Theory of mean E×B shear in a stochastic magnetic field for L-H transition: ambipolarity breaking and radial current

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Outline

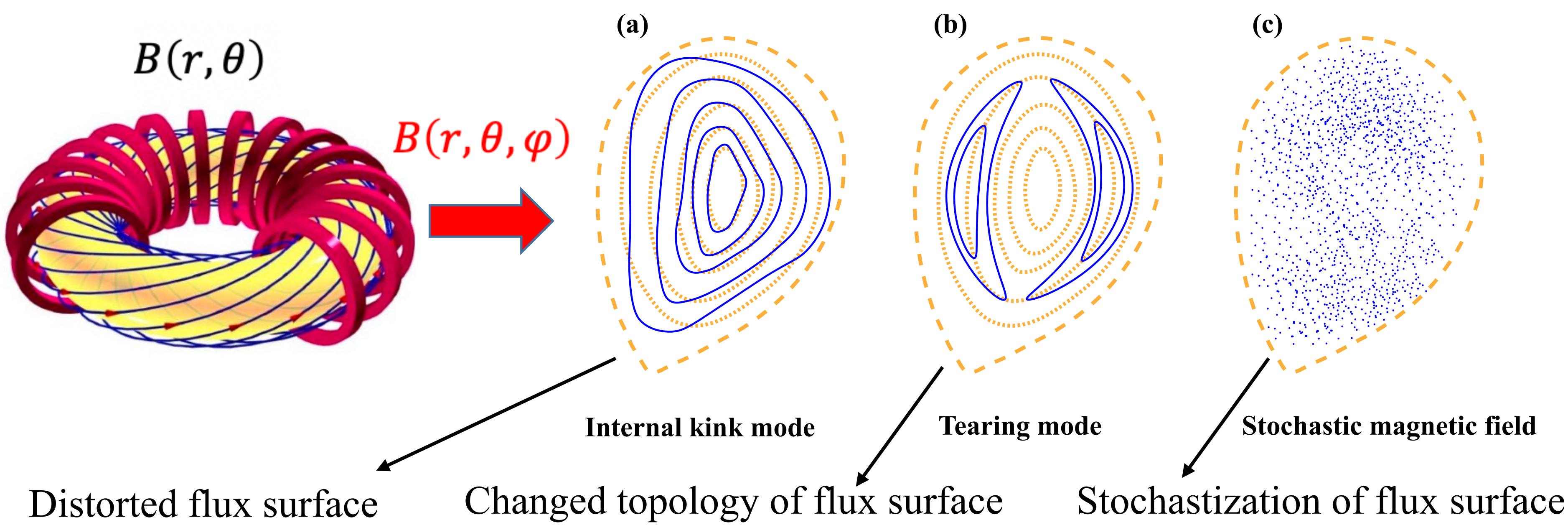
- Motivation and background
  - Why? → Interaction and co-existence of stochastic B field and turb.
  - Key issues
- Mean field model for turbulent transport induced by stochastic magnetic fields
  - How works? Stochastic field ⇒ radial current $\langle \boldsymbol{J_r} \rangle$
  - How calculate $\langle \boldsymbol{J_r} \rangle$? Ambipolarity breaking
  - Effects of stochastic field on
    - → particle transport via $\langle \tilde{b}_r J_\parallel \rangle$,
    - → $\langle V_\theta \rangle$ evolution via $\langle J_r \rangle B_t$
    - → $\langle V_\phi \rangle$ evolution via $\langle J_r \rangle B_\theta$
    - → also involve heat flux
  - Effects of $\langle E_r \rangle \leftrightarrow \langle \boldsymbol{J_r} \rangle$

⇒ $\langle E_r \rangle$ and shear

- Implications and conclusions

3D non-axisymmetric configuration

- 3D magnetic perturbations (MPs) like error field, toroidal field ripple, RMP, spontaneous MHD insta., is ubiquitous → break axisymmetry of ideal tokamaks → affect confinement → need study associated mechanism of confinement in 3D configuration
- Classification of magnetic configurations by 3D MPs

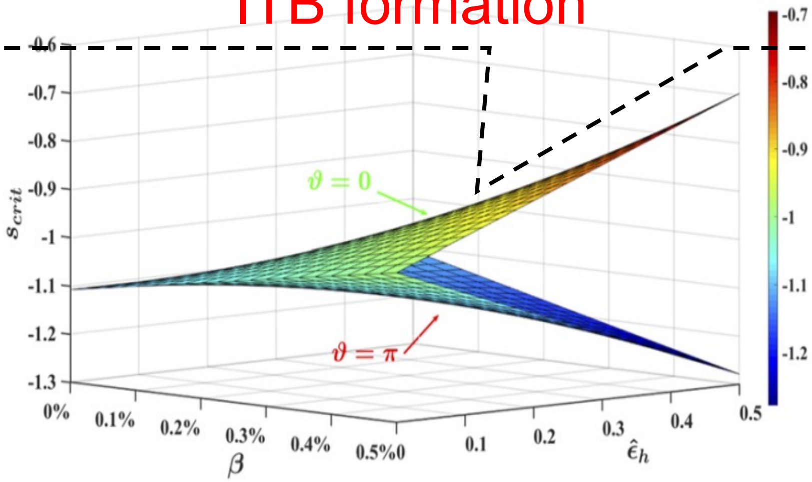


Results for 3D MPs with type (a), (b) configuration

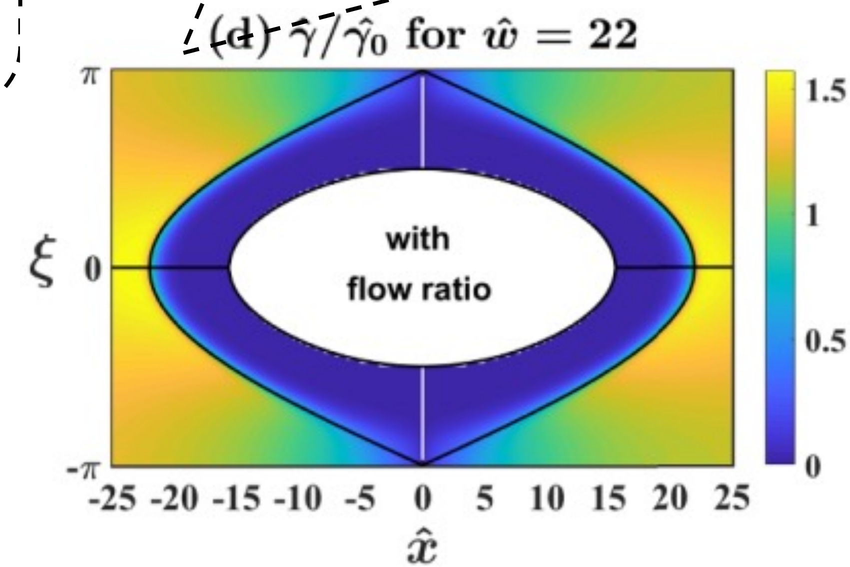
- 3D MPs' effects on micro-turbulence @ gyro-kinetic theory

Type (a): 3D MPs reduce the critical negative magnetic shear to completely stabilize CTEM tur.
→ reduce the needed power for ITB formation

Type (b): ITG turbulence is stabilized near the X point of the magnetic island



[Z Huang, W Guo and L Wang, NF 62 066044 (2022)] Cancelled Poster

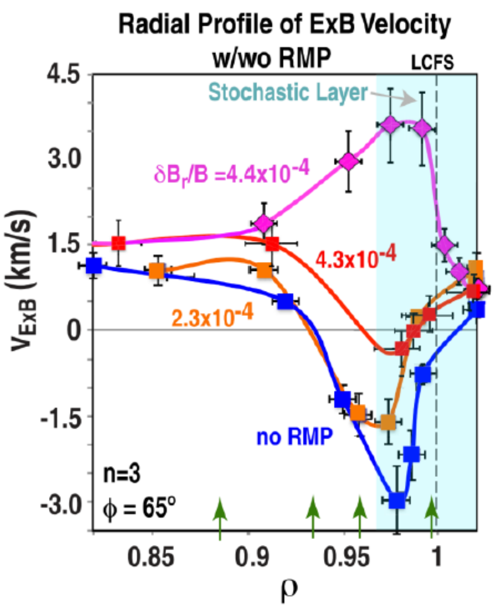


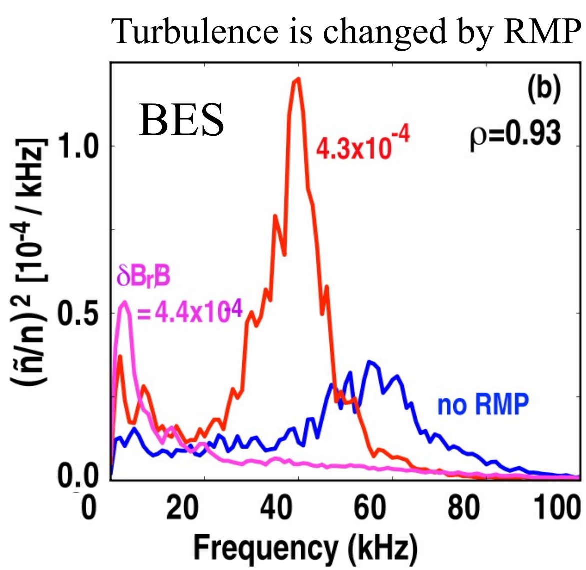
[G Zhang, W Guo and L Wang, PPCF 64 045006 (2022)]

Will focus on effects of stochastic field- type (c) configuration, in this talk

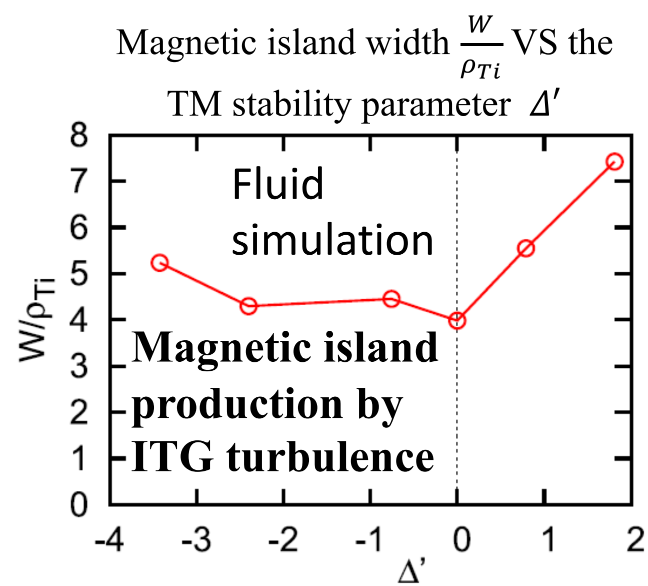
Stochastic magnetic field

- Stochastic field : chaos of magnetic field lines
- Important for boundary MHD control in fusion device
- Very fundamental problem:
  - ✓ **Interaction and co-existence** of stochastic magnetic field and turbulence, eg: RMP, island, stellarator ……





L. Schmitz et al, NF 2019



A Ishizawa et al, PPCF 2019

➡ Need theory for turbulent transport in stochastic B field!

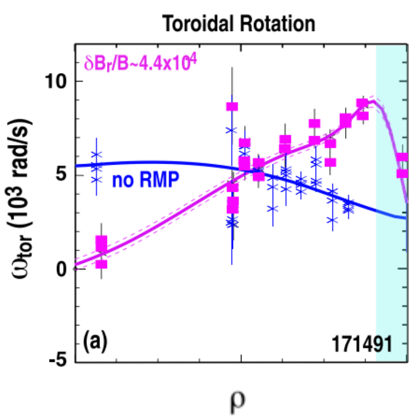
Effects of stochastic B fields in the literature

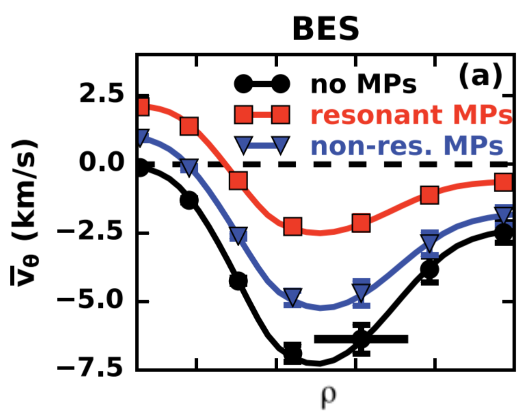
- From theory, stochastic B fields $\tilde{b}$ affects
  - Cast of thousands: electron heat transport[1]
  - Dephasing effect[2] $\rightarrow$ quenches poloidal Reynolds stress
  - Direct effect of stochastic field on turbulence[3]

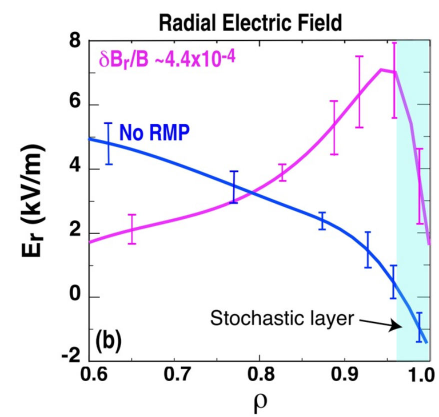
[1]Rechester, Rosenbluth PRL 1978
[2]C-C. Chen, PoP 2021
[3]M. Cao, PPCF 2022

- From experiments, RMP induces
  - Increase of toroidal (co)rotation[4], but reduce of mean poloidal velocity[5]
  - E_r well $\rightarrow$ E_r hill, and edge E_r shear layer sits in stochastic field region[4]

[4]L. Schmitz et al NF 2019
[5]Kriete, PoP 2020







- Point is that most $\tilde{b}$ theory is χ_e and electrons, but experiments demand that ion and flow physics be addressed

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    → also involve heat flux

    ⇒ $\langle E_r \rangle$ and shear
  - Effects of $\langle \boldsymbol{E_r} \rangle \leftrightarrow \langle \boldsymbol{J_r} \rangle$
- Implications and conclusions

Our goal – understand effects of stochastic field on $\langle E_r \rangle$

- More specific: how stochastic B-field affects $\langle v_E' \rangle$?

$$E_r = \frac{1}{enB}\frac{\partial}{\partial r}P_{\rm i} + v_\phi B_\theta - v_\theta B_\phi .$$

$\langle v_E' \rangle$ Heat, particles $\perp, \parallel$ flows $\rightarrow$ momentum

$\langle \boldsymbol{J_r} \rangle$ ↔️ $\langle \tilde{b}_r \tilde{b}_\theta \rangle$

- ✓ Study heat, particle and momentum transport to ascertain change of $\langle E_r \rangle$ due to stochastic B field in this work
- ✓ Goal is towards $\langle \boldsymbol{J_r} \rangle - \langle \boldsymbol{E_r} \rangle$ relation effective "Ohm's law"
- ✓ Stochastic B-field, which is externally excited but self-consistent within plasma, enters $\langle \boldsymbol{J_r} \rangle$

- How calculate $\langle \boldsymbol{J_r} \rangle$ induced by stochastic field ? And, what is its effect on turbulent transport? $\Longrightarrow V_E'$

Ambipolarity breaking $\Rightarrow \langle J_r \rangle$ induced by $\widetilde{\mathrm{B}}_r$ is a key

- Ambipolarity breaking due to stochastic field $\Rightarrow \langle J_r \rangle$

$$\langle J_r \rangle = \langle \vec{J}_\parallel \cdot \vec{e}_r \rangle = \frac{\langle \tilde{J}_\parallel \widetilde{\mathrm{B}}_r \rangle}{B} \qquad \langle J_\parallel \rangle = \langle J_{\parallel,e} \rangle + \langle J_{\parallel,i} \rangle$$

- From Ampere law:
  (Self- constraint)

$$\tilde{J}_\parallel = -\frac{c}{4\pi} \nabla^2 \tilde{A}_\parallel$$

Stochastic field produces currents in plasmas

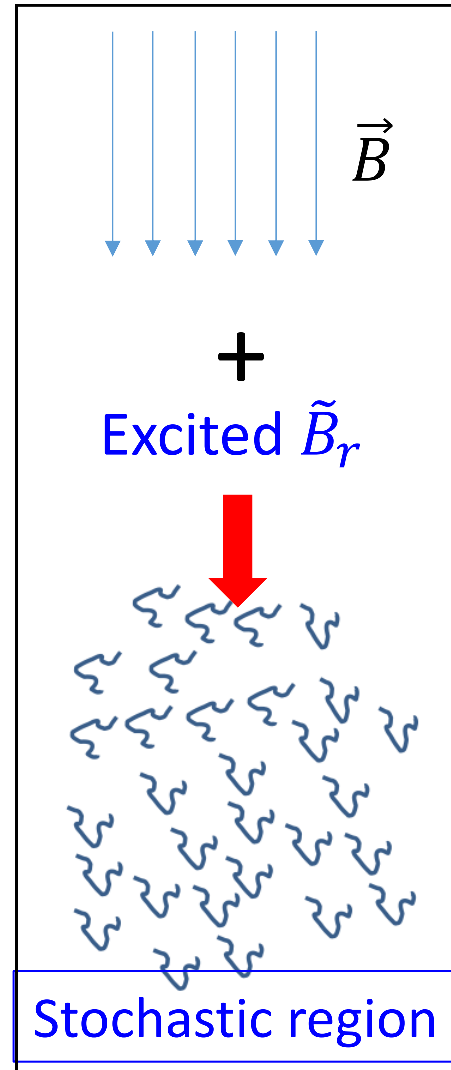
$$\langle J_r \rangle = \frac{\langle \tilde{J}_\parallel \tilde{B}_r \rangle}{B} = -\frac{c}{4\pi B} \left\langle \frac{\partial}{\partial y} \tilde{A}_\parallel \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \tilde{A}_\parallel \right\rangle$$

$$= -\frac{c}{4\pi B} \frac{\partial}{\partial x} \left\langle \left(\frac{\partial}{\partial x} \tilde{A}_\parallel \right) \left(\frac{\partial}{\partial y} \tilde{A}_\parallel \right) \right\rangle = \frac{c}{4\pi B} \frac{\partial}{\partial x} \langle \tilde{B}_x \tilde{B}_y \rangle$$

$$= \frac{cB}{4\pi} \frac{\partial}{\partial x} \langle \tilde{b}_x \tilde{b}_y \rangle \Rightarrow \frac{cB_0}{4\pi} \frac{\partial}{\partial r} \langle \tilde{b}_r \tilde{b}_\theta \rangle$$

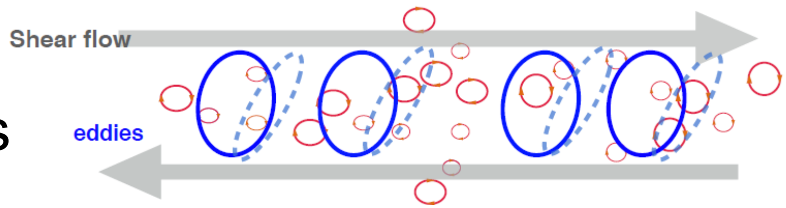
Maxwell Force

Note: $\langle J_r \rangle$ tracks momentum, not heat transport.
Recognized, but what set phases?



Phase in Maxwell stress

- Maxwell stress $\langle \tilde{B}_r \tilde{B}_\theta \rangle = \sum_k |\tilde{A}_k|^2 \langle k_r k_\theta \rangle$
  - ✓ phase set by k component correlation
  - ✓ $\boldsymbol{E}\times\boldsymbol{B}$ shear aligns phases, regardless of drive mechanisms



➢ Magnetic potential $\tilde{A}_k$ is tilted by developing $\boldsymbol{E}\times\boldsymbol{B}$ flow, and is scattered by fluctuations

$$\frac{\partial A}{\partial t} + V \cdot \nabla A = \mu \boldsymbol{J}$$

$$\frac{\partial A}{\partial t} + V_E' \frac{\partial A}{\partial y} + \tilde{V} \cdot \nabla A = \mu \boldsymbol{J}$$

Shear flow   Fluctuation scattering

$$k_r = k_r^{(0)} - k_\theta V_E' \tau_c$$

$$\tau_c : \begin{Bmatrix} shear \\ fluctuations \end{Bmatrix}$$

$k_r^{(0)}$ is the key difference with "test particle" calculation

➢ $k_r^0 k_\theta$ contribution appears to capture stochastic field effects on electrons, while the $k_\theta^2 \langle V_E \rangle' \tau_c$ bit is due mainly to ion effects.

Competition btw Reynolds and Maxwell stress

- If ignore $k_r^{(0)}$ due to focusing on ion dynamics, phases of Reynolds and Maxwell cross-phase align.

$$\langle \tilde{V}_\theta \tilde{V}_r \rangle - \langle \tilde{B}_r \tilde{B}_\theta \rangle = \sum_k \left(|\tilde{V}_k|^2 - |\tilde{B}_k|^2 \right) \langle V_E' \tau_c \rangle$$

Edge turbulence   Excited by RMP

with $\tau_c = \left(\frac{k_\theta^2 V_E'^2 D_T}{3}\right)^{-1/3}$, $\tilde{B}_r / B_0$ is around $10^{-4} \sim 10^{-3}$

✓ Stochastic field tends to **oppose** turbulent Reynolds stress due to the same phase

✓ $\boldsymbol{E}\times\boldsymbol{B}$ shear trends to align Reynolds stress of the turbulence with the Maxwell stress of the stochastic B field

Stochastic B-field affects electron density flux

- For electron density : $\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_e) = S_p$

with $\Gamma_e = -(D_{neo} + D_T)\frac{\partial n}{\partial r} + \Gamma_{e,stoch}$ $\longleftarrow$

- $D_{neo} = (m_e/m_i)^{1/2}\chi_{i,neo}$
- $D_T \sim bD_{GB}$ with b<1

$$S_p = \Gamma_a \frac{a - r + d_a}{L_{dep}^2}\exp\left(-\frac{(a + d_a - r)^2}{2L_{dep}^2}\right)$$

- The stochastic field can induce particle flux ($n_e = n_i$):

$$\Gamma_{e,stoch} = \underbrace{\frac{c}{4\pi eB}\langle \tilde{b}_r \nabla_\perp^2 \tilde{A}_\parallel \rangle}_{\langle \tilde{b}_r \tilde{J}_\parallel \rangle} + \underbrace{n\langle \tilde{V}_{\parallel,i} \tilde{b}_r \rangle}_{\langle \tilde{b}_r \tilde{J}_{\parallel,i} \rangle}$$

with

✓ $\frac{c}{4\pi eB}\langle \tilde{b}_r \nabla_\perp^2 \tilde{A}_\parallel \rangle = -\frac{cB}{4\pi e}\frac{\partial}{\partial r}\langle \tilde{b}_r \tilde{b}_\theta \rangle$ ✓ $\langle \tilde{b}_r \tilde{b}_\theta \rangle$ phasing via V_E' tilt.

✓ $n\langle \tilde{V}_{\parallel,i} \tilde{b}_r \rangle$: parallel ion flow along tilted field lines (hybrid)

$\boxed{\langle \tilde{b}_r \delta V_\parallel \rangle} \approx -D_{st}\partial \langle P \rangle / \partial r$

$$\boxed{D_{ST} = \sum_k c_s^2 |b_{r,k}|^2 / k_\perp^2 D_T}$$

Stochastic B-field affects $\langle V_\theta \rangle$

- Poloidal momentum balance

Turbulence Reynold stress

Maxwell stress of stochastic field perturbation

$B_\phi \langle J_r \rangle$

$$\frac{\partial \langle V_\theta \rangle}{\partial t} = -\mu(\langle V_\theta \rangle - V_{\theta,neo}) - \frac{\partial}{\partial r}\left(\langle \tilde{V}_\theta \tilde{V}_r \rangle - \frac{1}{4\pi\rho}\langle \tilde{B}_r \tilde{B}_\theta \rangle\right)$$

- For SS: $\langle V_\theta \rangle = V_{\theta,neo} - \frac{1}{\mu}\frac{\partial}{\partial r}\left(\langle \tilde{V}_\theta \tilde{V}_r \rangle - \frac{1}{4\pi\rho}\langle \tilde{B}_r \tilde{B}_\theta \rangle\right)$

$$= V_{\theta,neo} - \frac{1}{\mu}\frac{\partial}{\partial r}\left(\frac{1}{B^2}\tau_c V_E' \frac{I}{1+\alpha V_E'^2} - \frac{B^2}{4\pi\rho}\tau_c' V_E' |\tilde{b}_r|^2\right)$$

with $\mu = \mu_{00}(1 + \frac{\nu_{CX}}{\nu_{ii}})\nu_{ii} q^2 R^2$ $\quad V_{\theta,neo} \approx -1.17\frac{\partial T_i}{\partial r}$ $\quad \tau_c' = \tau_c$

- V_E' phasing via tilt tends to align turbulence and stochastic B-field, which counteracts the spin-up of $\langle V_\theta \rangle$.
- $\frac{\partial}{\partial r}|\tilde{b}_r|^2$, i.e., profile of stochastic enters → introduce stochastic layer width as novel scale

Stochastic B-field affects $\langle V_\phi \rangle$

- For V_ϕ: $\frac{\partial \langle V_\phi \rangle}{\partial t} + \nabla \cdot \langle \tilde{V}_r \tilde{V}_\phi \rangle = \frac{1}{\rho c} \langle J_r \rangle B_\theta + S_M$

$$\langle \tilde{V}_r \tilde{V}_\phi \rangle = -\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle, \quad \chi_\phi = \chi_T = \frac{\rho_s^2 c_s}{L_T}, \quad S_M = S_a \exp\left(-\frac{r^2}{2L_{M,dep}^2}\right)$$

Only consider diffusive term. G.B. Momentum source tracks heat (from core).

$$\Rightarrow \frac{\partial \langle V_\phi \rangle}{\partial t} = \frac{\partial}{\partial r}\left(\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle\right) + \frac{1}{4\pi\rho} \frac{B_\theta}{B} \frac{\partial}{\partial r} \langle \tilde{B}_r \tilde{B}_\theta \rangle + S_M$$

- For SS: $\frac{\partial}{\partial r}\left(\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle\right) = -\frac{V_{Ti}^2}{\beta} \frac{B_\theta}{B} \frac{\partial}{\partial r} \langle \tilde{b}_r \tilde{b}_\theta \rangle - S_M$

Stochasticity affects edge toroidal velocity, shear

$$\boxed{\frac{\partial}{\partial r} \langle V_\phi \rangle|_{r_{sep}} = -\frac{1}{\chi_\phi} \int_0^{r_{sep}} S_M dr - \frac{V_{Ti}^2}{\beta \chi_\phi} \frac{B_\theta}{B} \langle \tilde{b}_r \tilde{b}_\theta \rangle|_{r_{sep}}}$$

$B_\theta \langle J_r \rangle$

Integrated external torque

with $\langle \tilde{b}_r \tilde{b}_\theta \rangle = V_E' \tau_c' |\tilde{b}_r|^2$

- ✓ Force through radial current across separatrix.
- ✓ Shear affected by stochasticity.

Ion heat flux with stochastic field

- Heat flux induced by stochastic field :

$$Q_i = -(\chi_{i,neo} + \chi_{i,T})\nabla T_i + Q_{i,stoch}$$

- The stochastic field affects ion heat flux

• $\chi_{i,neo} = \varepsilon^{-3/2} q^2 \rho_s^2 \nu_{ii}$

• $\nu_{ii} = \frac{n_0 Z^4 e^4 \ln \Lambda}{\sqrt{3} 6\pi \varepsilon_0^2 m_i^{1/2} T_{i0}^{3/2}}$

• $\chi_{i,T} = \left(\frac{C_s^2 \tau_c}{1+\alpha V_E'^2}\right) * I$

$\sim \chi_{GB} * I$

$$Q_{i,stoch} = \int V_\parallel \langle \tilde{B}_r \delta f \rangle (V_\parallel^2 + V_\perp^2) = -\frac{\partial \langle T_i \rangle}{\partial r} \sqrt{\chi_{\parallel,i} \chi_{\perp,i}} \langle \tilde{b}_r^2 \rangle l_{ac} k_\perp$$

$$\propto -v_{th,i} D_{M,\,eff} \frac{\partial \langle T_i \rangle}{\partial r}$$

✓ Important as threshold power is directly related to heat flux.

✓ Power uptake determines turbulence and Reynolds force

Towards an Ohm's law for $\langle E_r \rangle$ and $\langle J_r \rangle$

- From Ohm's law, $\langle E_r \rangle$ and $\langle J_r \rangle$ are related :

$$\boxed{E_r = \frac{1}{enB}\frac{\partial}{\partial r}P_i + v_\phi B_\theta - v_\theta B_\phi.}$$

$$\langle \tilde{J}_\parallel \tilde{B}_r \rangle \rightarrow \langle J_r \rangle \begin{cases} n_e \text{ via } \langle \tilde{b}_r J_\parallel \rangle \\ \langle V_\theta \rangle \text{ via } \langle J_r \rangle B_t \\ \langle V_\phi \rangle \text{ via } \langle J_r \rangle B_\theta \end{cases}$$

- Elements for $\boldsymbol{E}\times\boldsymbol{B}$ shear:

✓ $n\frac{\partial T_i}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rQ_i) = S_H$

Ion temperature

✓ $\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\underbrace{\Gamma_e}_{\langle J_r \rangle}) = S_p$

Electron density

✓ $\langle V_\theta \rangle = V_{\theta,neo} + \frac{1}{\mu}\frac{\partial}{\partial r}\left(\frac{1}{B^2}\tau_c V_E' \frac{I}{1+\alpha V_E'^2} - \frac{B^2}{4\pi\rho}\tau_c' V_E' |\tilde{b}_r|^2\right)$ Poloidal flow

✓ $\frac{\partial}{\partial r}\langle V_\phi \rangle|_{r_{sep}} = -\frac{1}{\chi_\phi}\int_0^{r_{sep}} S_M dr - \frac{V_{Ti}^2}{\beta\chi_\phi}\frac{B_\theta}{B} V_E' \tau_c' |\tilde{b}_r|^2|_{r_{sep}}$ Toroidal flow

- $$\begin{aligned} \langle V_E \rangle' &= \frac{1}{eB}\frac{\partial}{\partial r}\langle \nabla P_i/n \rangle - \frac{\partial}{\partial r}\langle V_\theta \rangle + \frac{B_\theta}{B}\frac{\partial}{\partial r}\langle V_\phi \rangle \\ &= \frac{1}{eB}\frac{\partial}{\partial r}\langle \nabla P_i/n \rangle - \frac{\partial}{\partial r}\left[V_{\theta,neo} + \frac{1}{\mu}\frac{\partial}{\partial r}\left(\frac{1}{B^2}\tau_c V_E' \frac{I}{1+\alpha V_E'^2} - \frac{B^2}{4\pi\rho}\tau_c' V_E' |\tilde{b}_r|^2\right)\right] \\ &\quad + \frac{B_\theta}{B}\left[-\frac{1}{\chi_\phi}\int_0^{r_{sep}} S_M dr - \frac{V_{Ti}^2}{\beta\chi_\phi}\frac{B_\theta}{B} V_E' \tau_c' |\tilde{b}_r|^2|_{r_{sep}}\right] \end{aligned}$$

Five radial scales in this problem

- Multi-scale problem

Scale	Physics	Impact
L_n, L_T	Profile gradient	Drive of turbulence
u'/u	Flow damping profile scale	Rotation shear, $\langle V_E \rangle'$
$\ell_{env,\phi}$	Drift wave intensity	Reynolds stress drive
$\ell_{env,b}$	Stochastic field envelope scale	Magnetic stress scale
k_r	Stochastic field radial wavenumber	Magnetic stress phase

Conclusions

- The present conclusions:
  - ✓ Ambipolarity breaking $\Rightarrow \langle \tilde{b}_r \tilde{b}_\theta \rangle$, contribute to $\langle \boldsymbol{J_r} \rangle$
  - ✓ Both amplitude and profile of $|b_r|^2$ matter.
  - ✓ V_E' phasing $\Rightarrow$ stochastic $\langle \tilde{b}_r \tilde{b}_\theta \rangle$ opposes turbulence $\langle \tilde{V}_r \tilde{V}_\theta \rangle$, phase linked
  - ✓ Intrinsic toroidal torque, so that reversal or spin-up of edge $\langle V_\phi \rangle$ occur with RMP, $\langle \tilde{b}_r \tilde{b}_\theta \rangle$ enters edge $\langle V_\phi \rangle$
  - ✓ $|b_r|^2$ can modify T_i and n_e profiles
- Related to experiments: understand the relationship between the RMP effects on power threshold and micro-physics? RMP effects on evolution of the shear layer, LCO…

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Looking forward to your suggestions and comments.

Appreciate remote presentation!