Resiliency of Profile Staircases in a Fluctuating Cellular Array

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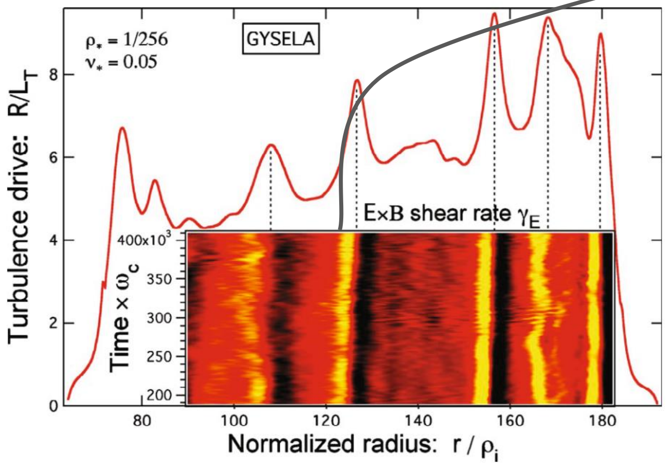
Outline

- A Review of Staircases
- Fixed Cellular Array (another way to get a **Staircase**)
- Relaxing Cellular Array with Vortex Array
  - Setup of Transport Problem
- Fluctuation from Marginal Point:
  - What happens to Staircase?
  - The Scalar Field
- Summary & Future work

A Review of Staircases

Background and Survey Results (cont.d)

ExB <u>staircase</u> current subject in M.F.E



Yellow and black colors are a rapid transition of the direction of flows around peaks in turbulence drive. This is the shear layer, which is interspersed with a regular pattern of shear layers and profile corrugations.

<u>Context</u>: Flat spots of high transport and nearly vertical layers acting as mini-barriers coexist. In plasmas, avalanches happen in flat spots and shear layers due to zonal flows occur in the areas of mini-barriers.

Suggested ideas:

- ExB shear feedback, predator-prey
  - Zonal flows predator and turbulence intensity prey
- Jams

Clue: **Staircase** formation, dynamics captured in ultra-simple mixing model with **two scales** (Ozmidov, Rhines). - Balmforth, et. al; Ashourvan and Diamond

- One of them is emergent.
- Feedback from presence of two scales.

<u>But</u>… is there an even **simpler** physical mechanism that can produce **layering?**
Answer: Yes (e.g., pattern of cells)

Some Questions

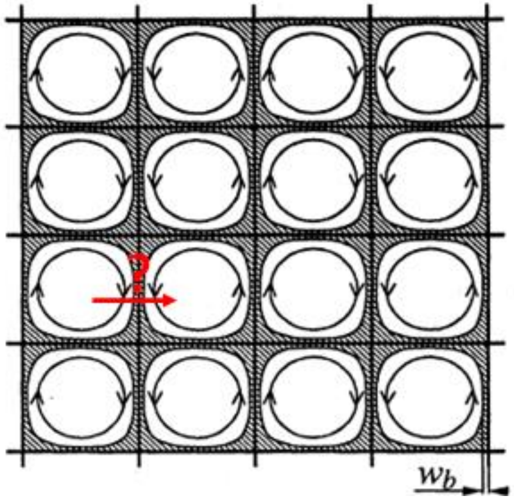
- How does staircase beat homogenization?
- Is the staircase a meta-stable state?
- What is the minimal set of scales to recover layering?

<u>Next</u>:
More on staircase!
But, <u>FIRST</u> let's discuss cell array…

Fixed Cellular Array
(another way to get a Staircase)

Fixed Cellular Array

Consider a **general** case of a system of eddies not overlapping but tangent → **Staircase**



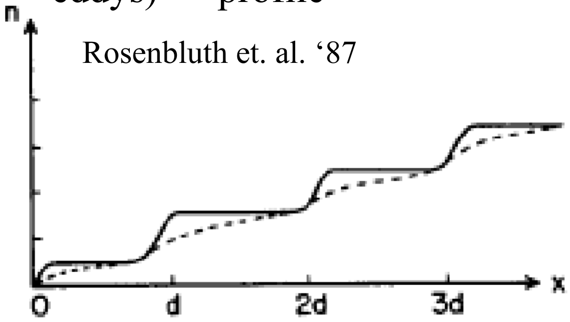
Transport? Answer: Deff ~ $D_o\, Pe^{1/2}$ {**Not a simple addition of process!**}

→ Two time rates: v_o / ℓ_o, D_o / ℓ^2_o

→ $Pe = v_o\, \ell_o / D_o >> 1$

Profile?

Consider concentration of injected dye (passive scalar transport in eddys) → profile



→ Layering!

→ **Simple** consequence of **two rates**

→ "Rosenbluth Staircase"

"Steep transitions in the density exist between each cell."

Staircase arises in an array of stationary eddies!

Important:

- **Staircase** arises in stationary array of passive eddies (Note that there is no FEEDBACK)
- Global transport hybrid:
  - → fast rotation in cell
  - → slow diffusion in boundary layer
- Irreversibility localized to inter-cell boundary.

Relevant to key question of "near marginal stability"

Fixed Cellular Array (cont.d)

The governing equation that produces layering is the **passive-scalar transport** equation.

$$\mathbf{u} = \hat{z} \times \nabla \psi$$

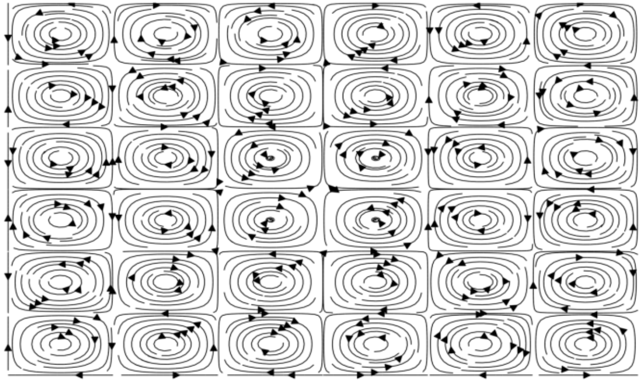
$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n - D\nabla^2 n = 0$$

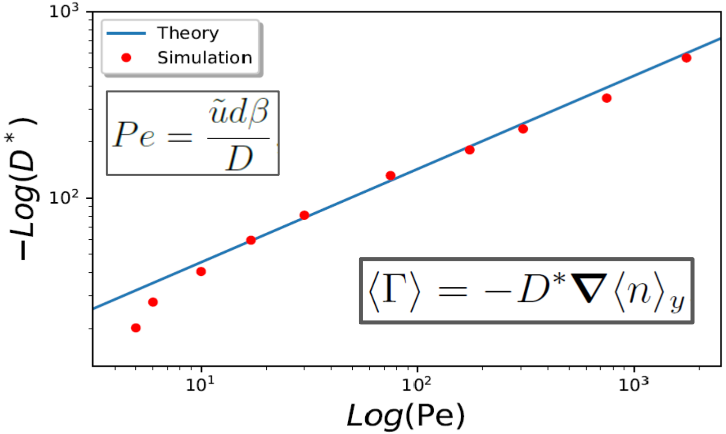
The Pe is defined as the ratio between the advective term and diffusion term!
(Pe >> 1)

A preliminary study is done to verify that code reproduces known theoretical results!

BUT, this setup is contrived, NOT self-organized!!!
Cellular array is severely constrained!

Constrained (6x6) cell array





What about the dynamics of a **less constrained** cell array (i.e., vortex array with fluctuations) ?

Next:

Relaxing Cellular Array w/ Vortex Array

Consider Another Approach

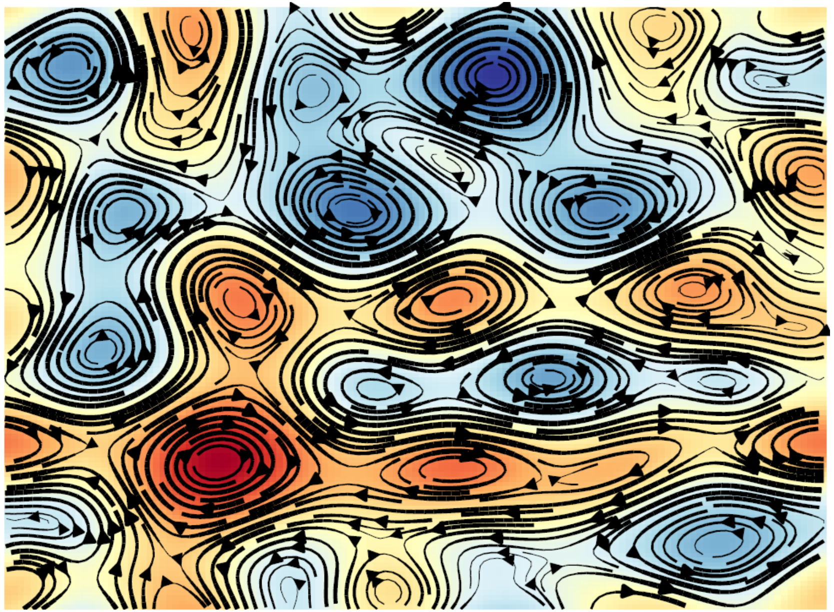
- We want to study a much more **general** and **less constrained** version of the cell array.
    - Consider a vortex array with fluctuations; jitters.
- How **resilient** is the staircase in the presence of these small variations to a fixed vortex array?

In the process of studying the **resilience** of the staircase, we aim to answer the following:

- What occurs to staircase steps as cells deviate from marginality? What about other cellular interactions?
- What does the scalar concentration path look like?
- How does increasing scattering affect transport?

To answer these questions, we use the idea of a **Melting Vortex Crystal**…

Example of **less constrained** cell array



Melting Vortex Crystal

Why are we doing this? We know that a system with two disparate time scales forms a staircase!

- Now consider fluctuations… → Will staircase survive?

Vortex crystal is an alternative way to view convection cells!

→ We begin with the 2D NS equation that can be written in nondimensional form (Perlekar and Pandit 2010),

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla}\right)\omega = \frac{1}{\Omega}\nabla^2\omega + F_\omega - \alpha\omega, \qquad \nabla^2\psi = \omega.$$

→ The "vortex crystal" is simply the array of cells and "melting" is related to turbulence induced variability in the structure. The melting vortex crystal allows us to study a **less constrained** version of the array!

Improved model of cells near marginality.

→ The melting flow structure is created by **slowly increasing the Reynolds number** in the NS equation

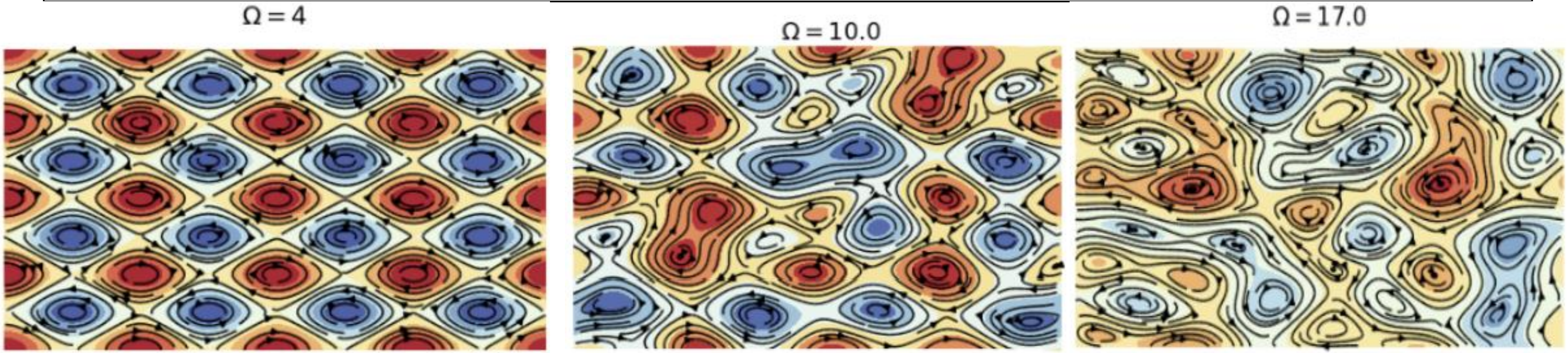
$$\Omega \equiv nRe$$

→ By increasing the Reynolds number this modifies the forcing and drag term, thus, **scattering** the vortex crystal. The **resilience** of the staircase is studied by **increasing disorder** in the vortex crystal through F_ω

$$F_\omega \equiv -n^3\left[\cos\left(nx\right) + \cos\left(ny\right)\right]/\Omega$$

The streamfunction, ψ, at different evolutionary stages of the "melting" vortex crystal is inserted into the passive scalar equation to study the resilience of the staircase structure.

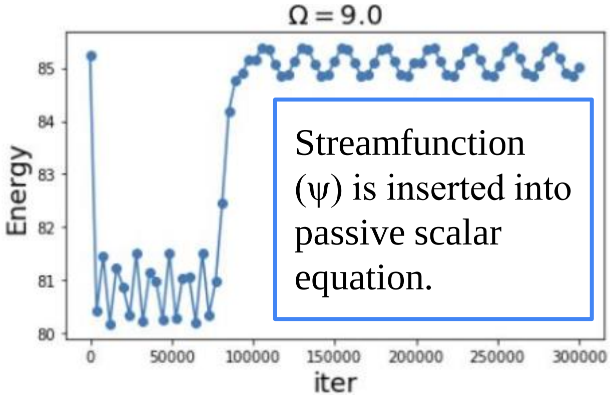
Fluctuation from Marginal Point (Melting)



- Contour plots of the streamfunction (ψ), illustrating the different stages of a “melting” vortex crystal.
- As Ω is slowly increased, there is a merger of vortices along with distortions of the crystal array.
  - Vibrating vortex crystal that goes into melting state!

We characterize different stages of the melting process by analyzing the contour plot of the crystal and the crystal’s energy trace during each different stage. There are five different stages:

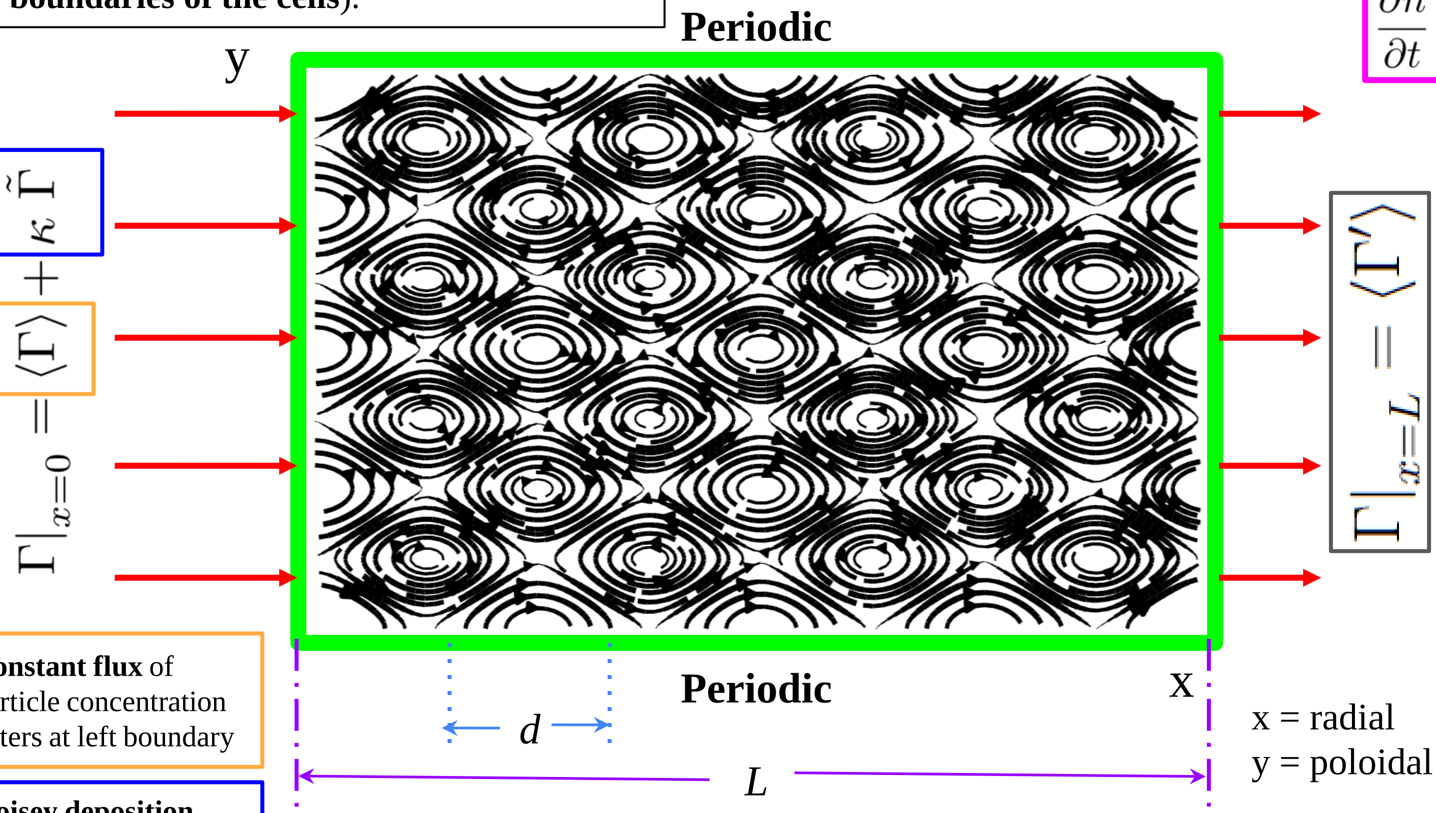
- Stable Crystal (SX) [$\Omega < 6.5$]
- Stable Distorted Crystal (SXA) [$6.5 < \Omega < 8$]
- Periodic Crystal (OPXA) [$8 < \Omega < 10$]
- Quasiperiodic Crystal (OQPXA) [$10 < \Omega < 13$]
- Spatiotemporal chaotic/turbulent crystal (SCT) [$13 < \Omega$]



Setup of Transport Problem

We are primarily concerned with Pe >> 1, where **layering** occurs (**physics explained by fast mixing within the cells and slow mixing across the boundaries of the cells**).

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D \nabla^2 n,$$



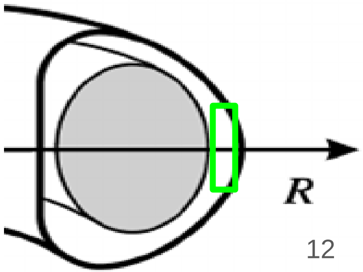
Combination of fixed flux and periodic boundary conditions are used to model the physics of **core to edge** in **fusion** devices.

Constant flux of particle concentration exits at right boundary

Constant flux of particle concentration enters at left boundary

Noisey deposition (Pulse Train)

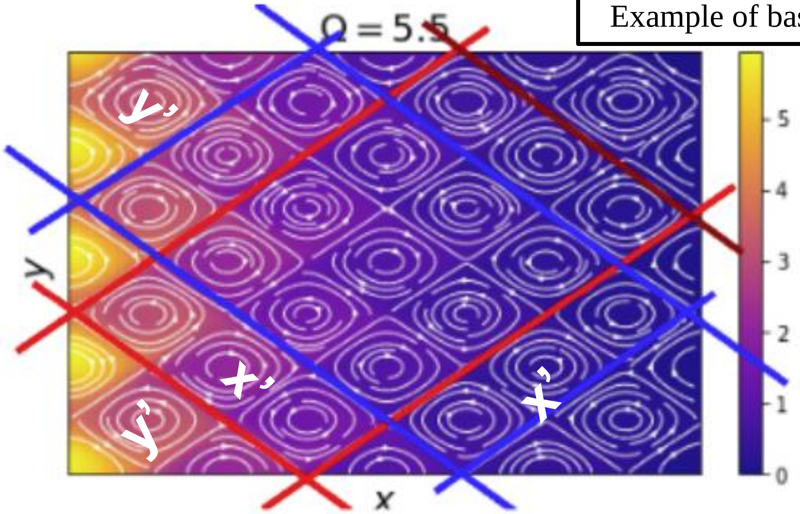
x = radial
y = poloidal

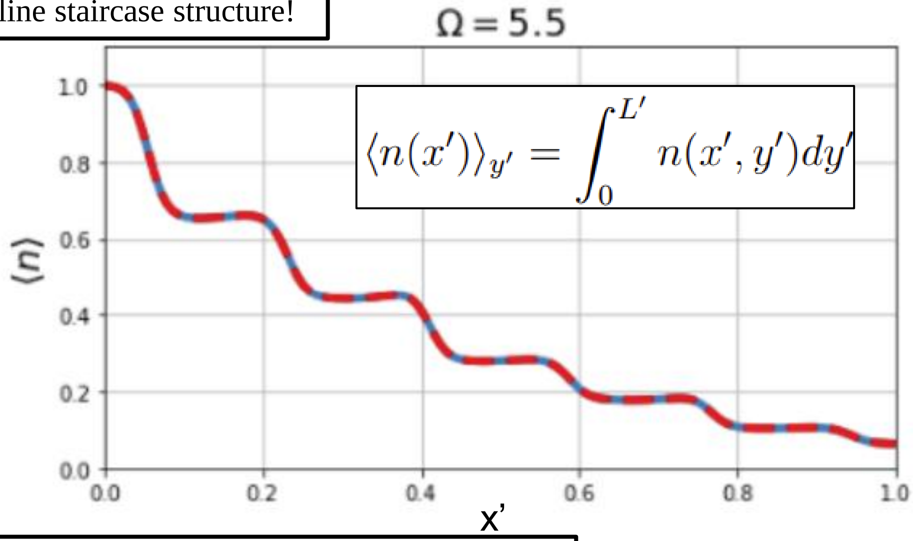


What Happens to Staircase?

The Staircase

Example of baseline staircase structure!



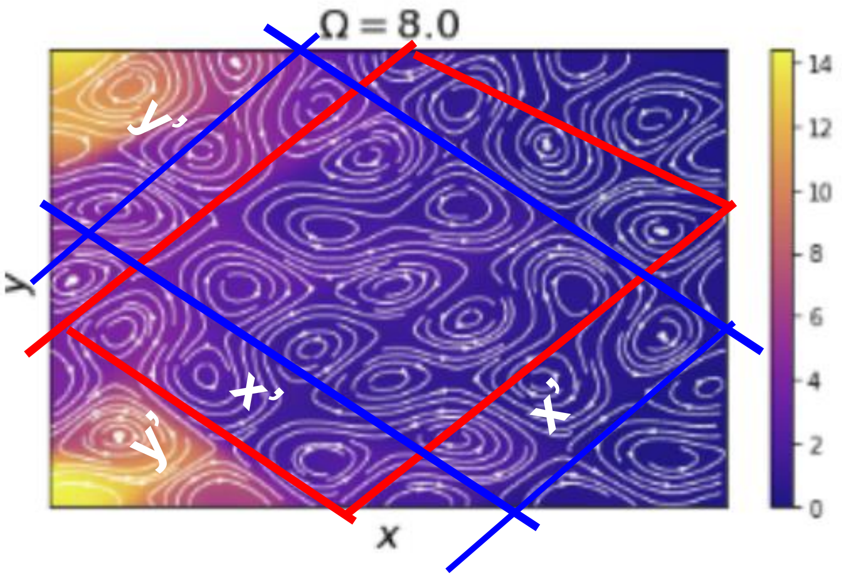


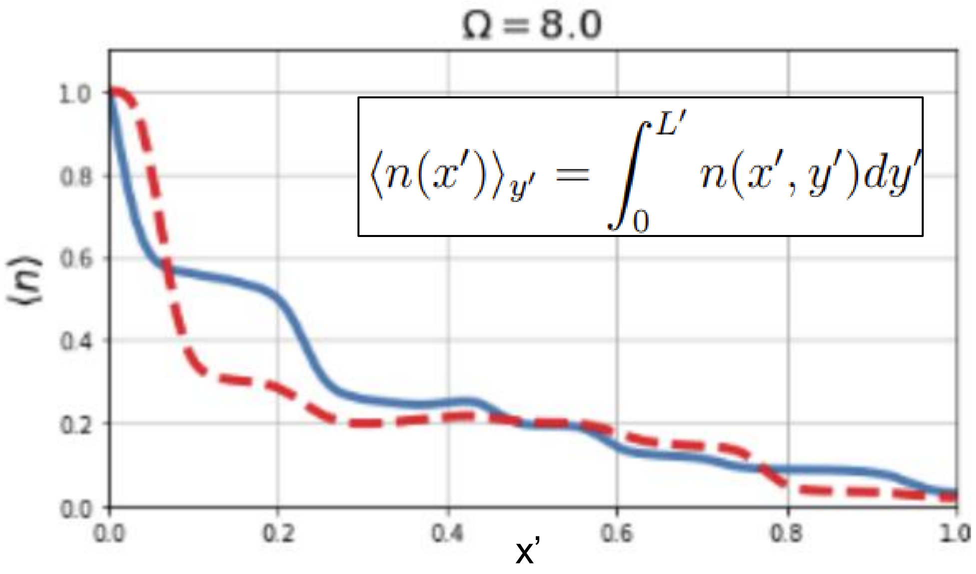
$$\langle n(x')\rangle_{y'} = \int_0^{L'} n(x', y')dy'$$

- For a weakly fluctuating crystal we get a **baseline staircase** structure.
- On the left figure the blue and red box correspond to the blue and red plot line on the right. Note that **steps** are **evenly** spaced!
  - Both blue and red average scalar concentration have the same profile in stable stage.

So what happens to the staircase if we increase the Reynolds number in the crystal?

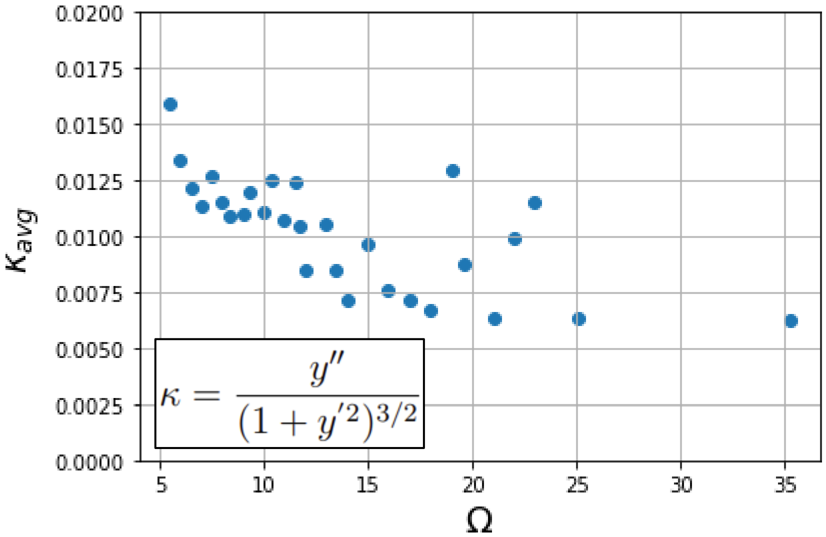
Staircase Resiliency to Fluctuations

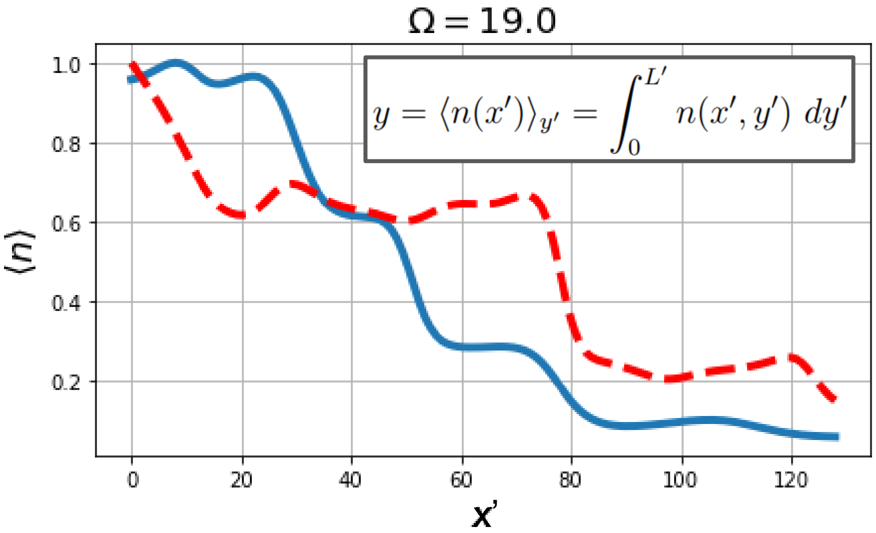




- As we **increase the degree of melting** (i.e., increase fluctuation from marginal point) through Ω, we can see **merger/connections** of vortex structures in the flow.
- These **vortex mergers** are shown in the scalar profile plot as **mergers in steps**.
  → As we increase the degree of melting, staircase steps start to merge together.

Behaviour of Staircase as Cells Deviate from Marginality





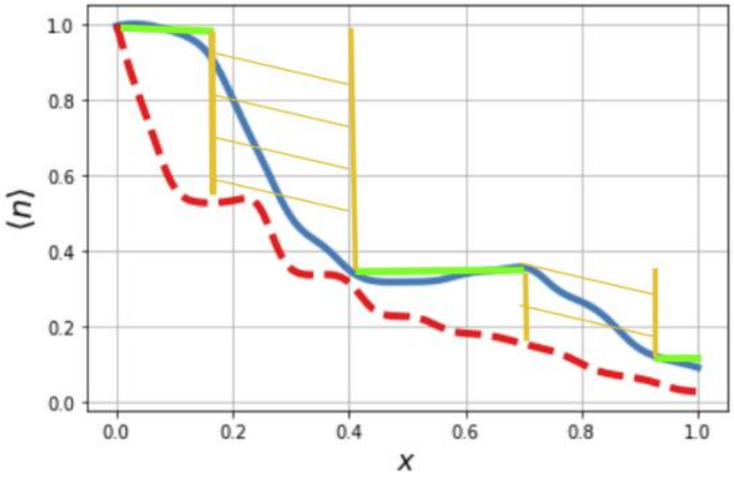
- To quantify the different stages of the melting process, we look at the **curvature** in scalar concentration.
- In general, as we **increase Ω**, the **curvature decreases**.
  - Steps are starting to merge together as we increase Ω, thus, scalar profile has less curvature.

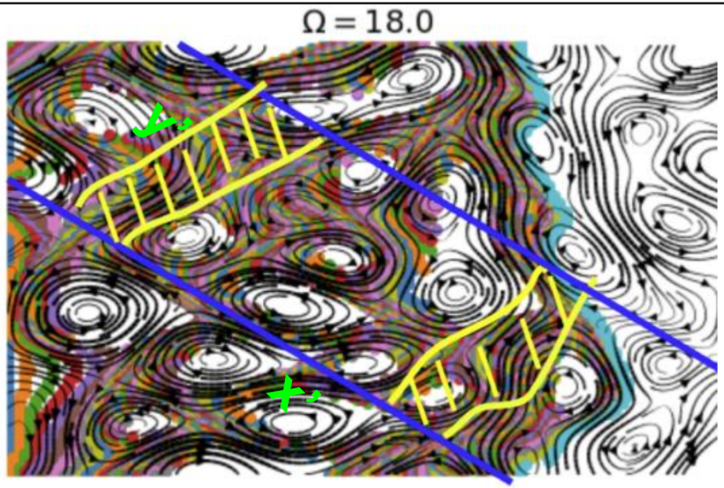
Main Point: Despite that vortex array becoming more turbulent, the staircase structure does not degrade.

- Staircase steps become **less regular**. They merge into longer steps.

The Scalar Field

Global Structure of the Scalar Field





Before the **staircase** structure forms, scalar concentration field forms a "**web**":

- The picture on the right represents the **scalar trajectory** during the first couple of time steps. We see that scalar flows quickly around saddle areas (strong shear).
  - The **holes** are shown to be **vortex structures**.
  - **Scalar quickly forms barriers** between vortex structures.
- Picture on the left shows the steady-state outcome of the web and scalar path dynamics.
  - **Yellow** represents the **barriers** between areas of strong mixing.
  - **Green** represents the **holes/vortices**, which are the areas of strong mixing.

Before reaching a steady state profile, we note the following observations:

- Scalar **slowly fills vortex**.
- Scalar **travels along and around vortex** structures.

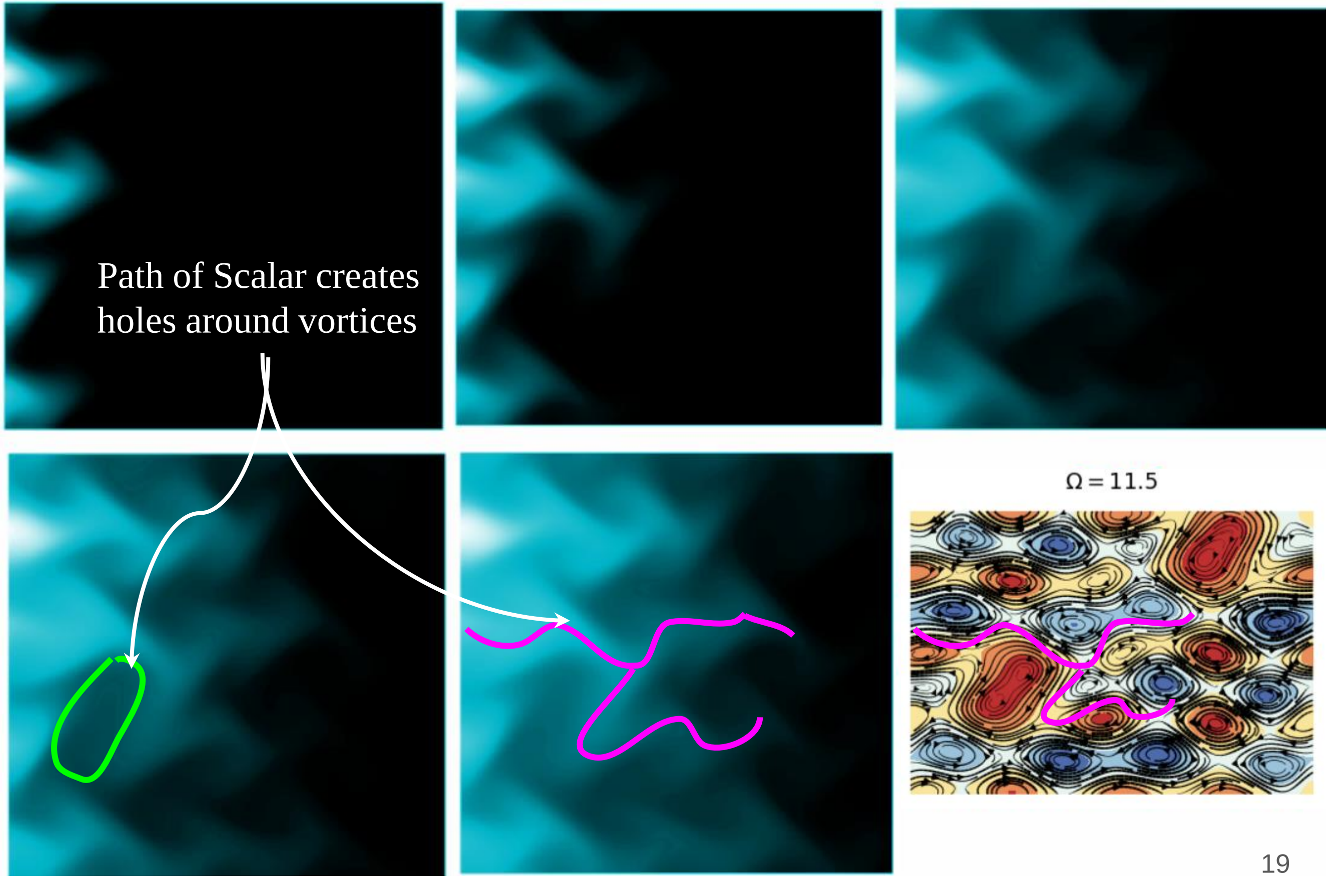
Next: the trajectory of the scalar concentration.

Trajectories through Vortex Array

- As the scalar concentration gets injected into the flow structure, a **flamelet network pattern** forms (Pocheau 2008).
  - **Fingers propagate through array**.

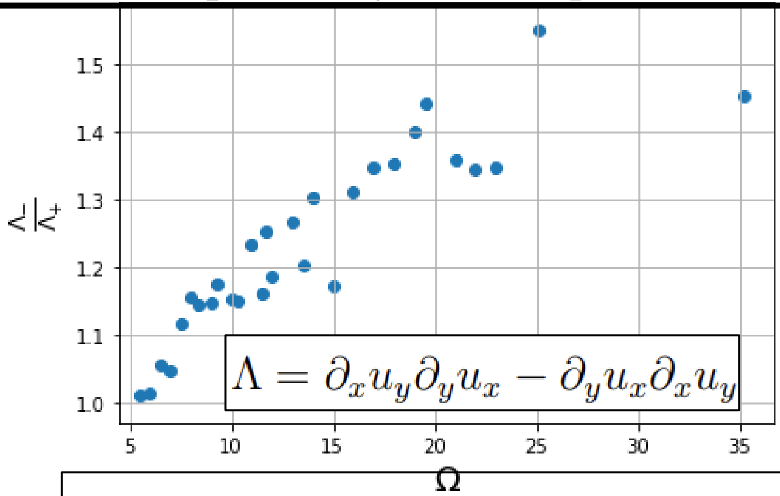
- **Scalar** concentration **flows along** and **around** areas of **vortex structures**.
  - Over time, the scalar slowly enters the vortex structure (Diffusion).

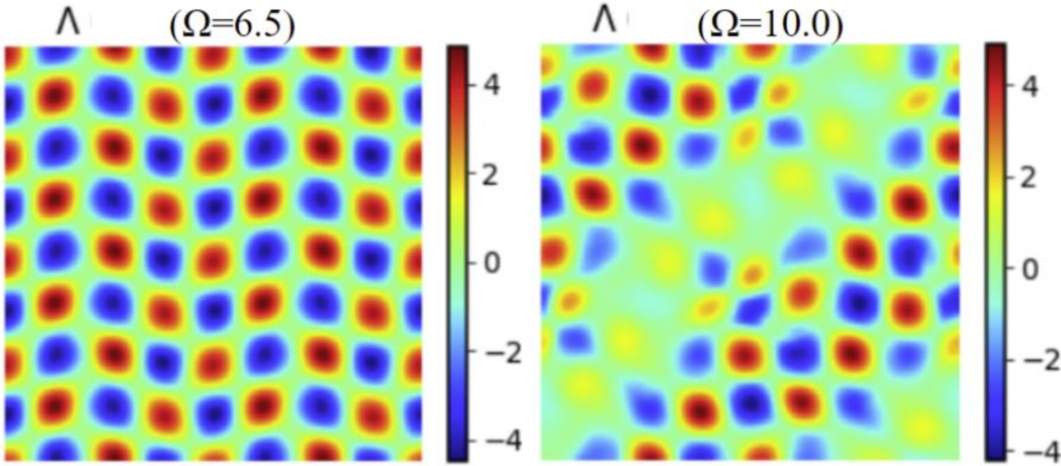
- **Scalar** concentration **flows** along areas of **strong shear** ($\Lambda < 0$).



Okubo-Weiss Field Determines Path of Scalar Concentration

Λ = mean sq. vorticity - mean sq. shear





We use **Okubo-Weiss field** (Λ) to study the **evolution** of the flow structure as we increase Ω. As we steadily increase Ω, the areas of saddles ($\Lambda < 0$) increases compared to areas of centers ($\Lambda > 0$).

- Saddles refer to areas of strong local shear and centers refers to areas of strong vorticity.

Some questions:
Does the increase in **<u>local shear</u>** affect the staircase structure? How does this affect the transport of scalar concentration? How does the Okubo-Weiss field describe the path of the scalar concentration?

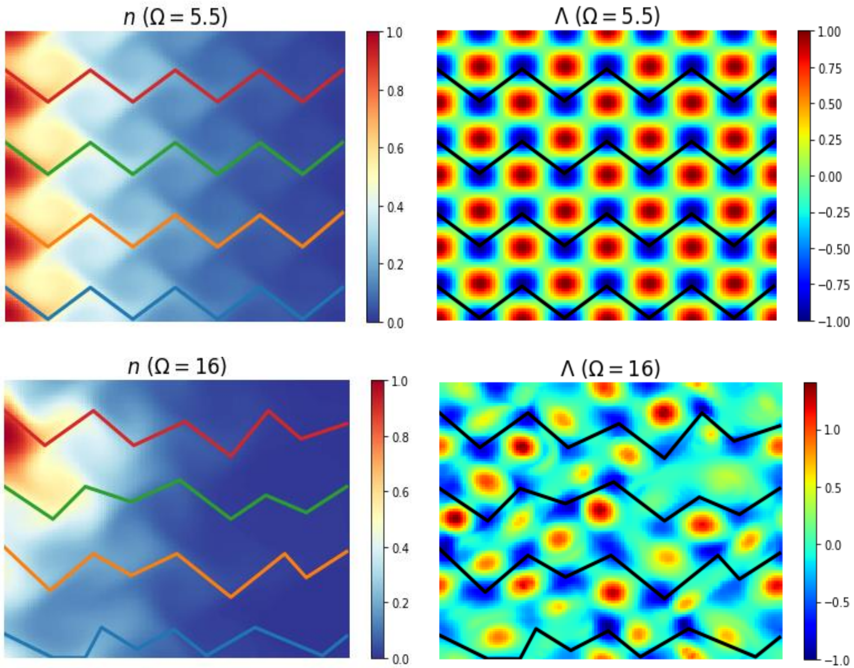
Trajectory in a Scattered Vortex Array

- Idea relevant here is the **<u>least time criterion</u>**. As the **vortex crystal melts**, the **path of least time would increase in length**.

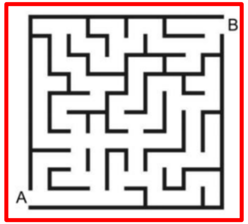
- We observe that the **scalar travels** fast along the areas of **strong shear** ($\Lambda < 0$).
  - **Travels along strong shear, but not across it! (conventional wisdom is missing part of the story…)**

- Similarities to percolation picture of infinite Kubo number.
  - How would this compare to percolation model? Can we reproduce dynamics?

- What is the **connection** between the **Web** and **Staircase**?
  - Is local shear beneficial? Goal is to reduce radial transport!

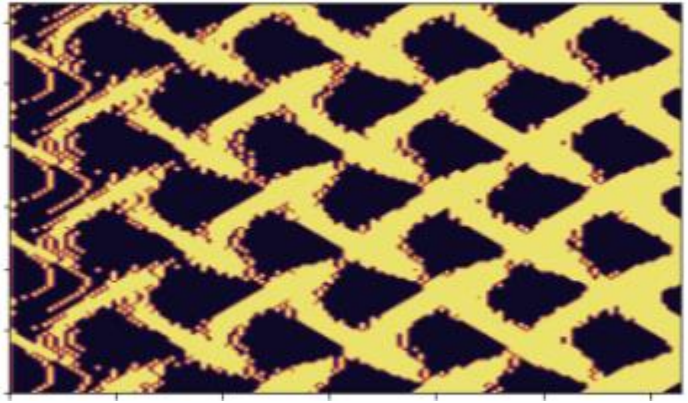


Can go from A to B if these two points are connected.

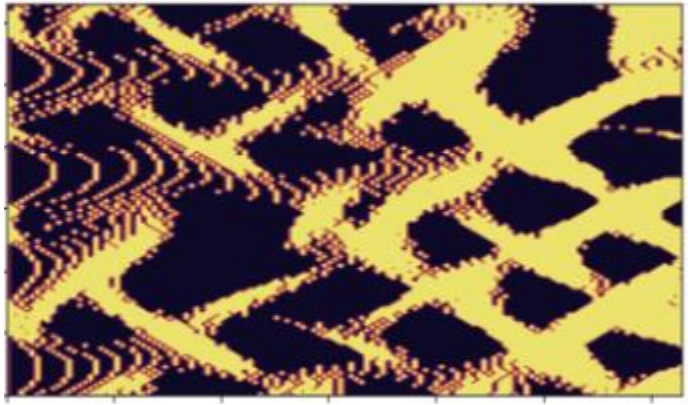


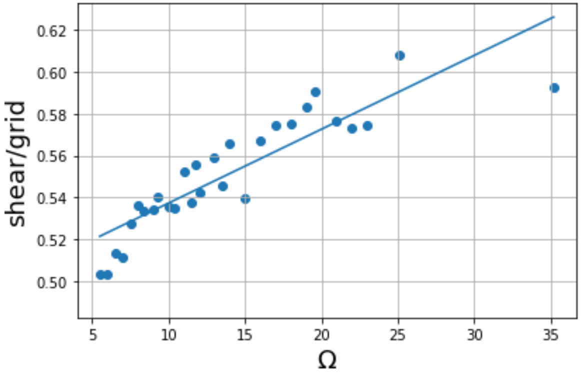
Structure of the Web Evolves

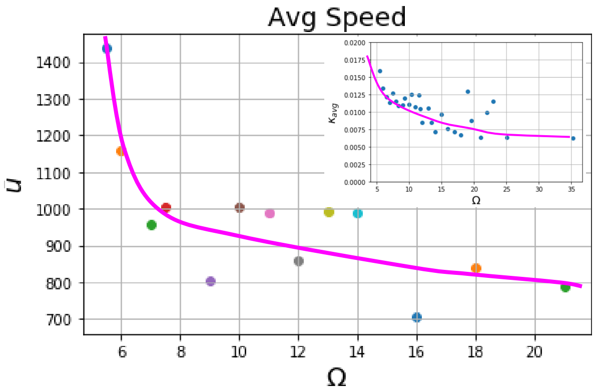
Ω = 5.5



Ω = 11.5







- As the cells deviate from marginality, the **<u>area of holes increases</u>**. The **web is not destroyed, it only degrades**.

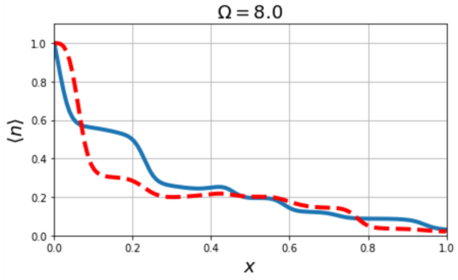
- Web area correlates with shear area increase! Web becomes thicker as we increase fluctuation from the marginal point!

- The **scattering of vortices** leads to an overall **decrease** in scalar concentration **velocity**! Agrees with least time criterion (similar idea to scattered path of light in atmosphere).
    - Connection between scalar velocity and scalar profile curvature? Plot implies there is a linear relationship between the two.

Summary & Future Work

In a much more general and less constrained version of a cell array, we study the behaviour and flow structure of a scalar concentration (AGAIN, all in a very simple model with no feedback). In this study we find the following:

- Staircase form and are **resilient** and **robust** to increasing Reynolds number (i.e., fluctuation from marginal point).
  - Mean **curvature decreases** with increase in Reynolds number.
  - Average **step size increases** due to cell mergers.
- Scalar concentration **travels along** regions of **strong shear** creating a "**web**" structure.
  - As cells deviate from marginality, the web is not **destroyed it only degrades**.
  - Web area correlates with local shear area increase.
    - Web becomes thicker as we increase fluctuation from marginal point.
  - **IMPORTANT**: Scalar travels along areas of strong shear, but not across them!
- The scattering of vortices leads to an overall decrease in scalar concentration velocity!
  - Agrees with **least time criterion**.
  - Plot of scalar concentration velocity and curvature imply there is a linear relationship between the two.
    - As curvature decreases, the scalar velocity decreases linearly.



Future: Flux expulsion

- Something w/ feedback interesting. Thread vortices w/ magnetic field (**J** x **B**), no longer passive!

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