What Limits Zonal Flow Shears in (nearly) Collisionless Drift-Wave Turbulence?

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Abstract

Drift wave-zonal flow turbulence is known to be self-regulating and has been modeled as a predator-prey system. This system is subject to zonal flow (tertiary) instabilities. However, the effects of these instabilities on zonal flow saturation aren't well understood. Here, we report on studies that analyze the effects of zonal flow stability criteria on zonal flow saturation through examination of the energy ratio between zonal flows and turbulence. This is a more direct probe of the impact of dynamics than the existence of a linear instability of the zonal flow.
The Rayleigh criterion, a flow inflection point theorem, is a classic condition for stability in fluid dynamics. However, for realistic values of electron adiabaticity in the Hasegawa-Wakatani model, this is replaced by the Rayleigh-Kuo (RK) criterion. The RK criterion states that the
total mean potential vorticity gradient (∇<PV>) vanishes for instability to occur ($\partial_r(\langle n\rangle - \rho_s^2\nabla_r^2\langle\varphi\rangle) = 0$). We analyze the effects of this criterion on saturated turbulence levels by calculating the local values of the ratio R = $E_{Zonal\ Flow}/E_{Turbulence}$, the zonal-to-turbulence energy ratio. This ratio was calculated for a set of cells which cover the region of the flow. Here, "zonal" means k_Θ, $k_z = 0$ and "turbulence" means k_Θ, $k_z \neq 0$. We would expect that in the care of zonal flow instability, there would be more turbulent energy than zonal flow energy. Using the BOUT++ framework, a set of plasma flow simulations were conducted. These were then processed and integrated to get values for zonal flow and turbulent energies and ∇<PV> to produce R vs. ∇<PV> distributions to examine correlations between the stability condition and actual dynamics. Our results indicate that RK is not a determining factor in the value of R. *Lower values of R (R < 1) are not co-located with regions where ∇<PV> ~ 0. This disagrees with RK.* At most, 3.3% of regions with ∇<PV> ~ 0 had R < 1. Higher values of R (R > 1) are co-located with regions where ∇<PV> ≠ 0. At least 70% of regions with R > 1 had ∇<PV> ≠ 0 across all simulations.Furthermore, collisionality has a significant impact on the value of R. Our simulations indicate that increasing drag leads to lower average zonal flow energy, which agrees with the conceptual understanding of damping. *This means that collisionless saturation does occur, but it is not primarily caused by tertiary instability.* Ongoing work is concerned with deeper analysis and visualization of the radial distribution of R in regions where the gradient of potential vorticity is 0.

Introduction

- Drift wave - zonal flow turbulence akin to predator prey model
- Zonal shear feedback → transport regulation
- Predator-prey model [3, 4, 10]:

$$\partial_t N = \gamma N - \alpha E_V N - \Delta\omega N^2$$

$$\partial_t E_V = \alpha N E_V - \nu_F E_V - \gamma_{nl}(N, E_V) E_V * E_V$$

- With $\gamma_{nl} = 0$, two fixed points appear:

No Flow: $E_V = 0$ and $N = \frac{\gamma}{\Delta\omega}$ Flow: $E_V = \frac{\alpha\gamma - \Delta\omega\nu_F}{\alpha^2}$ and $N = \frac{\nu_F}{\alpha}$

- $\nu_F \to 0$ leads to $E_{ZonalFlow} >> E_{DriftWave}$
- Problem of collisionless saturation → what else limits E_{ZF} ?

Critical Questions

- What criteria can be used to determine where zonal flow instability occurs?
- What effects do these criteria have on zonal flow and turbulent energies and are they significant?
- How do these effects vary given different density gradients and zonal flow damping parameters?
- Does R = $\frac{E_{ZonalFlow}}{E_{DriftWave}}$ show a correlation with the profile of mean potential vorticity (PV) and zonal flow stability?

Hasegawa-Wakatani

$$\partial_t\nabla_\perp^2\phi + \{\phi, \nabla_\perp^2\phi\} = \alpha(\phi - n) - \mu\nabla_\perp^2\phi - \nu\nabla_\perp^6\phi \to \partial_t\langle\nabla_\perp^2\phi\rangle - \partial_x\langle(\nabla_\perp^2\phi\partial_y\phi)\rangle = -\mu\langle\nabla_\perp^2\phi\rangle \text{ [1,9]}$$

$$\partial_t n + \{\phi, n\} = \alpha(\phi - n) - \kappa\partial_y\phi - D\nabla_\perp^4 n \to \partial_t\langle n\rangle - \partial_x\langle(n\partial_y\phi)\rangle = -D\langle\nabla_\perp^4 n\rangle$$

$\alpha_{eff} = \frac{\alpha}{\kappa}$ μ - flow-damping parameter κ - linear density gradient drive

- $\phi = \widetilde{\phi} + \langle\phi\rangle$ and $n = \widetilde{n} + \langle n\rangle$ with $\widetilde{n}$ = density fluctuation and $\langle n\rangle$ = zonally averaged density
- $\langle n\rangle = n_z + n_0$ with n_z = fluctuation in zonally averaged density and n_0 = background density = $\kappa * x$
- R = $\frac{E_{ZF}}{E_{DW}}$ calculated in a 10 x 5 region selected from the simulation space
- Zonal Flow Energy $= E_{ZF} = \int\int|\langle\nabla_\perp\phi\rangle|^2 dxdy$ for $\alpha_{eff} > 1$
- Drift Wave Energy $= E_{DW} = \int\int|\tilde{n}|^2 + |\nabla_\perp\tilde{\phi}|^2 dxdy \simeq \int\int|\tilde{\phi}|^2 + |\nabla_\perp\tilde{\phi}|^2 dxdy$ for $\alpha_{eff} > 1$

Rayleigh-Kuo

$$\partial_t[(\rho_s^2\nabla^2 - 1)\tilde{\phi}] + \langle v\rangle\cdot\nabla[(\rho_s^2\nabla^2 - 1)\tilde{\phi}] + \tilde{v}\cdot\nabla(\langle n\rangle - \langle\nabla^2\phi\rangle) = 0$$

- Using the HM equation above with proper normalization and letting $\zeta = \langle n\rangle - \langle\nabla_x^2\phi\rangle + \tilde{\zeta}$, and $\tilde{\zeta} = \tilde{\phi} - \nabla_x^2\tilde{\phi}$,

$$\partial_t\tilde{\zeta} + \{\phi, \zeta\} = 0$$

Letting $\tilde{\phi} = \varphi(x)e^{(ik_y y - i\omega t)}$ with k_y real and ω complex ($\omega = \omega_r + i\omega_i$), we get

$$(\partial_x^2 - k_y^2 - 1 - (\frac{\partial_x\langle\zeta\rangle}{-\partial_x\langle\phi\rangle - \frac{w}{k_y}}))\varphi = 0$$

Multiplying by φ^* and integrating the imaginary part of our equation for x from 0 to L,

$$\frac{\omega_i}{k_y}\int_0^L \frac{\partial_x\langle\zeta\rangle}{|-\partial_x\langle\phi\rangle - \frac{w}{k_y}|^2}|\varphi|^2 = 0$$

$$\boxed{\partial_x\langle\zeta\rangle = 0 \to \partial_x(\langle n\rangle - \nabla_x^2\langle\phi\rangle) = 0}$$

- Rayleigh-Kuo criterion is a necessary condition: $(\nabla(\langle PV\rangle) = 0) \to$ zonal flow instability
- Fixed $\nabla\langle n\rangle \to$ R-K sets condition on the zonal vorticity profile relative to the zonal density profile
- ∇n drives turbulence, via familar drift wave instability, but also limits shear flow instability
- Rayleigh $(\nabla(vort) = 0)$ is wrong; Rayleigh-Kuo $(\nabla(\langle PV\rangle) = 0)$ is correct

Setup

- Main Question: Does $\nabla(\langle PV\rangle)$ have any observable effect on R = $\frac{E_{ZF}}{E_{DW}}$?
- Produced BOUT++ simulations with varied density gradient drive [κ] (1 to 1.75) and flow damping [μ] (0.01 to 0.2)
- R calculated through integrating over a 10 x 5 region shown in Figure 1
  - Other region sizes (5x5, 7x7, 9x9) gave similar results
- Points are arbitrary selected to ensure impartial analysis of simulation space
  - Points near simulation border removed, as border cells are constrained by boundary conditions
- Gauged effects of altering κ and μ on $\nabla\langle PV\rangle$

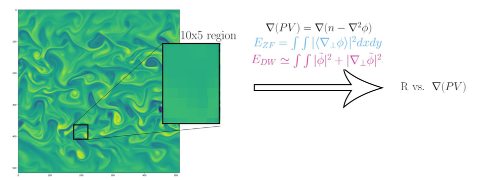

Figure 1: Analysis of BOUT++ Simulation

Results (Iterating over μ)

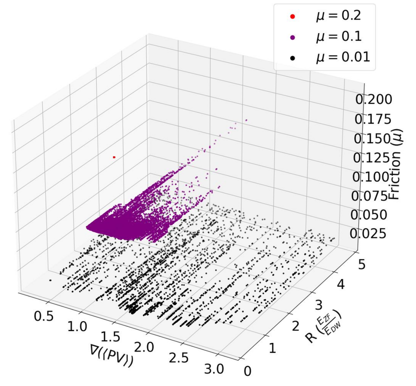

Figure 2: 3D Plot of R vs. $\nabla\langle PV\rangle$ vs. μ

- Variance in R = $\frac{E_{ZF}}{E_{DW}}$ and $\nabla(\langle PV\rangle)$ larger for lower μ
  - Less restriction on flow configuration
- Maximum value for R decreases as μ increases as expected
- For areas with $R < 1$, centralization occurs around $\nabla\langle PV\rangle = 1.5$

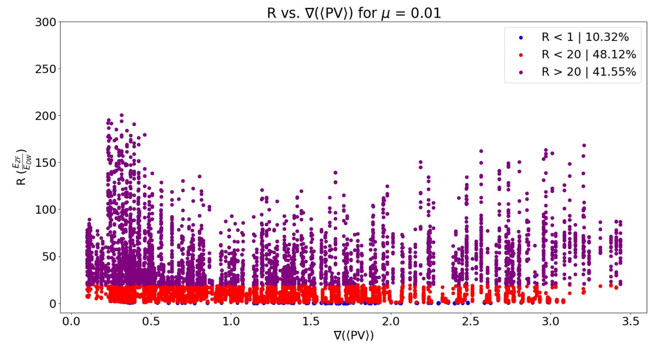

Figure 3: Distribution of R vs. $\nabla\langle PV\rangle$ for $\mu = 0.01$

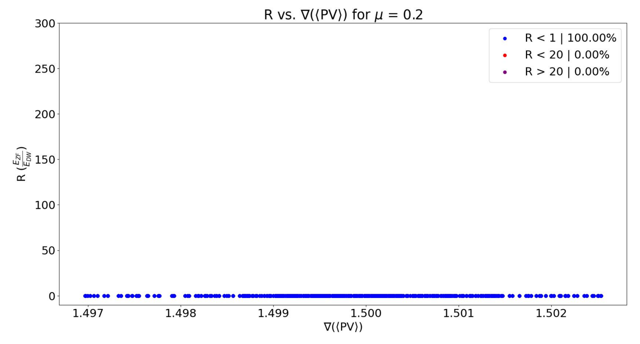

Figure 4: Distribution of R vs. $\nabla\langle PV\rangle$ for $\mu = 0.2$

- More zonal flow energy evident in lower damping conditions
- Dimits-like region visible in lower damping circled in black, disappears with higher damping
- For areas with $R < 1$, both damping scenarios show centralization around $\nabla\langle PV\rangle = 1.5$
- Most locations with low R values have $\nabla\langle PV\rangle \neq 0$, suggests RK stability isn't major player

Results (Iterating over κ)

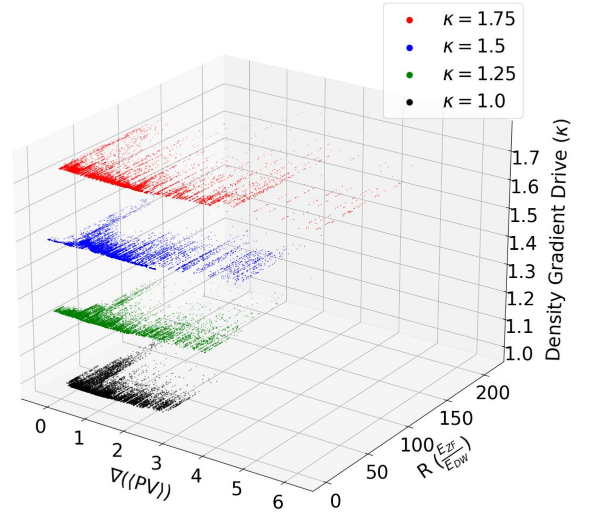

Figure 5: 3D Plot of R vs. $\nabla\langle PV\rangle$ vs. κ

- Keeping α_{eff} constant, R vs. $\nabla\langle PV\rangle$ graphs have similar shape independent of κ
- Larger value of κ translates the graph positively along the $\nabla\langle PV\rangle$ axis
- Stronger background gradient also produces a wider range of $\nabla\langle PV\rangle$

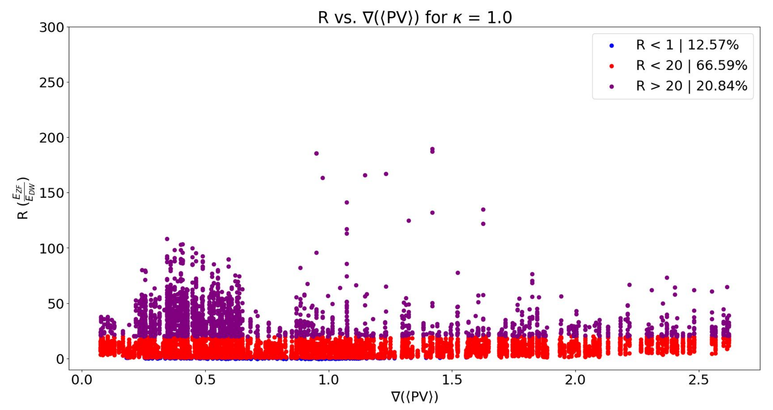

Figure 6: Distribution of R vs. $\nabla\langle PV\rangle$ for $\kappa = 1.0$

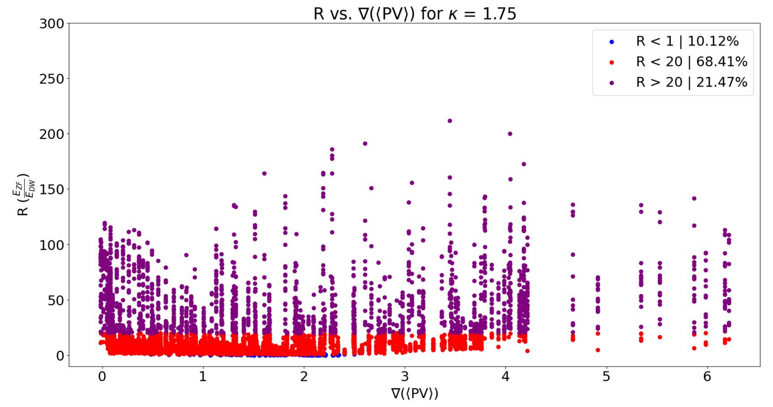

Figure 7: Distribution of R vs. $\nabla\langle PV\rangle$ for $\kappa = 1.75$

- Dimits-like regime are apparent, with two tails appearing with $R > 20$
- Increasing κ doesn't diminish the size or volume of these tails

Results (R Plot with ∇<PV> Overlay)

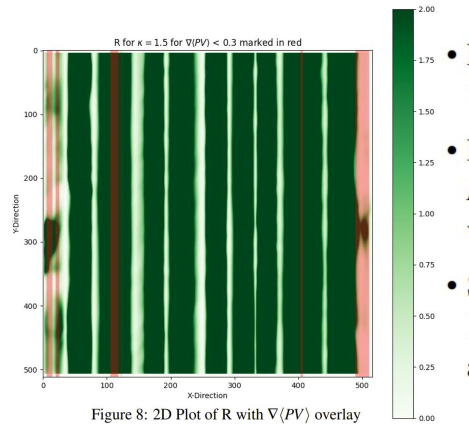

Figure 8: 2D Plot of R with $\nabla\langle PV\rangle$ overlay

- Red highlights indicate $|\nabla\langle PV\rangle| < 0.3$, $\kappa = 1.5$, $\mu = 0.01$
- Highlighted regions away from border have $R >> 1$ implies $\nabla\langle PV\rangle$ doesn't predict the value of R
- Some regions with $|\nabla\langle PV\rangle| >> 0$ have $R < 1$, indicating that RK doesn't have a perceivable effect on R

Key Results

- R $\left(\frac{E_{ZF}}{E_{DW}}\right)$ isn't correlated with the RK criterion $(\nabla\langle PV\rangle = 0)$
- Persistent Dimits-like regimes present in low friction damping scenarios and independent of kappa
- With α_{eff} constant, increasing density gradient drive (κ) shifts R vs. $\nabla\langle PV\rangle$ to the right
- Increasing frictional damping (μ) significantly reduces Zonal Flow Energy

Sources and Acknowledgements

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