How the Spatial Structure of Turbulent Mixing Determines Staircase Structure

Fredy R. Ramirez[1] and Patrick H. Diamond[1]

[1]Department of Physics, Univ. California San Diego

Motivations & Questions

- "E x B staircase" a current subject in Magnetic Fusion.
  - Global pattern of shear micro-barriers and avalanching zones creates a pressure profile resembling a **staircase**.
  - <u>Key feature:</u> The containment of avalanche activity by the series of micro-barriers, which is essential for confinement!
- **Suggested ideas**: E x B shear predator-prey.
- <u>**But**</u>… there is an even simpler physical mechanism to produce layering!
  - Staircase formation, dynamics captured in ultra-simple mixing model with **two disparate time scales**.
- **Questions**
  - How does a fixed global shear and spatial pattern of turbulent mixing affect the staircase structure?
  - How **resilient** is the staircase pattern?
  - How does the staircase **degrade**?

Simple Layering

- Staircase profile can be produced by solving the **passive scalar transport** equation.

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D \nabla^2 n, \qquad (1)$$

$$\mathbf{u} = (\tilde{u}d/\pi)\hat{z} \times \nabla\psi, \qquad (2)$$

- Consider cellular lattice of marginally overlapping cells.

$$\psi = \sin\pi\frac{x}{d}\sin\pi\beta\frac{y}{d}. \qquad (3)$$

- Global transport hybrid:
  - <u>Fast</u> rotation in cell & <u>slow</u> diffusion in boundary layer.
- Irreversibility localized to inter-cell boundary.

Dynamics

- **Transport?** $D^* \sim D\,Pe^{\frac{1}{2}}$ (<u>**Not a simple addition of process!**</u>)
  - Two time rates: τ_H and τ_D.
  - $Pe = \tau_D/\tau_H \gg 1$.
- **Profile?**
  - Consider concentration of injected dye → profile.
  - Staircase arises in stationary array of passive eddys.

$$D^* \sim DP^{1/2}$$

$$Pe = \frac{\tau_D}{\tau_H}$$

Back-of-Envelope Calculation

- Consider the following approximation,

$$D^* \approx f_{active}\frac{(\Delta x)^2}{\Delta t}$$

- f_{active} is the ratio of the boundary layer width and cell size and Δt is the circulation time.

$\delta \to$ BL width
$l_0 \to$ cell size

$$\delta^2 \sim D\Delta t \sim \quad Dl_o/v_o \quad \Longrightarrow \quad \boxed{D^* \sim DP^{1/2}}$$

Global Spatial Shear

$\psi_{shear}(m=1)$

$$\psi = \sin\pi(x/d)\sin\pi\beta(y/d) + \alpha\,\psi_{shear}$$

$$\psi_{shear} = -\cos\frac{mx}{2}$$

$$Pe_S = \frac{\tau_D}{\tau_S} = \frac{\alpha md}{12D}$$

$$\sigma_x^2 = \int_0^{L_x} (\langle n\rangle_y - \langle n_0\rangle_y)^2\,dx$$

As the shear strength increases, the staircase profile **breaks down**. Shear <u>**enhances**</u> mixing (i.e., increases irreversible process).

<u>Important:</u>
- Pe_{sh} (measure of shearing) introduces a shear dispersion time scale $\tau_{sh} = 12d/\alpha\,m$.
- Shear dispersion rate gives effective mixing rate faster than diffusion!
- Scalar concentration degrades as shear is increased.

Vortex Crystal

- Next, the staircase structure is studied at different stages of a "melting" vortex crystal.
- The "vortex crystal" is simply the array of cells and "melting" is related to turbulence mixing field structure.

$$(\partial_t + \mathbf{u}\cdot\nabla)\omega = \nabla^2\omega/\Omega + F_\omega - \alpha\,\omega, \qquad (4)$$

$$\omega = \nabla^2\psi.$$

- The stability of the crystal is broken by varying the β and η of a spatially periodic force term inside the 2D Navier-Stokes equation.

$$F_w = \beta\,n^3\left[\cos(\eta\,n\,x) + \cos(\eta\,n\,y)\right]/\Omega \qquad (5)$$

- The resilience of the staircase is studied by increasing disorder in the vortex crystal through F_w.

$\psi\,(\beta=1, \eta=1)$

$\psi\,(\beta=1, \eta=1.27)$

- Streamfunction ψ is output and inserted into passive scalar transport equation.
- Scalar concentration n is introduced on the left boundary of the crystal.

Fate of the Staircase

$\beta=1, \eta=1$

$\beta=3, \eta=1$

$\beta=4, \eta=1$

- As the degree of melting is increased, the staircase step size increases.
- The degree of melting is representative of the extent of deviation from marginality.

Conclusion

- Strong global spatial shear breaks down staircase profile, thus, shear enhances mixing.
- Deviation from marginality affects staircase step size. Staircase steps merge.
  - Staircase is robust and can survive in disordered flow.

Acknowledgements

References

[1] M. N. Rosenbluth *et al.* Effective Diffusion in Laminar Convective Flows. *The Physics of Fluids*, 30(9):2636-2647, 1987.

[2] G. Dif-Pradalier *et al.* Finding The Elusive **E** x **B** Staircase in Magnetized Plasmas. *Phys. Rev. Lett.*, 114:085004, Feb 2015.

Contact

Fredy R. Ramirez - f3ramire@ucsd.edu