On the Resilience of Staircase Structure in a Melting Vortex Crystal Flow

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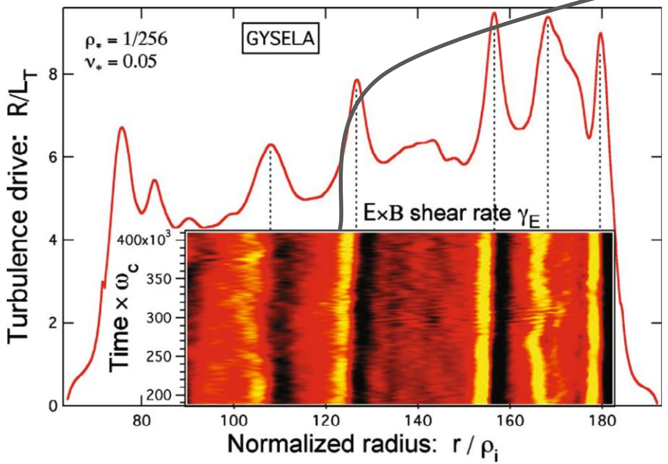
Outline

- Background and Survey Results
- Melting Vortex Crystal
- Passive Scalar Transport
  - System Setup
- Results:
  - Web and Path
  - Staircase
- Summary & Ongoing work

Background and Survey Results

ExB staircase current subject in M.F.E



Yellow and black colors are a rapid transition of the direction of flows around peaks in turbulence drive. This is the shear layer, which is interspersed with a regular pattern of shear layers and profile corrugations.

Context: Flat spots of high transport and nearly vertical layers acting as mini-barriers coexist. In plasmas, avalanches happen in flat spots and shear layers due to zonal flows occur in the areas of mini-barriers.

Suggested ideas:

- ExB shear feedback, predator-prey
  - Zonal flows predator and turbulence intensity prey
- Jams

  ...

But… is there an even **simpler** physical mechanism to produce **layering**

Clue: **Staircase** formation, dynamics captured in ultra-simple mixing model with **two scales**. - Balmforth, et. al; Ashourvan and Diamond

Some Questions

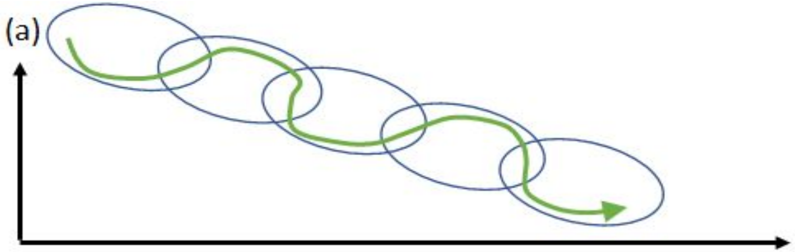
- How does staircase beat homogenization?
- Is the staircase a meta-stable state?
- What is the minimal set of scales to recover layering?

Next:
More on staircase!
But, FIRST let's discuss cell pattern…

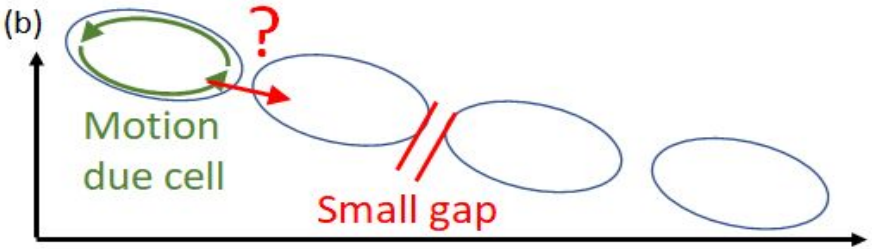
Background and Survey Results (cont.d)

Transport of particle between non-overlapping or marginally overlapping cells is an important topic in fusion plasma.

<u>Overlapping case</u>: particles can transport directly from cell to cell, wandering along streamlines



<u>Non-overlapping case</u> (cells sit at near overlap): transport is a synergy of motion due to cells and random kicks (Collisional diffusion, ambient scattering) thru gap regions.



Characteristic of **near marginal**.

The transport over gap is random kicks (ambient diffusion): collisions, micro-turbulence.

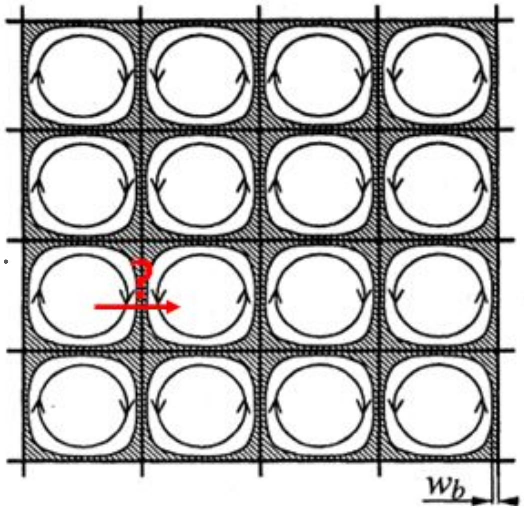
Coexistence of:
~ **Fast transport** - Mixing in cell
~ **Slow transport** - Kicks between cells

N.B.: "Profile stiffness" → Cells near overlap
→ Rapid increase in transport prevents strong overlap

Background and Survey Results (cont.d)

This relates to a classic problem studied by G.I. Taylor, Moffat, Rosenbluth, …, which discuss the laminar convective flows with the condition of infinite Kubo number.

Isichenko Review

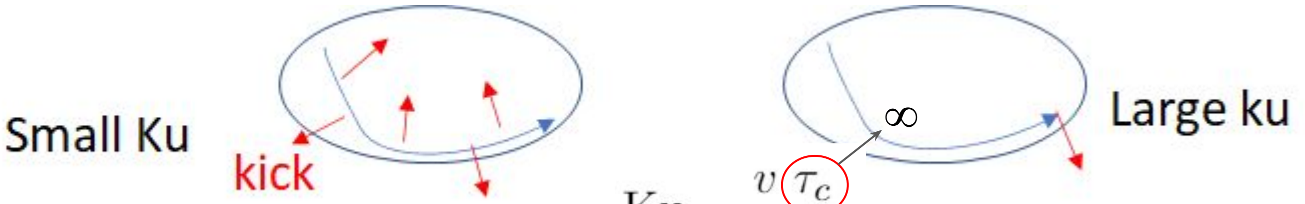


✔ The convective diffusion transport is equivalent to the transport by magnetic cells.

$$\mathbf{u} = \hat{z} \times \nabla\psi \quad \longleftrightarrow \quad \delta\mathbf{B} = \hat{z} \times \nabla\psi,$$

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n - D\nabla^2 n = 0 \quad \longleftrightarrow \quad \frac{\partial f}{\partial t} + v_{\parallel}\frac{\delta B}{|B|}\nabla f - D\nabla^2 f = 0.$$

✔ Large Kubo number means strong scattering and long correlation. The absence of kicks means particles stay on streamline, therefore, correlation time of the pattern is infinite.



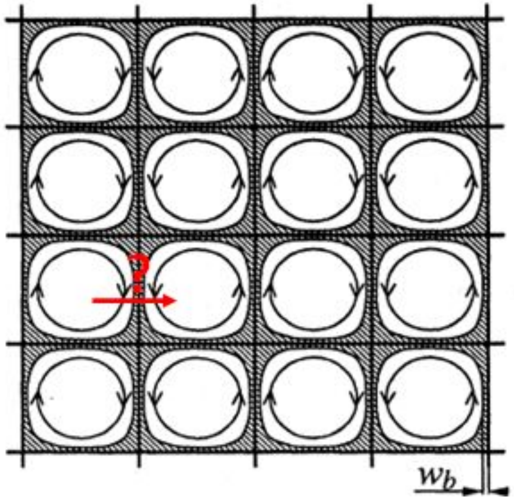
$$\mathrm{Ku} = \frac{v\,\tau_c}{\Delta}$$

If we were to inject dye into this cellular lattice, what does the profile look like? What about transport?

Next:

Background and Survey Results (cont.d)

Consider a **general** case of a cellular lattice of **marginally** overlapping cells.



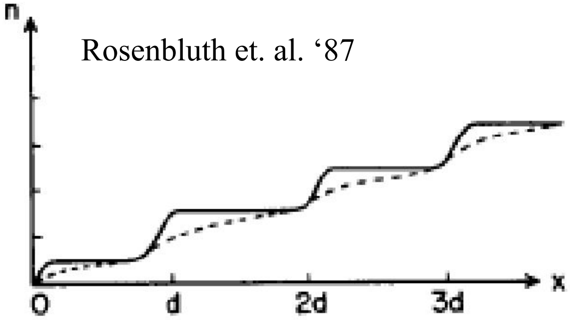
Transport? Answer: Deff ~ $D_o\, Pe^{1/2}$ {**Not a simple addition of process!**}

→ Two time rates: v_o / ℓ_o, D_o / ℓ^2_o

→ $Pe = v_o\, \ell_o / D_o >> 1$

Profile?

Consider concentration of injected dye → profile



"Steep transitions in the density exist between each cell."

→ Layering!

→ **Simple** consequence of **two rates**

→ "Rosenbluth Staircase"

Important:

- **Staircase** arises in stationary array of passive eddys.
- Global transport hybrid:
  - → fast rotation in cell
  - → slow diffusion in boundary layer
- Irreversibility localized to inter-cell boundary.

Relevant to key question of "near marginal stability"

What about the dynamics of a **less constrained** cell array (i.e., vortex array with fluctuations)?

Next: →

Melting Vortex Crystal

→ We begin with the 2D NS equation that can be written in nondimensional form (Perlekar and Pandit 2010),

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla}\right) \omega = \frac{1}{\Omega} \nabla^2 \omega + F_\omega - \alpha \omega, \qquad \nabla^2 \psi = \omega.$$

→ The "vortex crystal" is simply the array of cells and "melting" is related to turbulence induced variability in the structure. The melting vortex crystal allows us to study a **general less constrained** version of the array!

→ The melting flow structure is created by **slowly increasing the Reynolds number** in the NS equation

$$\Omega \equiv nRe$$

→ By increasing the Reynolds number this modifies the forcing and drag term, thus, **scattering** the vortex crystal. The **resilience** of the staircase is studied by **increasing disorder** in the vortex crystal through F_ω

$$F_\omega \equiv -n^3 \left[\cos\left(nx\right) + \cos\left(ny\right)\right] / \Omega$$

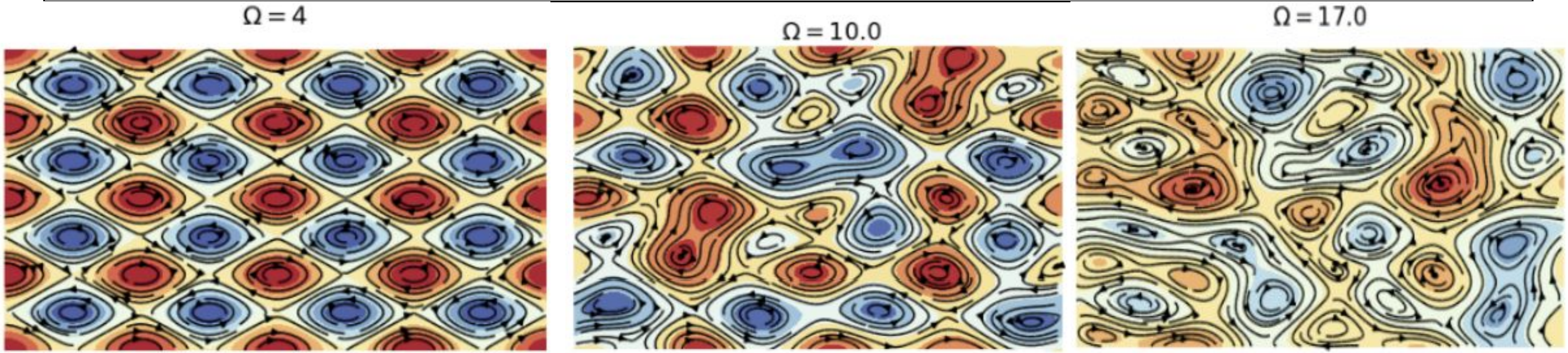
The streamfunction, ψ, at different evolutionary stages of the "melting" vortex crystal is inserted into the passive scalar equation to study the resilience of the staircase structure.

Why are we doing this? We know that a system with two disparate time scales forms a staircase!

- Now consider fluctuations… → Will staircase survive?

The melting vortex crystal will help answer this question!

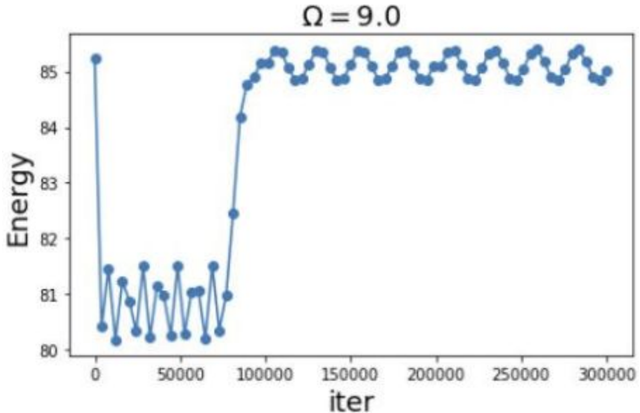
Melting Vortex Crystal (cont.d)



- Contour plots of the streamfunction (ψ), illustrating the different stages of a "melting" vortex crystal.
- As Ω is slowly increased, there is a merger of vortices along with distortions of the crystal array.

We characterize different stages of the melting process by observing a contour plot of the crystal and the crystal's energy trace during each different stage. There are five different stages:

- Stable Crystal (SX) [$\Omega < 6.5$]
- Stable Distorted Crystal (SXA) [$6.5 < \Omega < 8$]
- Periodic Crystal (OPXA) [$8 < \Omega < 10$]
- Quasiperiodic Crystal (OQPXA) [$10 < \Omega < 13$]
- Spatiotemporal chaotic/turbulent crystal (SCT) [$13 < \Omega$]



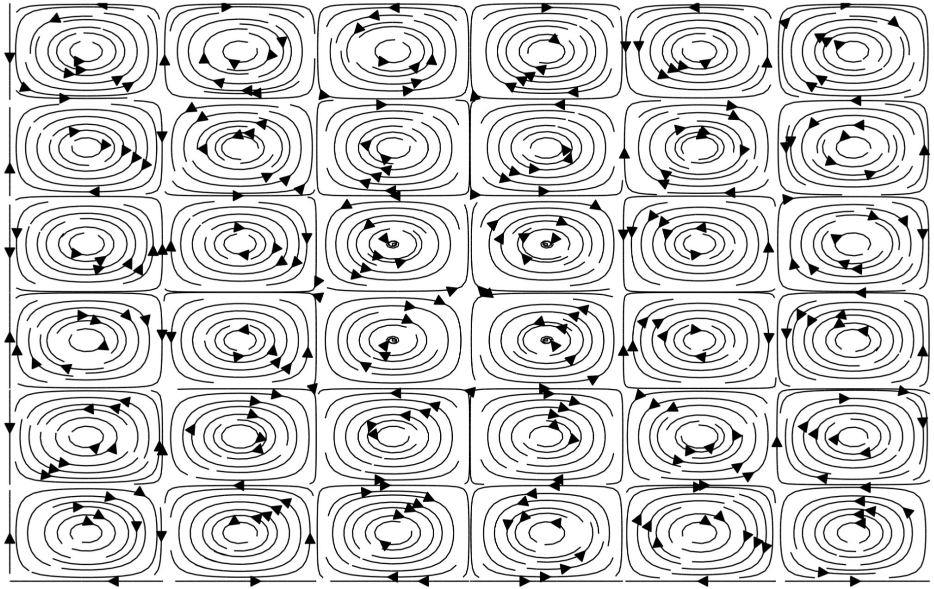
What is the Goal?

- How **resilient** is the staircase in the presence of vortex array melting fluctuations?
- Compared to previous work, we want to study a much more **general** and **less constrained** version of the cell array (consider vortex array with fluctuations; jitters).

In the process of studying the **resilience** of the staircase, we aim to answer the following questions:

- What occurs to staircase steps as vortices slowly begin to merge together? What about other cellular interactions?
- Does flux insertion orientation matter?
- What does the scalar/particle path look like?

Example of **constrained** cell array



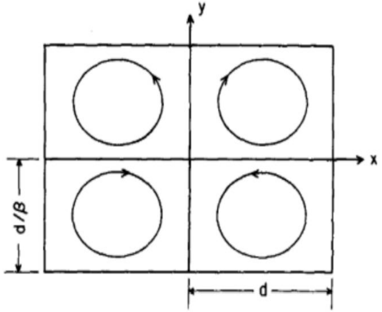
To answer these questions, we use a **Passive Scalar Transport** code.

Next

Passive Scalar Transport

→ The governing equation solved is the **passive scalar transport** equation,

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D\nabla^2 n,$$



We use the streamline function created by the 2D NS equation with forcing and drag,

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla}\right)\omega = \frac{1}{\Omega}\nabla^2\omega + F_\omega - \alpha\omega.$$

The fluid velocity **u** in the passive scalar transport equation is of the form

$$\mathbf{u} = \hat{z} \times \boldsymbol{\nabla}\psi$$

We use Pe~40-60 in these set of simulations.

We characterize the transport in this system by using the Peclet number, which is a nondimensional ratio of two time scales.

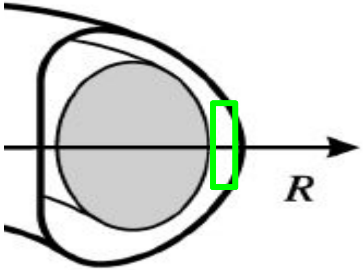
→ The two characteristic time-scales are the time for circulation around the roll ($\tau_H = d / \tilde{u}\beta$) and time for molecular diffusion of a particle through a roll ($\tau_D = d^2 / D$). The ratio of these two time-scales is

$$\mathrm{Pe} = \frac{\tau_D}{\tau_H} > 1$$

→ We are primarily concerned in the case of Pe > 1, where the **physics is explained by fast mixing within the cells and slow mixing across the boundaries of the cells**.

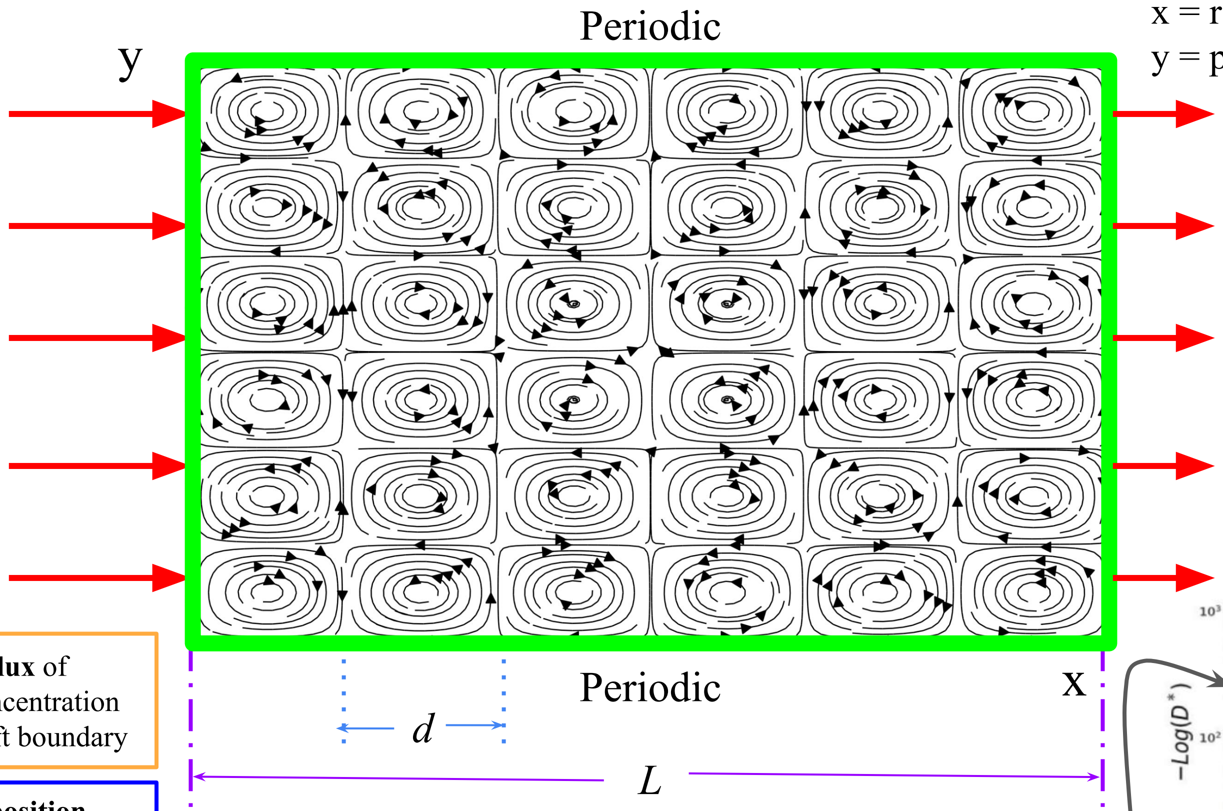
Next, let's discuss setup… →

System Setup



x = radial
y = poloidal

$\Gamma|_{x=0} = \langle\Gamma\rangle + \kappa\tilde{\Gamma}$



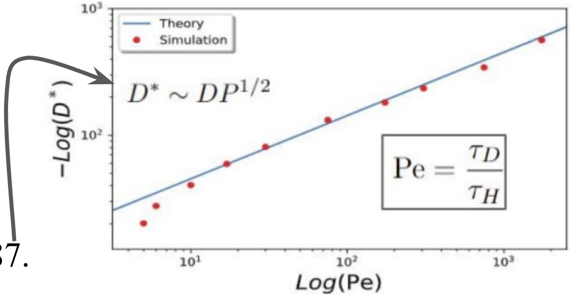
$\Gamma|_{x=L} = \langle\Gamma'\rangle$

Combination of fixed flux and periodic boundary conditions are used to model the physics of **core to edge** in **fusion** devices.

Constant flux of particle concentration exits at right boundary

Constant flux of particle concentration enters at left boundary

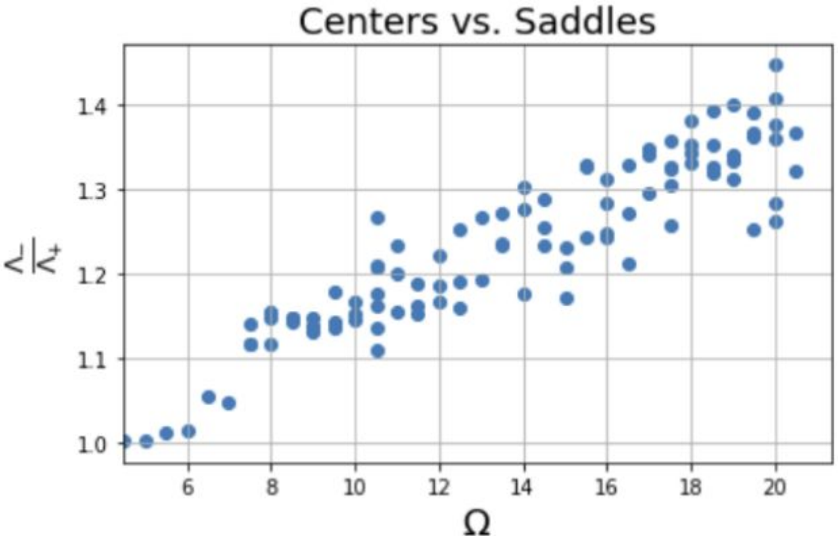
Noisey deposition (Future work)

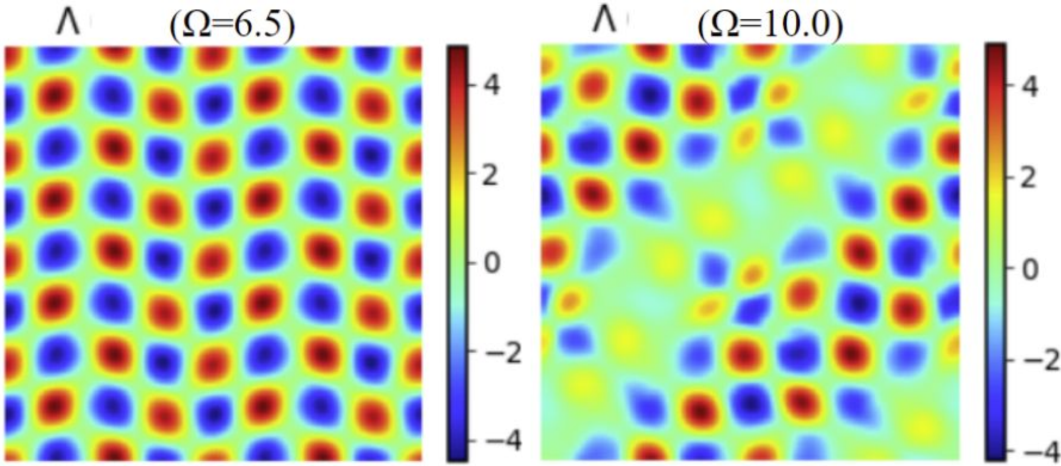


Initial simulation agrees with scaling given by Rosenbluth et al 1987.

Web & Path

$$\Lambda = \partial_x u_y \partial_y u_x - \partial_y u_x \partial_x u_y$$





Λ = mean sq. vorticity - mean sq. shear

We use **Okubo-Weiss field** (Λ) to study the **evolution** of the flow structure as we increase Ω. As we steadily increase Ω, the areas of saddles ($\Lambda < 0$) increases compared to areas of centers ($\Lambda > 0$).

- Saddles refer to areas of strong shear and centers refers to areas of strong vorticity.

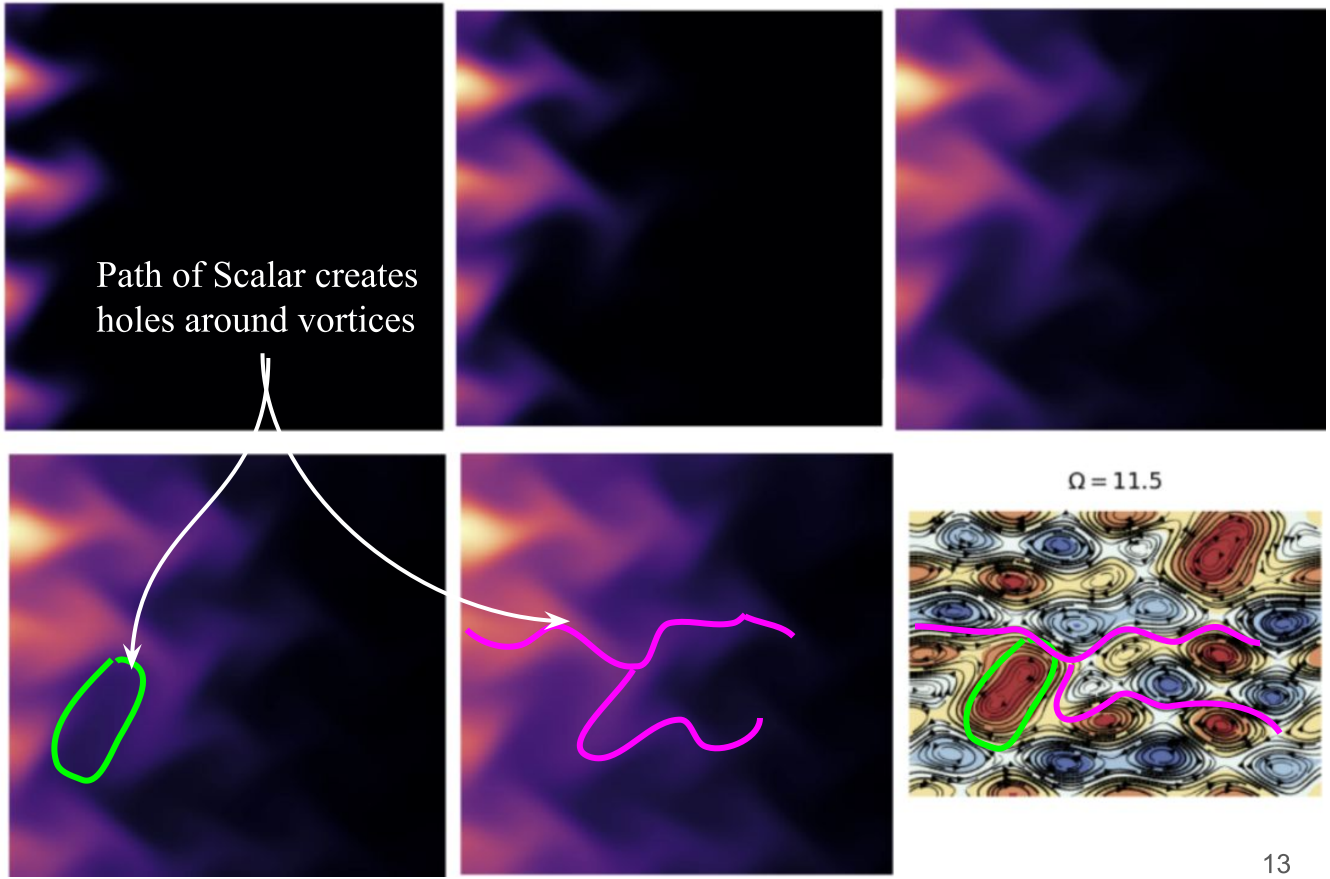
Some questions:
Does the increase in flow shear affect the staircase structure? How does this affect the transport of scalar concentration? How does the Okubo-Weiss field describe the path of the scalar concentration?

Web & Path (cont.d)

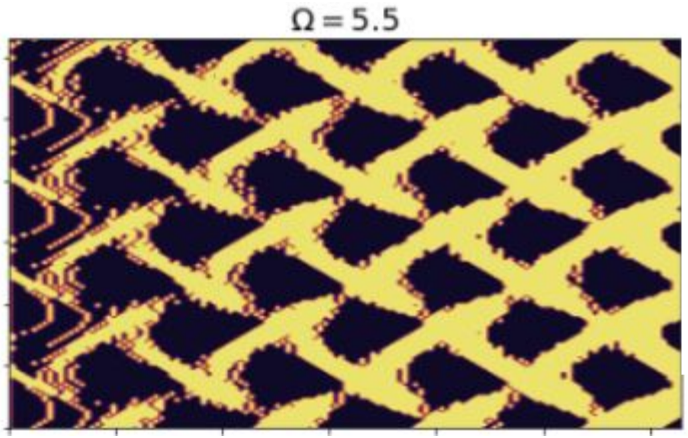
- As the scalar concentration gets injected into the flow structure, we see a **<u>flamelet network pattern</u>** (Pocheau 2008).

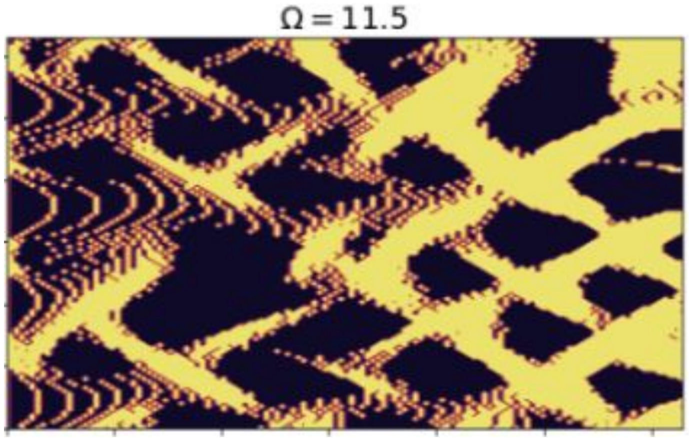
- **Scalar** concentration **flows along** and **around** areas of **vortex structures**.
    - Over time, the scalar slowly enters the vortex structure.

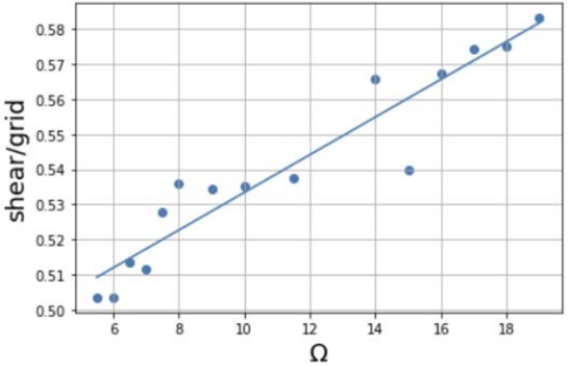
- **Scalar** concentration quickly **flows** along areas of **strong shear** ($\Lambda < 0$).

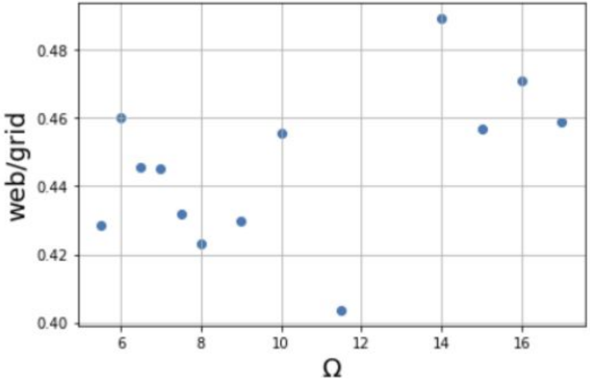


Web & Path (cont.d)









- The **path of the scalar** concentration **creates a web pattern** where the **<u>holes</u>** are vortices.
- As the degree of **<u>melting is increased</u>**, the **<u>area of holes increases</u>**. The **web is not destroyed, it only degrades**.
- Web area correlates with shear area increase! Web becomes thicker as we increase melting!

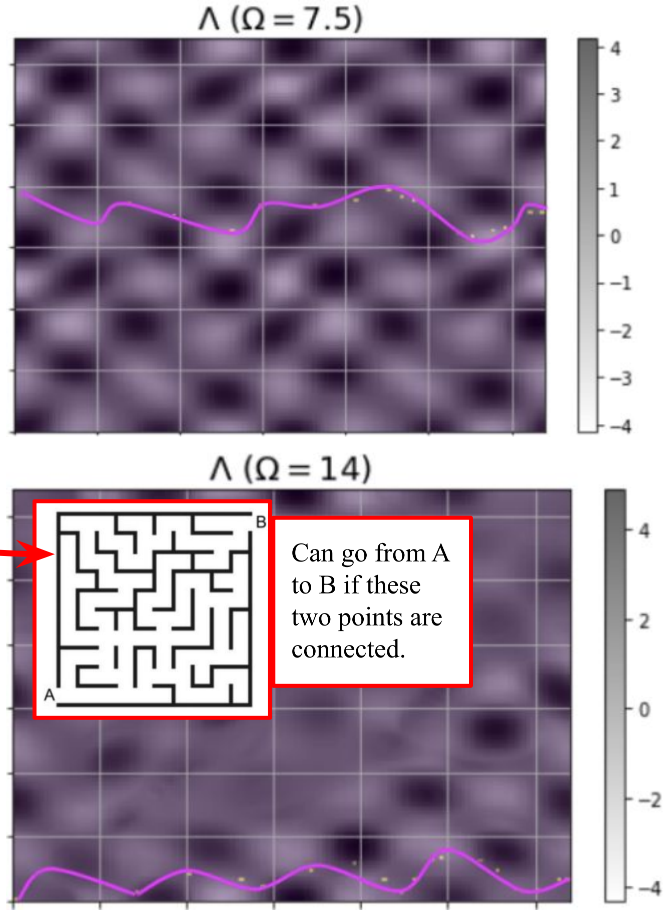
Web & Path (cont.d)

- Idea relevant here is the <u>**least time criterion**</u>. Here one might think that as the **vortex crystal melts**, the **path of least time would increase in length** (still work in progress).

- We observe that the **scalar travels** fast along the areas of **strong shear**.

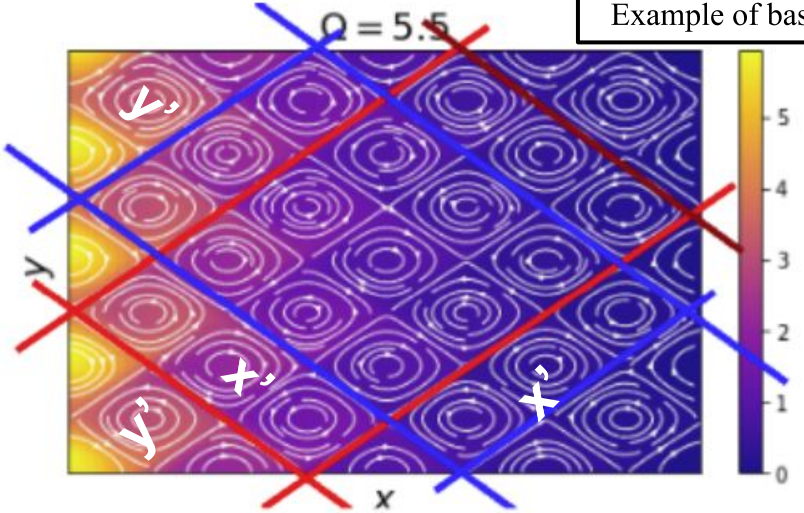
- Similarities to percolation picture of infinite Kubo number.
  - How would this compare to percolation model? Can we reproduce dynamics?

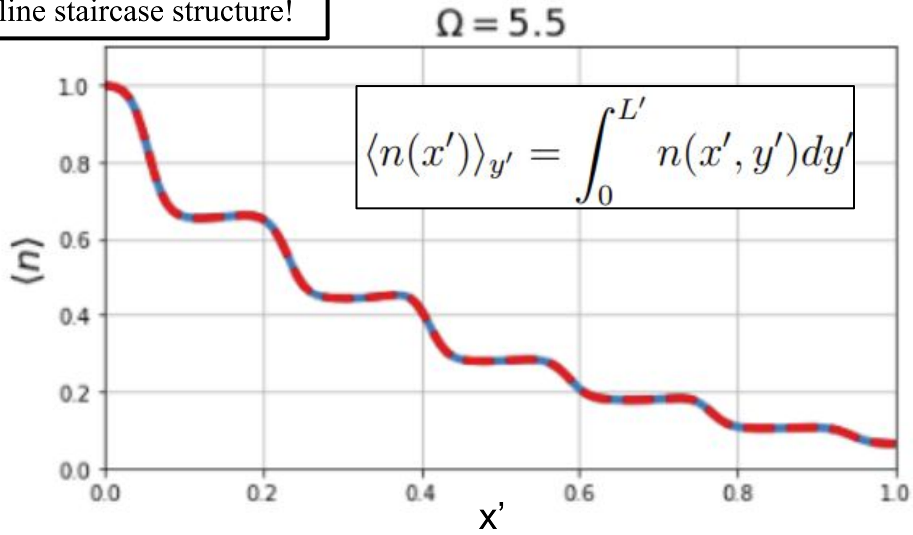
- What is the **connection** between the **Web** and **Staircase**? Next…



Staircase

Example of baseline staircase structure!

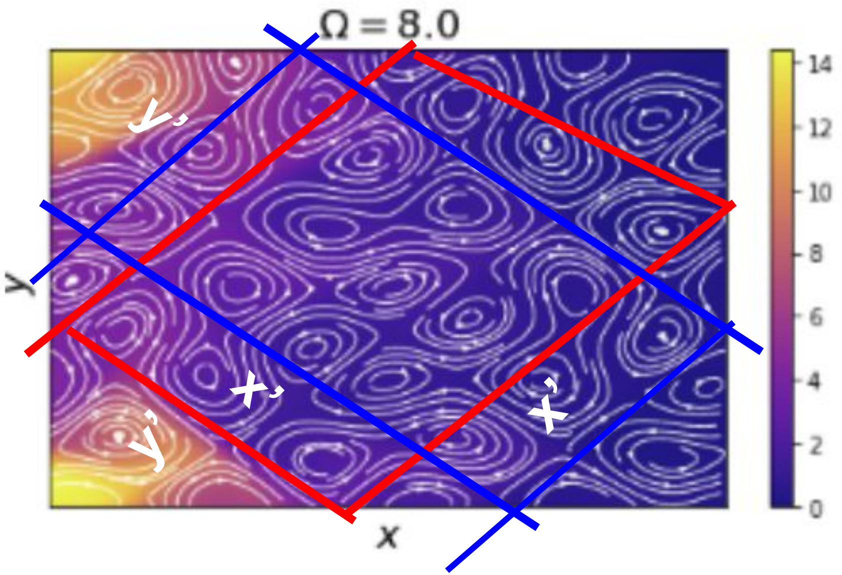


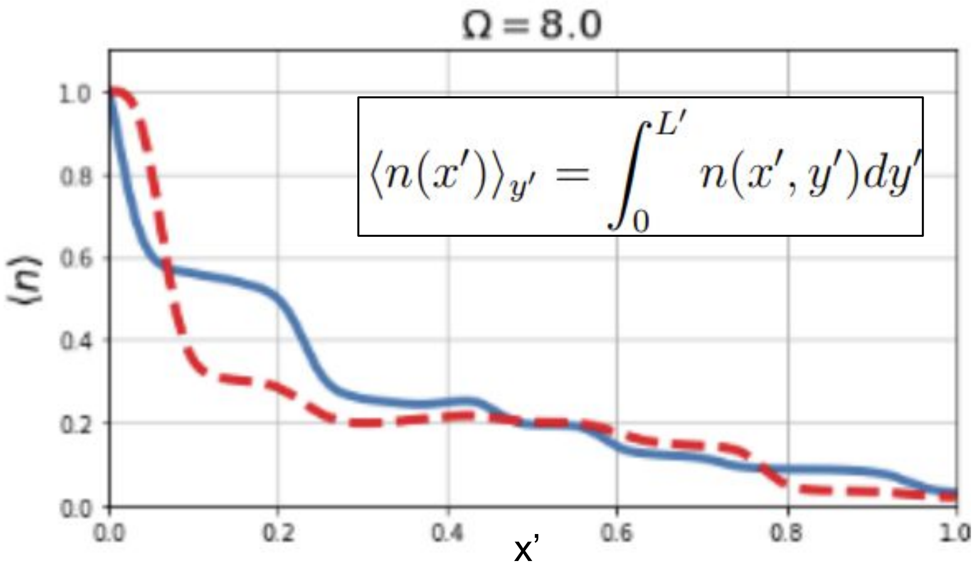


- For a stable crystal (SX) we get a **baseline staircase** structure.
- On the left figure the blue and red box correspond to the blue and red plot line on the right. Note that **steps** are **evenly** spaced!
  - Both blue and red average scalar concentration have the same profile in stable stage.

So what happens to the staircase if we increase the degree of melting in the crystal (i.e., increase Reynolds number)?

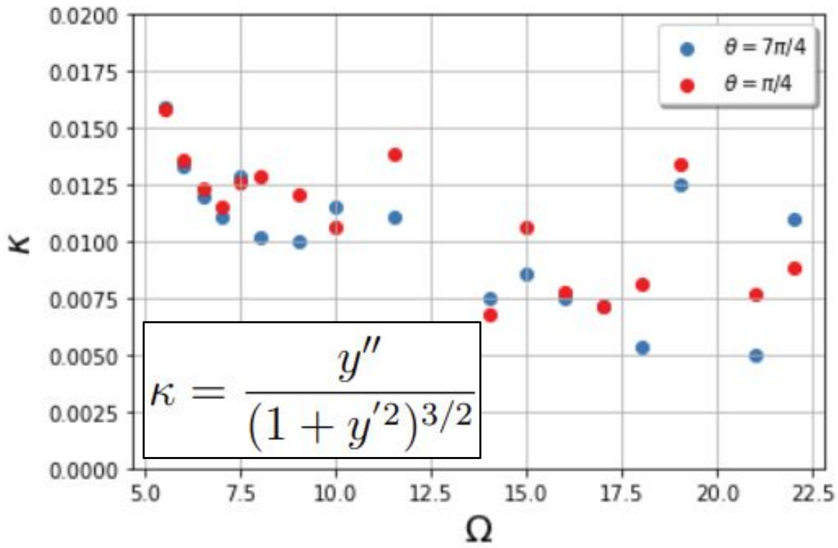
Staircase (cont.d)

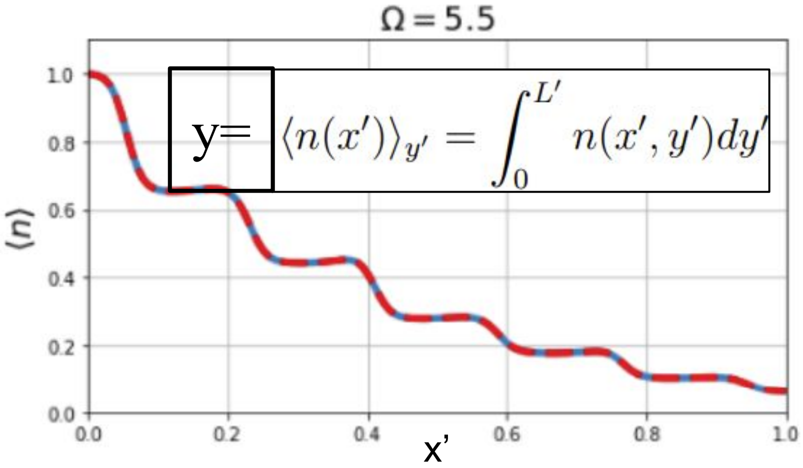


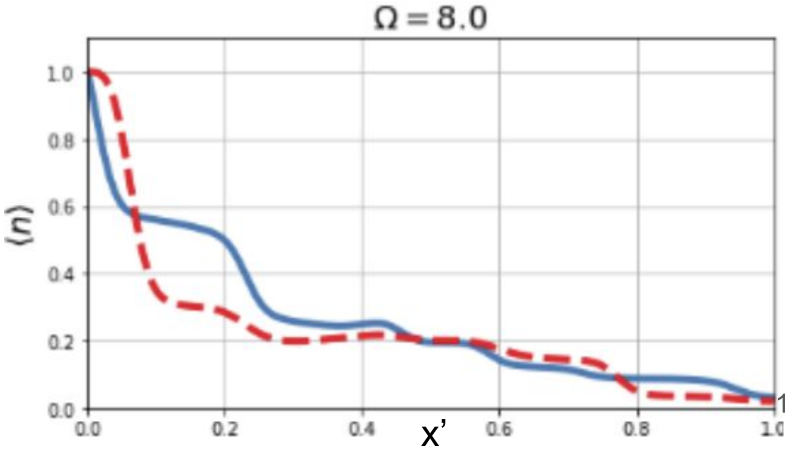


- As we **increase the degree of melting** through Ω, we can see **merger/connections** of vortex structures in the flow.
- These **vortex mergers** are shown in the scalar profile plot as **mergers in steps**.
  → As we increase the degree of melting, staircase steps start to merge together.

Staircase (cont.d)

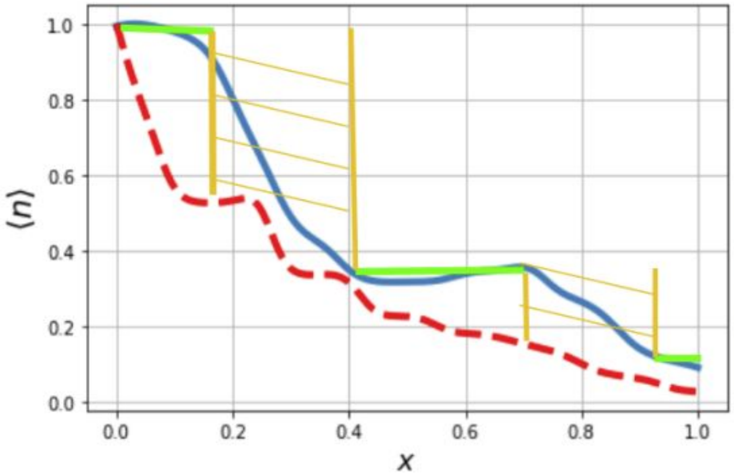


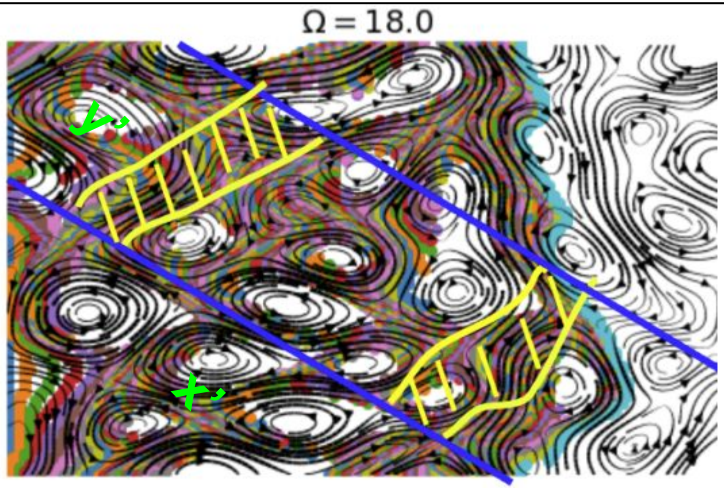




- To quantify the difference in the different stages of the melting process, we look at the **curvature** in scalar concentration.
- In general we see that as we **increase Ω**, the **curvature decreases**.
  - Make sense, since the steps are starting to merge together as we increase Ω.

Staircase + Web





We make a **<u>connection</u>** here between the **web** and the **staircase**:

- The picture on the right represents the **scalar trajectory** during the first couple of time steps. We see that scalar flows quickly around areas of strong shear.
  - The **holes** are shown to be **vortex structures**.
  - **Scalar quickly forms barriers** between vortex structures.
- Picture on the left shows the connection between the web and scalar path dynamics to the staircase structure.
  - **Yellow** represents the **barriers** between areas of strong mixing.
  - **Green** represents the **holes/vortices**, which are the areas of strong mixing.

<u>Main Point</u>: Despite that vortex array becoming more turbulent, the staircase structure does not degrade.

- Staircase steps become **less regular**. They merge into longer steps.

Summary of Work

In a much more general and less constrained version of a cell array, we study the behaviour and flow structure of a scalar concentration. In this study we find the following:

- As we increase the degree of melting in a cellular array, vortices connect/merge together becoming a longer vortex structure.
- By inserting the scalar concentration in a specific orientation, we see that the scalar flow along and around vortices from one boundary to the other.
    - Scalar fills the vortex at a slower rate leaving behind a web trajectory. This is clearly explain due to slow diffusion across boundary layers.
- As the degree of melting is increased, the area of holes increases.
    - The web is not destroyed, it only degrades.
    - Shear and web correlate. As the degree of melting is increases, the area of both increases. The path of the scalar trajectory widens.
- Despite the cellular array becoming more turbulent, the staircase structure persists! **BUT** steps become less regular.
    - The degree of melting causes vortices to connect/merge, which leads to merger of staircase steps in the scalar profile.
    - Scalar quickly forms barriers between areas of fast mixing as shown in the contour plots of the scalar path/web.

Ongoing Work

We want to further analyze and understand the trajectory of the scalar concentration. To do this, we plan the following:

- Modulate the flux on the LHS (Modulated pulse perturbations).
  - Single pulse, train pulse, etc.

→ Goal is to trace how the system response (i.e., see how the bright spot evolves in a steady state simulation).

In addition to this, we want to further quantify the path and time of trajectory between different melting stages…

- Does the scalar concentration follow a similar idea to that of the least time criterion?
- Do the number of paths decrease or increase?
- Does the scalar concentration take a longer or shorter time to travel to one end of the boundary?

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