac{L_s}{k_y c_s \tau_{c,k}}$ is the **acoustic width**—x_s defines the the location where the **parallel acoustic streaming rate** = **decorrelation rate.**

x_s is analogous to ion Landau resonant point.

Relevant cases

Results—Strong Turbulence Regime

In strong turbulence ($k_\perp^2 D_T \gg k_z c_s$ or $\lambda > 1$):

$$K \equiv \frac{1}{\rho}\langle \tilde{b}_x \tilde{p} \rangle \simeq -D_{st}\frac{\partial}{\partial x}\langle u_z \rangle$$, where

$$D_{st} = D_{st}(x) = \sum_{k_y k_z} |\tilde{b}_{x,k}|^2 \frac{c_s^2}{k_\perp^2 D_T}$$

Stochastic B field

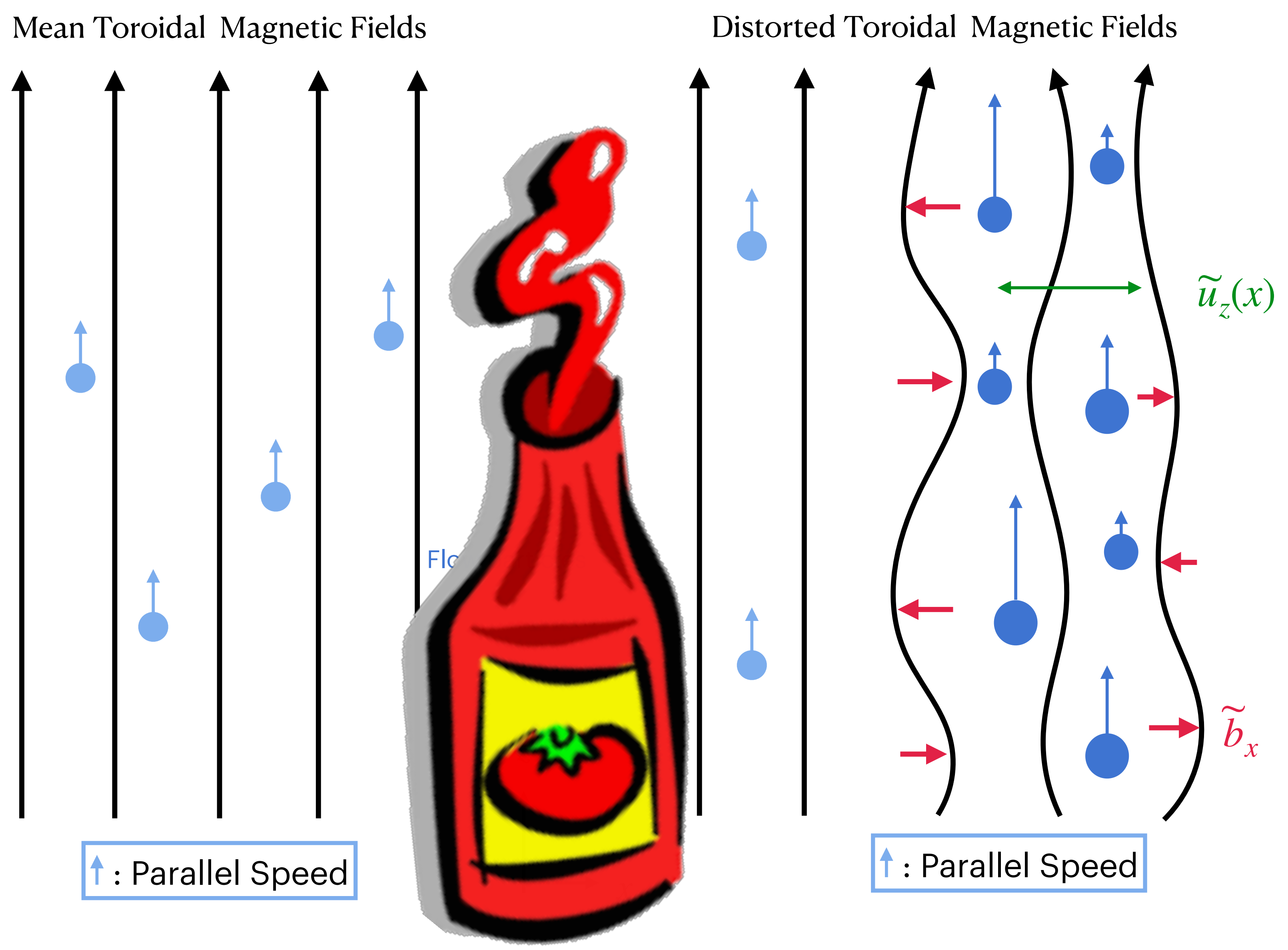
Turbulent scattering decorrelation rate

Turbulent fluid diffusivity $D_T \equiv \sum_k |\tilde{u}_{\perp,k}|^2 \tau_{ac}$

D_{st} : the **hybrid turbulent diffusivity**—explain how the kinetic stress is scattered by stochastic B fields and turbulence.

$$H \equiv \rho c_s^2 \langle \tilde{b}_x \tilde{u}_z \rangle \simeq -D_{st}\frac{\partial}{\partial x}\langle p \rangle$$

Strong Turbulence: $\tilde{b}_r \nabla_r \frac{\langle p \rangle}{\rho} \simeq D_T \nabla_\perp^2 \tilde{u}_z$



Pressure gradient $\partial\langle p \rangle/\partial x$ due to $\tilde{b}$ is balanced by turbulent mixing of parallel flow $\nabla_\perp^2 \tilde{u}_z$.

The pressure gradient in presence of tilted B lines balances with the *hybrid* turbulent diffusion.

Electron Particle Flux

Consider electron density evolution:

$$\frac{\partial\langle n_e\rangle}{\partial t} - \frac{\partial}{\partial x}\frac{\langle \tilde{b}_x \tilde{J}_{z,e}\rangle}{|e|} = 0$$

➕

Ampère's Law

$$-\nabla_\perp^2 A_z = \mu_0 (J_{z,e} + J_{z,i})$$

➡

$$\frac{\partial\langle n_e\rangle}{\partial t} = -\frac{1}{\mu_0 |e|}\frac{\partial}{\partial x}\langle \tilde{b}_x \nabla_\perp^2 \widetilde{A}_{z,e}\rangle - n_{0,i}\frac{\partial}{\partial x}\langle \tilde{b}_x \tilde{u}_{z,i}\rangle$$

Total current contribution

Ion current contribution

$$\langle \tilde{b}_x \tilde{u}_z\rangle = -D_M \frac{\partial}{\partial x}\langle u_z\rangle$$

$$D_M \equiv \sum_{k_y k_z} |\tilde{b}_{x,k}|^2 \tau_{d,k} c_s$$

Dispersal timescale of an acoustic wave packet along the stochastic magnetic field

➡

$$\frac{\partial\langle n_e\rangle}{\partial t} = -\frac{\partial}{\partial x}\Gamma_{e,s}$$

Electron particle flux

$$= \frac{\partial}{\partial x}\frac{B_0}{\mu_0 |e|}\frac{\partial}{\partial x}\langle \tilde{b}_x \tilde{b}_y\rangle + \frac{\partial}{\partial x} n_0 D_M \frac{\partial}{\partial x}\langle u_{z,i}\rangle$$

Familiar div. Maxwell stress

Ion flow along the tilted B line

Stochastic lines and parallel ion flow gradient drives a net electron particle flux, in additional to the Maxwell force contribution.

Conclusions

- We calculate the ***explicit form*** of the stochastic-field-induced transports—kinetic stress K and the compressive energy flux H—have different mechanisms in presence of strong/weak electrostatic turbulence.
- In practice, ***only* strong turbulent cases** $(k_\perp^2 D_T \gg k_z c_s$ or $\lambda > 1)$ **are relevant**. We found mean parallel flow and mean pressure are driven via the ***hybrid diffusivity*** that involves effect of stochastic field and turbulent scattering:

(Chen et al., PPCF, accepted (2021))

$$D_{st} = D_{st}(x) = \sum_{k_y k_z} |\tilde{b}_{x,k}|^2 \frac{c_s^2}{k_\perp^2 D_T}$$

Stochastic B field

Turbulent scattering decorrelation rate

Future Works

- Magnetic drift—effect of stochastic field and turbulence upon geodesic acoustic modes.
- One should include the effect of $\langle \tilde{b}\tilde{\phi} \rangle \neq 0$ in the future (Cao & Diamond 2021, submitted).
- Relevant problems: cosmic ray acceleration and propagation.