Momentum/Energy Transport in Predator–Prey Model Associated with Pressure Bump and Shear Flow Background

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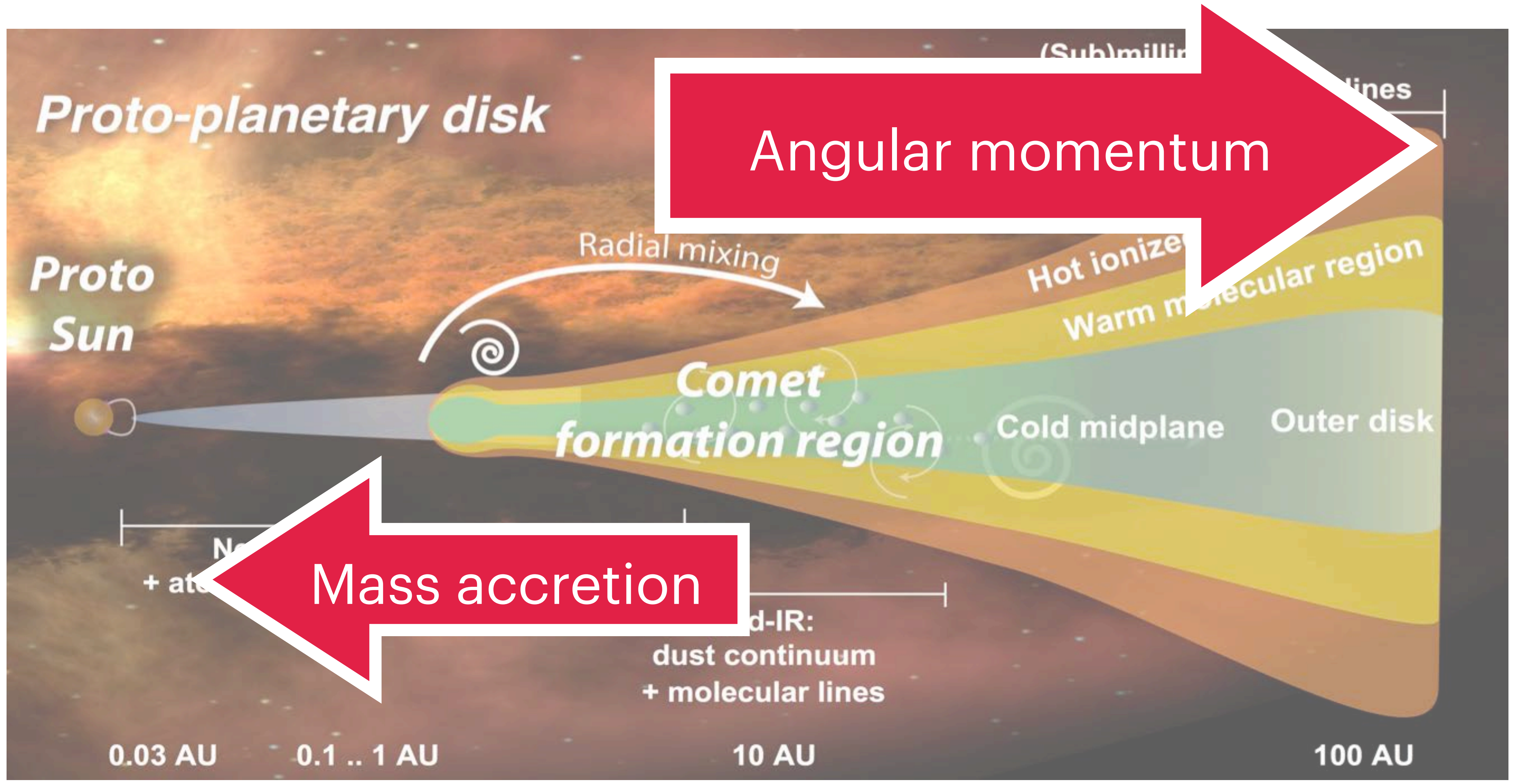
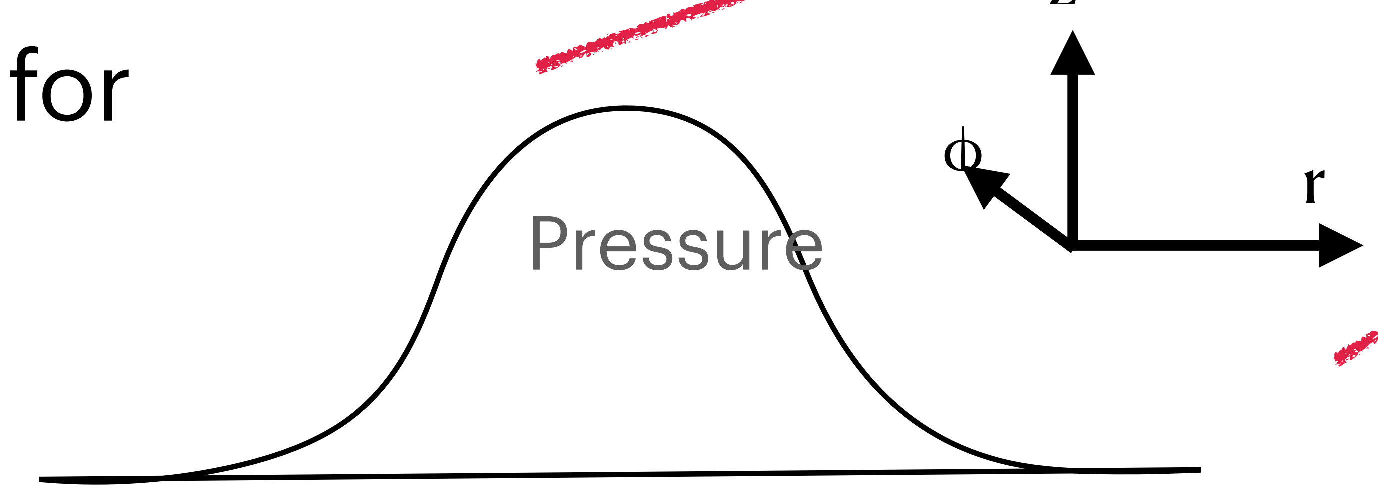
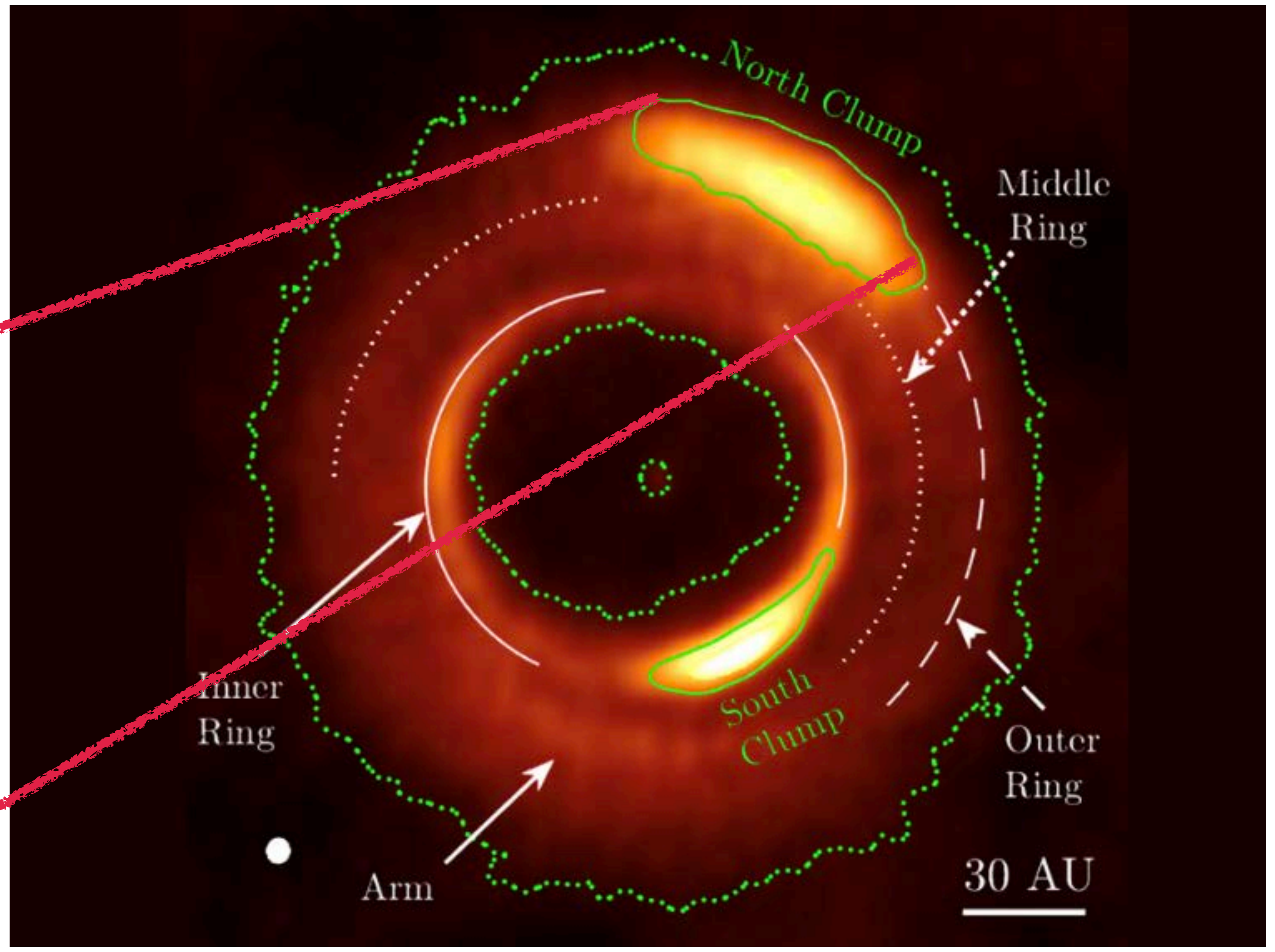
Why it matters?—Proto–Planetary Disks



- **Momentum transport**: angular momentum re-distribution results in mass accretion
- Weakly magnetized
  $\rightarrow$ compressible: $\nabla \cdot \mathbf{u} \neq 0$

- Observations founds **ring structures:** Pressure bumps traps particles for planet formation.



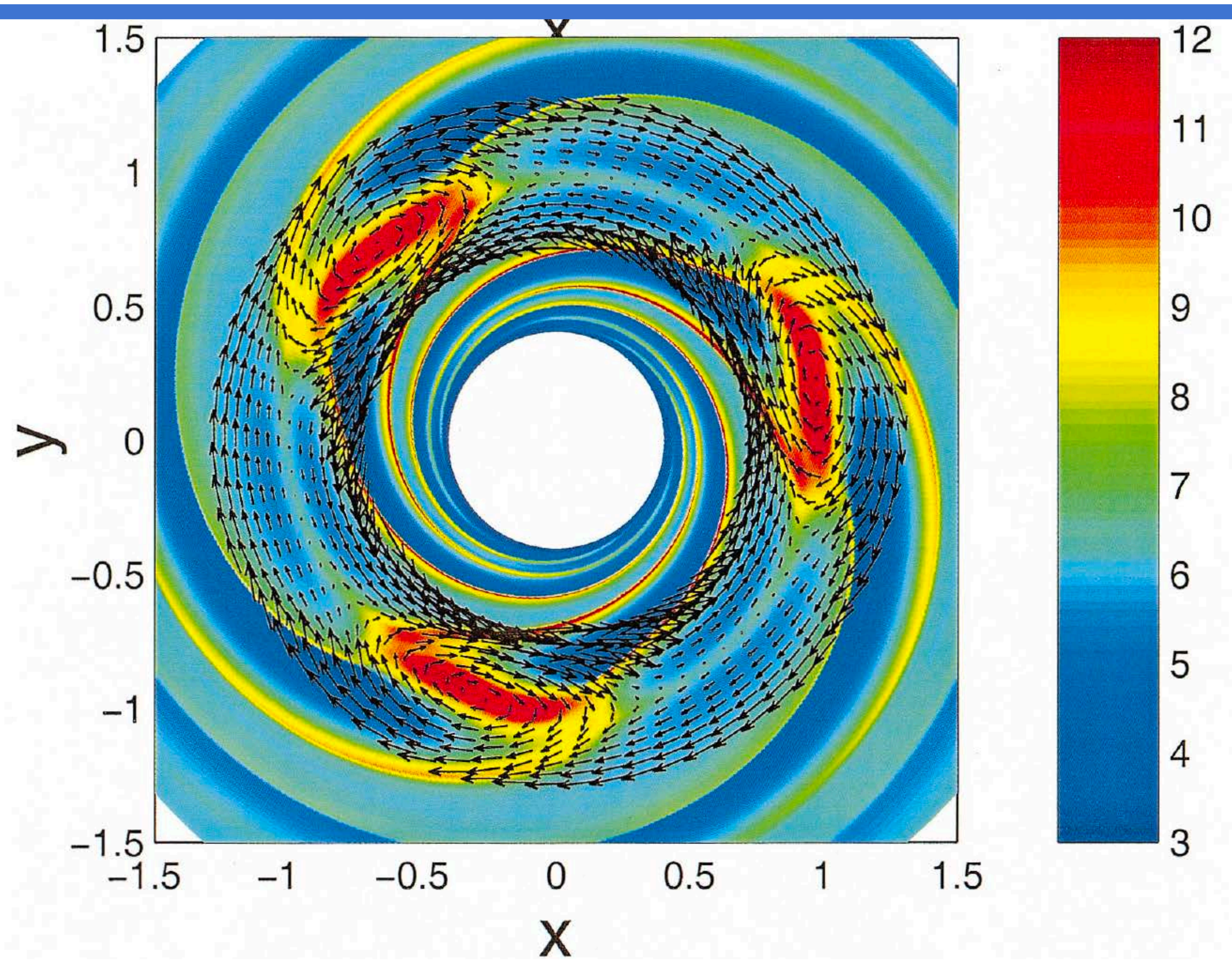


ALMA 0.87 mm continuum emission from MWC 758
(Dong et al., ApJ ,860:124 (2018))

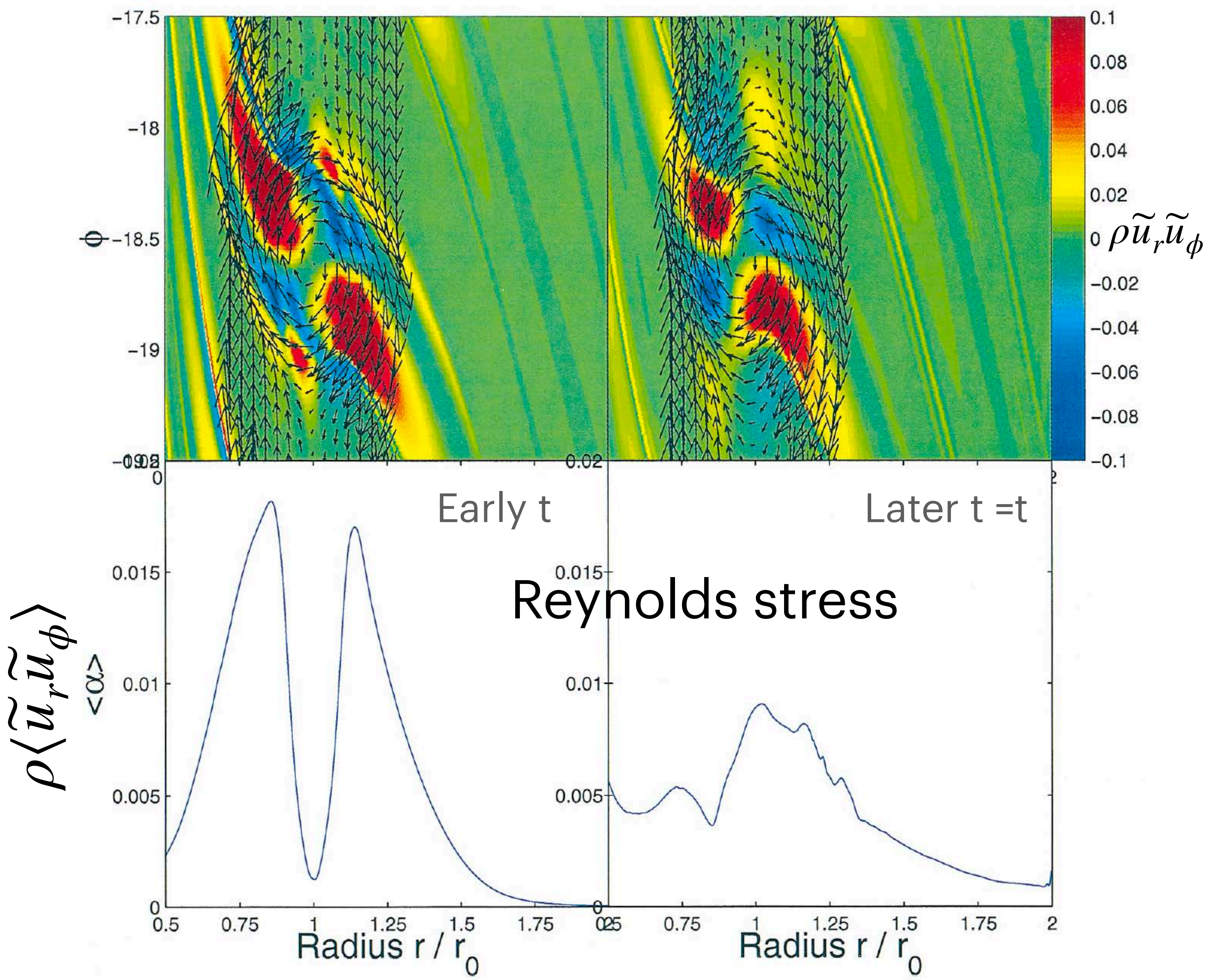
Reynolds Stress and Momentum Transport

- Rotation: Rossby waves Instability
- Vortices triggered by the Rossby wave instability (RWI).

Vortices constrain the properties of the disk and embedded planets.



- Reynolds Stress and angular momentum transport: $\rho\langle\tilde{u}_r\tilde{u}_\phi\rangle$



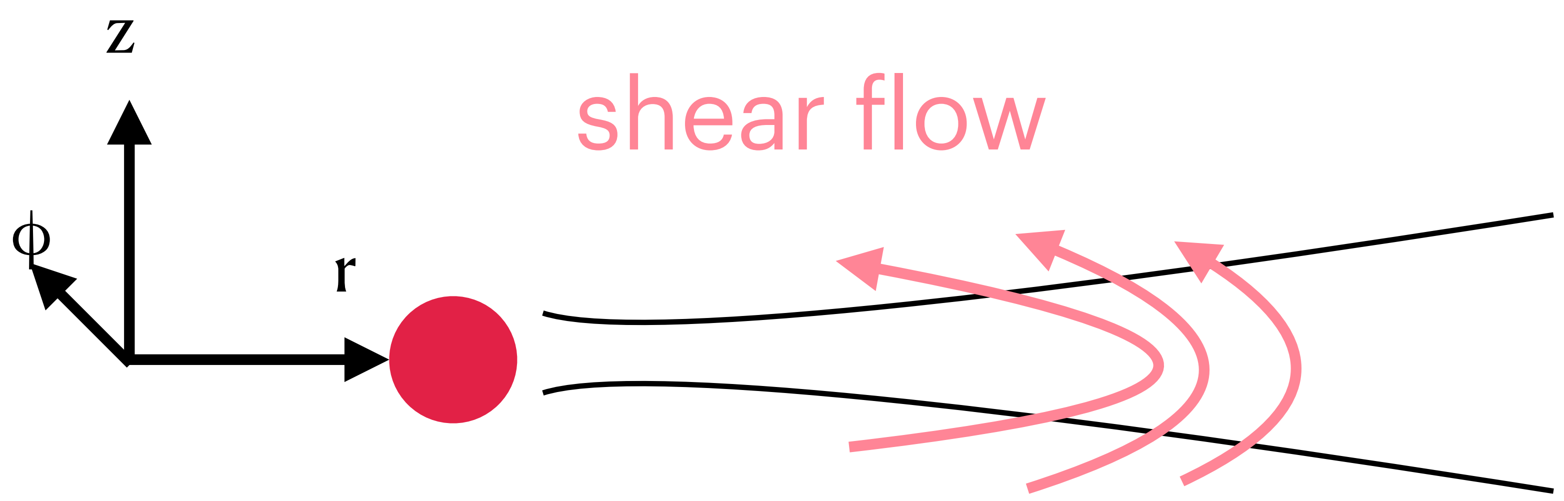
Reynolds stress and momentum transport (Li et al., ApJ , 551:874 (2001))

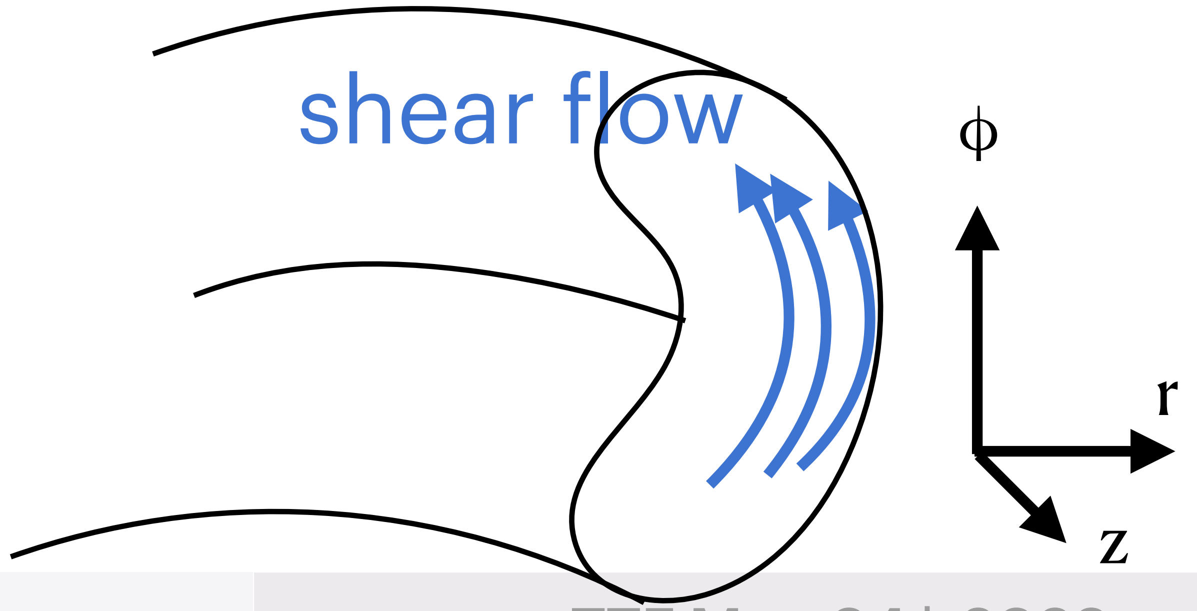
Key question:

Will the turbulence generate **zonal flow** in a compressible disk, via NL momentum transport (or gradient Reynolds stress)?

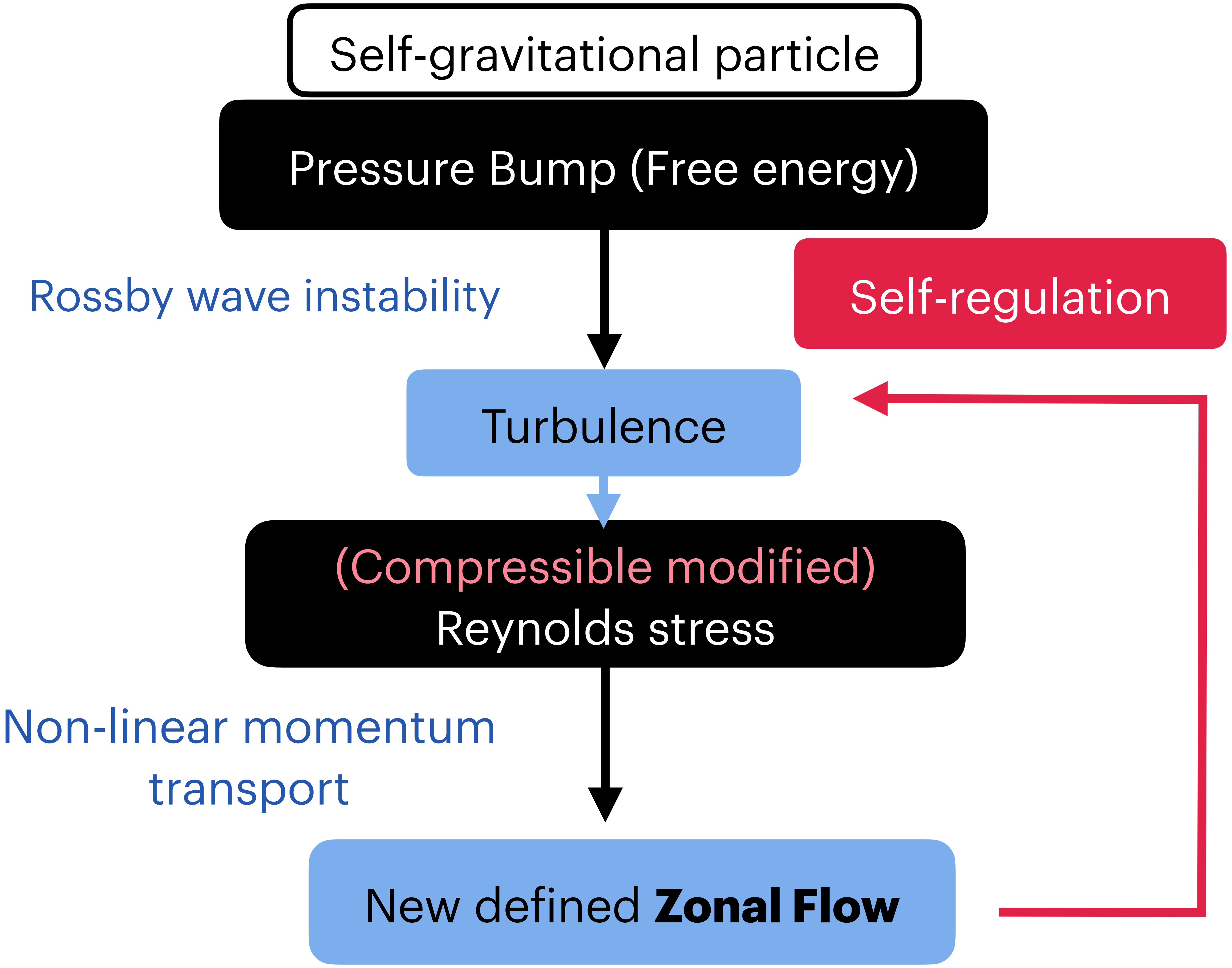
Introduction—Astrophysical disk and Edge Plasma Physics

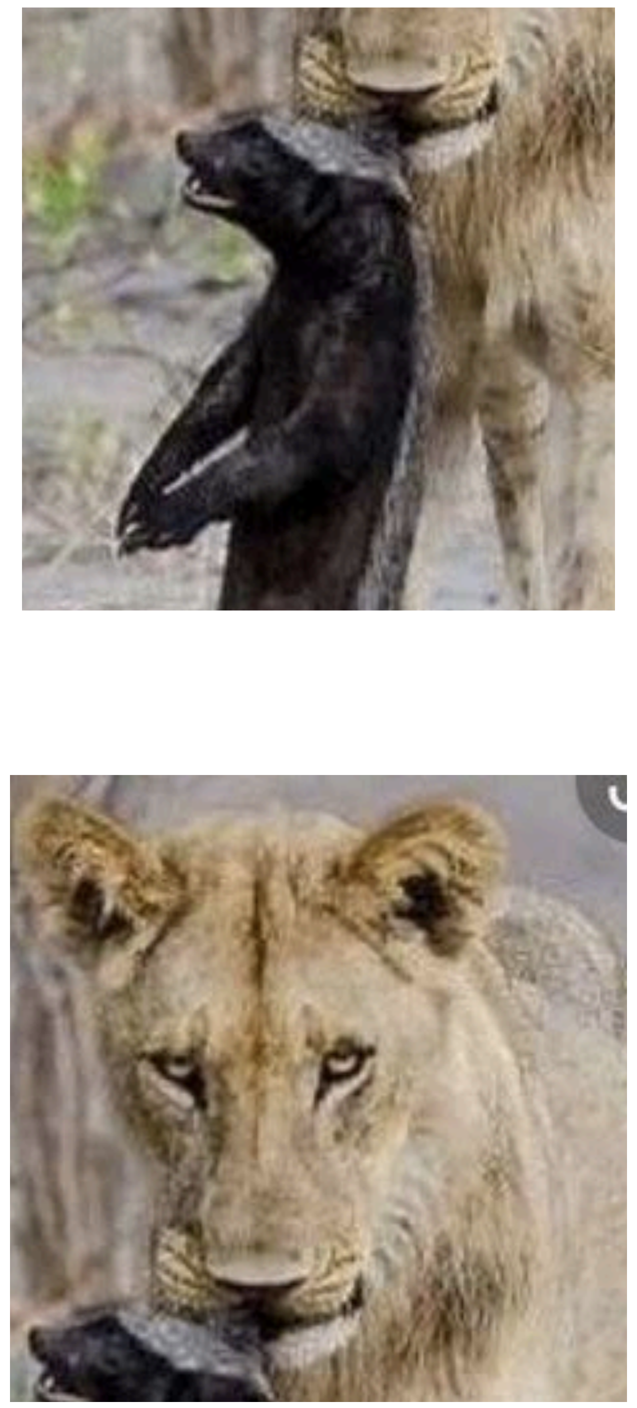
	Accretion Disk	Edge Plasma Physics
Linear wave	Rossby (-Alfven) wave	Drift wave
Pressure Bump (Free energy source)	Self-gravity: Trapped particles—> planet formation	Pedestal formation
B field	Weakly magnetized, compressible	Strong B field in Toroidal direction, Incompressible in $\perp$ direction
Conserved PV	(Compressible) PV = $\nabla \times \mathbf{u}/\rho$.	$PV = -\nabla^2\phi + n$
Shear flow	Keplerian Shear $\sqrt{\frac{GM}{r}}$ (toroidal).	ExB shear flow (poloidal)

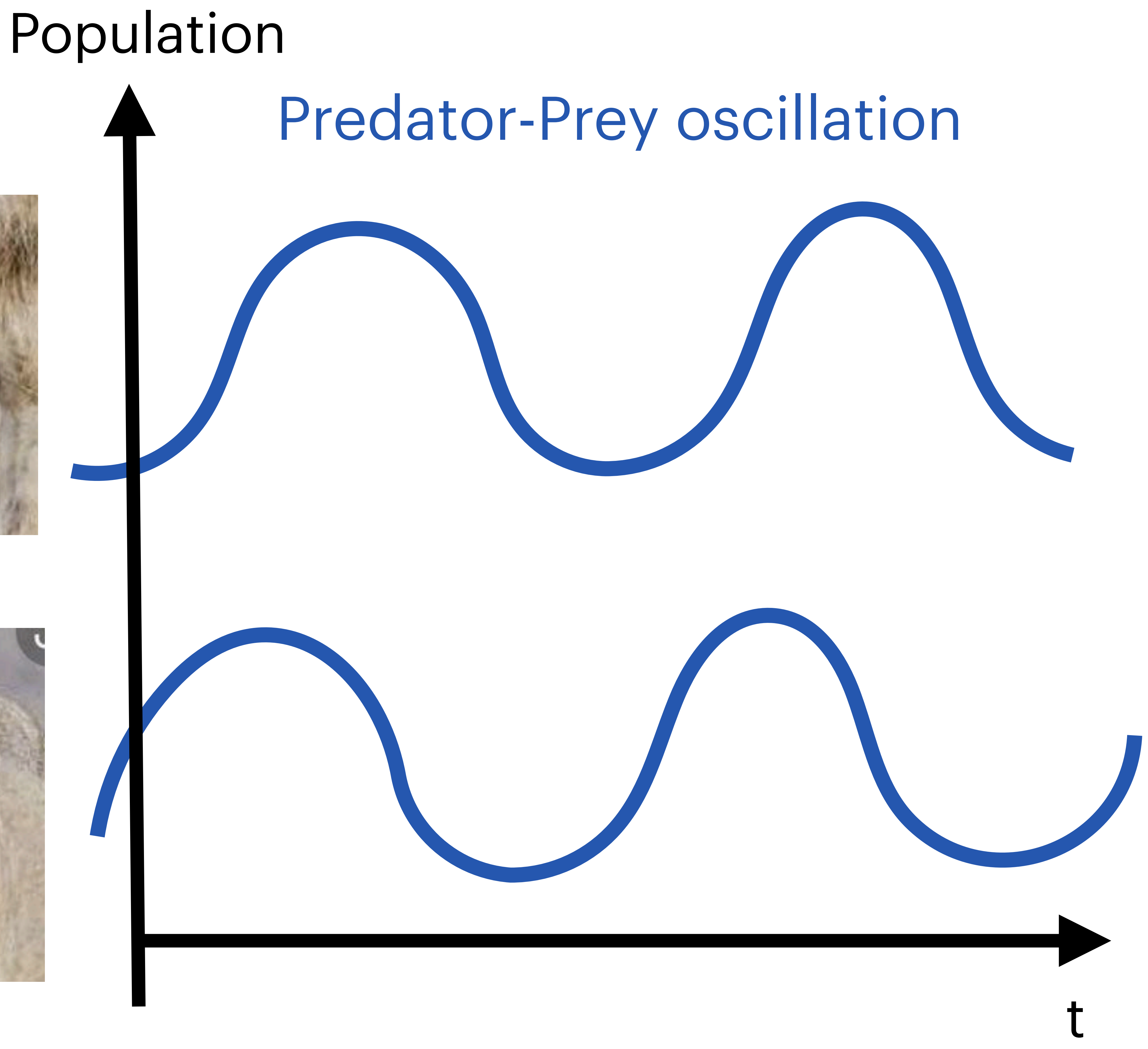




Predator–Prey/Limit Cycle Oscillation







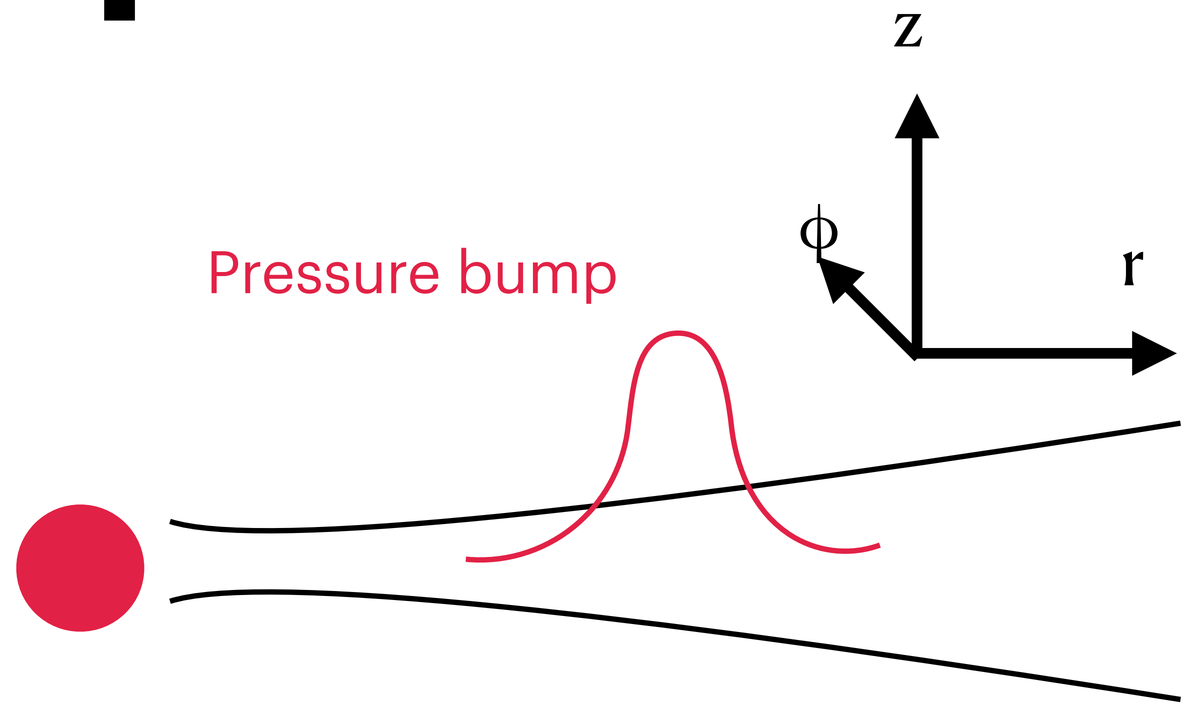
Is there a self-regulation mechanism that can explain the physics of a steady state in an accretion disk, and sustain the environment for planetary formation?

Find the Momentum Transport

- **Basic Equations** (cylindrical coordinate):

  1. Continuity eq.: $\frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla)\rho + \rho(\nabla \cdot \mathbf{u}) = 0$

  2. Momentum eq. in rotating fluid: $\begin{cases} \frac{\partial u_r}{\partial t} + (\mathbf{u} \cdot \nabla)u_r = -\frac{1}{\rho}\frac{\partial p}{\partial r} \\ \frac{\partial u_\phi}{\partial t} + (\mathbf{u} \cdot \nabla)u_\phi = 0, \\ \frac{\partial p}{\partial t} + (\mathbf{u} \cdot \nabla)p = -\gamma p(\nabla \cdot \mathbf{u}) \end{cases}$



$$c_s^2 = \frac{\gamma p}{\rho}$$

- **Compressible**: basic equations— Induction eq. + continuity equation

  => $\frac{D}{Dt}(\frac{\zeta}{\rho}) = 0$

- **Pressure bumps**: Condensation of mass due to self gravitation (or planet formation).

- **Shear background**—Pressure modified Keplerian Velocity

$$\cancel{\frac{\partial u_r}{\partial t}} + \cancel{u_r\frac{\partial u_r}{\partial r}} + \cancel{\frac{u_\phi}{r}\frac{\partial u_r}{\partial \phi}} - \frac{u_\phi^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + f_r$$

$$f_r = -\frac{GM}{r^2}$$

$$\langle u_\phi \rangle_{pmv}(r,t) = \sqrt{r\frac{\nabla_r \langle p \rangle(t)}{\langle \rho \rangle(t)} + \frac{GM}{r}}.$$

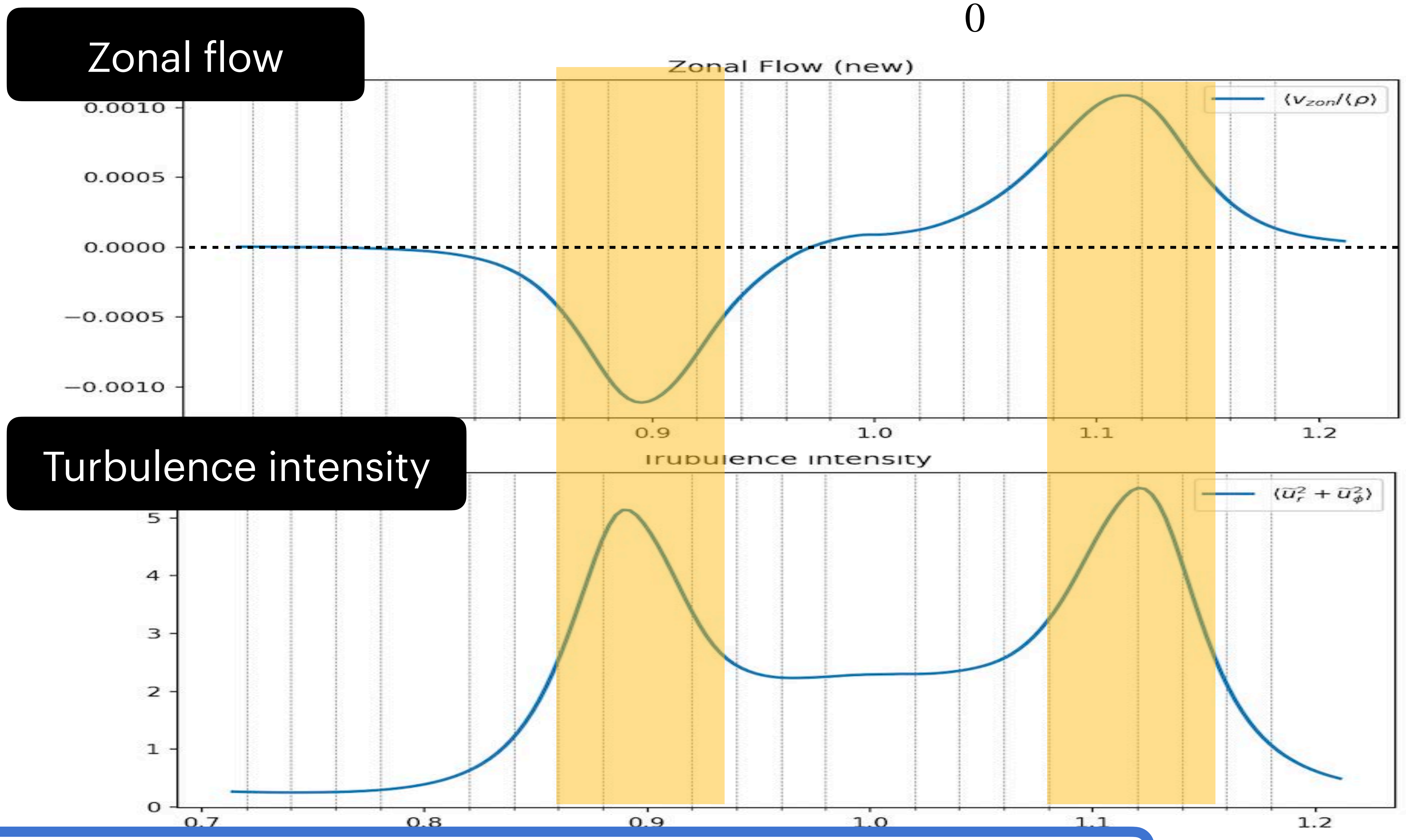
Weak Turbulence—QL approximation

$$\begin{cases} \dfrac{\partial u_r}{\partial t} + (\mathbf{u}\cdot\nabla)u_r = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial r} \\ \dfrac{\partial u_\phi}{\partial t} + (\mathbf{u}\cdot\nabla)u_\phi = 0, \\ \dfrac{\partial p}{\partial t} + (\mathbf{u}\cdot\nabla)p = -\gamma p(\nabla\cdot\mathbf{u}) \end{cases}$$

- $u_r = \langle u_r\rangle + \tilde{u}_r$
  $u_\phi = \langle u_\phi\rangle + \tilde{u}_\phi$
  $p = \langle p\rangle + \tilde{p}$

- Zonal flow definition:
  $\langle u_\phi\rangle = \langle u_\phi\rangle_{pmv}(r,t) + u_{ZF}$

$$\langle\rangle = \text{along azimuthal} = \frac{1}{2\pi}\int_0^{2\pi} d\phi$$



Zonal flow generated by the turbulence exist in an accretion disk!

Detailed Calculations

PV conservation and Zonal flow generation

- Derive from PV conservation:

$$\frac{D}{Dt}\left(\frac{\nabla\times\mathbf{u}}{\rho}\right) = \mathcal{O}(\nu, \mathbf{B}, \ldots),$$

$$\frac{\partial}{\partial t}\langle PV\rangle = \nabla_r(\langle PV\rangle\langle u_r\rangle) - \nabla_r\langle \tilde{u}_r\widetilde{PV}\rangle + \underbrace{\langle \widetilde{PV}\nabla\cdot\tilde{\mathbf{u}}\rangle}_{\text{odd, negligible}}$$

Compressibility

PV flux

<PV>1

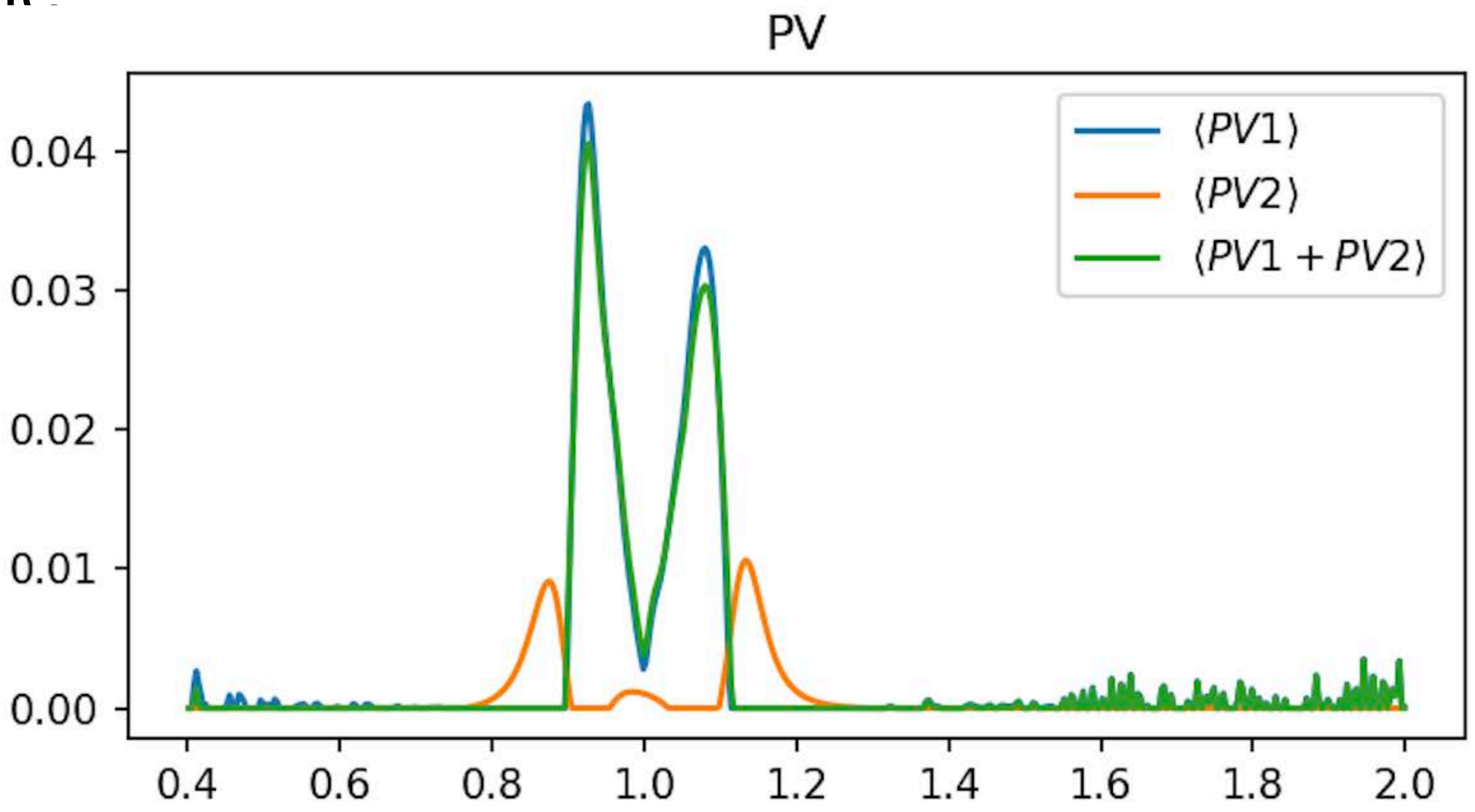
<PV2>

$$\text{LHS} = \frac{\partial}{\partial t}\left[\nabla\times\langle\frac{u_{zon}}{\rho}\rangle + \langle\mathbf{u}\times\nabla\frac{1}{\rho}\rangle\right]$$

$$\simeq \nabla\times\frac{\partial}{\partial t}\langle\frac{u_{zon}}{\rho}\rangle$$

Vorticity Freezing-in law

Incompressible: $\frac{D}{Dt}(\nabla\times\mathbf{u}) = 0$ → Compressible: $\frac{D}{Dt}\left(\frac{\nabla\times\mathbf{u}}{\rho}\right) = 0$



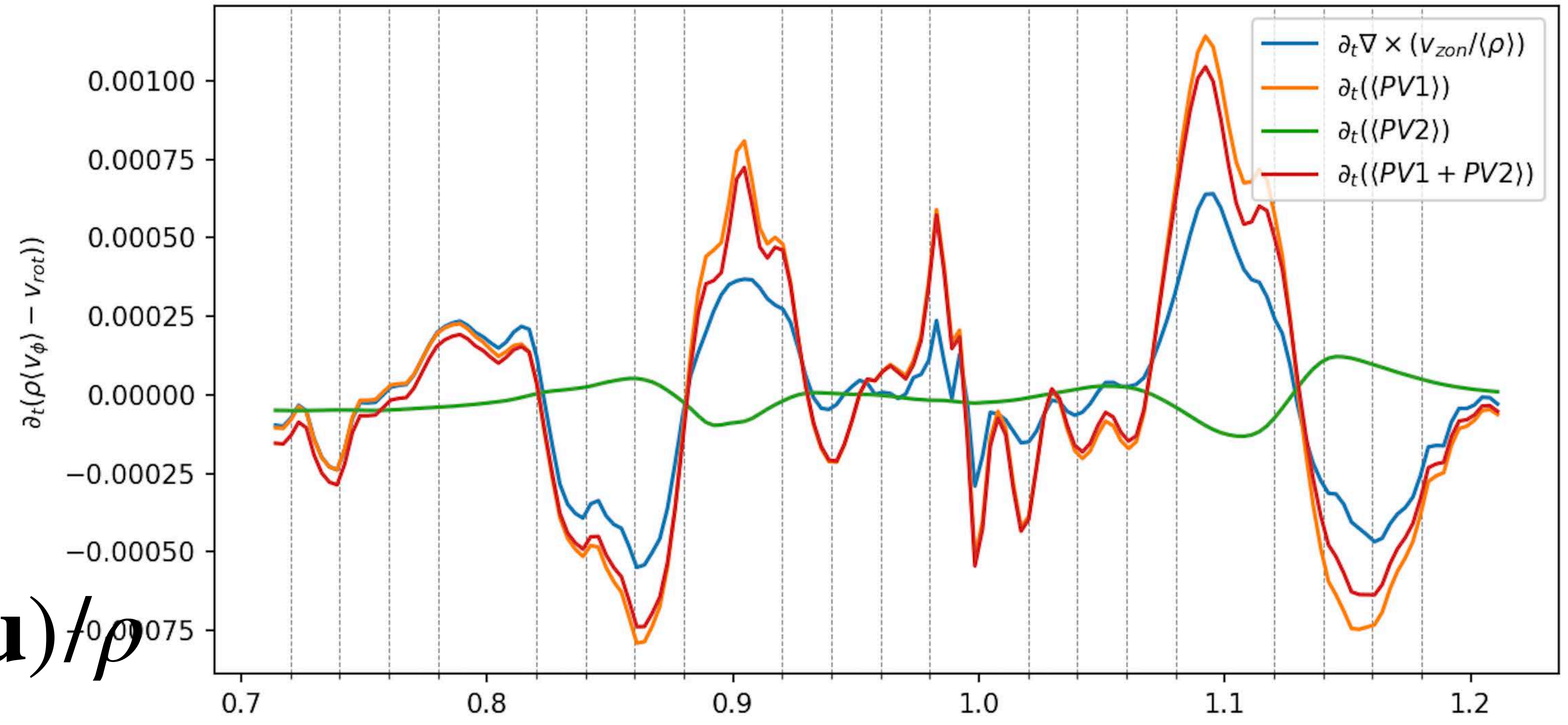
Detailed Calculations

Zonal Flow Evolution

- Decomposed BG mean flow and zonal flow:

$$\frac{\partial}{\partial t}\langle PV\rangle = \frac{\partial}{\partial t}\left(\nabla \times \frac{u_{pmv} + u_{zon}}{\rho}\right) + \frac{\partial}{\partial t}(u_{pmv} + u_{zon}) \times \nabla\frac{1}{\rho}$$

$$= -\nabla_r(\langle PV\rangle\langle u_r\rangle) - \nabla_r\langle\widetilde{u}_r\widetilde{PV}\rangle$$



- Derive from PV conservation: $PV \equiv (\nabla \times \mathbf{u})/\rho$

small

$$\frac{\partial}{\partial t}\left\langle\frac{u_{zon}}{\rho}\right\rangle \simeq -\frac{1}{\langle\rho\rangle}\langle\widetilde{u}_r\partial_r\widetilde{u}_\phi\rangle - \frac{1}{r\langle\rho\rangle}\langle\widetilde{u}_r\widetilde{u}_\phi\rangle + \frac{1}{\langle\rho\rangle^2}\langle\widetilde{u}_r\widetilde{\rho}\rangle\partial_r\langle u_{zon}\rangle + \frac{1}{r\langle\rho\rangle^2}\langle\widetilde{u}_r\widetilde{\rho}\rangle\langle u_{zon}\rangle$$

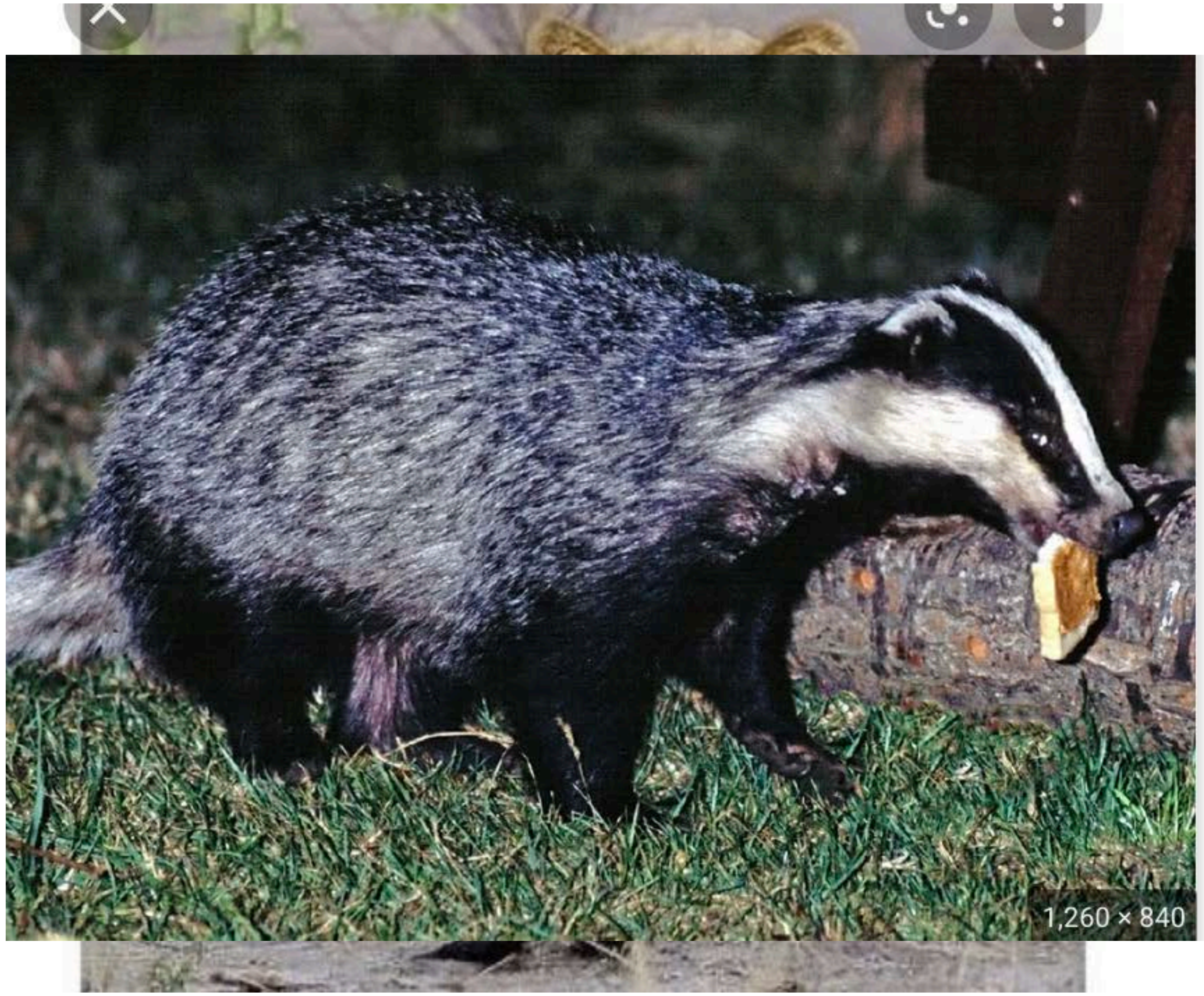
$$= -\langle\widetilde{u}_r\widetilde{PV}\rangle$$

$$-\int^r dr\left(\frac{\partial}{\partial t}u_{zon}\nabla\left(\frac{1}{\langle\rho\rangle}\right)\right)$$

THE analog "Reynolds stress" | geometric Reynolds stress | Mass transport | Geometric Mass transport

This can be viewed as the Compressible Taylor Identity.

What are new Cats/Rats?



- Compressibility:
  Mass density ρ must be play a role.

Incompressible: $\frac{D}{Dt}(\nabla \times v) = 0$ → Compressible: $\frac{D}{Dt}(\frac{\nabla \times v}{\rho}) = 0$

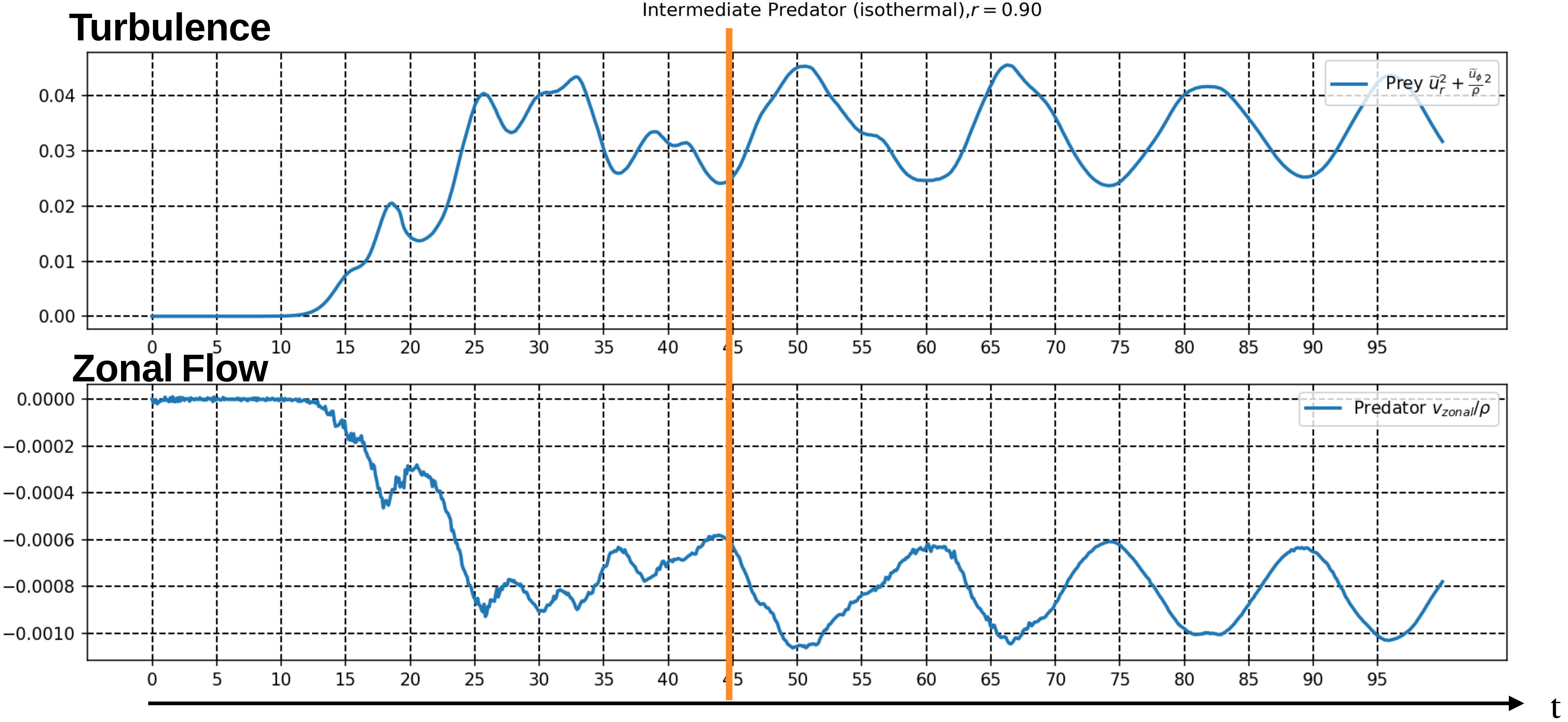
- Candidates:

new Predator: $\frac{u_{zon}}{\rho}$,

new Prey:

$|\widetilde{u}_r|^2 + |\widetilde{\frac{u_\phi}{\rho}}|^2$

PNP Cycle in Turbulence and Zonal Flow



Predator-Prey oscillations explain the turbulence saturation on a 2D pure fluid disk!

Phenomeno-logically

Simulation Results— Zonal/Mean flow generation

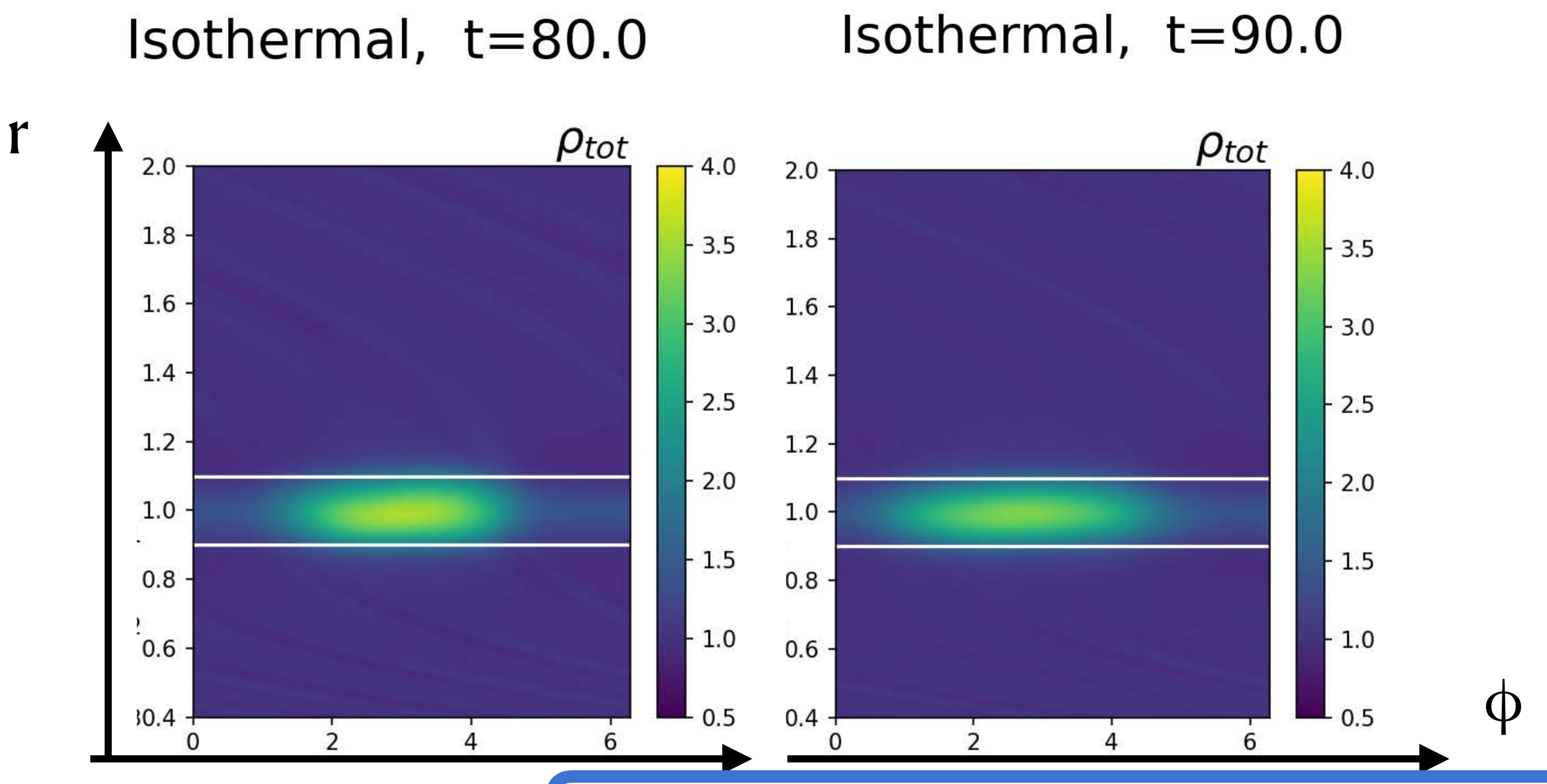
- From pressure-modified Keplerian velocity increase mean flow:

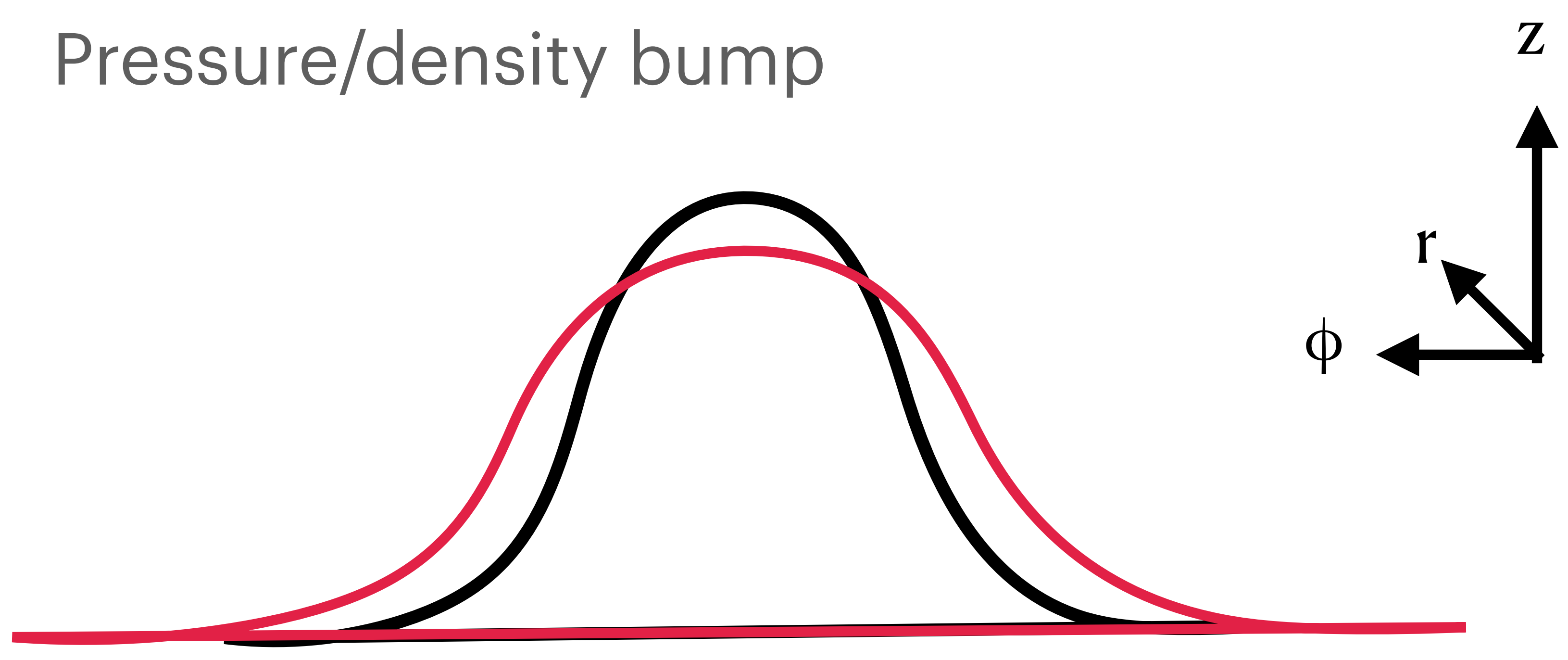
$$\text{mean flow}\langle u_\phi \rangle = \langle u_{\phi,tot}(r,t) - \langle u_\phi \rangle_{pmv}(r,t) \rangle$$

$$= \langle \frac{1}{2} v_k (\frac{c_s}{v_k})^2 r \frac{\partial}{\partial r} \left[\ln \frac{\rho(r,\phi,t)}{\langle \rho \rangle (r,t)} \right] \rangle .$$

$$= \quad \widetilde{\ln \rho(r,\phi,t)} \quad \propto \text{Mean flow}$$

Axis-symmetry → Asymmetry



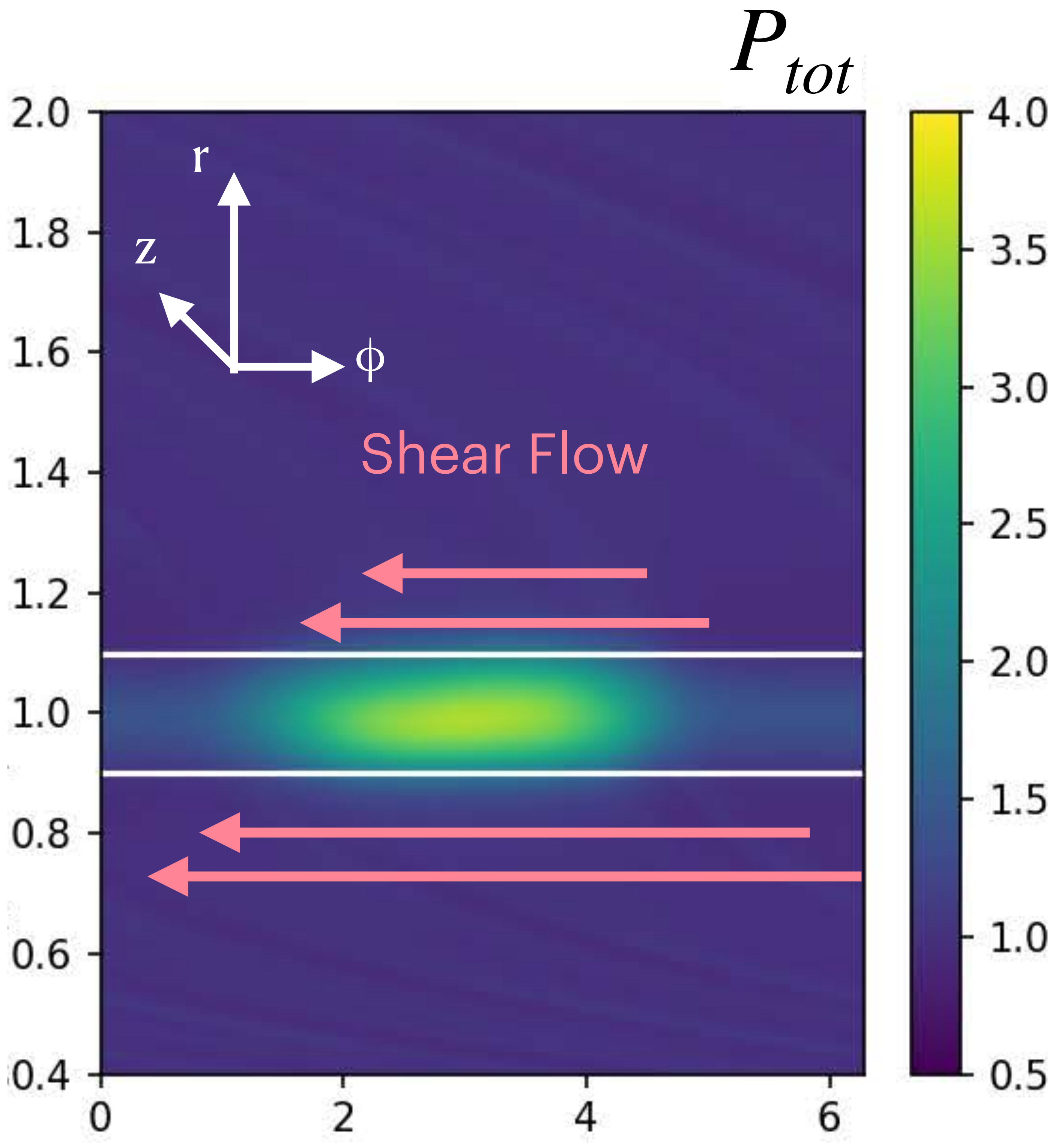


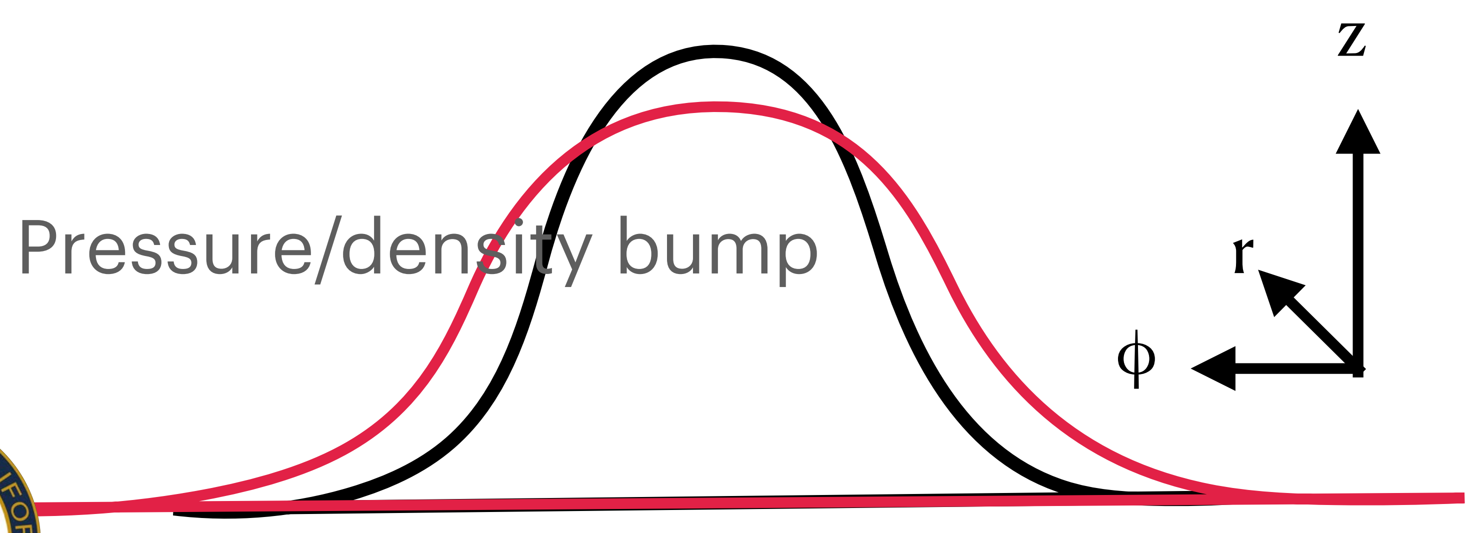
"Breathing" Blobs

Mean flow is proportional to the log density perturbation.

Phenomeno-
logically

Simulation Results—
Bump (Blob) on Shear Flow





Simulation Results: **Self-binding** (coherency) of the blob

- Critical Balance:

A. E. Gill (2005)

$$(\omega - kV_0)\widetilde{P}_{k,\omega} = i\tilde{u}_{r,k,\omega}\frac{\partial}{\partial r}[\langle P \rangle + \delta P]$$

$$\frac{\partial_r P}{V_0'} \propto [(V_{ph} - V_0)^{-1} V_0' \Delta r]$$

$$\text{shear}^2 \propto \partial_r P \rightarrow \text{shear}^2 \propto \frac{\beta + \delta\beta}{\Delta r_{blob}} \propto \mathcal{N}$$

Rossby parameter:

$$\beta \rightarrow \beta_{0,(r)} + \delta\beta$$

$$\mathcal{N} \equiv L_{0,P}/L_P \propto \frac{1}{\Delta r_{blob}}$$

More Related work: **Patrick Diamond**—> Today 2:15pm:
Blob/void on shear, Self-coherence

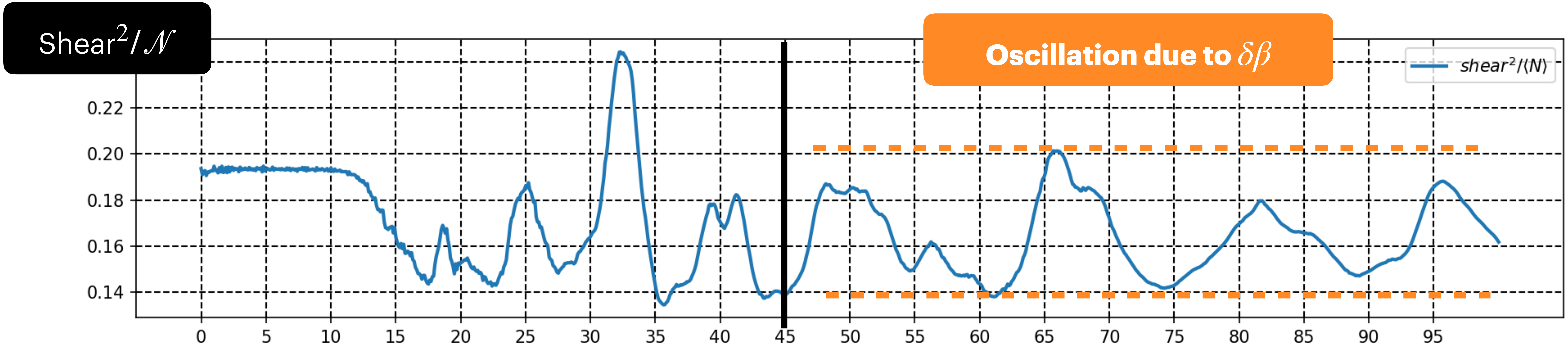
Simulation Results—
Bump (Blob) on Shear Flow

- Critical Balance:

$$\text{shear}^2 \propto \partial_r P \rightarrow \boxed{\frac{\text{shear}^2}{\mathcal{N}} \propto \beta + \delta\beta}$$

Rossby parameter:

$\beta \rightarrow \beta_{0,(r)} + \delta\beta$ (due to the pressure bump oscillation)



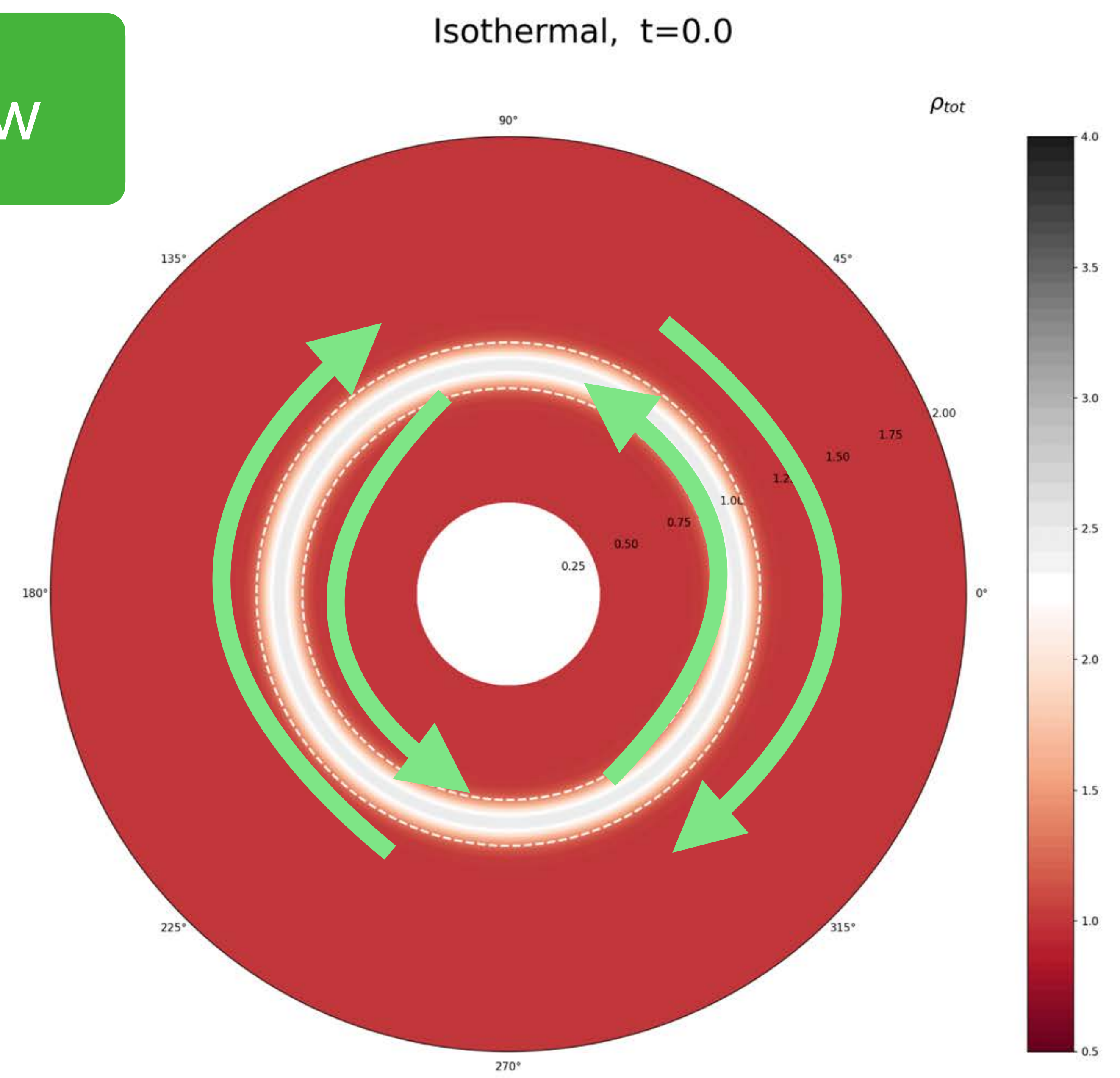
Self-binding (coherency) of the blob explains the pressure bump's length and its oscillation.

Limit Cycle in Turbulence and Zonal Flow

Vortices

Zonal flow constrain the properties of pressure bump (where planets embedded), sustaining the planet-forming region in a steady state!

Zonal flow



Takeaways

- Adding to Balbus & Hawley (1998), we now consider the pressure/density variation in transport, and found that in a compressible disk, $\widetilde{\rho}$ will play an important role.
- We derive the **compressible Taylor Identity**:

$$\langle \widetilde{u}_r \widetilde{PV} \rangle \simeq \frac{1}{\langle \rho \rangle} \langle \widetilde{u}_r \partial_r \widetilde{u}_\phi \rangle + \frac{1}{r \langle \rho \rangle} \langle \widetilde{u}_r \widetilde{u}_\phi \rangle - \frac{1}{\langle \rho \rangle^2} \langle \widetilde{u}_r \widetilde{\rho} \rangle \partial_r \langle u_{zon} \rangle - \frac{1}{r \langle \rho \rangle^2} \langle \widetilde{u}_r \widetilde{\rho} \rangle \langle u_{zon} \rangle$$

  - Momentum/Energy transport via Reynolds stress at edge plasma physics.
- We analytically explain the **self-binding mechanism** of a blob on a shear flow background.
  - Self-binding pressure → Gradient Relaxation/SOL Broadening
- We found the **Predator-Prey oscillation**. This explains the **turbulence/instability saturation** on an proto-planetary disk.
  - Experiments show the self-regulating mechanism—the predator-prey cycle in DIII-D (Schmitz et al.,PRL 108, 155002 (2012)).