Neutrals and Drift–Rossby–Alfvén turbulence: Drag, Entrainment, and Ambipolar Diffusion

Chang-Chun Chen[1], Patrick Diamond[1], Mikhail Malkov[1] and Steven Tobias[2]

[1]University of California San Diego, USA
[2]University of Leeds, UK

This work is supported by the U.S. Department of Energy under award number DE-FG02-04ER54738

2022 US-EU JOINT TRANSPORT TASK FORCE WORKSHOP, April 7th 2022

Outline

- Introduction

  1. Why neutrals? Zonal flow damping.

  2. Non-trivial Problems: Ambipolar effect in electromagnetism.

- Results

  1. Drift-Rossby wave and zonal flow with neutrals

  2. Neutrals and ambipolar diffusion.

  3. Critical parameter for the Ambipolar effect on zonal flow → Regimes of zonal flow.

- Conclusions

Why we study neutrals in fusion devices?

Neutrals influx

↓

Charge exchange damping or friction

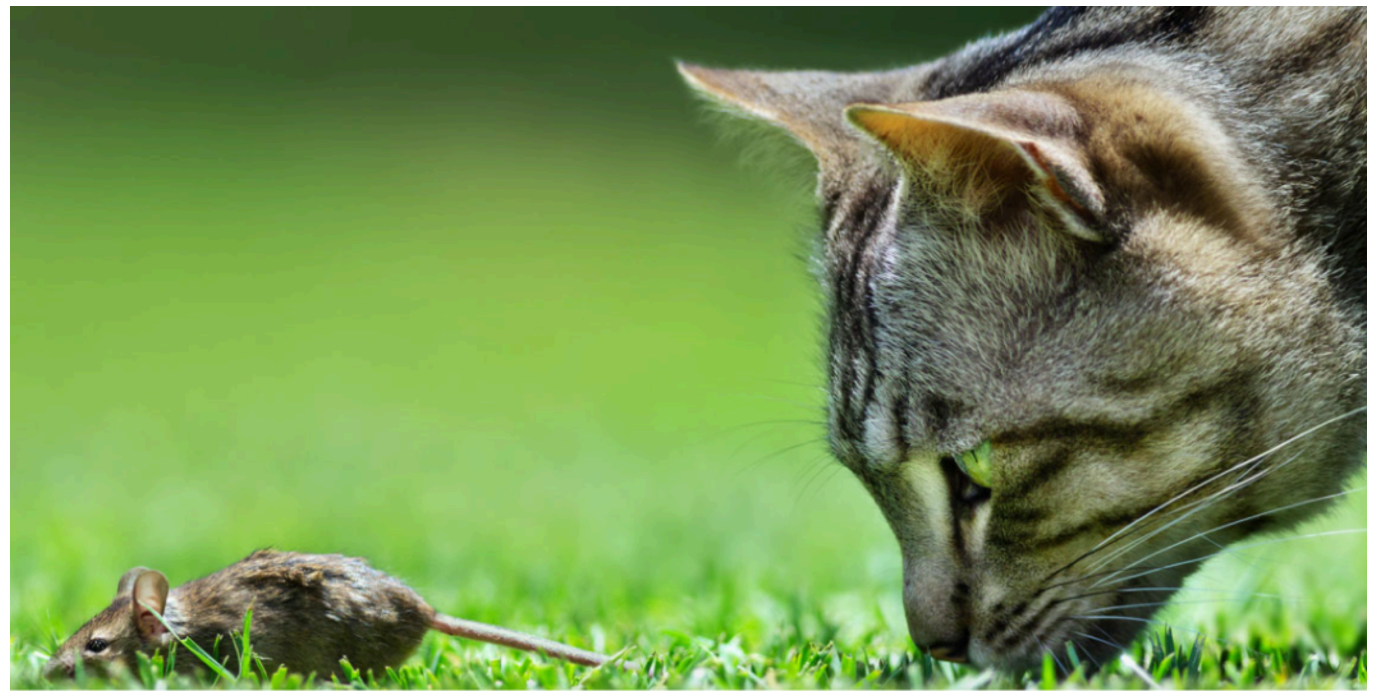
↓

Predator prey model:

predator: zonal flow
prey: turbulence

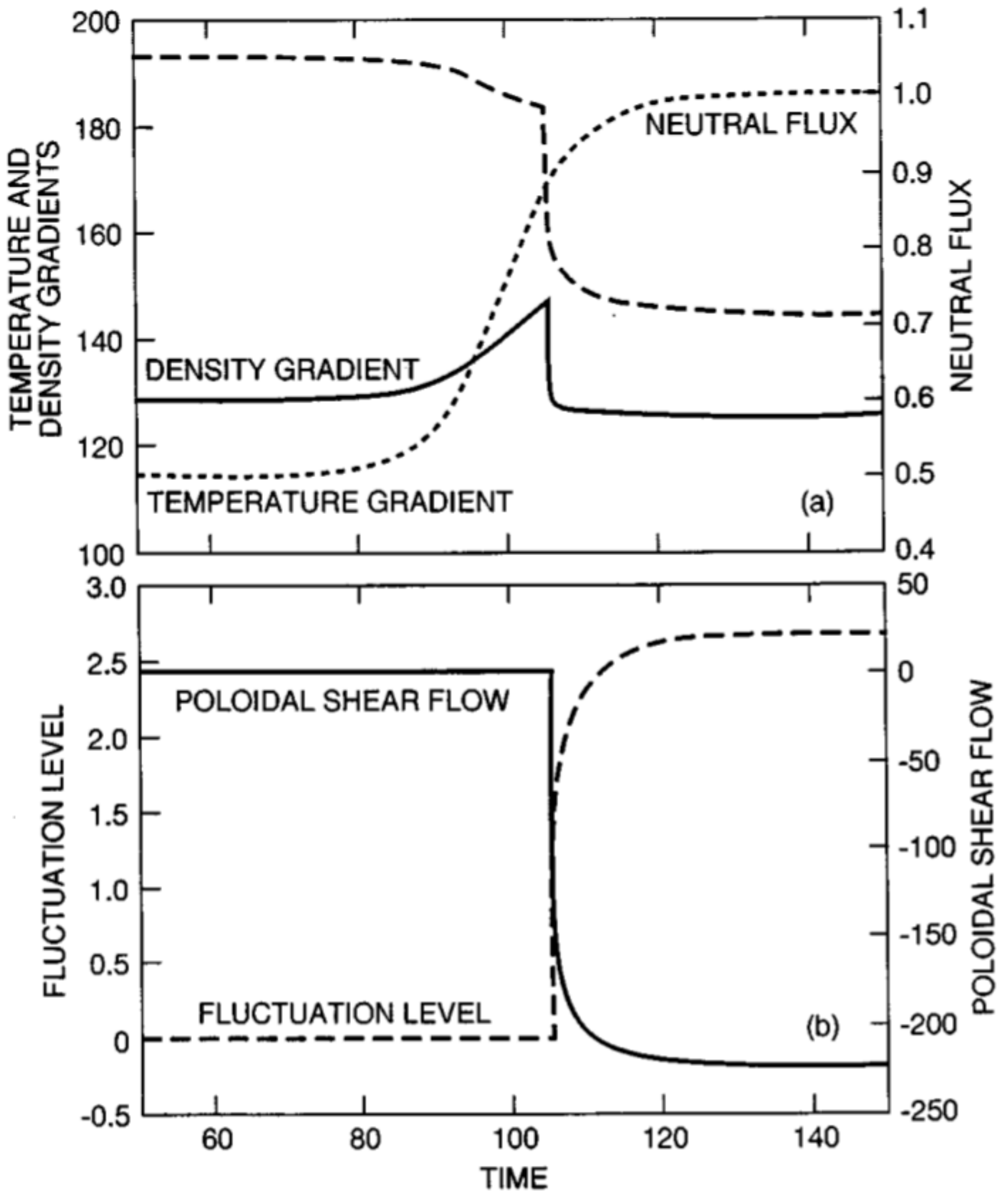
↓

Increase the flow shear threshold



Conventional wisdom:

1. Neutrals damp the poloidal (zonal) flow, increase fluctuation level.
2. Increase effective fluctuation energy.



(Carreras et al., PoP **3**, 4106 (1996))

The neutral density is important in studies of L-H transition power threshold in fusion device.

Non-trivial: 1. E-M effect →Ambipolar diffusion (classic in Astrophys). ✓
2. Entrainment of neutral particles.

Drift–Rossby Waves and Zonal Flow

(Simplest possible model)

- Evolution of zonal flow is from the competition btw the Reynolds and Maxwell Stress:

Zonal flow evolution:

$$\frac{\partial}{\partial t}\langle u_y\rangle = -\frac{\partial}{\partial x}\left(\langle \tilde{u}_x \tilde{u}_y\rangle - \frac{\langle \tilde{B}_x \tilde{B}_y\rangle}{\mu_0 \rho}\right)$$

Reynolds stress ⟷ Maxwell stress

Competition

Induction equation:

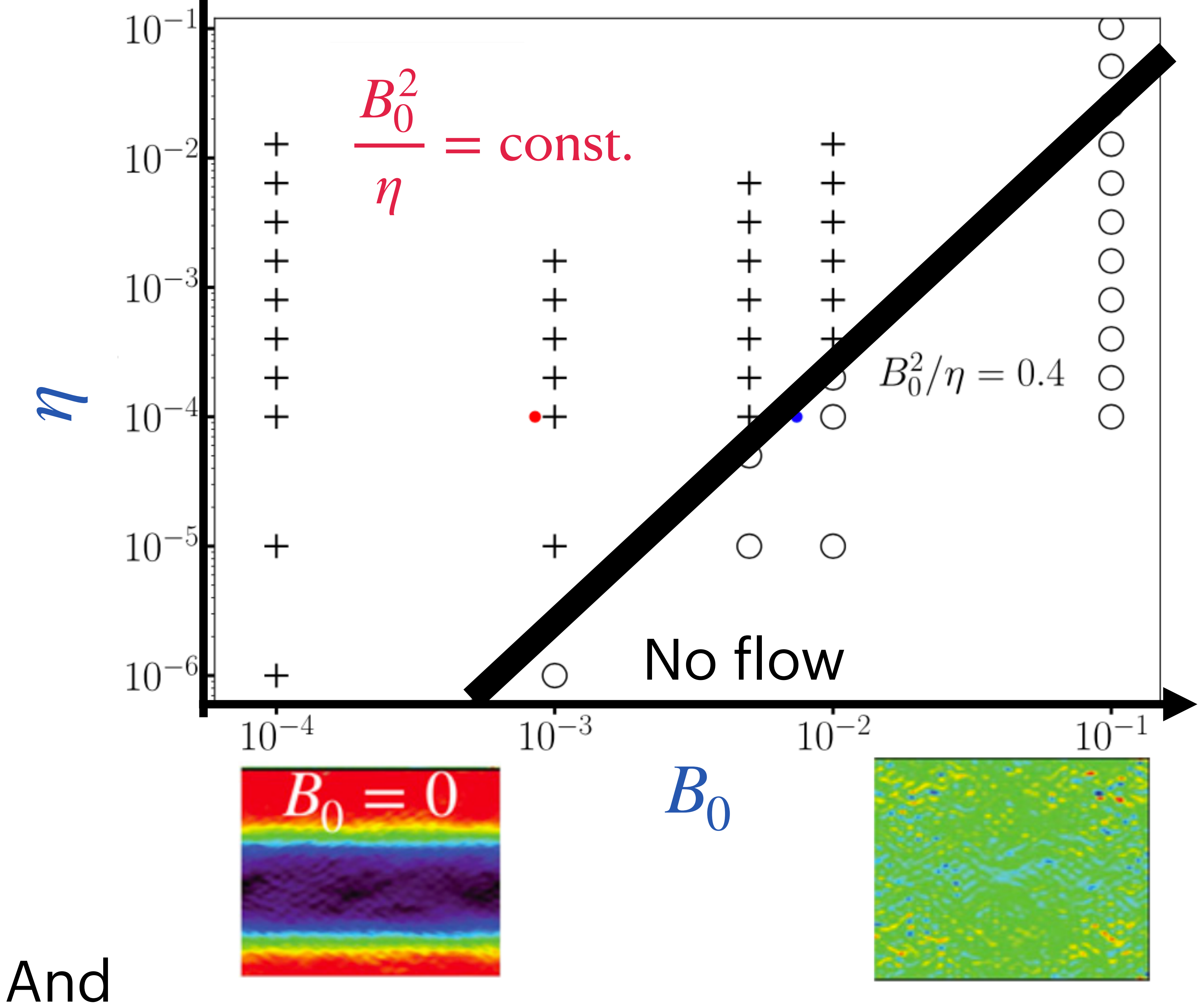
$$\frac{\partial \tilde{\mathbf{B}}}{\partial t} = \nabla \times (\tilde{\mathbf{u}}_\mathbf{i} \times \mathbf{B}) + \eta \nabla^2 \tilde{\mathbf{B}}$$

η: increase the zonal flow

B_0: suppress the zonal flow

$\rightarrow \frac{B_0^2}{\eta}$: regulates Mag. And Reynolds stress competition

(Chen et al., PoP, 28, 032306 (2021))



→Zel'dovich Theorem

What if we add neutrals? Neutrals will enter this competition and have a **production** on zonal flow.

Multi-Fluid Model

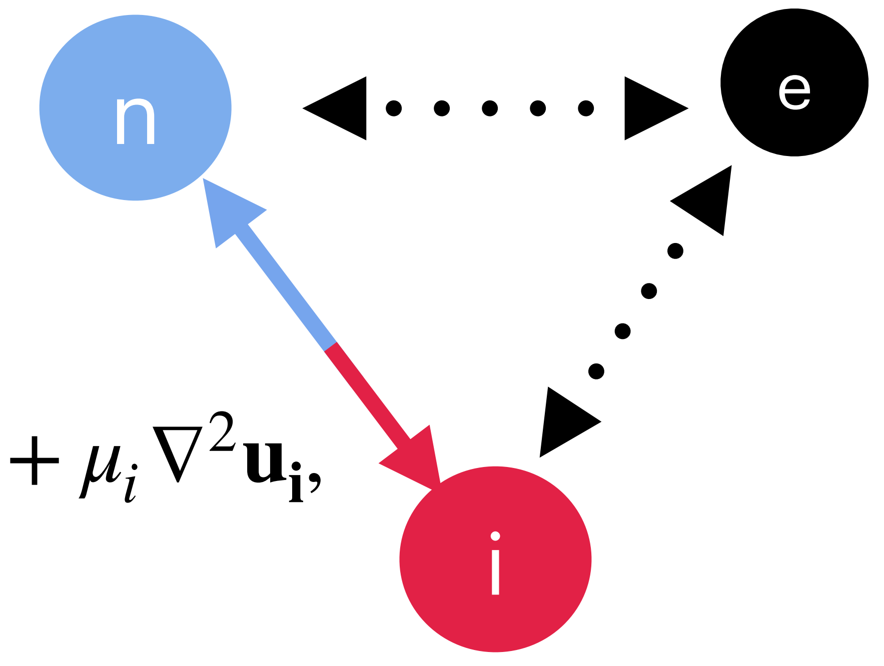
- Equation of motion of three species on **β-plane** (rel. to planetary system):

n
$$\rho_n\left(\frac{\partial}{\partial t}+\mathbf{u_n}\cdot\nabla\right)\mathbf{u_n} = -\nabla p_n^* - \underbrace{\rho_n\nu_{ni}\left(\mathbf{u_n}-\mathbf{u_i}\right)}_{\text{i-n drag}} - \underbrace{\rho_n\nu_{ne}\left(\mathbf{u_n}-\mathbf{u_e}\right)}_{\text{e-n drag}} - \underbrace{2\rho_n\mathbf{\Omega}\times\mathbf{u_n}}_{\text{Coriolis force}} + \mu_n\nabla^2\mathbf{u_n}$$

e
$$0 = \rho_e\left(\frac{\partial}{\partial t}+\mathbf{u_e}\cdot\nabla\right)\mathbf{u_e} = -\nabla p_e^* - \underbrace{\rho_e\nu_{ei}\left(\mathbf{u_e}-\mathbf{u_i}\right)}_{\text{i-e drag}} - \underbrace{\rho_e\nu_{en}\left(\mathbf{u_e}-\mathbf{u_n}\right)}_{\text{n-e drag}} - ne(\mathbf{E}+\mathbf{u}_e\times\mathbf{B}) + \mu_e\nabla^2\mathbf{u_e}$$

i
$$\rho_i\left(\frac{\partial}{\partial t}+\mathbf{u_i}\cdot\nabla\right)\mathbf{u_i} = -\nabla p_i^* - \underbrace{\rho_i\nu_{in}\left(\mathbf{u_i}-\mathbf{u_n}\right)}_{\text{n-i drag}} - \underbrace{\rho_i\nu_{ie}\left(\mathbf{u_i}-\mathbf{u_e}\right)}_{\text{n-e drag}} + ne(\mathbf{E}+\mathbf{u}_i\times\mathbf{B}) - \underbrace{2\rho_i\mathbf{\Omega}\times\mathbf{u_i}}_{\text{Coriolis force}} + \mu_i\nabla^2\mathbf{u_i},$$

Ignore coupling btw e-i and e-n $m_e\nu_{en} \ll m_i\nu_{in}$



Strength of coupling

$$\rho_i\nu_{in} = \rho_n\nu_{ni} \equiv \alpha\rho_i\rho_n$$

- Ion and neutral field equation:

$$\rho_i\left(\frac{\partial}{\partial t}+\mathbf{u_i}\cdot\nabla\right)\mathbf{u_i} = -\nabla p_i^* + \frac{1}{\mu_0}(\mathbf{B}\cdot\nabla)\mathbf{B} + \rho_i\nu_{in}\left(\mathbf{u_n}-\mathbf{u_i}\right) - 2\rho_i\mathbf{\Omega}\times\mathbf{u_i} + \mu_i\nabla^2\mathbf{u_i}$$

$$\rho_n\left(\frac{\partial}{\partial t}+\mathbf{u_n}\cdot\nabla\right)\mathbf{u_n} = -\nabla p_n^* + \rho_n\nu_{ni}\left(\mathbf{u_i}-\mathbf{u_n}\right) - 2\rho_n\mathbf{\Omega}\times\mathbf{u_n} + \mu_n\nabla^2\mathbf{u_n}$$

In high coupling regime, ion has force balance:

$$\mathbf{J}\times\mathbf{B} + \text{drag} = 0$$

So the neutrals will also feel the $\mathbf{J}\times\mathbf{B}$ force.

Electromagnetism: Ambipolar Diffusion

JxB force acts on ions

↓

Ambipolar Diffusion

Magnetic diffusion: $\eta \rightarrow \eta + \eta_{am}$

Damps the pert. magnetic field $\widetilde{B}$

↓

The Maxwell Stress decreases

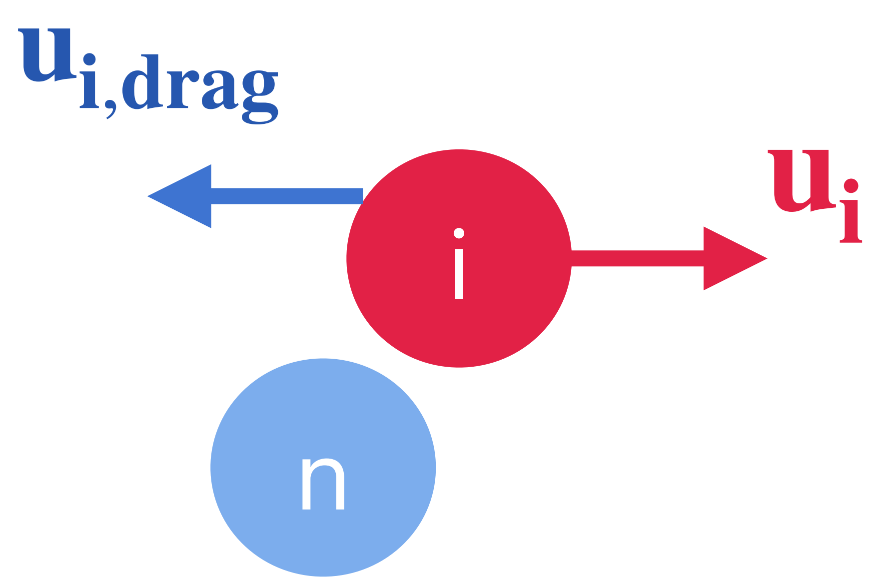
↓

Increase zonal flow and damps drift-wave turbulence

Ohm's Law:

$$\mathbf{E} = \eta \mathbf{J} - \mathbf{u_i} \times \mathbf{B} - \frac{m_e \nu_{en}}{e}(\mathbf{u_i} - \mathbf{u_n}),$$

JxB due to the ion-neutral coupling:



$$\mathbf{u_{i,drag}} = \mathbf{u_i} - \mathbf{u_n} = \frac{\mathbf{J} \times \mathbf{B}}{\alpha \rho_i \rho_n}$$

B field frozen into the neutrals, but now the magnetic diffusion is **ambipolar diffusion** η_{am} .

Key question:

How does the **ambipolar diffusion** (non-linear B-diffusivity from neutral effect) affect the zonal flow? What's the effect on magnetic perturbation $\widetilde{B}$.

Linear Modes of Coupled System

- In the linear theory, we obtain scalar equations:

Vorticity equation:

$$\rho_i\left(\frac{\partial}{\partial t}+\langle \mathbf{u_i}\rangle\cdot\nabla\right)\widetilde{\zeta}_i=-\rho_i\widetilde{u}_{i,y}\frac{\partial}{\partial y}\langle\zeta_i\rangle+B_{0,x}\frac{\partial\widetilde{J}_z}{\partial x}+\widetilde{B}_y\frac{\partial\langle J_z\rangle}{\partial y}+\alpha\rho_i\rho_n\left(\widetilde{\zeta}_n-\widetilde{\zeta}_i\right)+\mu_i\nabla^2\widetilde{\zeta}_i.$$

$$\rho_n\left(\frac{\partial}{\partial t}+\langle \mathbf{u_n}\rangle\cdot\nabla\right)\widetilde{\zeta}_n=-\rho_n\widetilde{u}_{n,y}\frac{\partial}{\partial y}\langle\zeta_n\rangle+\alpha\rho_i\rho_n\left(\widetilde{\zeta}_i-\widetilde{\zeta}_n\right)+\mu_n\nabla^2\widetilde{\zeta}_n$$

Induction equation:

$$\widetilde{A}_z=\frac{-\widetilde{\zeta}_n}{k^2}\frac{B_0k_x}{\omega+i\eta k^2+i\frac{B_0^2}{\mu_0\alpha\rho_i\rho_n}k^2}$$

Coupling effect

$$\eta_{eff,k}\equiv\eta+\frac{B_0^2}{\mu_0\alpha\rho_i\rho_n}$$

$$\widetilde{\zeta}_{n,k}=\frac{D_n}{D}=\frac{S_1\rho_i(-i\omega_i+\nu_ik^2)+\alpha\rho_i\rho_n(S_1+S_2)}{\rho_i\rho_n(-i\omega_i+\nu_ik^2)(-i\omega_n+\nu_nk^2)+\alpha\rho_i\rho_n\left(\rho_i(-i\omega_i+\nu_ik^2)+\rho_n(-i\omega_n+\nu_nk^2)\right)+\frac{i\alpha\rho_i\rho_n}{\mu_0}\frac{B_0^2k_x^2}{\omega+i\eta_{eff}k^2}}$$

- We obtain a critical dimensionless parameter:

$$\gamma=\frac{(\alpha\rho_i)\omega_{eff}^2}{\omega+i\eta_{eff}k^2}\cdot\tau_{eddy}\cdot\frac{1}{\omega_{ci}}$$

→ Important Time scales

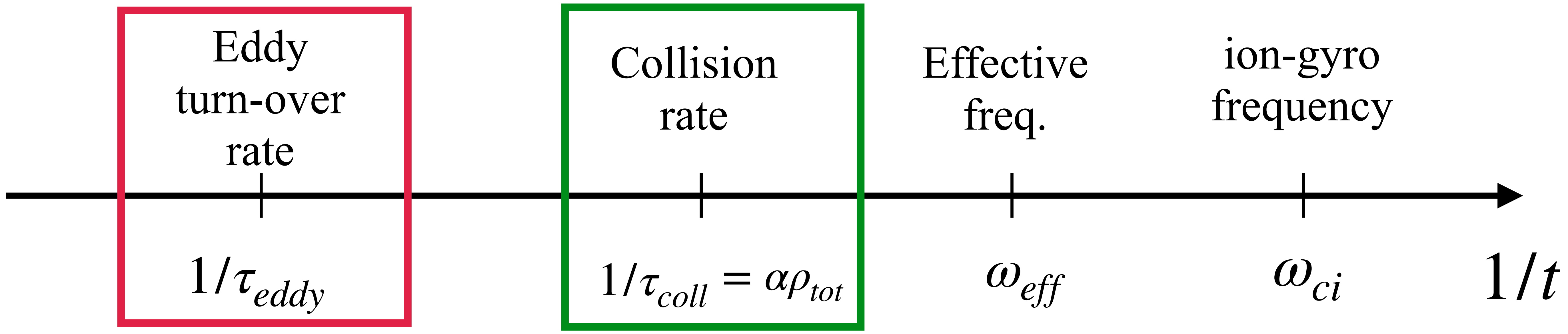
$$\omega_{eff}^2=\equiv\frac{B_0^2k_x^2}{\omega+i\eta_{eff}k^2}$$

$$\begin{cases}\gamma\ll 1 & \text{weak coupling regime}\\ \gamma\gg 1 & \text{strong coupling regime}\end{cases}$$

Important Time Scales

- We consider rates:

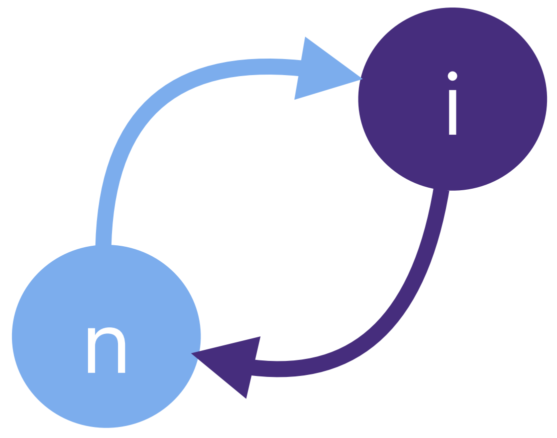
$$\gamma = \frac{(\alpha\rho_i)\omega_{eff}^2}{\omega + i\eta_{eff}k^2} \cdot \tau_{eddy} \cdot \frac{1}{\omega_{ci}}$$



- A simplified coupling parameter:

$$\tilde{\alpha} \equiv \frac{\tau_{eddy}}{\tau_{coll}} = \frac{l_\perp \rho_{tot}\alpha}{\tilde{u}}$$

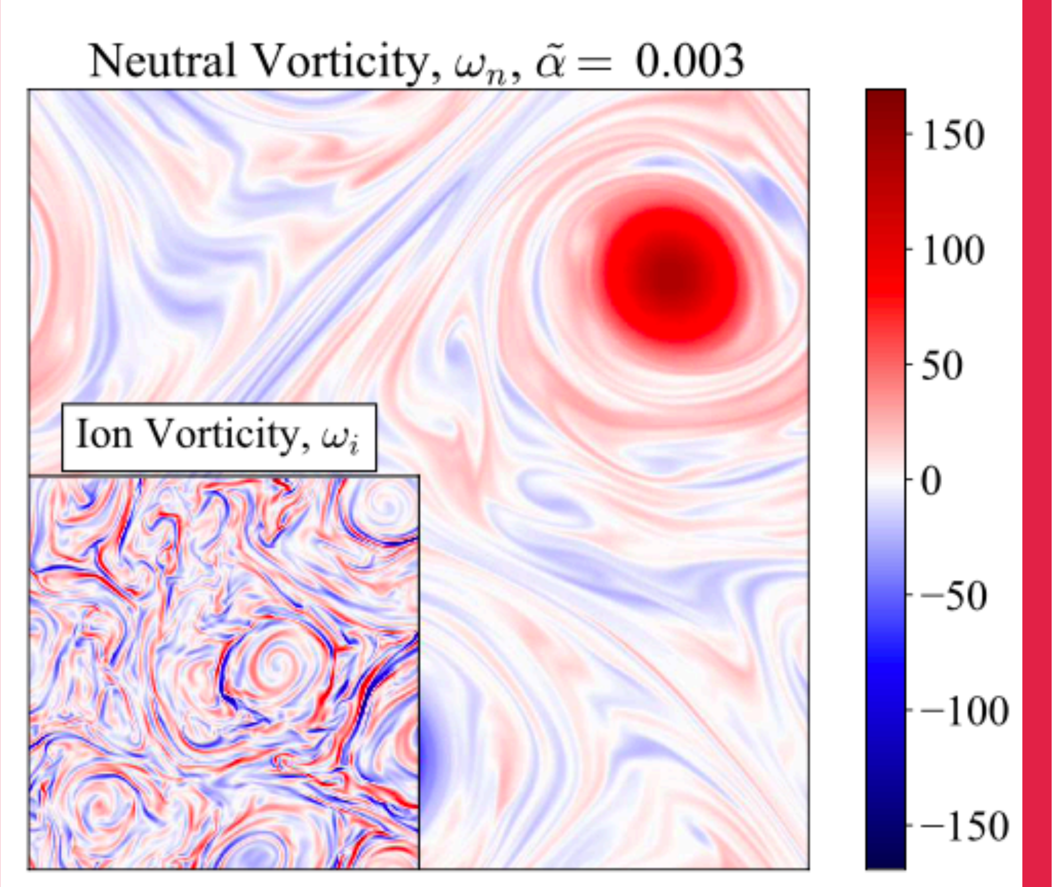
→ Strength of coupling

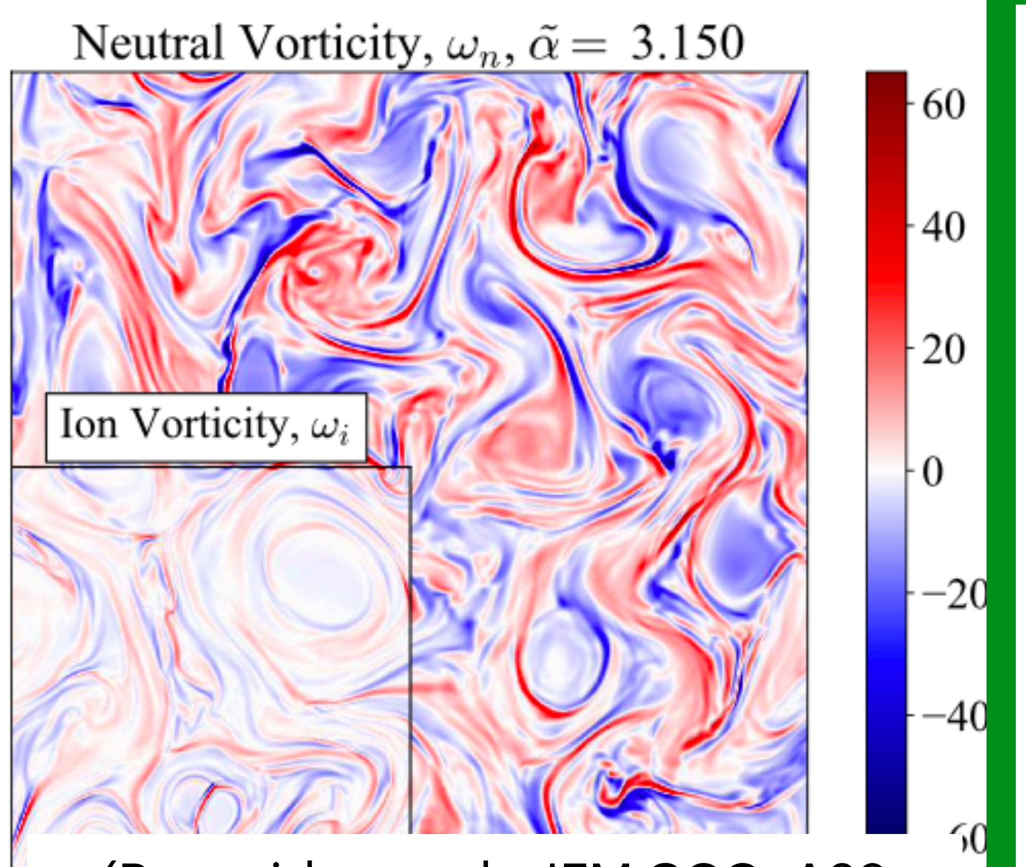


- Dynamics of neutrals and ions:

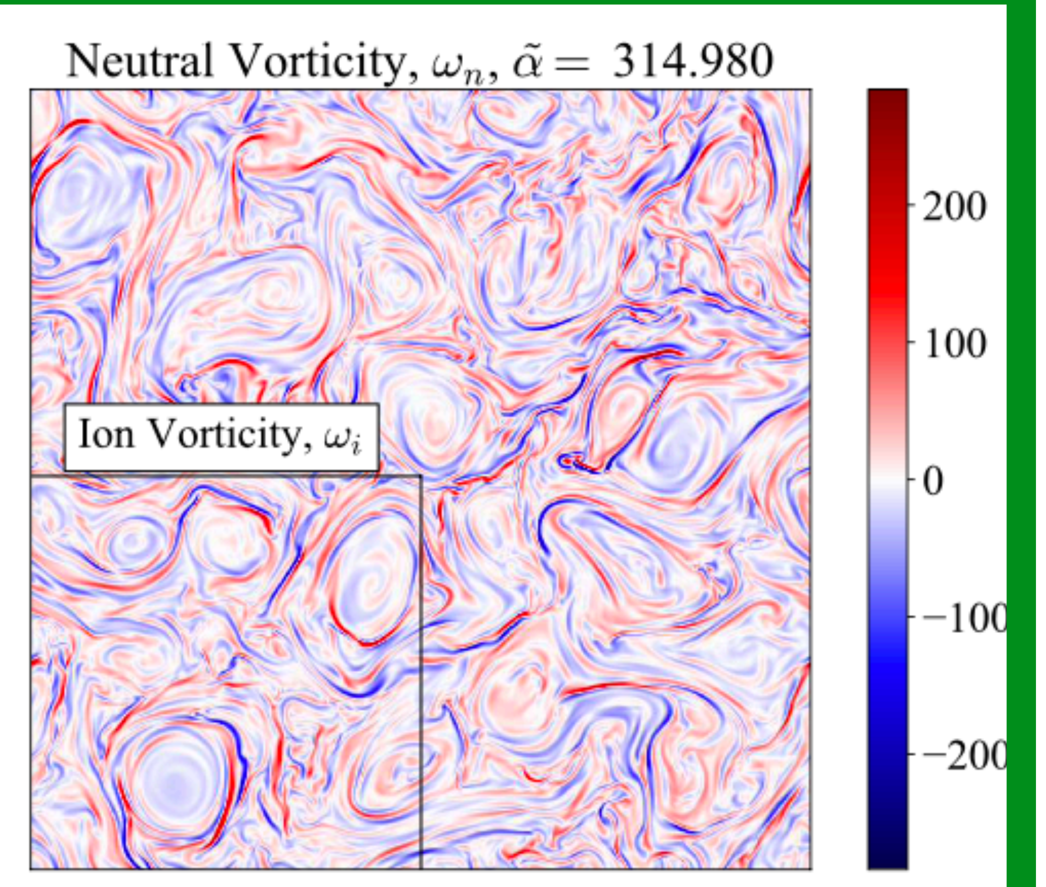
Weak coupling:

Neutrals and ions not well-coupled —> Two fluid.





(Benavides et al., JFM **900**, A28 (2020))



Strong coupling:

Neutrals couple to ions and behaves as **one MHD fluid**.

small α large α

Strong Coupling

Deriving Mag. Potential Equation

- The induction equation, and consider **strong coupling regime**: $\alpha\rho_i \gg 1/\tau_{eddy}$:

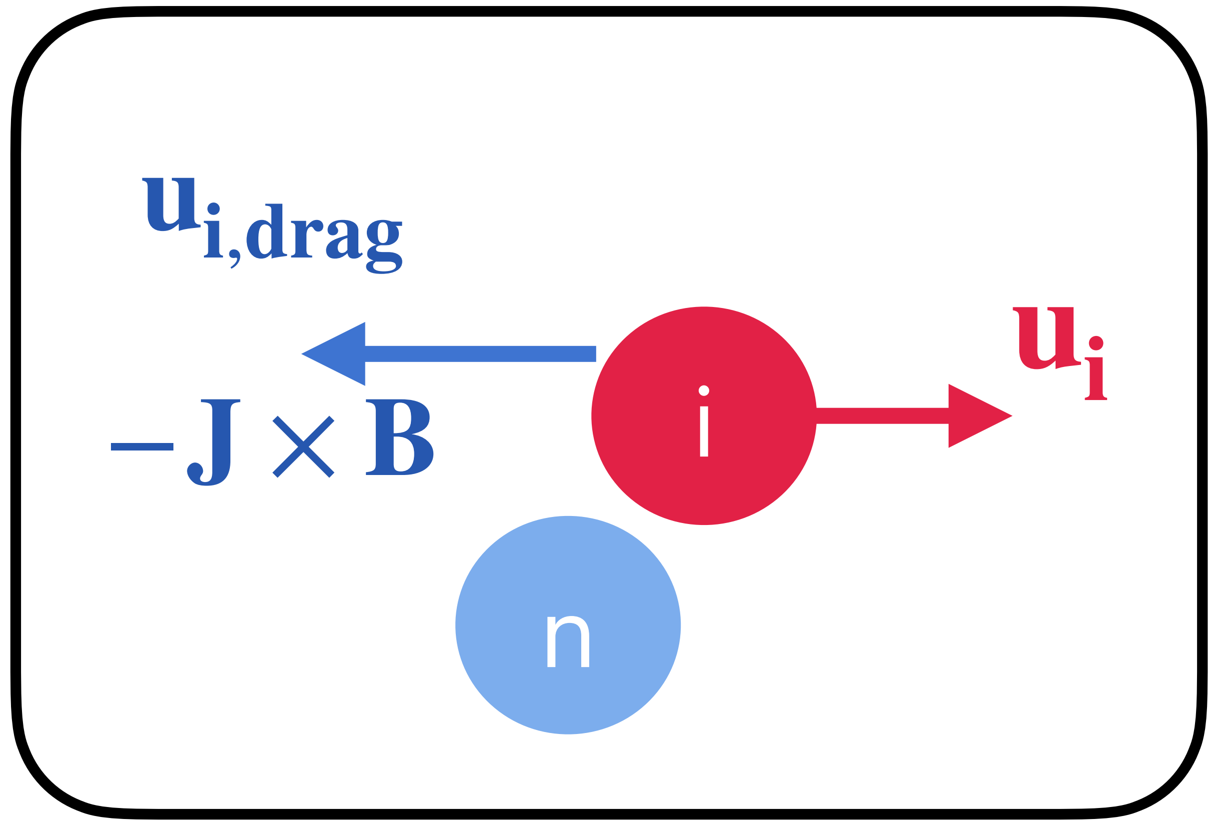
JxB-drag balance on ion: $-\mathbf{J}\times\mathbf{B} = \mathbf{f}_{\mathbf{d,i}}$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u_i} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}\,.$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u_n} \times \mathbf{B}) + \nabla \times (\frac{\mathbf{J}\times\mathbf{B}}{\alpha\rho_i\rho_n} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

$$\mathbf{u_i} = \mathbf{u_n} - \frac{2\mathbf{u}_{i,drag}}{1+\rho_i/\rho_n}$$

$$\mathbf{u_i} = \mathbf{u_n} + \frac{\mathbf{J}\times\mathbf{B}}{\alpha\rho_i\rho_n}$$



- The **magnetic potential** equation becomes

$\eta \rightarrow \eta + \underline{\underline{\eta}}_{am}$

Resistivity correction due to the ion-neutral drag

$$\frac{\partial A_z}{\partial t} + (\mathbf{u_n}\cdot\nabla)A_z = \frac{1}{\mu_0\alpha\rho_i\rho_n}\underline{\nabla}\cdot\left[\underline{\underline{\eta}}_{am}\cdot\underline{\nabla}A_z\right] + \eta\nabla^2 A_z + C$$

$$= \frac{1}{\mu_0\alpha\rho_i\rho_n}\begin{pmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y}\end{pmatrix}\begin{pmatrix}\mathscr{F} & \mathscr{G}\\ \mathscr{H} & \mathscr{I}\end{pmatrix}\begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\end{pmatrix}A_z + \eta\nabla^2 A_z + C$$

Ambipolar diffusion tensor for A:

$$\underline{\underline{\eta}}_{am} = \begin{pmatrix}\mathscr{F} & \mathscr{G}\\ \mathscr{H} & \mathscr{I}\end{pmatrix} = \begin{pmatrix}\frac{1}{2}(B_y^2 - B_x^2) & -B_xB_y\\ -B_xB_y & \frac{1}{2}(B_x^2 - B_y^2)\end{pmatrix}$$

Additional terms:

$$C = \frac{1}{\mu_0\alpha\rho_i\rho_n}\left[\frac{-B^2}{2}\nabla^2 A_z + B_x^2\frac{\partial^2}{\partial x^2}A_z + B_y^2\frac{\partial^2}{\partial y^2}A_z + 2B_xB_y\frac{\partial^2}{\partial x\partial y}A_z\right]$$

Results — Two-Fluid Rossby-Alfvén Wave

By solving the linear equations for ions, neutrals, and magnetic potential, we have

- Strong coupling regime γ>>1 ($1/\tau_{eddy} \ll \alpha\rho_i \ll \omega_{eff} \ll \omega_{ci}$):

$$(\omega - \omega_R + i\nu k^2)(\omega + i\eta_{eff} k^2) = \frac{\rho_i}{\rho_{tot}} \frac{B_0^2 k_x^2}{\omega + i\eta_{eff} k^2} \qquad \eta_{eff,k} \equiv \eta + \frac{B_0^2}{\mu_0 \alpha \rho_i \rho_n}$$

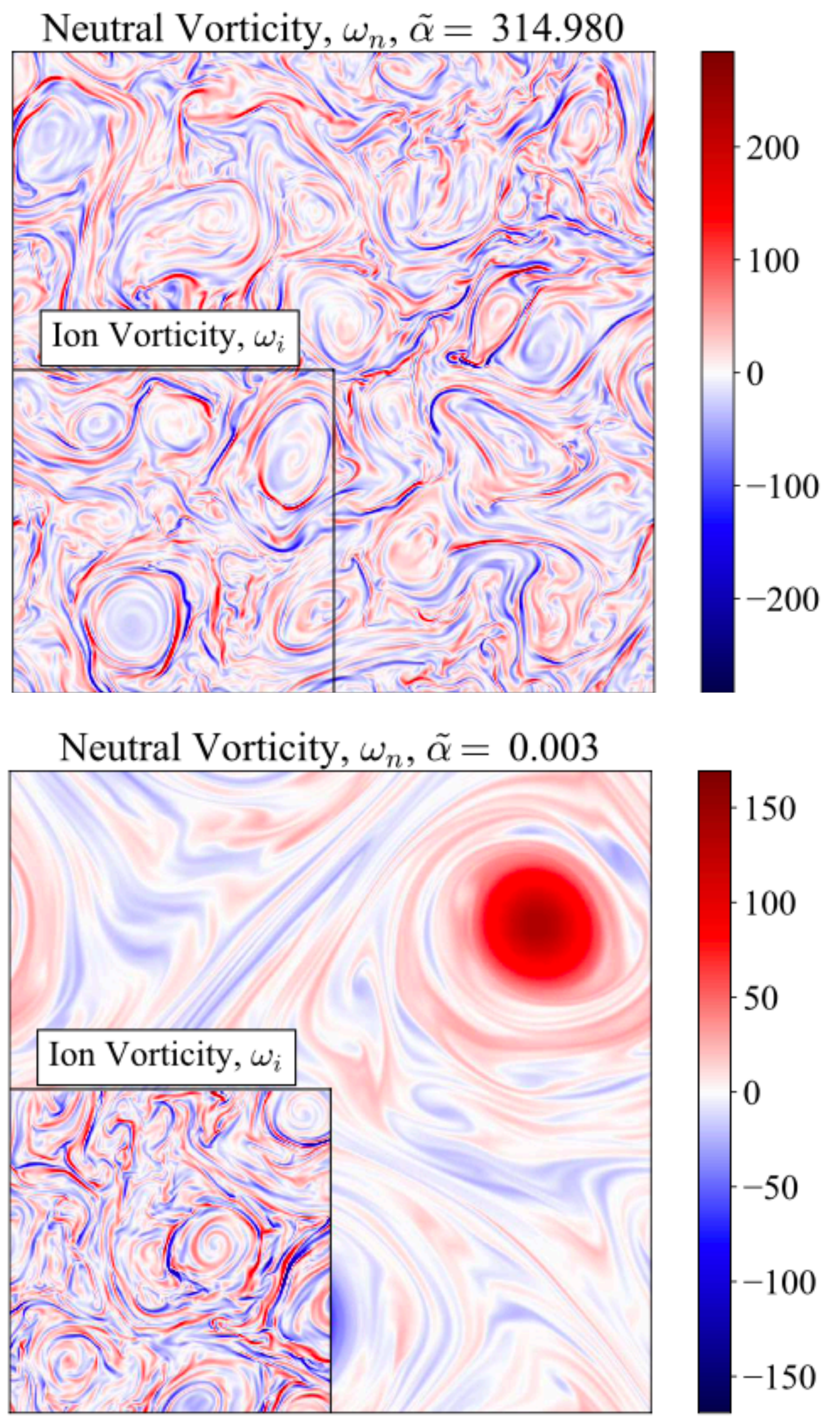
In strong collision regime, the ions and neutrals are strongly coupled and behave like **single MHD fluid**.

But contains **ion-neutral coupling effect**.

- Weak coupling regime γ<<1 ($\alpha\rho_i \ll 1/\tau_{eddy} \ll \omega_{eff} \ll \omega_{ci}$):

$$(\omega_i - \omega_R + i\nu k^2 + i\alpha\rho_n)(\omega_n - \omega_R + i\nu k^2 + i\alpha\rho_i) = -\alpha^2 \rho_i \rho_n$$

Ions and neutrals evolve separately but have a weak mutual drag on each other.



(Benavides et al., JFM **900**, A28 (2020))

How to Determine Maxwell Stress?

- Zeldovich Theorem:

Magnetic diffusion η damps the pert. magnetic field $\widetilde{B}$ →

$$Rm \simeq \frac{\langle \widetilde{B}^2 \rangle}{B_0^2} = \frac{\langle \widetilde{u}^2 \rangle \tau_c}{\eta}$$

$$\langle \widetilde{B}^2 \rangle = \langle \widetilde{u}^2 \rangle \tau_c \frac{B_0^2}{\eta} \propto \frac{B_0^2}{\eta}$$

enhance

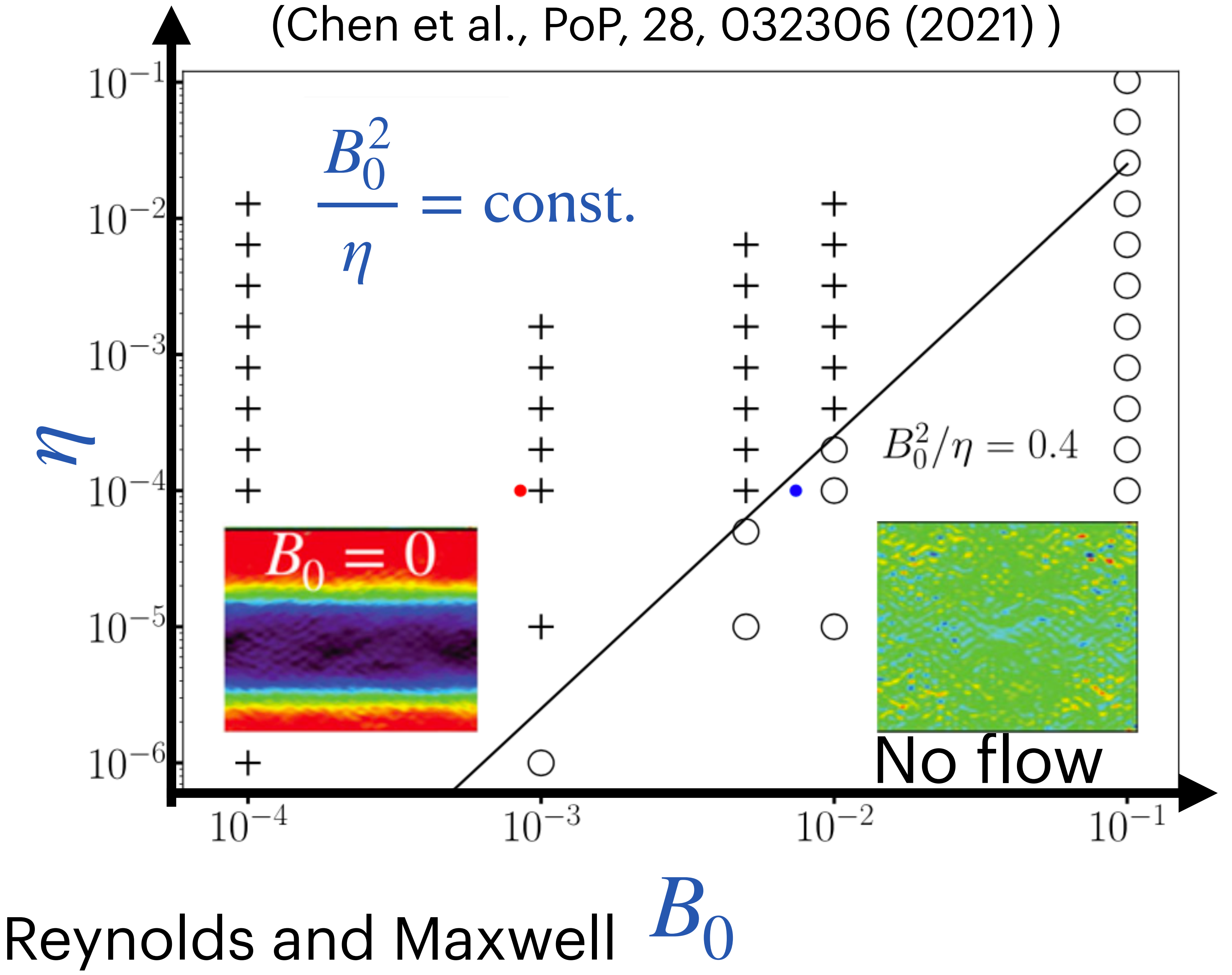
suppress

B_0^2/η is a regulator in the competition btw Reynolds and Maxwell stress competition.

Evolution of zonal flow →

$$\frac{\partial \langle u_y \rangle}{\partial t} = -\frac{\partial}{\partial x}\left(\langle \widetilde{u}_x \widetilde{u}_y \rangle - \frac{\langle \widetilde{B}_x \widetilde{B}_y \rangle}{\mu_0 \rho} \right)$$

(Chen et al., PoP, 28, 032306 (2021))



Reynolds and Maxwell Stress competition

Mean diffusivity η damps while B_0^2 increase the Maxwell stress $\langle \widetilde{B}_x \widetilde{B}_y \rangle$. And hence B_0^2/η regulates the growth of zonal flow.

What is the effect of neutrals on the Zel'dovich Theorem?

Modified Zel'dovich Theorem

- Without neutrals, magnetic stress is proportional to B_0^2/η :

$$\langle\widetilde{B}^2\rangle = \langle\widetilde{u}^2\rangle\tau_c\frac{B_0^2}{\eta} \propto \frac{B_0^2}{\eta}$$

- Now, consider the neutral-ion drag effect:

$$\eta \rightarrow \eta + \eta_{am} \qquad Rm \rightarrow Rm + Rm_{am}$$

$$Rm_{tot} = Rm + Rm_{am} = \frac{\langle\widetilde{B}^2\rangle}{B_0^2} + \frac{\langle\widetilde{B}^2\rangle}{\eta\mu_0\alpha\rho_i\rho_n}\frac{\langle\widetilde{B}^2\rangle}{B_0^2}$$

Assumption: $\langle\widetilde{B}^4\rangle \simeq \langle\widetilde{B}^2\rangle\langle\widetilde{B}^2\rangle$

$$\eta\nabla^2\mathbf{B} \quad \text{v.s} \quad \nabla\times\left(\frac{\mathbf{J}\times\mathbf{B}}{\alpha\rho_i\rho_n}\times\mathbf{B}\right)$$

- In the limit where ambipolar diffusion dominated ($Rm \ll Rm_{am}$) we have:

$$Rm_{tot} \simeq Rm_{am} = \frac{\langle\widetilde{B}^2\rangle}{\eta\mu_0\alpha\rho_i\rho_n}\frac{\langle\widetilde{B}^2\rangle}{B_0^2} = \frac{\langle\widetilde{u}^2\rangle\tau_c}{\eta}$$

Maxwell Stress intensity: $$\langle\widetilde{B}^2\rangle = \left(\langle\widetilde{u}^2\rangle\tau_c\mu_0\rho_i\frac{B_0^2}{A}\right)^{1/2} \propto \frac{B_0}{A^{1/2}}$$

$$A \equiv \frac{1}{\alpha\rho_n} = \frac{1}{\nu_{in}}$$

Change in Scaling

	Maxwell Stress Intensity
No Neutrals	$\langle\widetilde{B}^2\rangle \propto B_0^2/\eta$
Ambipolar diffusion effect	$\langle\widetilde{B}^2\rangle \propto B_0/A^{1/2}$

...alters the competition

The magnetic stress effect on zonal flow generation with neutrals is less sensitive to B_0 .

Conclusions

- We study Non-trivial neutral effect on DW-ZF turbulence: Ambipolar diffusion
- We derive the **key parameter γ** for the Drift-Alfvén + Neutral effect:

$$\gamma = \frac{(\alpha\rho_i)\omega_{eff}^2}{\omega + i\eta_{eff}k^2} \cdot \tau_{eddy} \cdot \frac{1}{\omega_{ci}}$$

- We study the Drift-Alfvén wave with neutrals and found—
  In strong coupling regime: **one MHD fluid**.
  In weak coupling regime: **two fluid.**
- **Modified Zel'dovich Theorem**. We derive the key parameter that **regulates** Maxwell and Reynolds stress competition:

$$\langle \widetilde{B}^2 \rangle \propto B_0^2/\eta \text{ (original)}$$

$$\langle \widetilde{B}^2 \rangle \propto B_0/A^{1/2} \text{ (ambipolar effect)}$$

Future Works

- Calculate $\widetilde{B}$ evolution for arbitrary $\gamma \rightarrow$ Pouquet + neutrals (clarify asymptotic regime).
- Study the physics of neutral entrainment.