Power Scaling of The L mode Density Limit: Physics and Predictions

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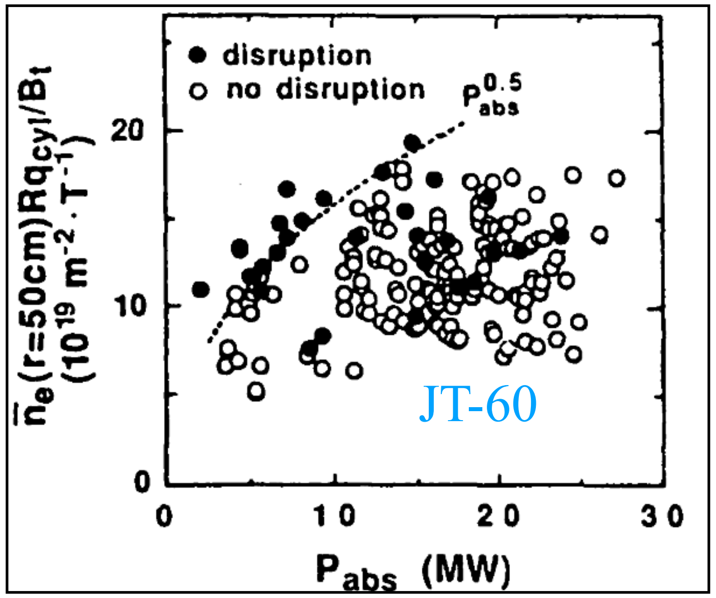
What's the issue?

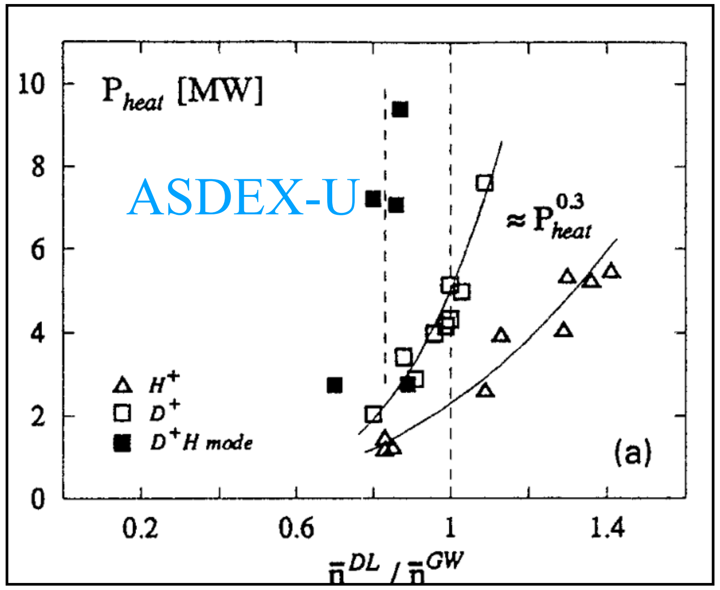
- Discharge terminates when line integrated density exceeds a critical value called Greenwald density $\bar{n}_g = I_p/\pi a^2$.

- Greenwald density limit scaling is Power (Q) Independent !(?) . [Greenwald PPCF 2002]

- Contradictions: Q dependence observed in many experiments !

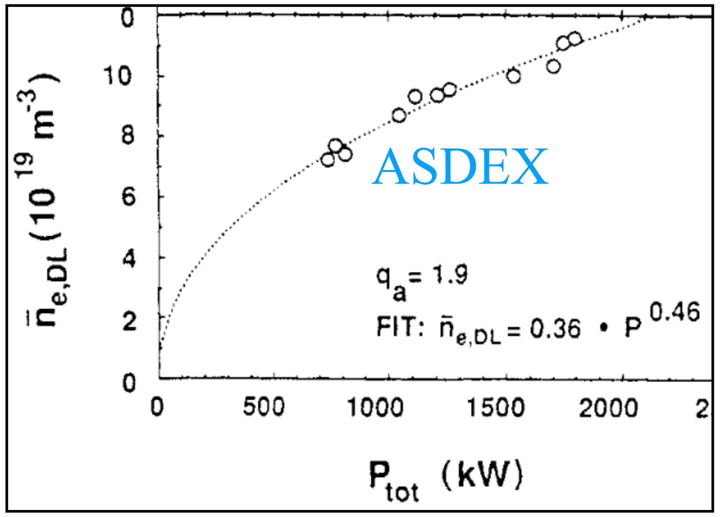
Kamada *et al* NF 1991

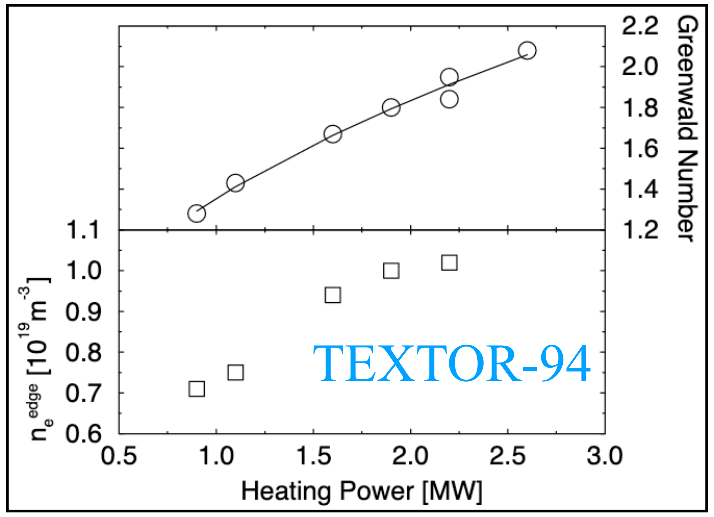




Mertens *et al* NF 1997

Stabler *et al* NF 1992



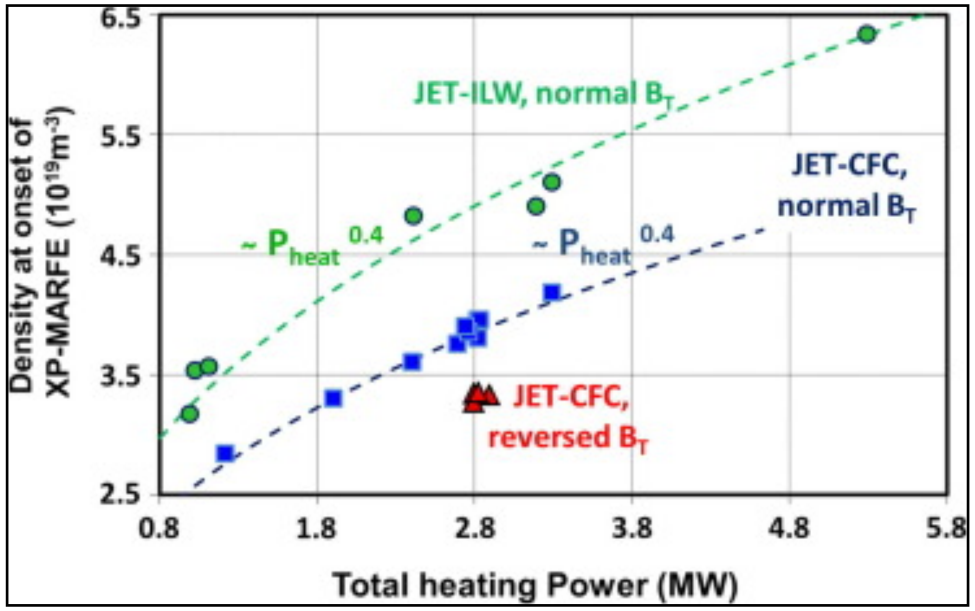


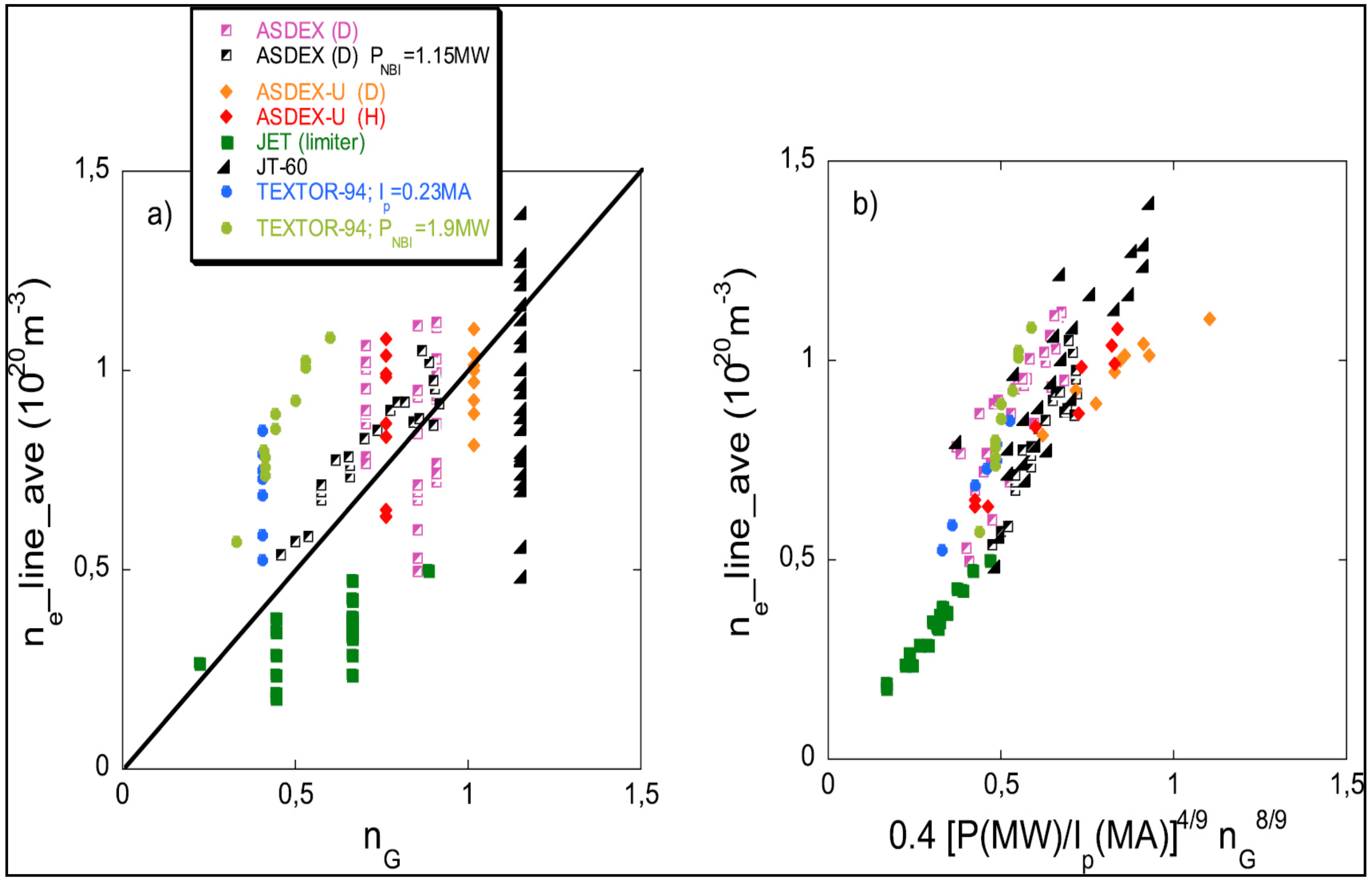
Rapp *et al* NF 1999

Power scaling observed after Greenwald 2002

- Density limit in JET $n_{crit} \sim Q^{0.4}$

Huber *et al* JNM 2013





- New analysis based on radiative power balance $n_{crit} \sim Q^{4/9}$
- Notice significantly less scattering in data points on inclusion of power dependence!

Zanca *et al* NF 2019

Preview of results

- Density limit phenomenology is linked with *shear layer collapse*. Radiative effects are *secondary* to transport bifurcation.

- We show that *power scaling of L mode density emerges from zonal shear layer collapse dynamics*. Shear layer strength increases with power (Q) →improved particle confinement → $n_{crit}\uparrow$

- Predictions beyond scalings:

  - Zonal shear layer collapse is hysteretic. Hysteresis is due to *dynamical delay in bifurcation* due to critical slowing down. *Fundamentally different* from L-H hysteresis associated with bi-stable states.

- In the primacy hierarchy, scalings are at the 'surface'. *Fluctuations and dynamics are more fundamental.* Hysteresis is a manifestation of the underlying transport bifurcation process. Experimental observation of shear layer and fluctuation hysteresis would *validate* the linkage of density limit to transport bifurcation.

Conventional Wisdom associates Density Limit to radiative events

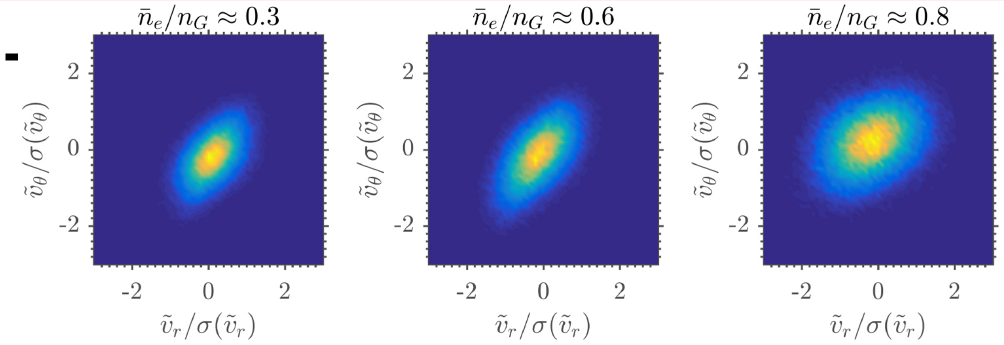
Density limit is often associated with macroscopic events

- Global thermal collapse, Radiative condensation / MARFEs. Poloidal detachment, Divertor detachment, MHD activity -radiation driven islands.

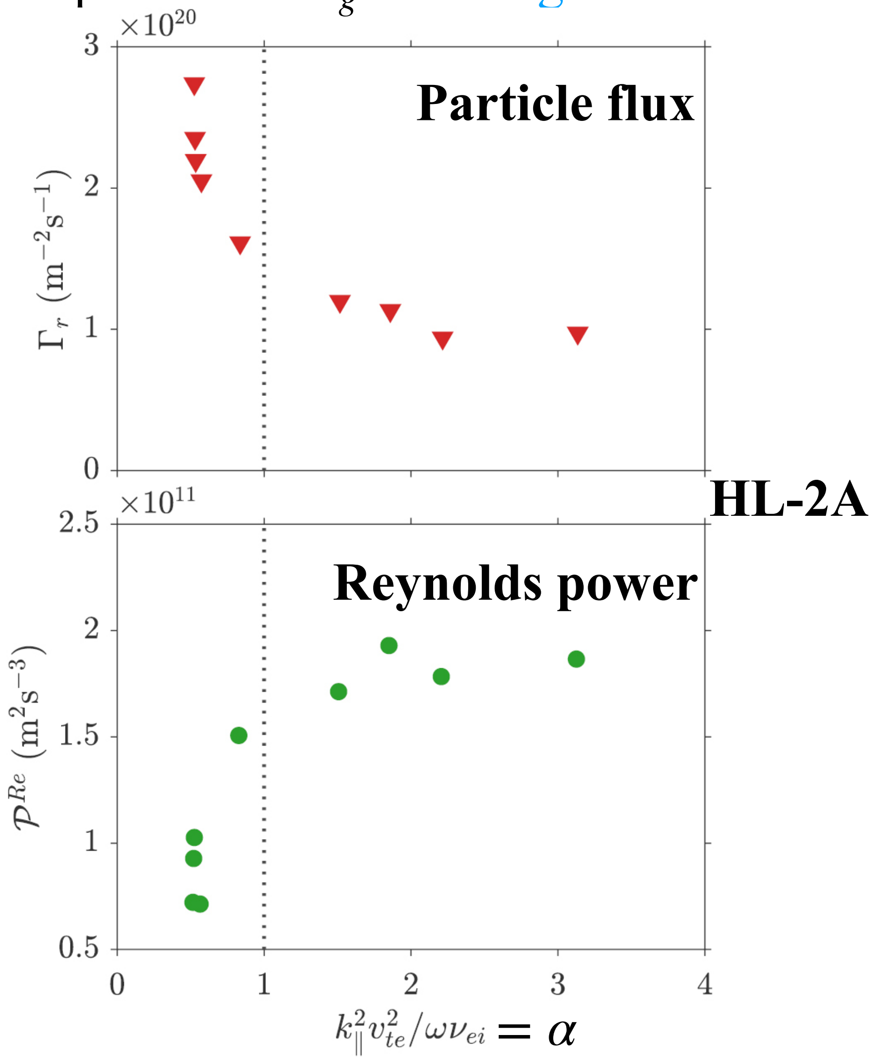
- High density→Edge cooling→MARFEs→Current profile shrinkage → Tearing→Islands …→Disruption!

What's the role of microscopic transport physics?

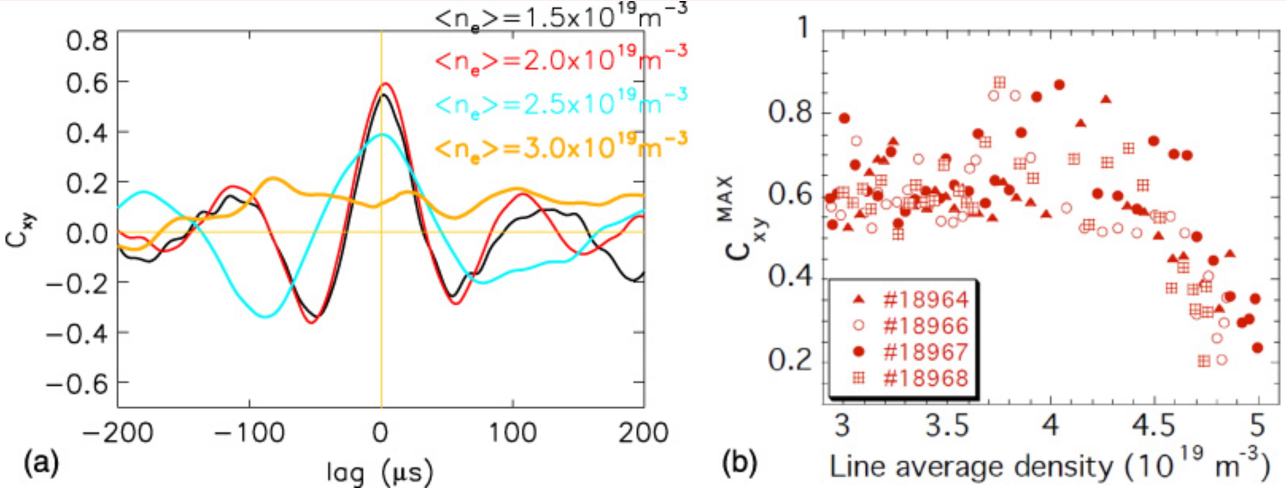
Recent experiments show density limit is related to shear layer decay



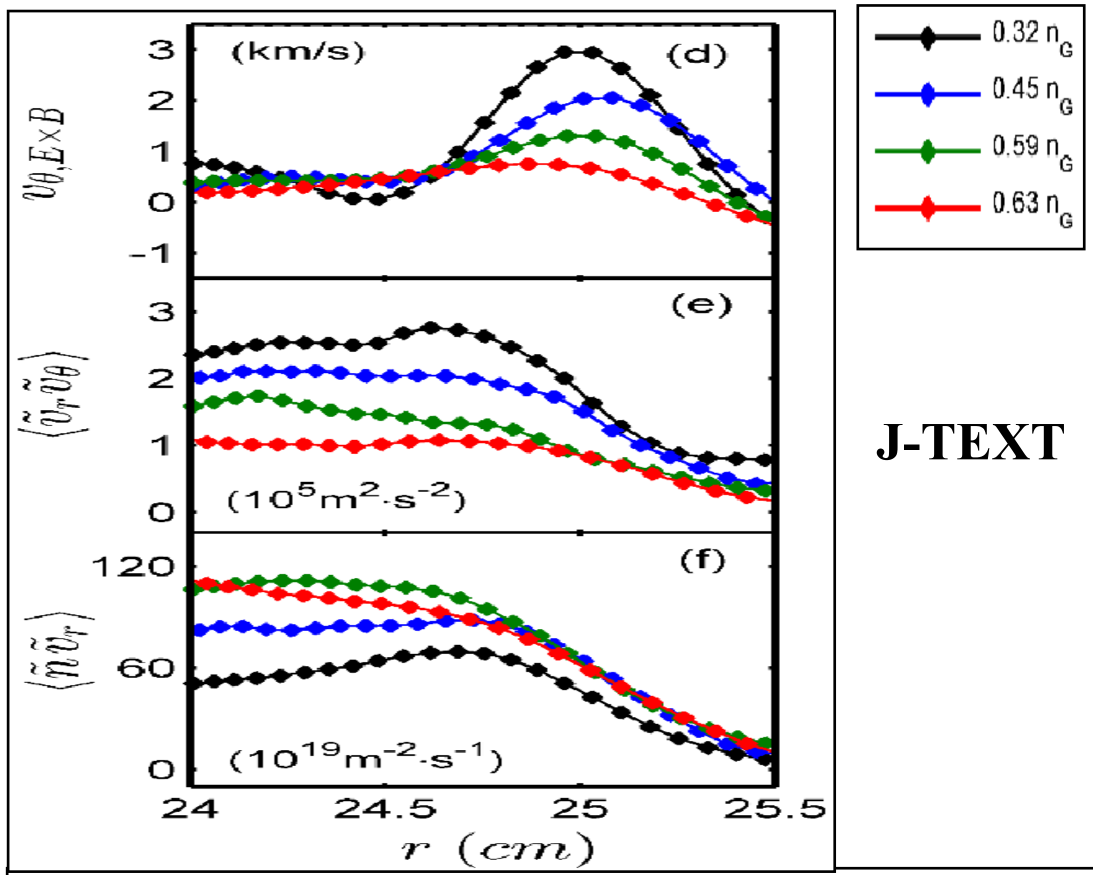
Radial and poloidal velocity correlations drops as $n \rightarrow n_g$ Hong *et al* NF 2018



Reynolds power $P_{Re} = -\langle v_\theta \rangle \partial_r \langle \tilde{v}_r \tilde{v}_\theta \rangle$↓and particle flux↑as α drops below 1.



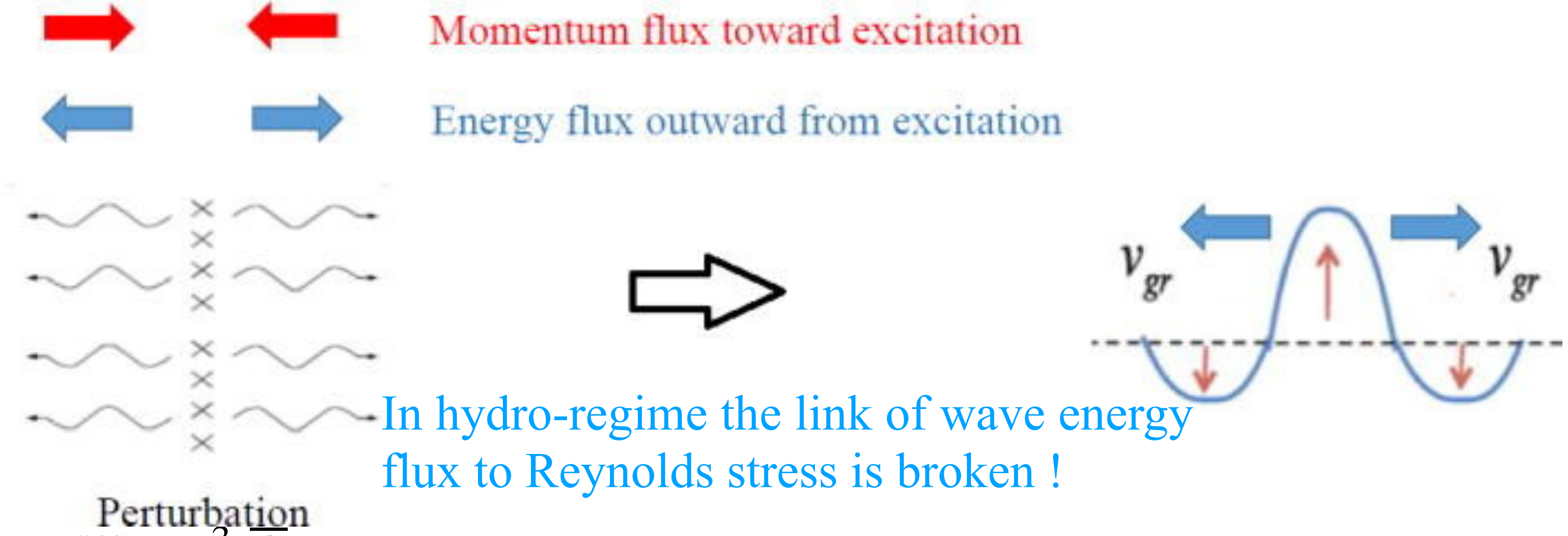
Long range correlations (LRC) decrease as the line averaged density increases in both TEXTOR and TJ-II. LRC ↔ ZF strength Y. Xu *et al* NF 2011



Particle flux ↑ and Reynolds stress, ExB flow and flow shear ↓ as $n \rightarrow n_g$. T Long *et al* NF 2021 @ this meeting

Shear layer collapse in hydro- regime and origin of current scaling

- Plasma response for Hasegawa - Wakatani :- HDM Theory[Hajjar, Diamond, Malkov 2018]



In hydro-regime the link of wave energy flux to Reynolds stress is broken !

- $\Gamma_n, \chi \uparrow$ and Π^{res}, $\nabla_\perp^3 \bar{\phi} \downarrow$ as the electron response passes from adiabatic to hydrodynamic regime.

- Weak zonal flow production for $\alpha \ll 1 \rightarrow$ weak regulation of turbulence and enhancement of particle transport and turbulence.

- **Origin of current scaling:** zonal flow drive is "screened" by neoclassical dielectric [Rosenbluth - Hinton 1998]. ;

- Poloidal gyro-radius ρ_θ emerges as screening length ρ_{sc} ! Effective ZF inertia ↓ as $I_p \uparrow \rightarrow$ <u>ZF strength increases with I_p</u>, for fixed drive.

- Favorable I_p scaling persist in plateau regime (edge of interest). NO I_p scaling in P-S regime. [Singh & Diamond NF 2021]

$$\frac{\partial}{\partial t}\left\langle \left|\phi_k\right|^2 \right\rangle = \frac{2\tau_c \left\langle \left|S_k\right|^2 \right\rangle}{\left|\varepsilon(q)\right|^2};$$

Emission from polarization interaction

Neoclassical response

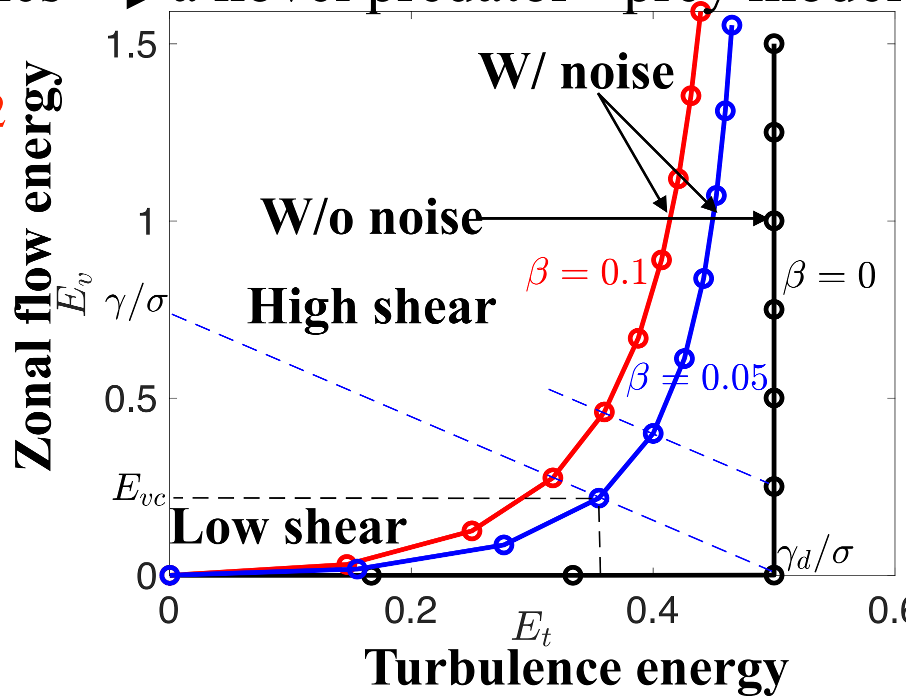
$$\varepsilon = \varepsilon_{cl} + \varepsilon_{neo} = \frac{\omega_{pi}^2}{\omega_{ci}^2}\left\{1 + \frac{q^2}{\epsilon^2}\right\} k_r^2 \rho_i^2$$

Zonal wave #

Shear layer collapse in adiabatic regime

Singh and Diamond NF 2021

Neoclassical screening + drift wave - zonal flow dynamics ➔ a novel predator - prey model.



$$\partial_t E_t = \gamma E_t - \sigma E_v E_t - \eta E_t^2;\ \partial_t E_v = \sigma E_t E_v - \gamma_d E_v + \beta E_t^2$$

Notice, $\sigma \sim \varepsilon^{-1} \sim B_\theta^2 \sim I_p^2$ and $\beta \sim \varepsilon^{-2} \sim B_\theta^4 \sim I_p^4$

$\Longrightarrow I_p$ jacks up modulational growth and zonal noise → stronger feedback on turbulence.

- Noise eliminates threshold for zonal flow production.

- Criterion for zonal flow collapse with noise tracks that for collapse of zonal flow without noise i.e.,

$$\gamma < \eta\frac{\gamma_d}{\sigma} \Longrightarrow \underset{\textbf{Dimensionless parameter}}{\boxed{\frac{\rho_s}{\sqrt{\rho_{sc}L_n}}}} < \left[\frac{\eta}{\Omega_i}\frac{\gamma_d}{2k_x^2\rho_s^2\Theta\Omega_i^2}\frac{\hat{\alpha}}{q_\perp^2\rho_s^2}\frac{\left(1+q_\perp^2\rho_s^2\right)^3}{q_y^2\rho_s^2}\right]^{1/4}$$

Particle source S

$\rho_s/\sqrt{\rho_{sc}L_n} < crit \rightarrow S < S_c \sim B_\theta^{-3} \sim I_p^{-3}$

Local edge density n	**Zonal flow damping**
$n > n_c \sim I_p S^{1/3}$	**Viscosity dominated**
$n > n_c \sim I_p^2 S^{2/3}$	**Charge exchange friction**

Extended model for power scaling of density limit from shear layer collapse scenario (I)

- A variant of KD03 model - used for L-H studies, Power (Q) limited to the **L mode**
- Coefficients derived for ITG mode, Includes **neoclassical zonal flow screening** response
- In Gyro-Bohm normalization

Normalized Turbulence energy $\mathscr{E}$:

$$\frac{\partial \mathscr{E}}{\partial t} = \frac{a_1 \gamma(\mathscr{N}, \mathscr{T}) \mathscr{E}}{\left(1 + a_3 \mathscr{V}^2\right)} - \underset{\underset{\text{Nonlinear Damping}}{\uparrow}}{a_2 \mathscr{E}^2} - \frac{a_4 \mathscr{E}_z \mathscr{E}}{\left(1 + b_2 \mathscr{V}^2\right)}$$

Normalized Zonal flow energy $\mathscr{E}_z$:

$$\frac{\partial \mathscr{E}_z}{\partial t} = \underset{\underset{\text{Mod. Growth}}{\uparrow}}{\frac{b_1 \mathscr{E} \mathscr{E}_z}{\left(1 + b_2 \mathscr{V}^2\right)}} - \underset{\underset{\text{Coll. Damping}}{\uparrow}}{b_3 \hat{n} \mathscr{E}_z} + \underset{\underset{\text{Zonal noise}}{\uparrow}}{b_4 \mathscr{E}^2}$$

Normalized Temperature gradient $\mathscr{T}$:

$$\frac{\partial \mathscr{T}}{\partial t} = - \underset{\underset{\text{Turb. Flux}}{\uparrow}}{c_1 \frac{\mathscr{E} \mathscr{T}}{\left(1 + c_2 \mathscr{V}^2\right)}} - \underset{\underset{\text{Neo. Flux}}{\uparrow}}{c_3 \mathscr{T}} + \underset{\underset{\text{Input power}}{\uparrow}}{Q}$$

Normalized Density $\hat{n}$:

$$\frac{\partial \hat{n}}{\partial t} = - \underset{\underset{\text{Turb. Flux}}{\uparrow}}{d_1 \frac{\mathscr{E} \hat{n}}{\left(1 + d_2 \mathscr{V}^2\right)}} - \underset{\underset{\text{Neo. Flux}}{\uparrow}}{d_3 \hat{n}} + \underset{\underset{\text{Source}}{\uparrow}}{S}$$

Control knobs

Normalized Mean ExB shear $\mathscr{V}$ (from radial force balance):

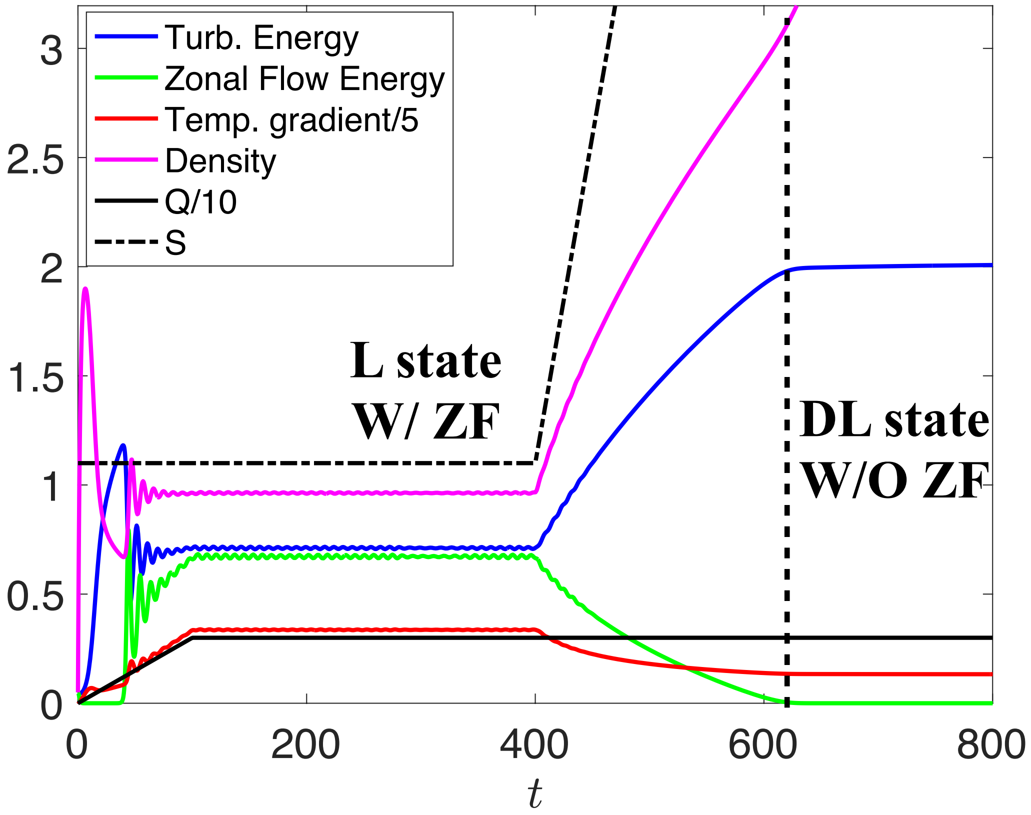
$$\mathscr{V} \equiv \frac{V_E' a}{\rho^\star v_{thi}} = - \frac{1}{\hat{n}} \mathscr{N} \left(\frac{1}{\hat{n}} \mathscr{N} + \frac{1}{\hat{T}} \mathscr{T} \right).$$

Density gradient $\mathscr{N}$ remains frozen.

Notice, Modulational growth $b_1 \sim \varepsilon^{-1} \sim B_\theta^2 \sim I_p^2$ and zonal noise $b_4 \sim \varepsilon^{-2} \sim B_\theta^4 \sim I_p^4$

Extended model for power scaling of density limit from shear layer collapse scenario (II)

- Power (Q) ramp up followed by particle source (S) ramp-up
- Two states: L & DL
- Zonal flows (ZF) excited when $Q > Q_{th}$ (Power threshold)
- Q_{th} set by turbulence level and zonal flow damping
- Oscillations- Predator-Prey competition- self-regulation
- ZF energy set by power and density
- Source ramp up→density ↑ → ZF damping ↑→ZF energy ↓
- At a critical density(n_{crit}) ZF energy →0 and the system bifurcates to DL state



Time evolution of turbulence energy, zonal flow energy, density and temperature gradient in a successive power Q and particle source S ramp.

L state (W/ ZF)

$$\mathscr{E} = b_3\hat{n}\left(1 + b_2\mathscr{V}^2\right)/b_1$$

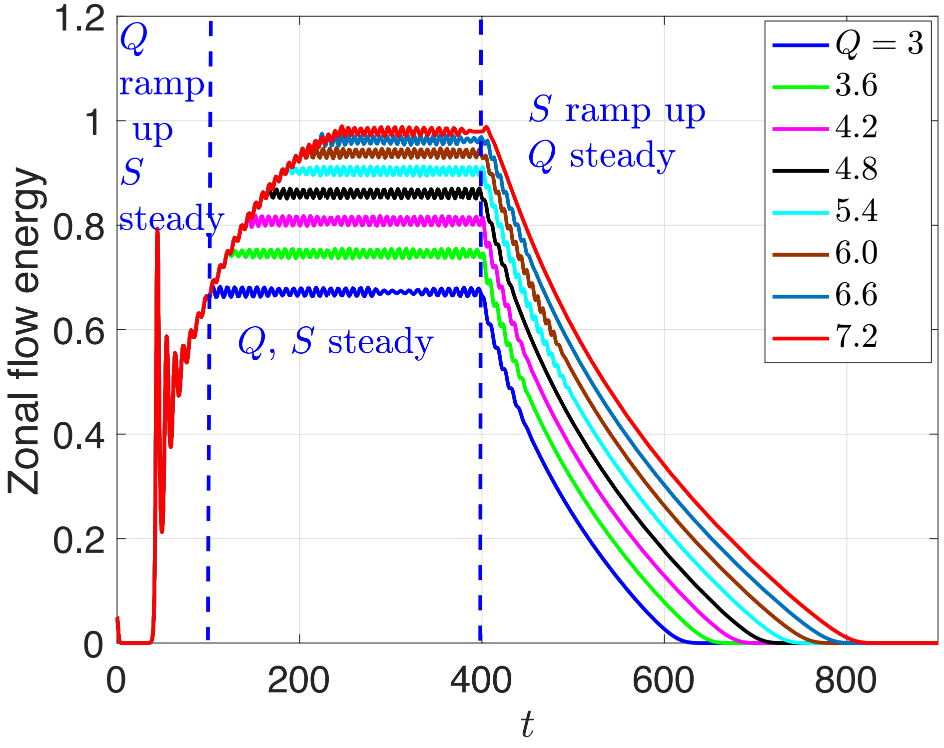
$$\mathscr{E}_z = \frac{a_2}{a_4}\left(1 + b_2\mathscr{V}^2\right)\left[\frac{a_1}{a_2}\frac{\gamma}{\left(1 + b_2\mathscr{V}^2\right)} - \frac{b_3}{b_1}\hat{n}\left(1 + b_2\mathscr{V}^2\right)\right].$$

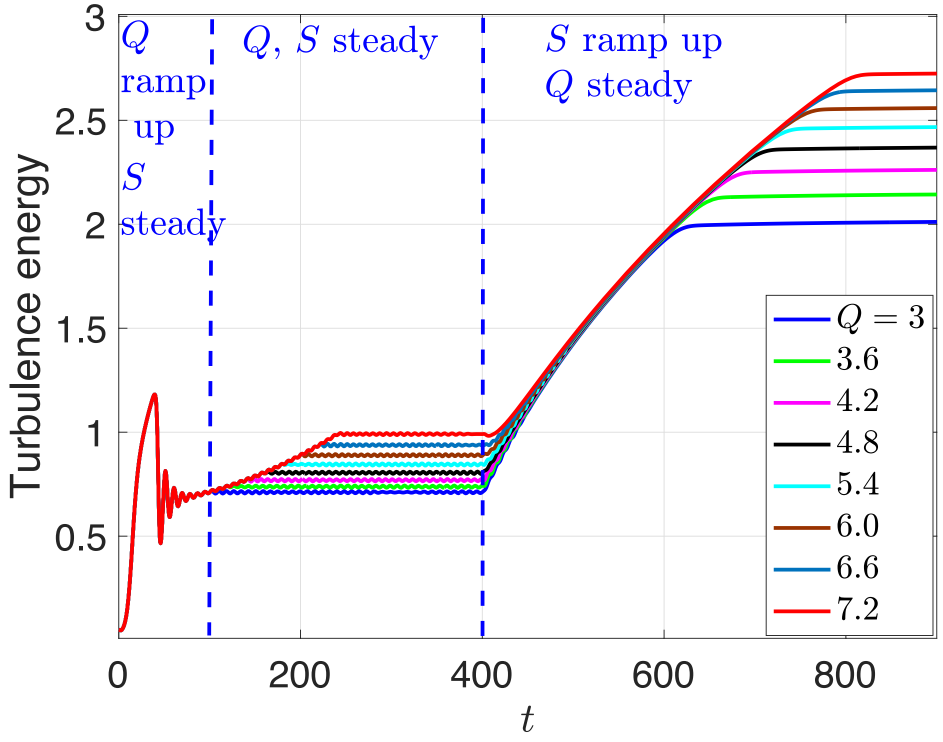
DL state (W/O ZF)

$$\mathscr{E} = \frac{a_1}{a_2}\frac{\gamma}{(1 + b_2\mathscr{V}^2)} \qquad \mathscr{E}_z = 0$$

L →DL transition studies: Power scan

- Observe shear layer collapse
- Power (Q) ramp up followed by particle source (S) ramp-up
- Zonal flow increases with power (Q)
- Oscillations→Predator - prey
- Higher power (Q) → longer ZF damping time →higher n_{crit}
- Higher power (Q) → higher Turb. Energy $\left(\frac{\partial \mathscr{E}}{\partial Q}\right)_L < \left(\frac{\partial \mathscr{E}}{\partial Q}\right)_{DL}$





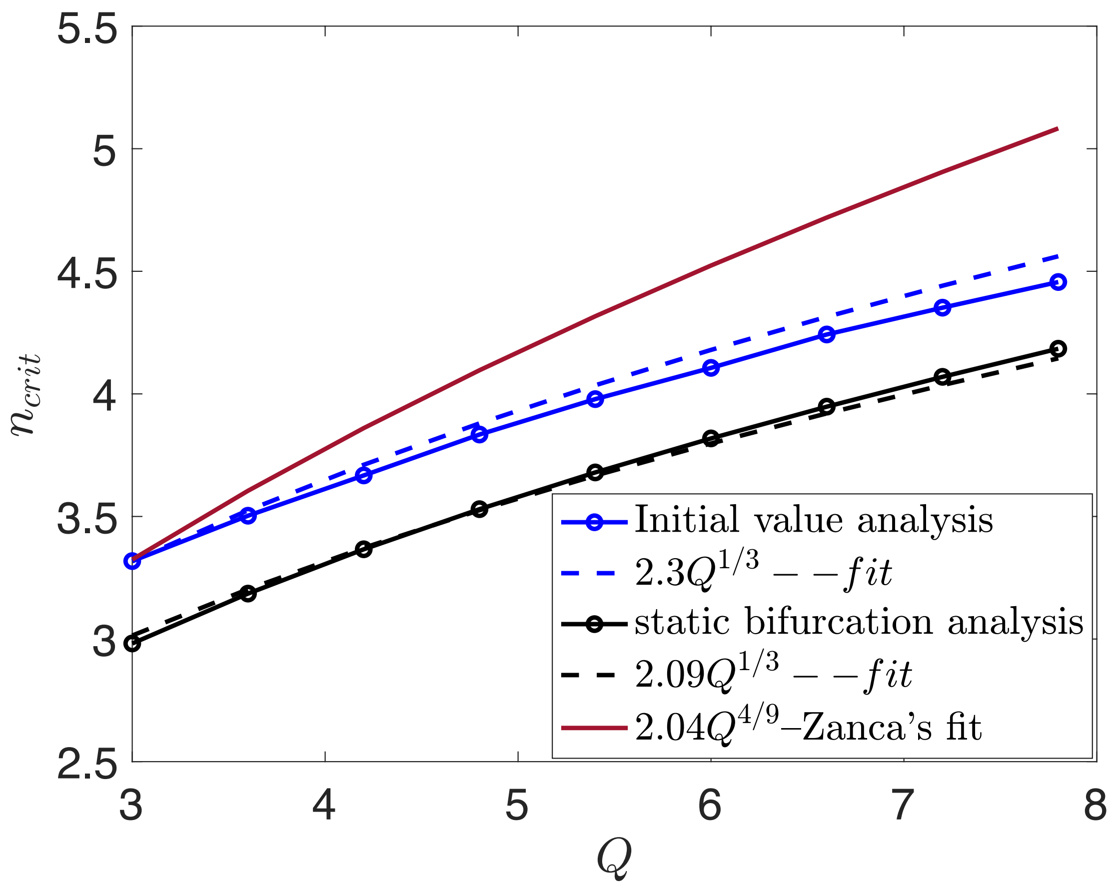
Power Scaling

- Favorable power scaling of density limit $n_{crit} \sim Q^{1/3}$

- Physics: $\gamma(\nabla T)$ vs ZF damping + saturation by ZF
  - Shear layer strength increases with power (Q) → improved particle confinement → $n_{crit} \uparrow$

- Close to Zanca's radiative collapse density $n_{crit} \sim Q^{4/9}$ from radiative power balance

- n_{crit} from initial value > n_{crit} from static bifurcation → bifurcation delay due to critical slowing down



Power scaling of critical density

L→DL transition studies: Stochastic field scan

- RMP→ breaking of nested magnetic surfaces→ magnetic islands at rational surfaces→overlapping of islands→Stochastic B

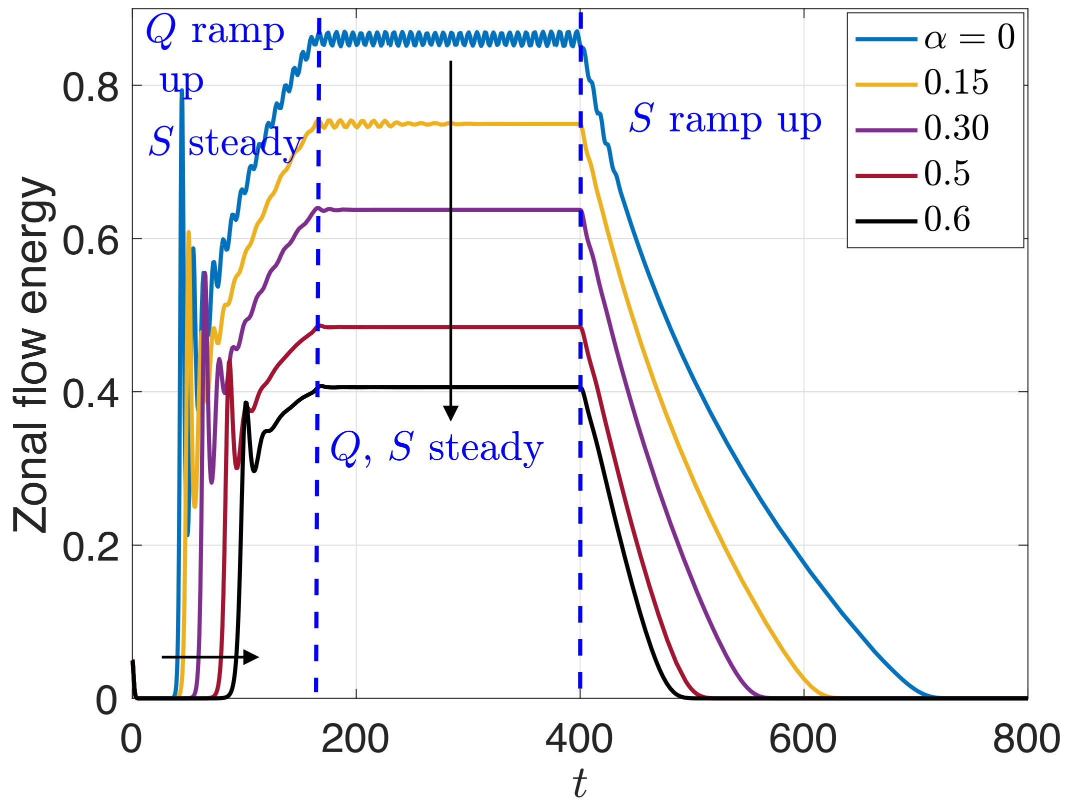
- Model extensions: [Chen *et al* PoP 2021]
  - Growth rate reduction $\gamma \rightarrow \gamma/(1+\alpha)$,
  - Reynolds stress dephasing $\langle v_r v_\theta \rangle \rightarrow \langle v_r v_\theta \rangle/(1+\alpha)$,
  - Particle and heat flux dephasing $\langle v_r n \rangle \rightarrow \langle v_r n \rangle/(1+\sqrt{\beta}\alpha)$, $\langle v_r T \rangle \rightarrow \langle v_r T \rangle/(1+\sqrt{\beta}\alpha)$
  - $\alpha = q\left(\dfrac{\delta B_r}{B}\right)^2/\sqrt{\beta}\rho_\star^2\epsilon$ is normalized stochastic B intensity

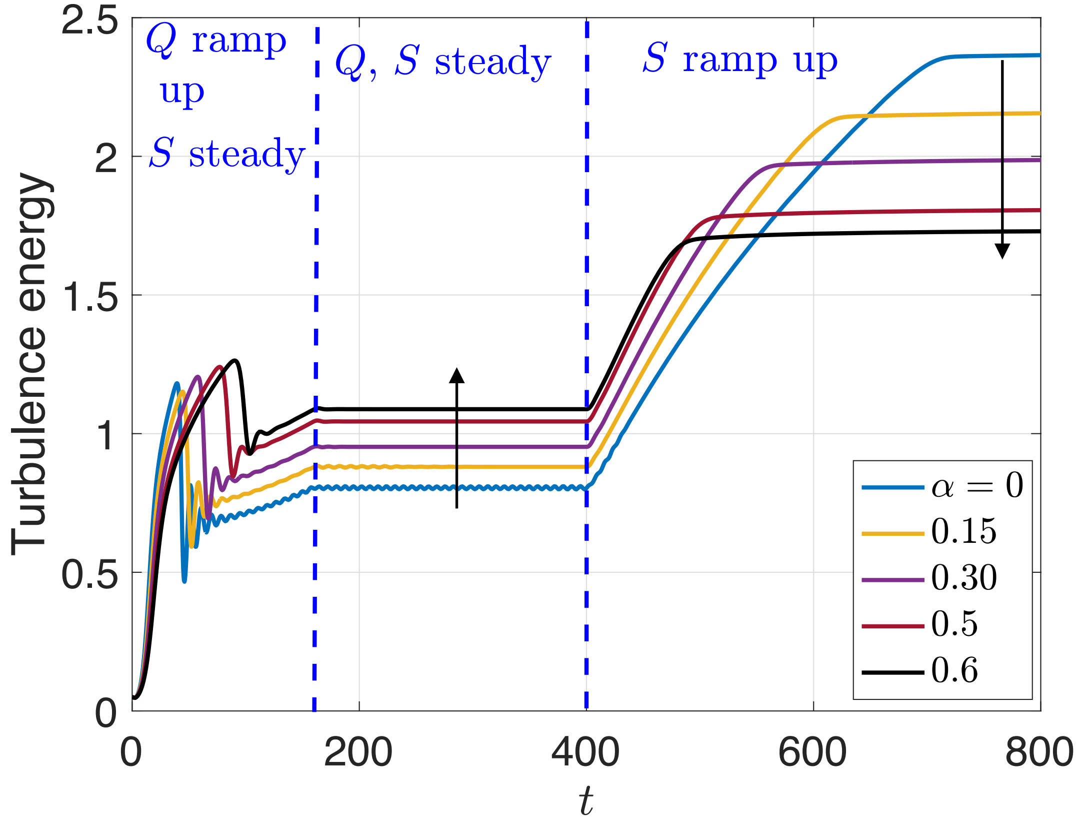
- Power threshold Q_{th} for zonal flow excitation ↑ when $\alpha\uparrow$

- ZF energy ↓ and Turb. Energy ↑ in steady Q and S region

- ZF damping time ↓ when $\alpha\uparrow$
  $\Longrightarrow n_{crit}$ decreases with stochastic B

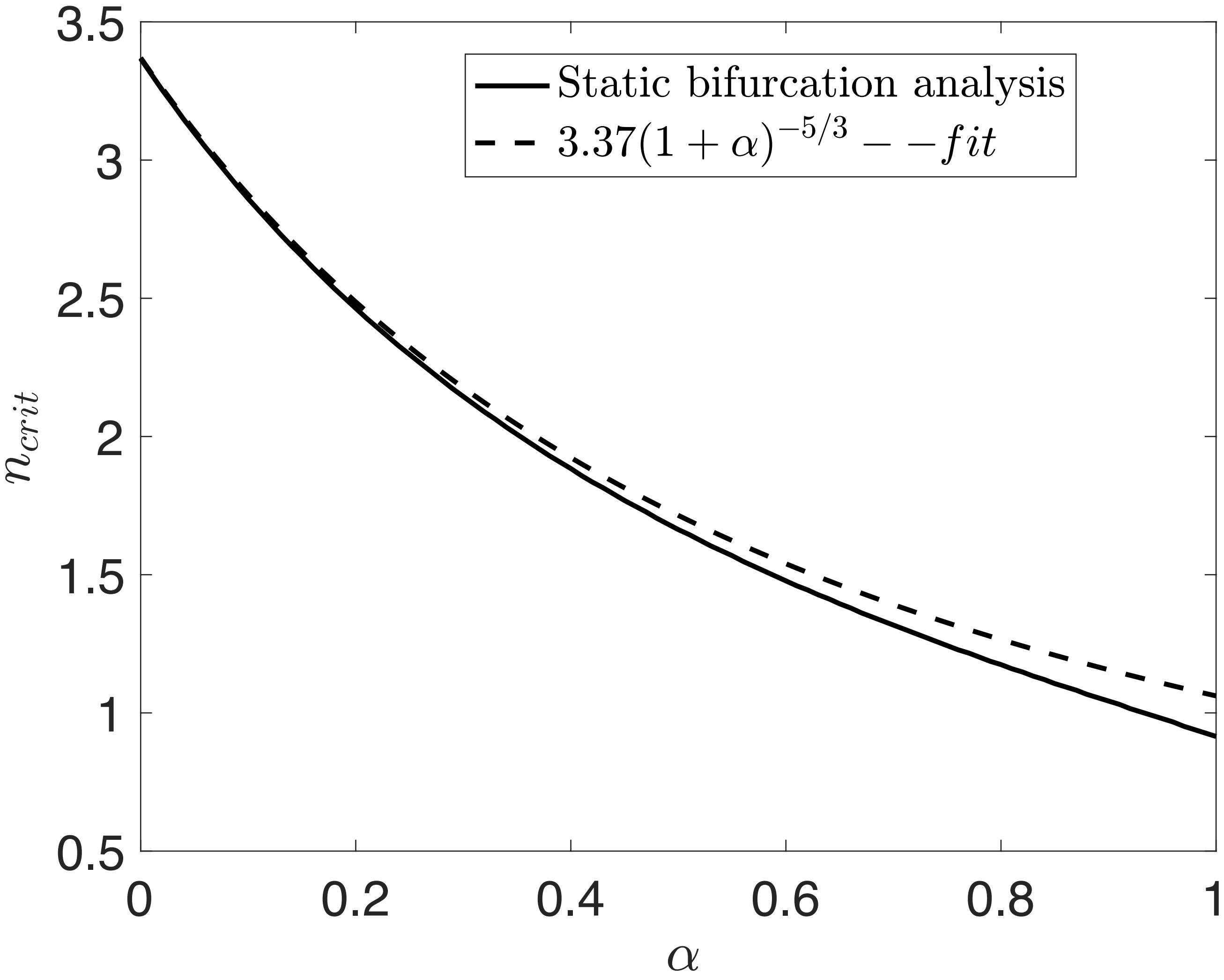
- After shear layer collapse, Turb. energy decreases with stochastic B





Stochastic field scaling

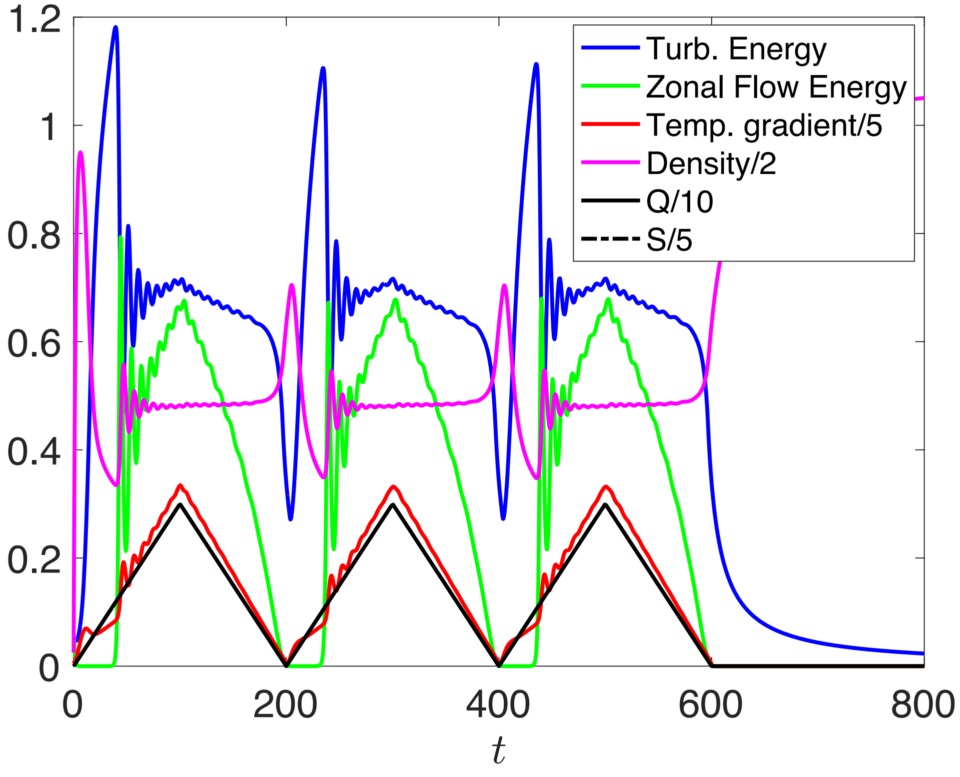
- Unfavorable stochastic field scaling $n_{crit} \sim (1+\alpha)^{-5/3}$
- Physics: Stochastic fields erode the shear layer by dephasing the Reynolds stress
- Ambient stochastic B accelerate the shear layer decay on increasing n.

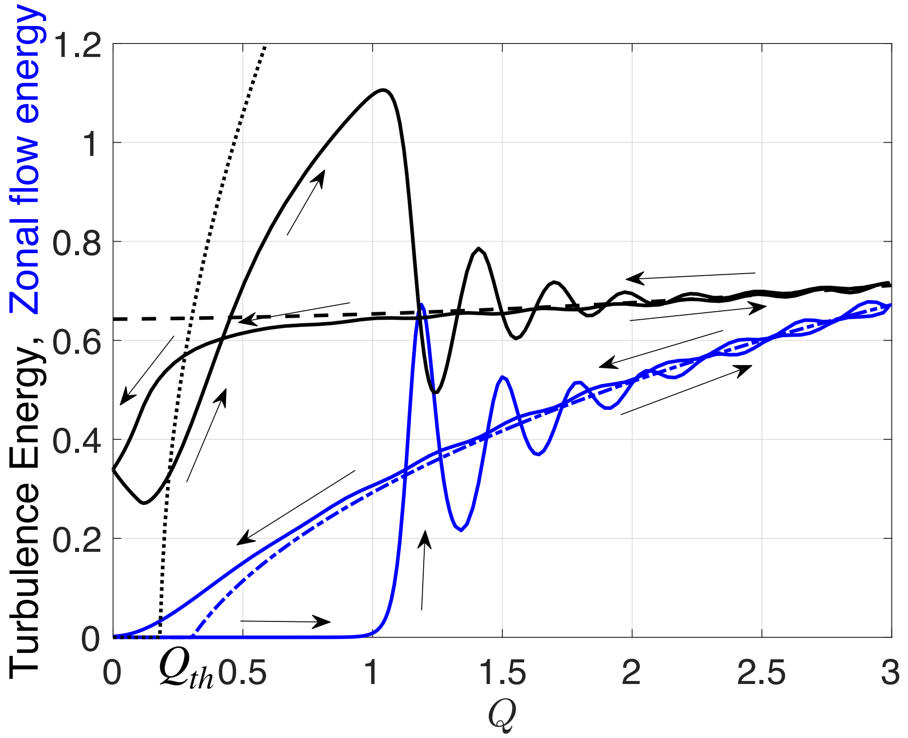


Stochastic field α scaling of critical density

Hysteresis with cyclic power ramp

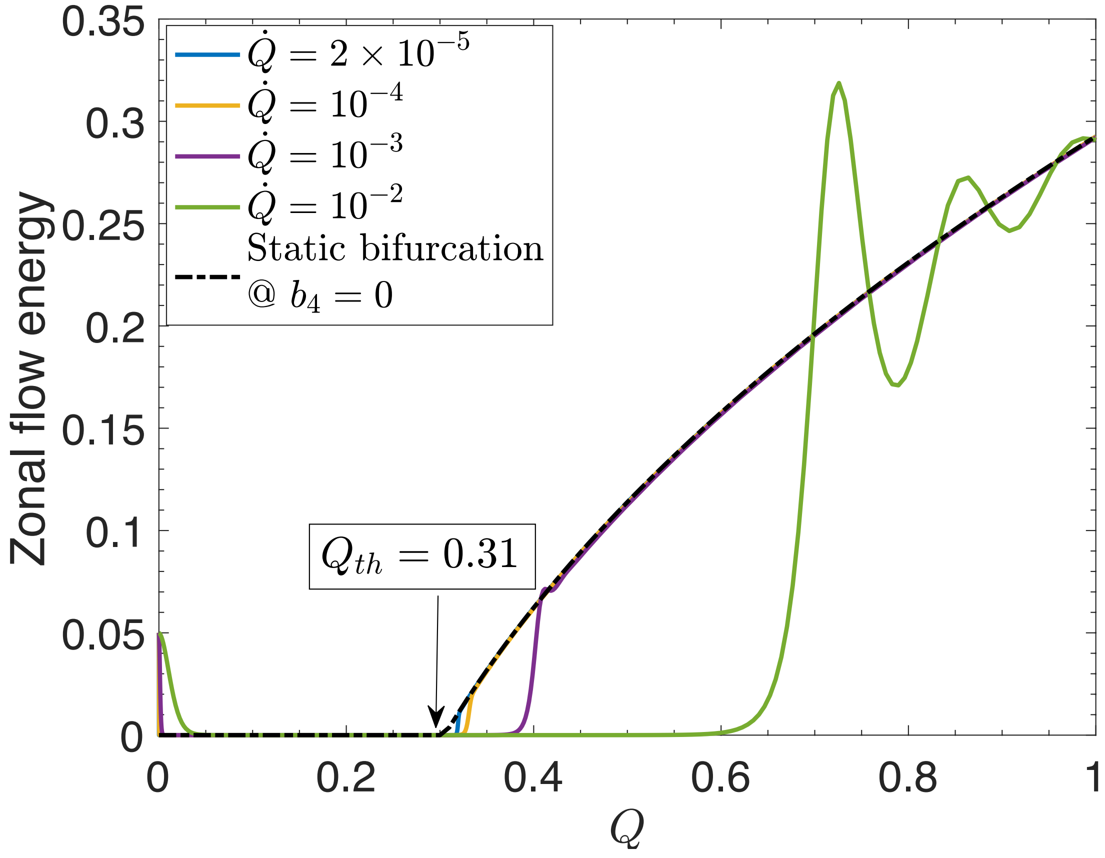
- All fields exhibit hysteresis in cyclic Q ramp!
- Physics prediction - - - beyond scalings !
- Physics:
  - Two states L(w/ ZF), DL(w/o ZF)
  - But only one stable state at any moment —then why hysteresis?
  - Delay in (transcritical) bifurcation due to critical slowing down at the static bifurcation point Q_{th}.
  - Linearized dynamics near bifurcation point reveals zonal flows begin to grow when $\int_{t_0}^{t} dt' \left(b_1(t') \frac{a_1 \gamma(t')}{a_2} - b_3 \hat{n}(t') \right) > 0$
  - **Dynamical bifurcation** at time $t^\star$ when $\int_{t_0}^{t^\star} dt' \left(b_1(t') \frac{a_1 \gamma(t')}{a_2} - b_3 \hat{n}(t') \right) = 0$
  - Static bifurcation at time $\bar{t}$ when $\left(b_1(\bar{t}) \frac{a_1 \gamma(\bar{t})}{a_2} - b_3 \hat{n}(\bar{t}) \right) = 0$





Hysteresis plot in t=[200 400]

Effect of ramp speed on bifurcation delay

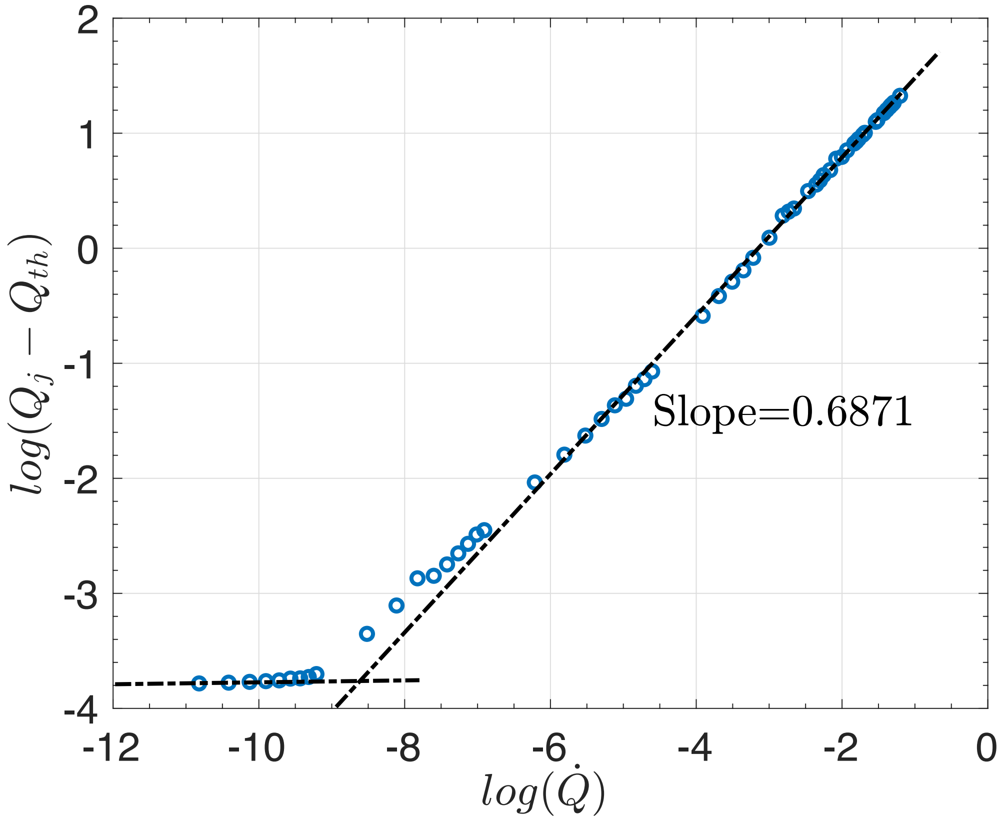


- Passage through static bifurcation point (Q_{th}) exhibits a delay that depends on the ramp speed $\dot{Q}$!

- $Delay = Q_j - Q_{th}$

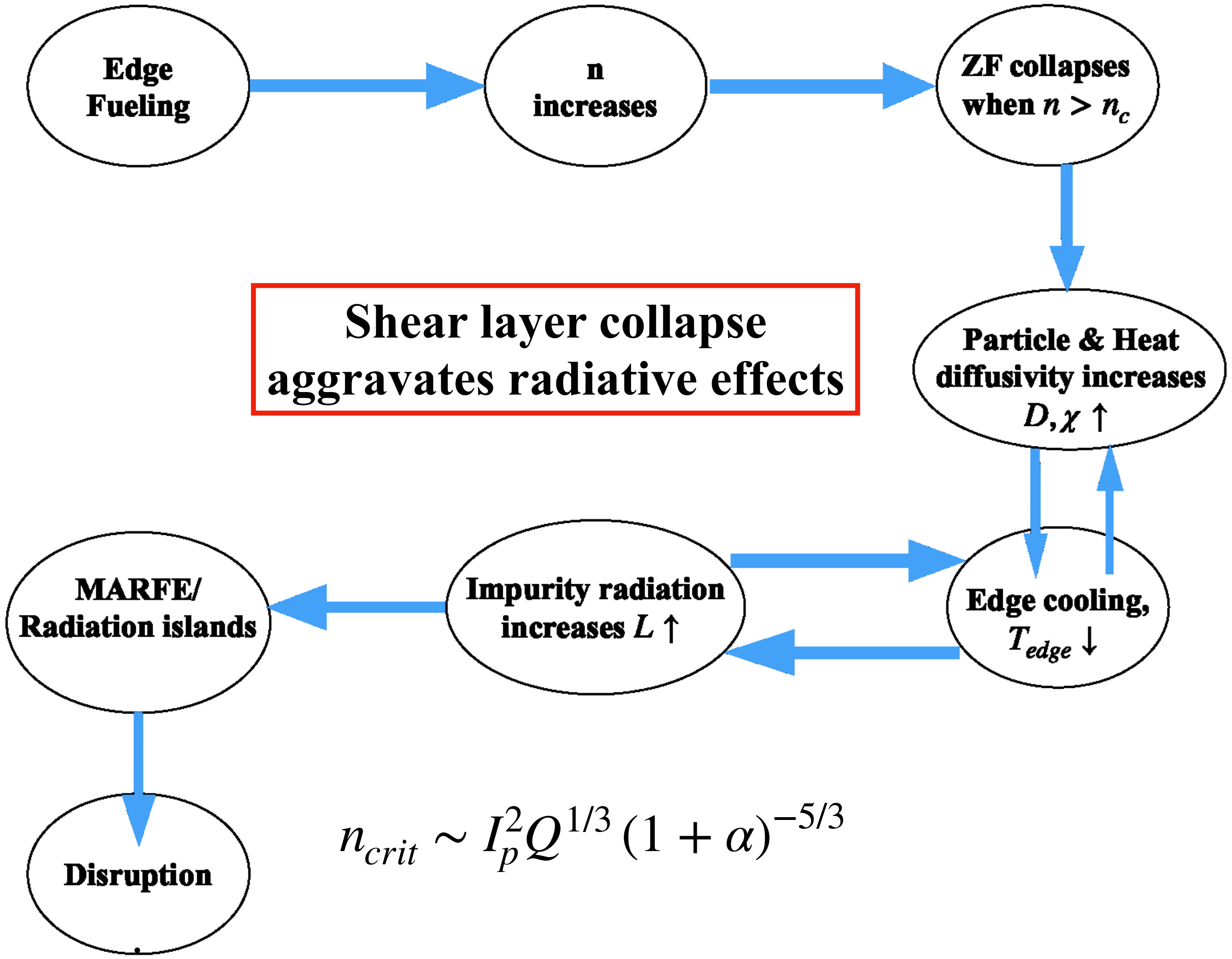
- For low $\dot{Q}$, Delay is independent of $\dot{Q}$

- For high $\dot{Q}$, Delay follows a power law $\dot{Q}^{0.69}$



Conclusions

- Radiative cooling is secondary (i.e., a consequence of) to the transport bifurcation.
- Density limit is a back transition L→DL phenomena
- L→DL associated with shear layer collapse



Shear layer collapse aggravates radiative effects

$$n_{crit} \sim I_p^2 Q^{1/3} (1+\alpha)^{-5/3}$$

- Scalings emerge from shear layer dynamics
  - I_p scaling from neoclassical zonal flow screening
  - Favorable Q scaling due to strengthening of shear layer on increasing Q, thus preventing shear layer collapse
  - Unfavorable stochastic B scaling due to erosion of shear layer by Reynolds stress dephasing
- L→DL hysteresis due to dynamical delay in bifurcation is a testable prediction of the model beyond scalings ! Hysteresis is a manifestation of the underlying transport bifurcation process associated with shear layer collapse.

Future directions

- H-mode density limit: H-L back transition by mean ExB shear collapse? Mechanism of mean shear collapse at high density? Role of SOL or pedestal ? Role of spreading?

- How flux surface shaping effects the shear layer collapse criterion? Can negative triangularity sustain $n > n_g$? Triangularity scaling of n_{crit} ?

  - Analytic calculations and GENE simulations show that residual zonal flow ϕ_{res} increases with δ^+ but $\phi_{res}(\delta^-) < \phi_{res}(\delta^+)$. Does it mean n_{crit} increases with δ^+ but $n_{crit}(\delta^-) < n_{crit}(\delta^+)$?

  - More experiments on density limit studies in δ^+ and δ^- ?