Staircase Formation and Evolution by an Array of Stationary Convective Cells

F.R. Ramirez[1] and P.H. Diamond[1]

[1]Department of Physics, University of California San Diego

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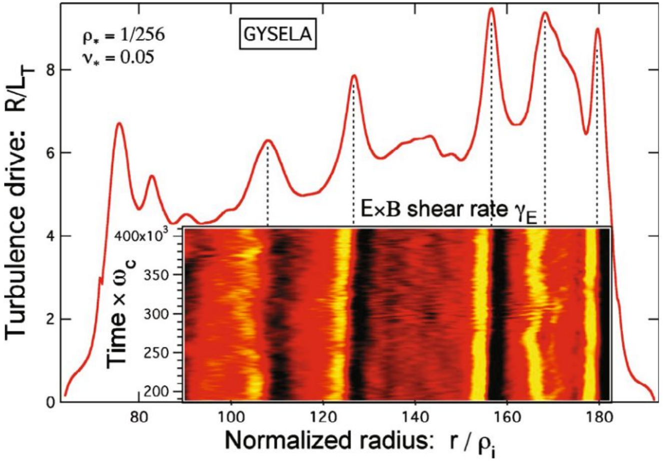
Outline

- Background and Survey Results
- System
- Results:
  - Staircase with No Shear
  - Staircase with Shear
- Summary & Ongoing work

Background and Survey Results

ExB <u>staircase</u> current subject in M.F.E



Suggested ideas:

- Zonal flow eigenmode
- ExB shear feedback, predator-prey
- Jams

...

<u>But</u>… is there an even simpler physical mechanism to produce layering

Clue: Staircase formation, dynamics captured in ultra-simple mixing model with two scales. - Balmforth, et. al; Ashourvan and Diamond

Some Questions

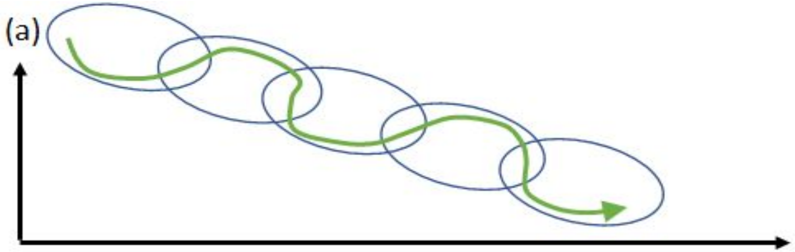
- How does staircase beat homogenization?
- Is the staircase a meta-stable state?
- What is the minimal set of scales to recover layering?

<u>Next</u>:
More on staircase!
But, <u>FIRST</u> let's discuss cell pattern…

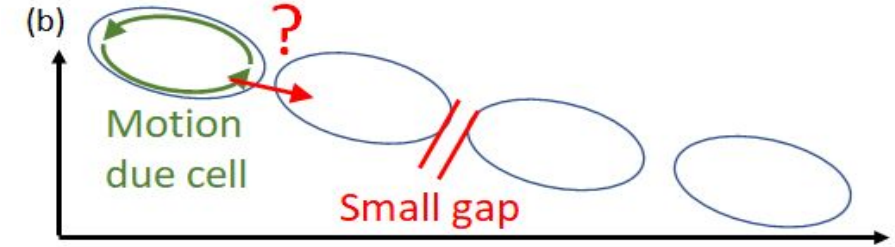
Background and Survey Results (cont.d)

Transport of particle between non-overlapping or marginally overlapping cells is an important topic in fusion plasma.

<u>Overlapping case</u>: particles can transport directly from cell to cell, wandering along streamlines



<u>Non-overlapping case</u> (cells sit at near overlap): transport is a synergy of motion due to cells and random kicks (Collisional diffusion, ambient scattering) thru gap regions.



→ Characteristic of near marginal.

→ The transport over gap is random kicks (ambient diffusion): collisions, micro-turbulence.

→ Coexistence of:
~ Fast transport - Mixing in cell
~ Slow transport - Kicks between cells

N.B.: "Profile stiffness" → Cells near overlap
→ Rapid increase in transport prevents strong overlap

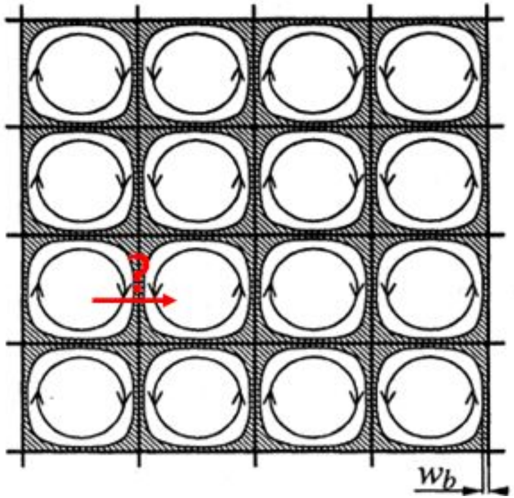
Background and Survey Results (cont.d)

Consider cellular lattice of marginally overlapping cells.

Transport? <u>Answer</u>: Deff ~ $D_o\, Pe^{1/2}$ {**<u>Not a simple addition of process!</u>**}

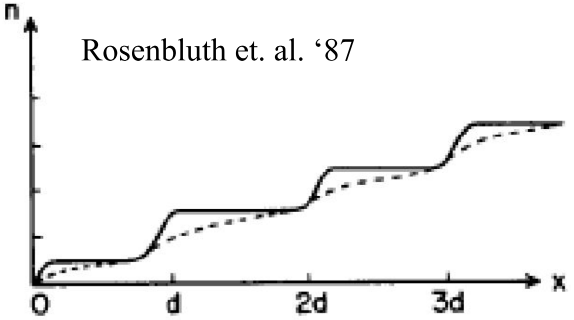
→ Two time rates: v_o / ℓ_o, D_o / ℓ^2_o

→ $Pe = v_o\, \ell_o / D_o \;>>\; 1$



Profile?

Consider concentration of injected dye → profile



"Steep transitions in the density exist between each cell."

Relevant to key question of "near marginal stability"

→ Layering!

→ Simple consequence of two rates

→ "Rosenbluth Staircase"

Important:

- Staircase arises in stationary array of passive eddys.
- Global transport hybrid:
  - → fast rotation in cell
  - → <u>slow</u> diffusion in boundary layer
- Irreversibility localized to inter-cell boundary.

<u>Next</u>: Work… →

System

→ The governing equation solved in this study is the **passive scalar transport** equation,

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D\nabla^2 n,$$

The streamline function used to create the Bénard convection patterns in the fluid flow is,

$$\psi = \sin \pi(x/d) \sin \pi\beta(y/d) + \alpha\ \psi_{\text{shear}}.$$

Discuss later...

The fluid velocity **u** is of the form

$$\mathbf{u} = (\tilde{u}d/\pi)\hat{z} \times \nabla\psi,$$

here d gives the size of the roll, β its aspect ratio, and $\tilde{u}$ is the maximum flow of the velocity.

→ Two characteristic time-scales, time for circulation around the roll ($\tau_H = d / \tilde{u}\beta$) and time for molecular diffusion of a particle through a roll ($\tau_D = d^2 / D$). The ratio of these two time-scales,

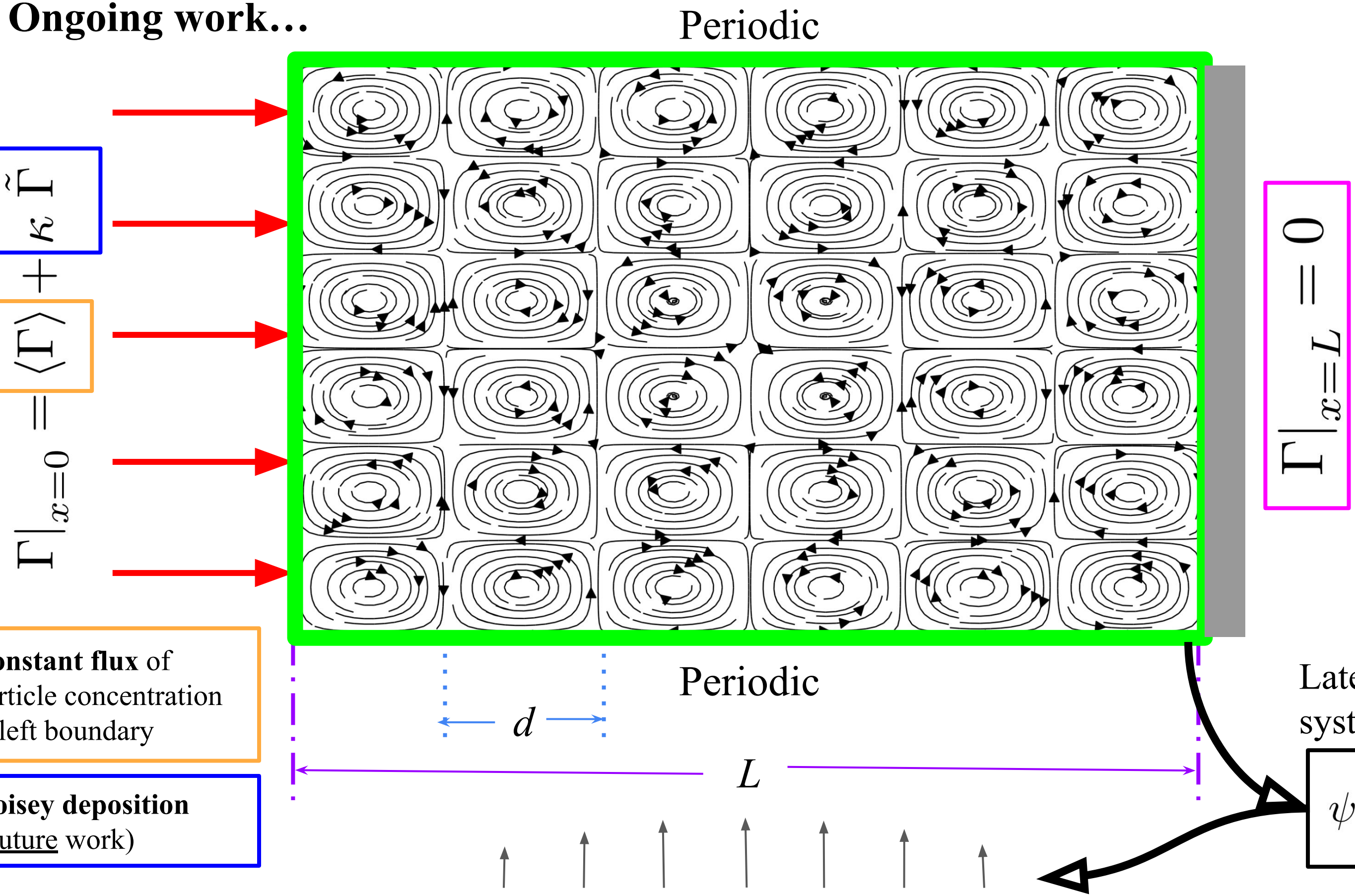
$$\text{Pe} = \frac{\tau_D}{\tau_H} >> 1$$

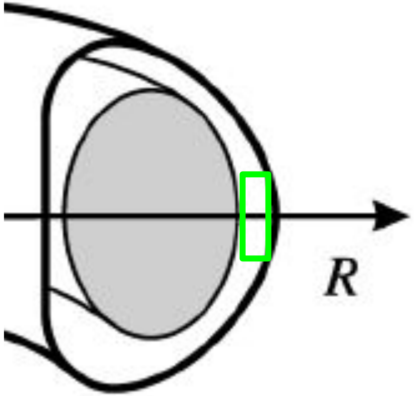
→ Primarily concerned in the case of Pe >> 1, where the **physics is explained by fast mixing within the cells and slow mixing across the boundaries of the cells**.

→ Later will discuss the Pe_{sh} which introduces an additional characteristic time-scale (τ_{sh}).

System (cont.d)

Ongoing work…





Constant flux of particle concentration at left boundary

Noisey deposition (Future work)

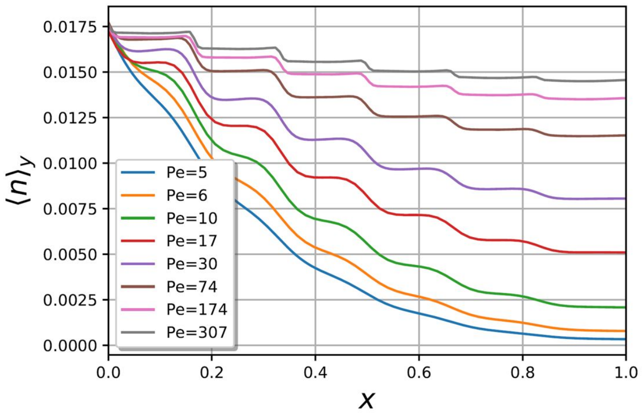
Insulated boundary condition on right boundary.

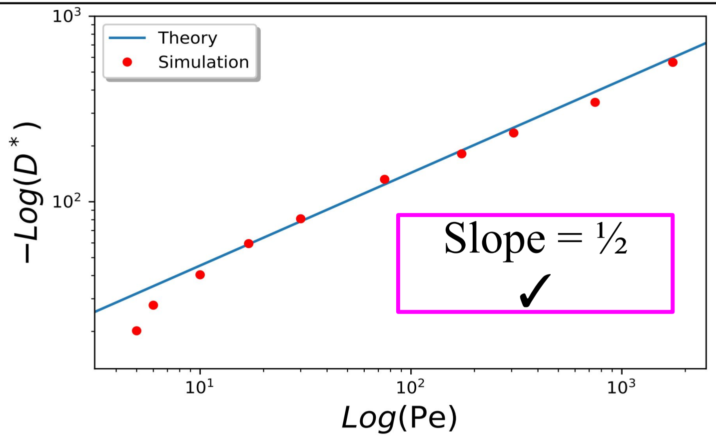
In the future we will explore **fixed flux** conditions (ejected at edge)!

Later we add shear to the system in the form of,

$$\psi_{\text{shear}} = -\cos\frac{mx}{2}$$

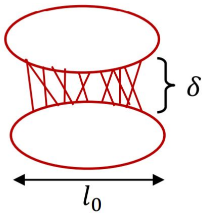
Staircase with No Shear (α=0)





Note: $\langle n \rangle_y$ is the scalar concentration averaged with respect to *y*.

Back-of-Envelope Calculation



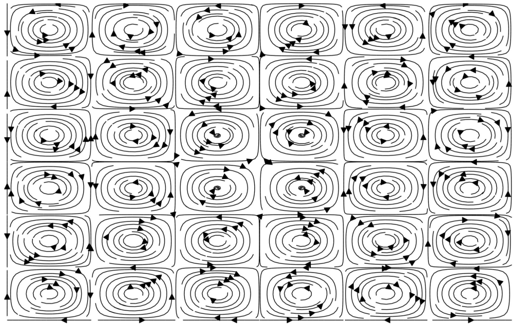
$\delta \rightarrow$ BL width
$l_0 \rightarrow$ cell size

$D^* \approx f_{active}((\Delta x)^2 / \Delta t)$; $f_{active} \equiv$ active fraction $\sim \delta / \ell_o$

$\Delta t \sim \ell_o / v_o \rightarrow$ cell circulation time

So, $\delta^2 \sim D\,\Delta t \sim D\,\ell_o / v_o$

$D^* \sim [(D\,\ell_o / v_o)^{1/2} 1 / \ell_o]\,(\ell_o^2 / \ell_o)\,v_o \sim [D\,D_{cell}]^{1/2} \sim D\,Pe^{1/2}$

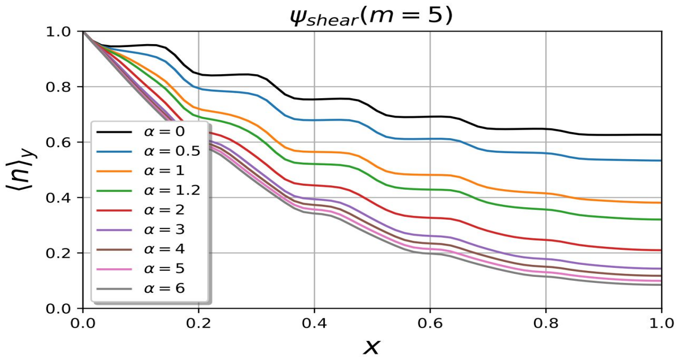


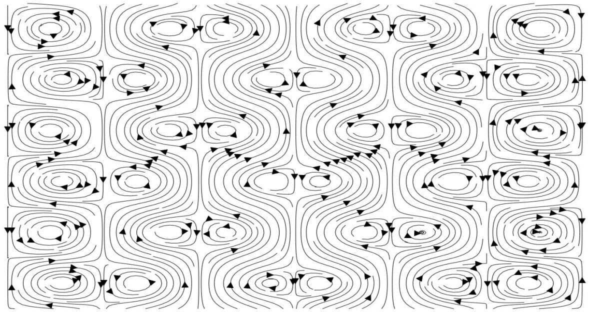
Again, fast mixing in cell and slow diffusion across boundary layer!

Next: Shear…

Simulation agrees with theoretical result scaling $D^* \propto Pe^{1/2}$

Staircase with Shear (α≠0)





Now shear is introduced

$$\psi_{\text{shear}} = -\cos\frac{mx}{2}$$

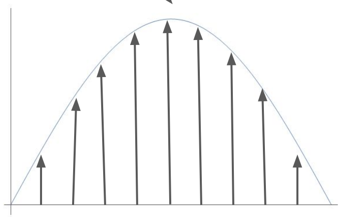
The strength of the shear is controlled by α. Recall,

$$\psi = \psi_0 + \alpha\,\psi_{\text{shear}}$$

A scan of averaged scalar concentration at fixed D as a function of x for different α values.

$\alpha = 4$

$m = 1$

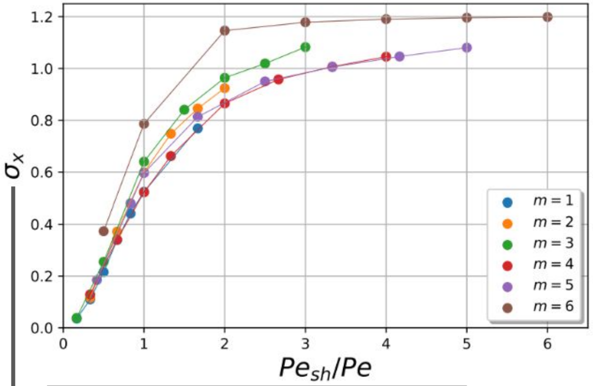


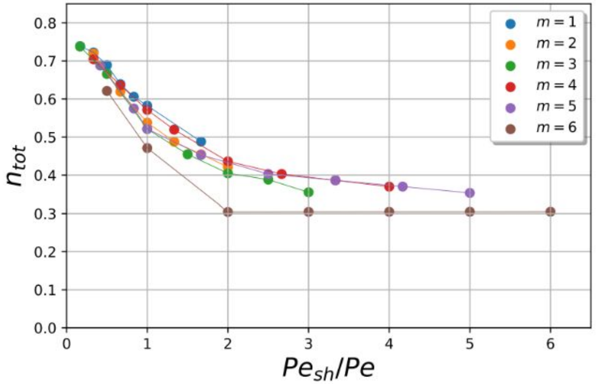
As the shear strength increases, the staircase profile **breaks down**. Shear **enhances** mixing (i.e., increases irreversible process).

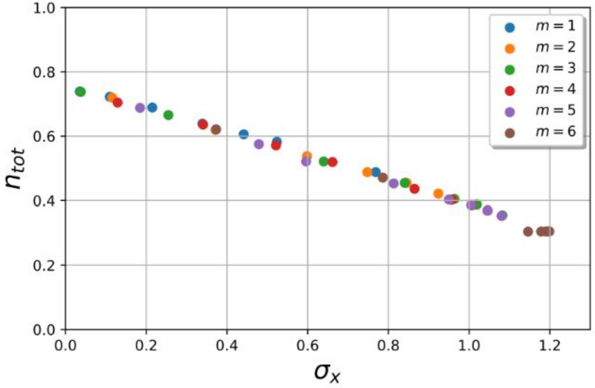
Important:

- Corrugation breaks down!
- There is critical α and m where the staircase begins to breaks down!
  - → Let's introduce a new Peclet number, $Pe_{sh} = v' / (D / \ell_o^2) \Rightarrow \alpha\, m^2\, d^2 / D$ (measure of shearing)
    - → In addition, there will be a shear dispersion time scale, $\tau_{sh} = (\ell_o^2 / D\, v^2)^{1/3}$
- Shear dispersion rate gives effective mixing rate faster than diffusion!

Staircase with Shear (α≠0) (cont.d)

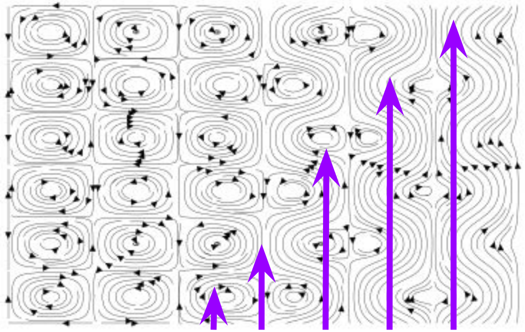






$$\sigma_x^2 = \int_0^{L_x} \left(\langle n\rangle_y - \langle n_0\rangle_y\right)^2 dx$$

$$\mathrm{Pe_{sh}} \sim \frac{\alpha m^2 d^2}{D_0}$$



- ($\tau_{sh} = (\ell_o^2 / D\, v'^2)^{1/3}$) shear dispersion gives effective mixing rate faster than diffusion.
- Then supercritical shear → irreversible mixing outside inter-cell boundary layer.
- **Results suggest** here that shear flow actually weakens staircase, by **reducing slow-fast time scale ratio**!

IMPORTANT: For $\mathrm{Pe_{sh}} >> 1$ **corrugation decays**!

Next: What about localized shear? Ongoing…

Summary

- Staircase appears in stationary, passive cellular array with diffusion at Pe >> 1.

  → Fast mixing in cell, slow mixing across cell boundary is sufficient!

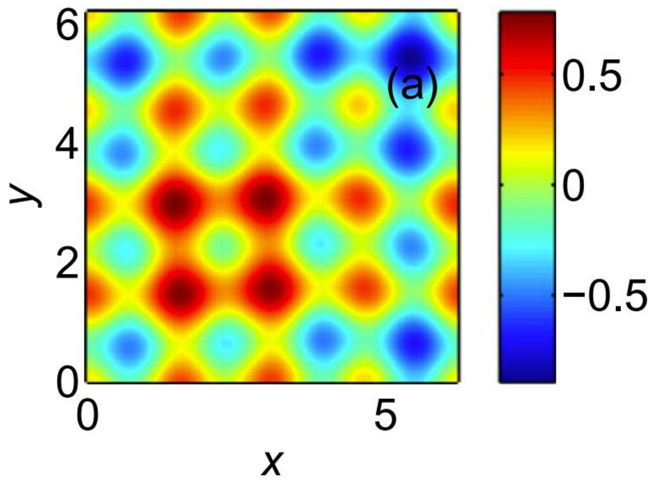
- Simple consequence of two time scales, well separated, and their interplay in transport. Slow time scale → transport barrier.
- Relevant to nearby overlapping cells, near marginality.
- Enhanced shear mixing reduces $\tau_{slow} / \tau_{fast}$ → degrades corrugation, staircase.
- No dynamical feedback in this system.

  → Simpler then shearing feedback loop scenario.

Ongoing

- Localized shear
- Fixed flux boundary conditions
- Noisey deposition
- Irregular cells
- Noisey cell pattern
    - Vortex crystal + forcing
    - ‘Melting crystal’

$\Rightarrow$ consider <n> profile in melting crystal flow.



Streamfunction showing turbulence-induced melting of a vortex crystal (Perlekar and Pandit, 2010)

How **resilient** is staircase pattern?

How does staircase **degrade**?

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