This research was supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, under Award Number DE-FG02-04ER54738

UC San Diego

Theory of Pedestal Microturbulence with RMP-induced Stochasticity [1]

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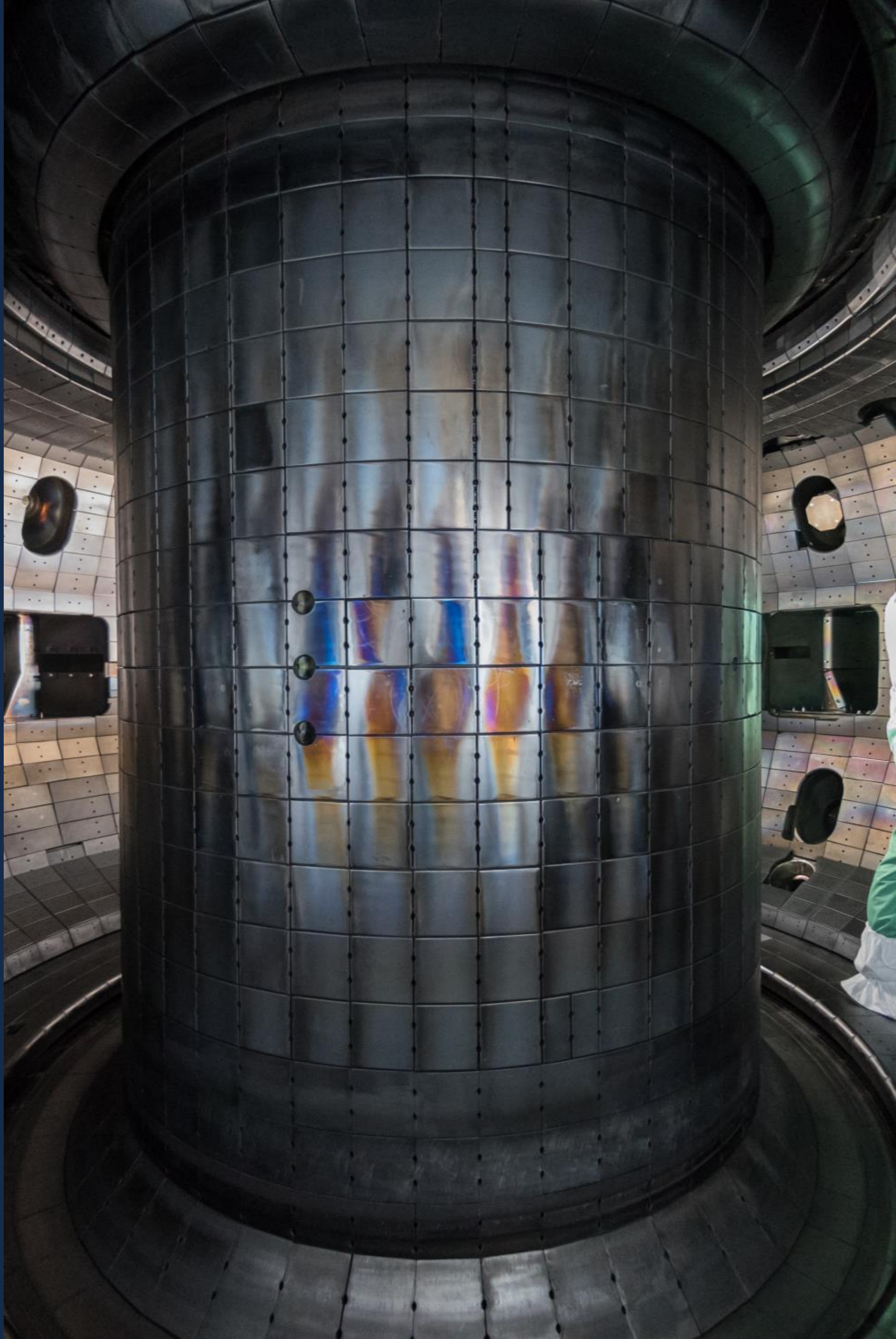
AAPPS-DPP 2022

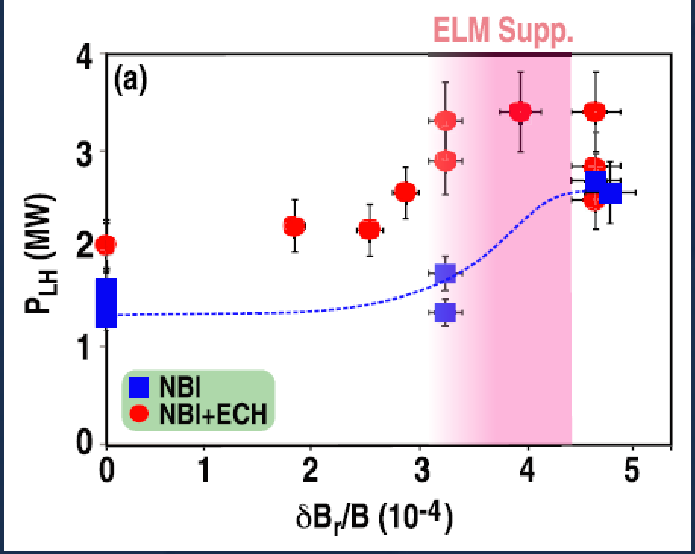
1. Cao, Mingyun, and Patrick H. Diamond. *Plasma Physics and Controlled Fusion* 64, no. 3 (2022): 035016.

OUTLINE

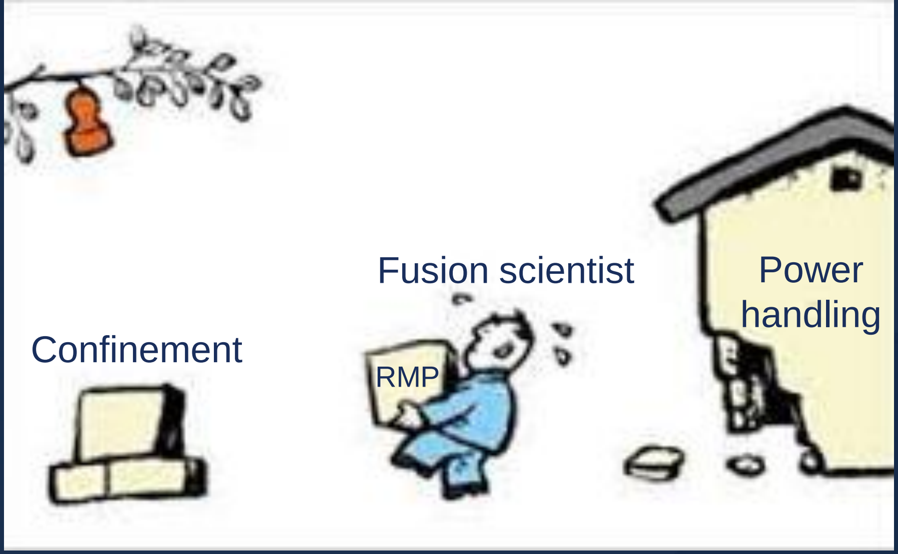
- Motivation: the Application of RMP
- Observations : Questions arising from Sims & Expts
- Model Development: A Multi-scale Feedback Loop
- Results: Theoretical Predictions & Answers to Questions
- Conclusion: What We Have Learned



Motivation: the Application of RMP



RMP raises the L-H transition power threshold [1]



Improve confinement at the expense of power handling

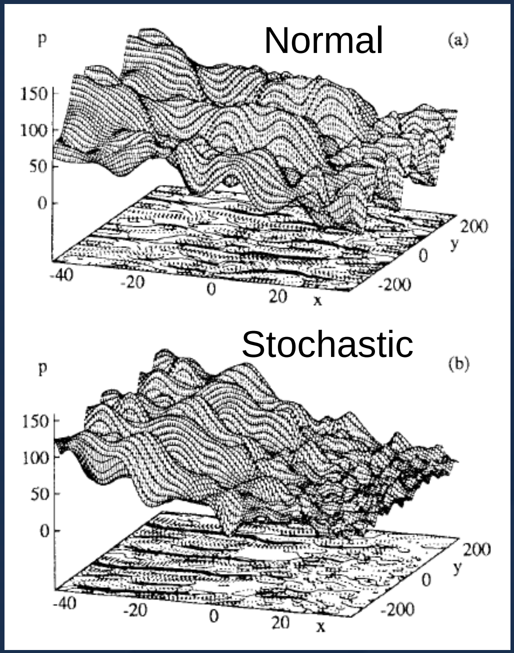
- **A new trend:** A trade-off between good confinement and good power handling.

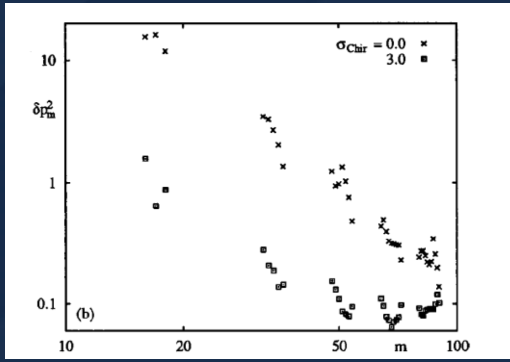
Stochastic Magnetic Fields → (modify) → Plasma Instabilities → (drive) → Plasma Turbulence

- **A basic question:** How does a stochastic magnetic field modify the instability process?

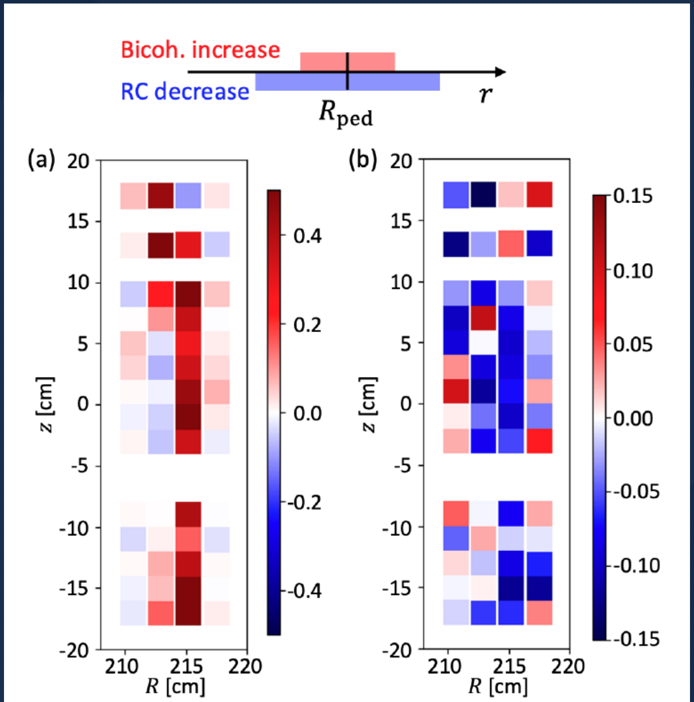
1. Schmitz, L., et al. *Nuclear Fusion* 59, no. 12 (2019): 126010.

Questions from Sims and Expts





- Simulations of resistive ballooning modes in a stochastic magnetic field. [1]
- Increased small-scale structures and spatial roughness.
- Stronger suppression of Large-scale fluctuations.



- Experimental study on the fluctuations with the stochastic magnetic field. [2]
- An increase in the bicoherence of the temperature fluctuation.
- A reduction in the Jensen-Shannon complexity.

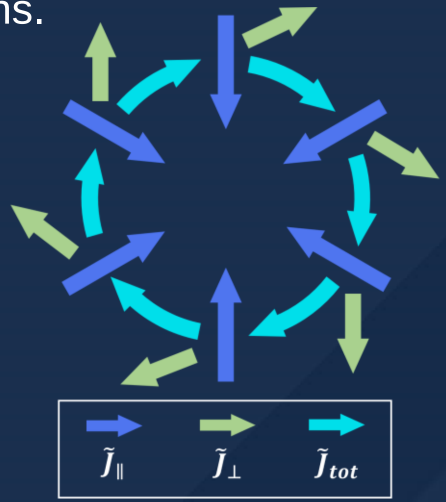
1. Beyer, P., Xavier Garbet, and Philippe Ghendrih. *Physics of Plasmas* 5, no. 12 (1998): 4271-4279.
2. Choi, Minjun J., et al. *arXiv preprint arXiv:2102.10733* (2021).

Why?

Possible Answer: A Microturbulence

- **Constraint:** Quasi-neutrality **(**$\nabla \cdot \boldsymbol{J} = 0$**)** at all scales!
- **Effect:** Introduction of $\tilde{\boldsymbol{b}}$ leads to parallel current density fluctuations.

$$\nabla_{\parallel}\widetilde{J_{\parallel}} = -\frac{1}{\eta_{\parallel}}\left\{\nabla_{\parallel}^{(0)}\left[\left(\tilde{\boldsymbol{b}}\cdot\nabla_{\perp}\right)\bar{\varphi}\right] + \left(\tilde{\boldsymbol{b}}\cdot\nabla_{\perp}\right)\nabla_{\parallel}^{(0)}\bar{\varphi}\right\} \neq 0$$



- **Insights from the classic:** Kadomtsev and Pogutse'78 [1]:

$$\nabla \cdot \boldsymbol{q} = 0 \rightarrow \tilde{T}$$

- To us:

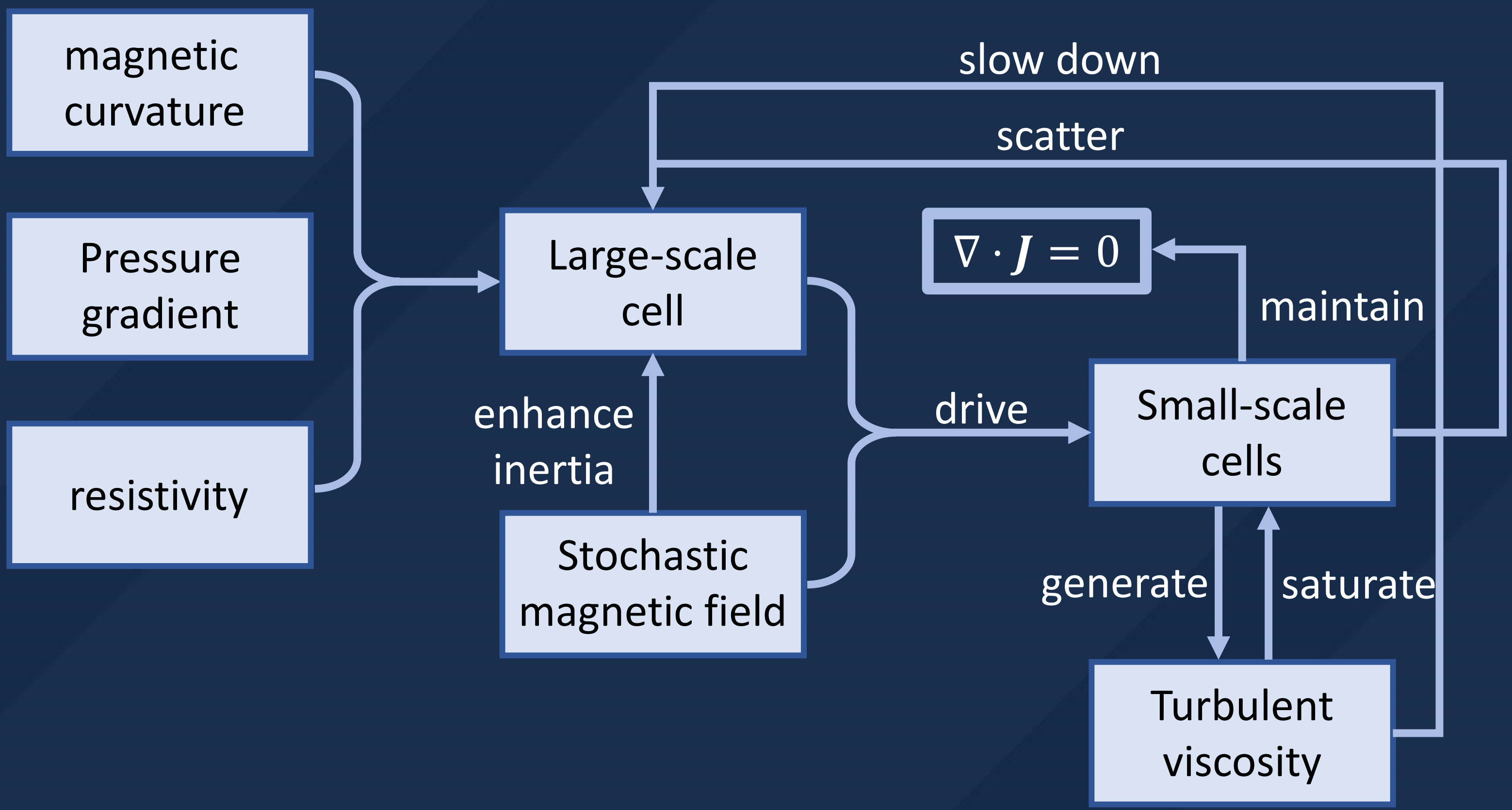
$$\nabla \cdot \boldsymbol{J} = 0 \rightarrow \tilde{\varphi}$$

- **Implication:** A current density fluctuation $\tilde{\boldsymbol{J}}_{\perp}$ must be driven to balance $\tilde{\boldsymbol{J}}_{\parallel}$, so that the total current density fluctuation $\tilde{\boldsymbol{J}}_{\boldsymbol{tot}}$ is divergence free.
- $\tilde{\varphi}$: **Intrinsic Multi-scale Microturbulence**.
- Our model is supposed to
  - ➢ maintain $\nabla \cdot J = 0$ at all scales
  - ➢ be tractable ⟶ resistive interchange mode
  - ➢ connect micro and macro scales
  - ➢ be generic

1. B. B. Kadomtsev, and O. P. Pogutse, 1979.

Model Development

A Multi-scale Feedback Loop



Model Development

- **Modified model:**

$$\left(\frac{\partial}{\partial t} + \tilde{\boldsymbol{v}} \cdot \nabla\right)\nabla_\perp^2(\bar{\varphi} + \tilde{\varphi}) = \frac{\eta S}{\tau_A}\nabla_\parallel J_\parallel - \frac{\kappa B_0}{\rho_0}\frac{\partial(\bar{p}_1 + \tilde{p}_1)}{\partial y}, \quad \longrightarrow \quad \boxed{\nabla \cdot \boldsymbol{J} = 0}$$

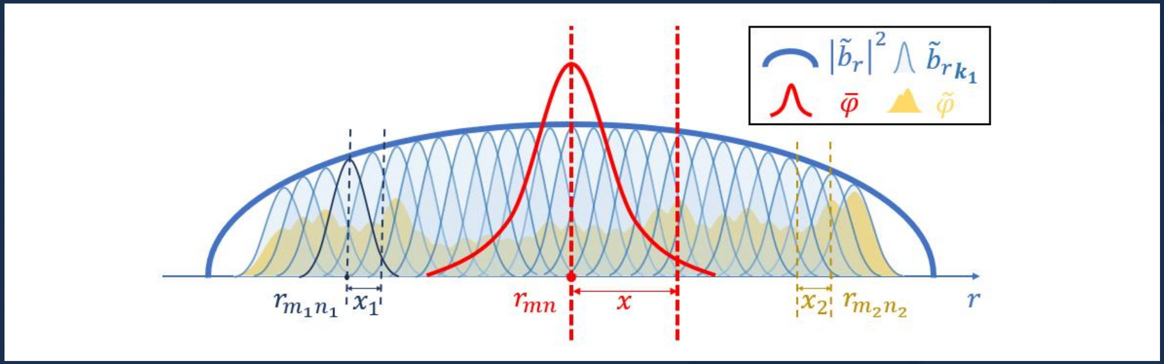
$$\left(\frac{\partial}{\partial t} + \tilde{\boldsymbol{v}} \cdot \nabla\right)(\bar{p}_1 + \tilde{p}_1) - \frac{\nabla(\bar{\varphi} + \tilde{\varphi}) \times \hat{\boldsymbol{z}}}{B_0} \cdot \nabla p_0 = 0,$$

$$\eta J_\parallel = -\nabla_\parallel(\bar{\varphi} + \tilde{\varphi}).$$

- **Parallel gradient operator:** $\nabla_\parallel = \tilde{\boldsymbol{b}}_{\boldsymbol{0}} \cdot \nabla \longrightarrow \nabla_\parallel = \nabla_\parallel^{(0)} + \tilde{\boldsymbol{b}}_\perp \cdot \nabla_\perp$, where

$$\tilde{\boldsymbol{b}} = \tilde{\boldsymbol{B}}_\perp / B_0 = \sum_{m_1 n_1} \tilde{\boldsymbol{b}}_{\boldsymbol{k_1}}(x')e^{i(m_1\theta - n_1\phi)} . \, (x' = r - r_{m_1 n_1})$$

- **Technique:** Method of averaging



Model Development

- Separation of different scales:

$$① \left[\frac{\partial}{\partial t}+\tilde{\boldsymbol{v}}\cdot\nabla\right]\nabla_\perp^2\bar{\varphi} = -\frac{S}{\tau_A}\Big[\nabla_\parallel^{(0)^2}\bar{\varphi} + \underbrace{\left(\nabla_\perp\cdot\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}\rangle\right)\cdot\nabla_\perp\bar{\varphi}}_{(a)} + \underbrace{\left\langle\nabla_\parallel^{(0)}\tilde{b}\cdot\nabla_\perp\tilde{\varphi}\right\rangle}_{(b)} + \underbrace{\left\langle\left(\tilde{b}\cdot\nabla_\perp\right)\nabla_{\parallel'}^{(0)}\tilde{\varphi}\right\rangle}_{(c)}\Big] - \frac{gB_0}{\rho_0}\frac{\partial\bar{p}_1}{\partial y},$$

$$② \left[\frac{\partial}{\partial t}+\tilde{\boldsymbol{v}}\cdot\nabla\right]\nabla_\perp^2\tilde{\varphi} = -\frac{S}{\tau_A}\Big[\nabla_\parallel^{(0)^2}\tilde{\varphi} + \underbrace{\left(\tilde{b}\cdot\nabla_\perp\right)\nabla_\parallel^{(0)}\bar{\varphi}}_{(\alpha)} + \underbrace{\nabla_\parallel^{(0)}\left(\tilde{b}\cdot\nabla_\perp\right)\bar{\varphi}}_{(\beta)}\Big] - \frac{gB_0}{\rho_0}\frac{\partial\tilde{p}_1}{\partial y},$$

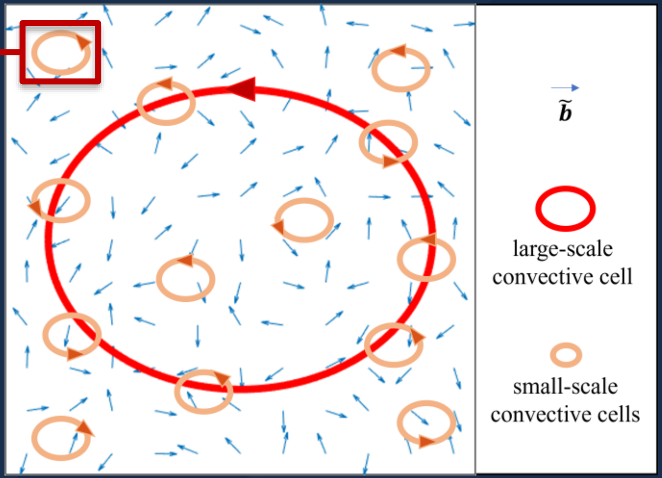
→ Relate $\tilde{\varphi}$ to $\tilde{\boldsymbol{b}}$

$$③ \left[\frac{\partial}{\partial t}+\tilde{\boldsymbol{v}}\cdot\nabla\right]\bar{p}_1 - \frac{\nabla\bar{\varphi}\times\hat{\mathbf{z}}}{B_0}\cdot\nabla p_0 = 0,$$

$$④ \left[\frac{\partial}{\partial t}+\tilde{\boldsymbol{v}}\cdot\nabla\right]\tilde{p}_1 - \frac{\nabla\tilde{\varphi}\times\mathbf{z}}{B_0}\cdot\nabla p_0 = 0,$$

$\tilde{\boldsymbol{v}}\cdot\nabla$ replaced by $-\nu_T\nabla_\perp^2$ or $\chi_T\nabla_\perp^2$ → **Very Important!**

leads to



- Some assumptions/observations:
  1. *$\bar{\varphi}$: low $\boldsymbol{k}$, slow interchange approximation*
  2. *$\tilde{\varphi}$: high $\boldsymbol{k}_2$, fast interchange approximation*
  3. *The beat of $\tilde{\boldsymbol{b}}$ and $\bar{\varphi}$ drives $\tilde{\varphi}$*
  4. *$\tilde{\varphi}$ reacts on the evolution of $\bar{\varphi}$*

  } A feedback loop

$$\boldsymbol{B_0} = B_\phi\hat{\boldsymbol{\phi}} + B_\theta(r)\hat{\boldsymbol{\theta}}, \quad \tau_A = a(4\pi\rho_0)^{1/2}/B_0, \quad S = \tau_R/\tau_A, \quad \tau_R = 4\pi a^2/\eta$$

$$L_p = |(1/p_0)(dp_0/dr)|^{-1}, \quad L_s = s/Rq, \quad s = |(r/q)(dq/dr)|, \quad k_\theta = m/r_{mn}$$

Results: $\tilde{v}_r$ *'locks on'* to $\tilde{b}_r$ & 1st Order Correction

$$\left[\frac{\partial}{\partial t} - \boxed{\nu_T}\nabla_\perp^2\right]\nabla_\perp^2\tilde{\varphi} + \frac{S}{\tau_A}\nabla_\parallel^{(0)^2}\tilde{\varphi} + \frac{gB_0}{\rho_0}\frac{\partial\tilde{p}_1}{\partial y} = \big[\underbrace{(\tilde{b}\cdot\nabla_\perp)\nabla_\parallel^{(0)}\bar{\varphi}}_{(\alpha)} + \underbrace{\nabla_\parallel^{(0)}(\tilde{b}\cdot\nabla_\perp)\bar{\varphi}}_{(\beta)}\big]$$

$\tilde{\boldsymbol{b}}$ is static!

fast interchange — slow interchange

ν_T is required to saturate the growth of $\tilde{\varphi}$ on a short time scale

- **Implication:** A non-trivial correlation

$$\langle\tilde{b}_r\tilde{v}_r\rangle = \pi^{\frac{1}{2}}\frac{\tilde{k}_\theta R r_{mn}}{L_S^3 B_0}\frac{S}{\tau_A}\bar{\varphi}_k(0)\times\int dk_{2\theta}|k_{2\theta}|k_{2\theta}\frac{c^2Z^2(k_\theta - k_{2\theta})w_{k_2}o_{k_2}^2}{\Lambda_{k_2}^0 - \Lambda_{k_2}},$$

- **Reminder:** The statistics of the turbulence is affected by $\tilde{\boldsymbol{b}}$ (Minjun Choi).

- The first-order growth rate correction:

$$\gamma_k^{(1)} = \frac{\int_{-\infty}^{\infty}\hat{\tilde{\varphi}}_k^{(0)}\hat{H}_1\hat{\tilde{\varphi}}_k^{(0)}dk_x}{\int_{-\infty}^{\infty}\hat{\tilde{\varphi}}_k^{(0)}\left[\partial_{\gamma_k^{(0)}}\hat{H}_0\right]\hat{\tilde{\varphi}}_k^{(0)}dk_x} = -\frac{5}{6}\underbrace{\hat{v}_T}_{TBD}\left(\frac{\tau_p\tau_\kappa}{\tau_A^2}\right)^{\frac{1}{3}}S^{\frac{2}{3}}\tilde{k}_\theta^{\frac{2}{3}} - \frac{1}{3}\frac{S}{\tau_A}\left|\tilde{b}_r\right|^2 - \frac{2\sqrt{2}}{3}\frac{\hat{I}S^{\frac{4}{3}}\tilde{k}_\theta^{\frac{4}{3}}}{\left(\tau_p\tau_\kappa\tau_A^4\right)^{\frac{1}{3}}} < 0,$$

where $\hat{v}_T = \nu_T/L_s^2$, $\hat{I} = IRr_{mn}/L_s^3$, $\tilde{k}_\theta = k_\theta L_s$.

Physics?

- **Reminder:** Stronger suppression of large-scale fluctuations. (P. Beyer)

Results: Magnetic Braking Effect & Scaling of ν_T

Retaining the effect of the blue term, main equation reduces to

$$-\frac{S}{\tau_{\mathrm{A}}}\frac{k_\theta^2}{L_{\mathrm{s}}^2}\frac{d^2}{dk_x^2}\bar{\varphi}_{\boldsymbol{k}} + \left[\frac{S}{\tau_{\mathrm{A}}}\left|\tilde{b}_r\right|^2 + \boxed{\gamma_{\boldsymbol{k}}}\right]k_x^2\bar{\varphi}_{\boldsymbol{k}} - \frac{\kappa p_0}{L_{\mathrm{p}}\rho_0}\frac{k_\theta^2}{\gamma_{\boldsymbol{k}}}\bar{\varphi}_{\boldsymbol{k}} = 0.$$

→ Inertia term $\rho\partial_t\nabla_\perp^2\bar{\varphi}$

- **Effect:** Enhances the plasma inertia → ***Magnetic braking effect*** [1].
- The critical width of magnetic islands

$$o_{\boldsymbol{k}_2} \sim \left[\boxed{k_\theta^2/k_{2\theta}^2}(\Delta x)^4\right]^{1/4}$$

k_θ: large-scale cell
$k_{2\theta}$: small-scale cells

- In the limit of $\nu k_{2\theta}^2 - \left(1/\tau_p\tau_\kappa\right)^{1/2} \gg 0$, the scaling of the turbulent viscosity is

$$\nu_T = \left[\pi^{\frac{1}{2}}\frac{Rr_{mn}}{B_0^2}\frac{\tilde{k}_\theta^2}{L_{\mathrm{s}}^5}\left(\frac{S}{\tau_{\mathrm{A}}}\right)^2\bar{\varphi}_{\boldsymbol{k}}^2(0)\int dk_{2\theta}\frac{c^2Z^2w_{\boldsymbol{k}_2}o_{\boldsymbol{k}_2}^2}{|k_{2\theta}|^5\gamma_{\boldsymbol{k}_2}^{(0)}}\right]^{1/3}.$$

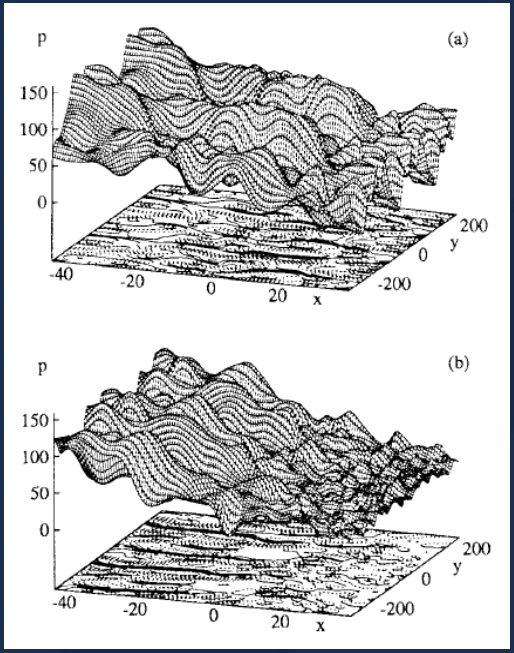
- **Analysis:** Equation (2) can be simplified to

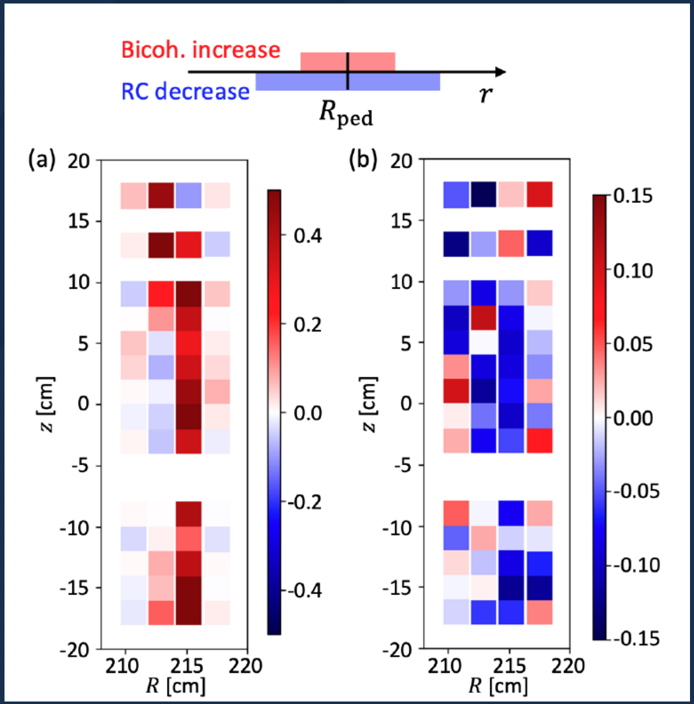
$$\frac{\partial\tilde{\varphi}}{\partial t} + \underbrace{\lambda}_{\downarrow}\tilde{\varphi} = \underbrace{\widehat{D}\left[\tilde{b}_r\bar{\varphi}\right]}_{drive}. \longrightarrow \text{Langevin equation!}$$

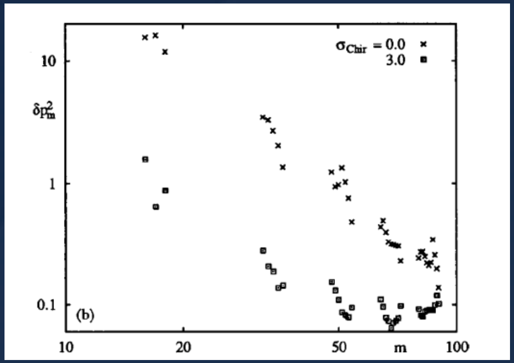
$$\lambda: \nu_T k_{2\theta}^2 - \left(1/\tau_p\tau_\kappa\right)^{1/2} > 0$$

1. Rutherford, Paul Harding. *The Physics of Fluids* 16, no. 11 (1973): 1903-1908.

Questions from Sims and Expts





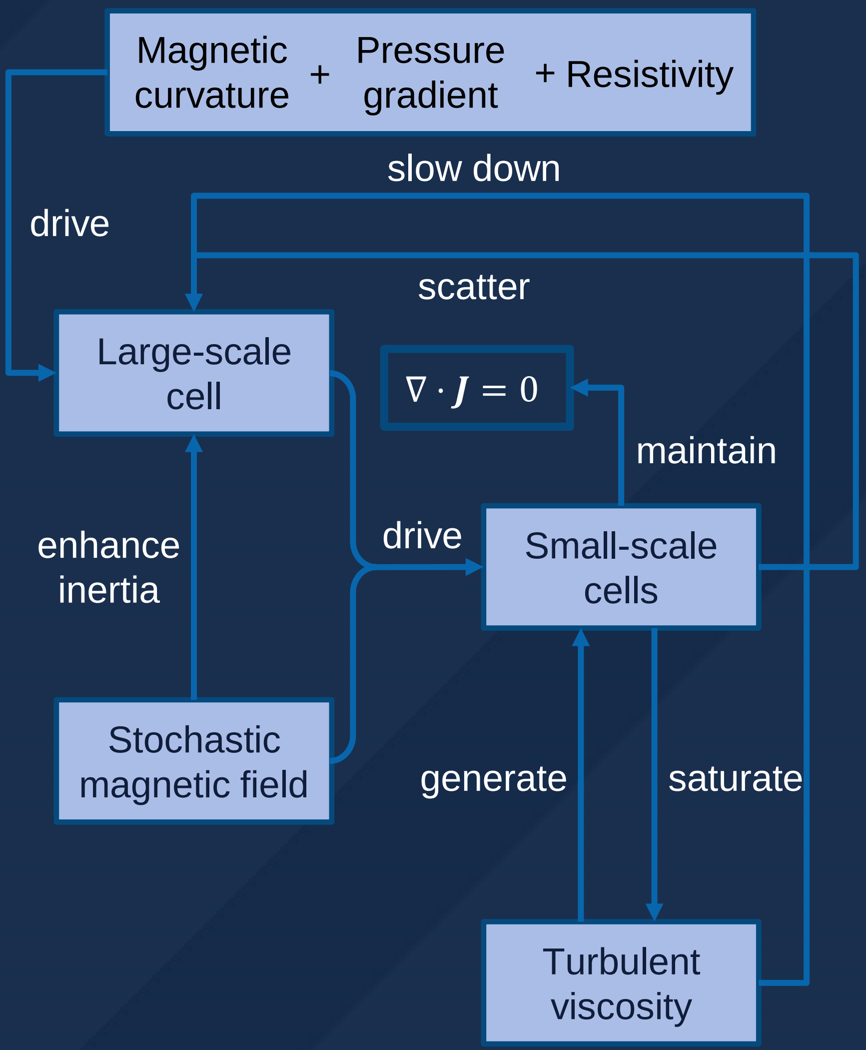


- Increased bicoherence in the pedestal turbulence → small-scale convective cells increase the number of triad interactions.
- Reduced C_{JS} → *electrostatic turbulence phase 'locks on to the stochastic magnetic field'*.

- Appearance of small-scale structures → appearance of the small-scale convective cells;
- Stronger suppression of large-scale fluctuations → stabilization by the stochastic magnetic field.

Conclusion: What We Have Learned

- $\nabla \cdot \boldsymbol{J} = 0$ is maintained at all scales, which reveals that electrostatic convective cells must be driven by $\tilde{\boldsymbol{b}}\bar{\varphi}$ beat.
- Large scale and small scale are connected. As small-scale convective cells are modulated by large-scale mode, large-scale mode is modified by small-scale cells through turbulent viscosity ν_T and electrostatic scattering.
- Stochastic magnetic field produces a magnetic braking effect, which enhances the plasma inertia and exerts a drag on large-scale mode. This is similar in structure to Rutherford's nonlinear $\boldsymbol{J} \times \boldsymbol{B}$ forces[1], but in our case, it's produced by stochastic magnetic perturbations.
- We get a non-trivial $\langle \tilde{b}_r \tilde{v}_r \rangle$. The velocity fluctuations $\tilde{\boldsymbol{v}}$ 'lock on' to the magnetic perturbations $\tilde{\boldsymbol{b}}$.



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Thank you