Persistence and Evolution of "Staircase" Profiles

(Relevant to near-marginal systems)

F.R. Ramirez & P.H. Diamond

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Outline

1) **Background**

2) **Fixed Cellular Array (FCA) Problem**

3) **Relaxing FCA with Fluctuating Vortex Array**

4) **Passive Scalar Dynamics**

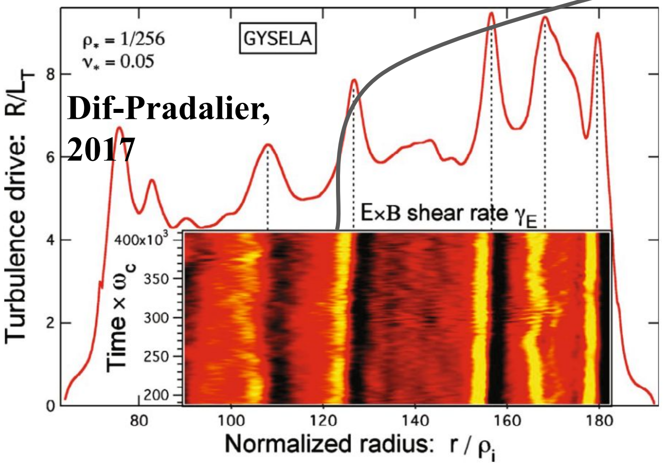
   * Summary of results

5) **Active Scalar Dynamics**

   * Ongoing work

$E \times B$ Staircase

***E* × *B* <u>staircase</u> current subject in M.F.E**



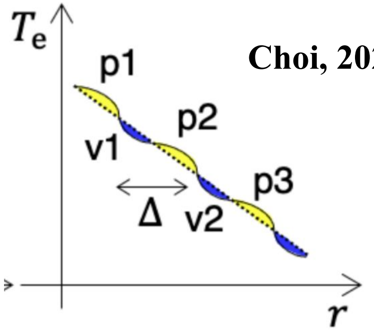
Dif-Pradalier, 2017

Yellow and black colors are a rapid transition of the direction of flows around peaks in turbulence drive.

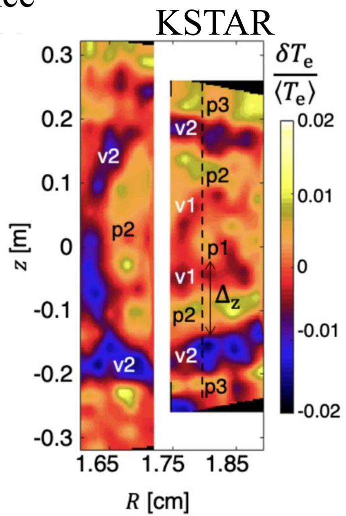
<u>Context</u>: Flat spots of high transport and nearly vertical layers acting as mini-barriers coexist. In plasmas, avalanches happen in flat spots and shear layers due to zonal flows occur in the areas of mini-barriers.

Suggested ideas:

- ***E* × *B*** shear feedback, predator-prey
  - Zonal flows predator and turbulence intensity prey
- Jams (time-delay between temperature modulations and local heat flux)



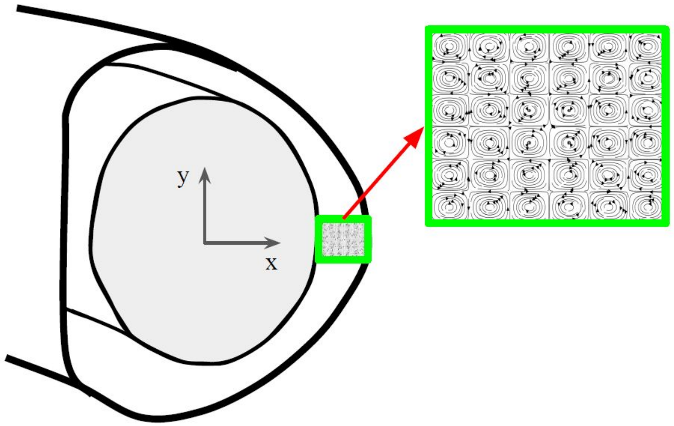
Choi, 2022



Jet like patterns in δT/<T> image correspond to staircase corrugations

Some Questions

- How does staircase beat homogenization?
- Is the staircase a meta-stable state?
- What is the minimal set of scales to recover layering?



Fixed Cellular Array Problem

(another way to get a Staircase)

<u>But</u>… is there an even **simpler** physical mechanism that can produce **layering?**
Answer: Yes (e.g., pattern of cells)

$$Pe = \frac{\tau_D}{\tau_H}$$

Fixed Cellular Array

Consider a **general** case of a system of eddies not overlapping but tangent → **Staircase**

Transport? Answer: Deff ~ D $Pe^{1/2}$ {**Not a simple addition of process!**}

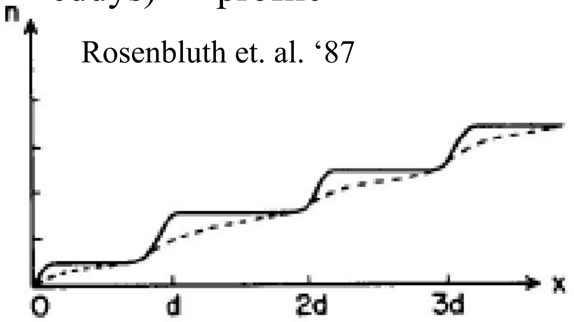
→ Two time rates: v / ℓ, D / ℓ²

→ Pe = v ℓ / D >> 1

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D \nabla^2 n,$$

Profile?

Consider concentration of injected dye (passive scalar transport in eddys) → profile

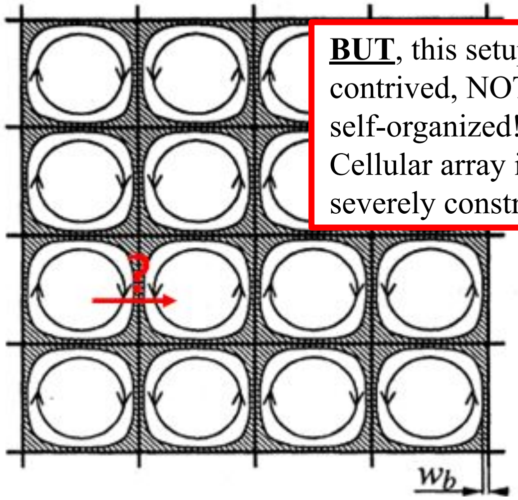


→ Layering!

→ **Simple** consequence of **two rates**

"Steep transitions in the density exist between each cell."

Relevant to key question of "near marginal stability"



BUT, this setup is contrived, NOT self-organized!!! Cellular array is severely constrained!

Staircase arises in an array of stationary eddies!

Important:

- **Staircase** arises in stationary array of passive eddies (Note that there is no FEEDBACK)
- Global transport hybrid:
  - → fast rotation in cell
  - → slow diffusion in boundary layer
- Irreversibility localized to inter-cell boundary.

What about the dynamics of a **less constrained** cell array (i.e., vortex array with fluctuations) ?

Relaxing Fixed Cellular Array with Fluctuating Vortex Array

Consider a Broader Approach

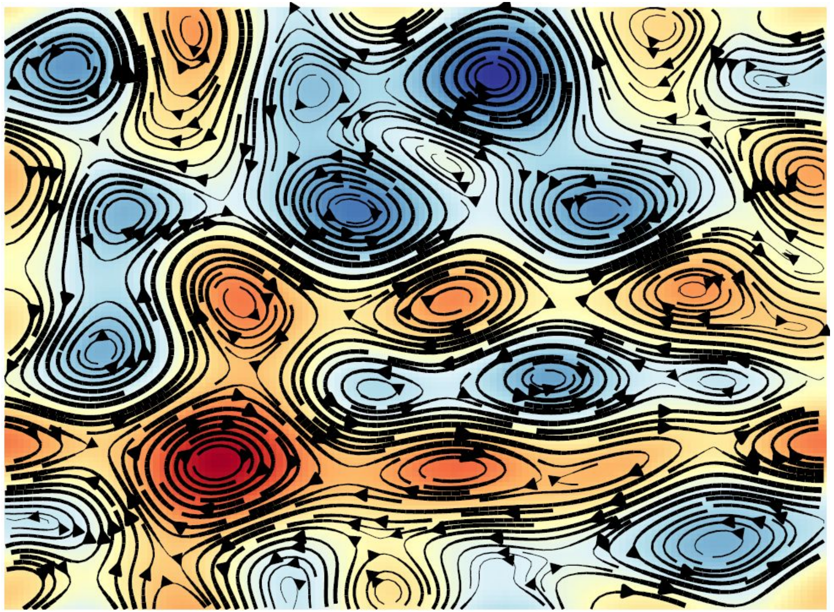
- We want to study a much more **general** and **less constrained** version of the cell array.
  - Consider a vortex array with fluctuations; jitters.
- How **resilient** is the staircase in the presence of these small variations to a fixed vortex array?

In the process of studying the **resilience** of the staircase, we aim to answer the following:

1. What happens to interspersed regions of strong scalar concentration mixing as cells relax? What about general cell interactions/behavior?
2. What is the behavior of the scalar trajectory through the vortex array?
3. How does the increase of scattering in the vortex array affect the transport of scalar concentration?

To answer these questions, we use the idea of a **Melting Vortex Crystal**…

Example of **less constrained** cell array



Fluctuating Vortex Array

Why are we doing this? We know that a system with two disparate time scales forms a staircase!

- Now consider fluctuations… → Will staircase survive?

Vortex array is an alternative way to view convection cells!

→ We begin with the 2D NS equation that can be written in nondimensional form (Perlekar and Pandit 2010),

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla}\right)\omega = \frac{1}{\Omega}\nabla^2\omega + F_\omega - \alpha\omega, \qquad \nabla^2\psi = \omega.$$

→ The "vortex array" is simply the array of cells and "fluctuation" is related to turbulence induced variability in the structure. The fluctuating vortex array (FVA) allows us to study a **less constrained** version of the array! **Improved model of cells near marginality.**

→ The fluctuating flow structure is created by **slowly increasing the Reynolds number** in the NS equation

$$\Omega = \frac{\tau_\nu}{\tau_H}$$

→ By increasing the Reynolds number this modifies the forcing and drag term, thus, **scattering** the vortex array. The **resilience** of the staircase is studied by **increasing disorder** in the vortex crystal through F_ω

$$F_\omega \equiv -n^3\left[\cos(nx) + \cos(ny)\right]/\Omega$$

The streamfunction, ψ, at different evolutionary stages of the "fluctuating" vortex array is inserted into the passive scalar equation to study the resilience of the staircase structure.

Comparison of Vortex Array model to Drift-wave Turbulence in fusion devices

	Vortex Field	Drift-Wave Turbulence (tokamak)
Inhomogeneity (free energy source)	$\boldsymbol{\nabla} n$	B_0, $\boldsymbol{\nabla} n$, and $\boldsymbol{\nabla} T$
Reynolds number	$\Omega = 0 - 40$	$Re = 10^1 - 10^2$ (Landau Damping)
Flux	Scalar	Heat
Zonal Flow	Boundary layer between cells	$\mathbf{E} \times \mathbf{B}$ shear flow (poloidal)

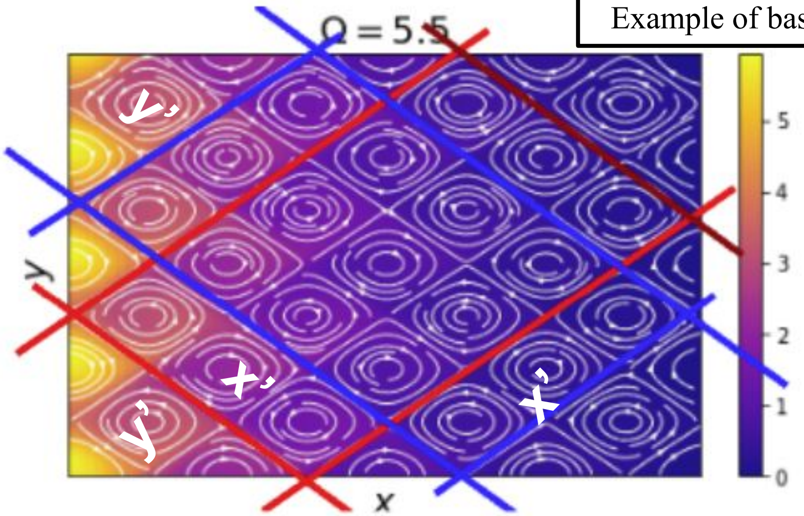
What Happens to Staircase? (Passive Scalar Dynamics)

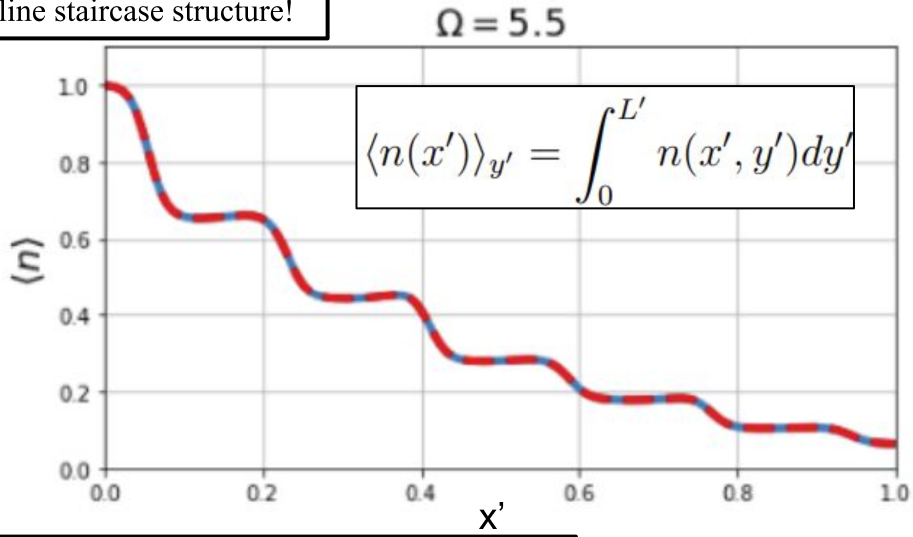
$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D\nabla^2 n,$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla}\right)\omega = \frac{1}{\Omega}\nabla^2\omega + F_\omega - \alpha\omega, \qquad \nabla^2\psi = \omega.$$

The Staircase

Example of baseline staircase structure!

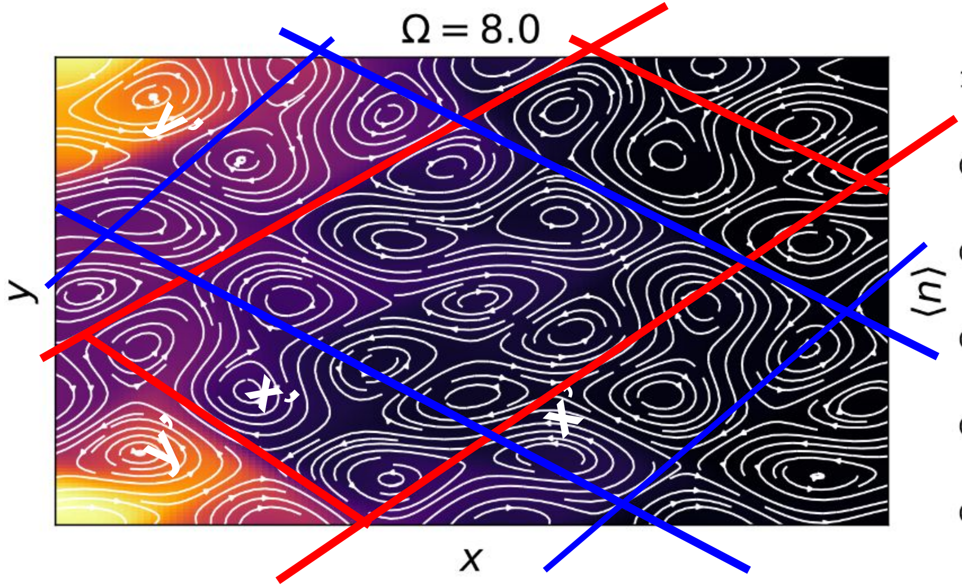


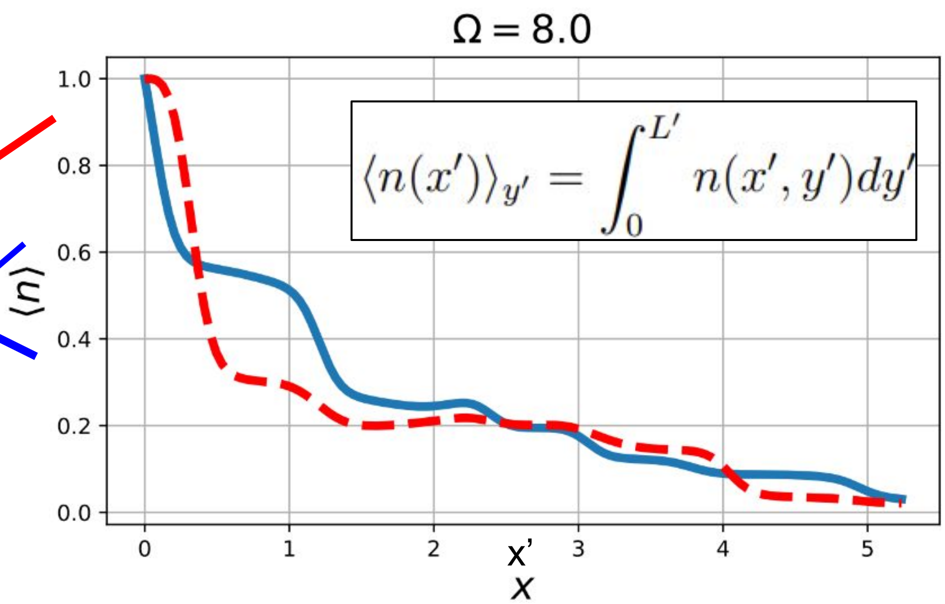


- For a weakly FVA we get a **baseline staircase** structure.
- On the left figure the blue and red box correspond to the blue and red plot line on the right. Note that **steps** are **evenly** spaced!
  - Both blue and red average scalar concentration have the same profile in stable stage.

So what happens to the staircase if we increase the Reynolds number in the VA?

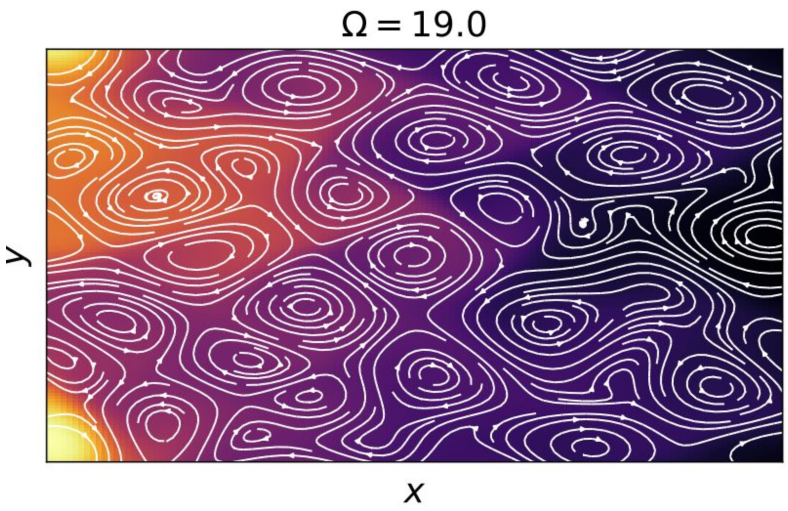
Staircase Resiliency to Fluctuations

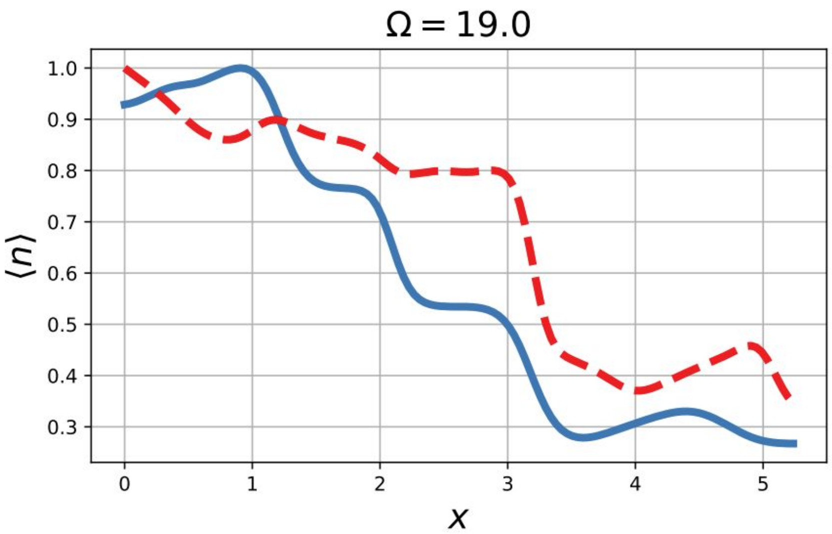




- As we **increase fluctuations in vortex array** through Ω, we can see **merger/connections** of vortex structures in the flow.
- These **vortex mergers** are shown in the scalar profile plot as **mergers in steps**.
  → As we increase jittering, staircase steps merge together.

Staircase Resiliency to Fluctuations (cont.d)



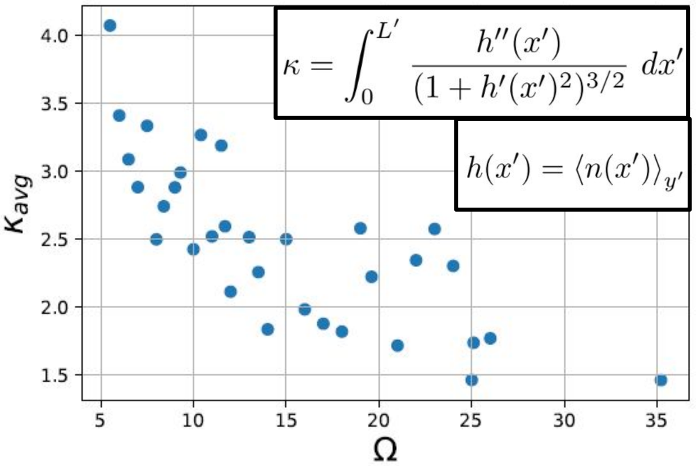


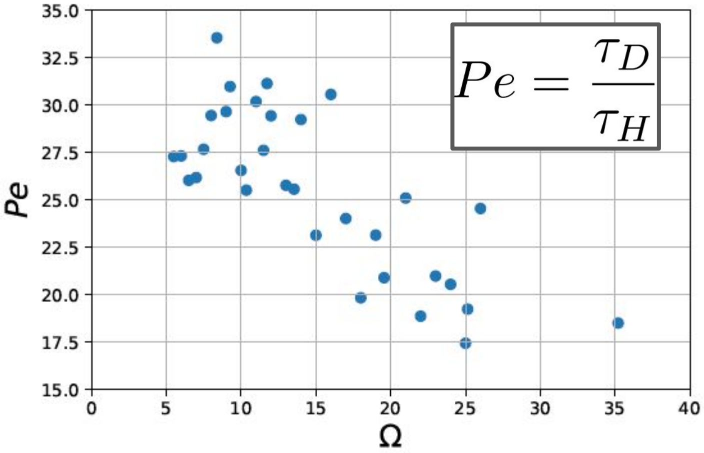
<u>Main Point</u>: Despite that vortex array becoming more turbulent, the staircase structure does not collapse.

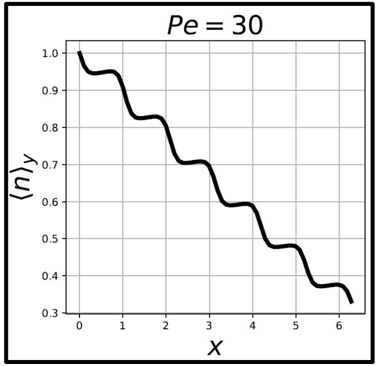
- Staircase steps become **less regular**. They merge into longer steps.

→ **Okay, but how to quantify?**

Criteria for Staircase Resiliency





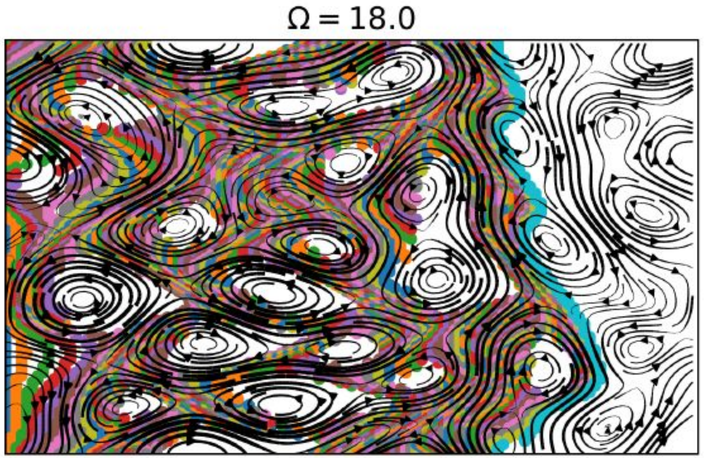


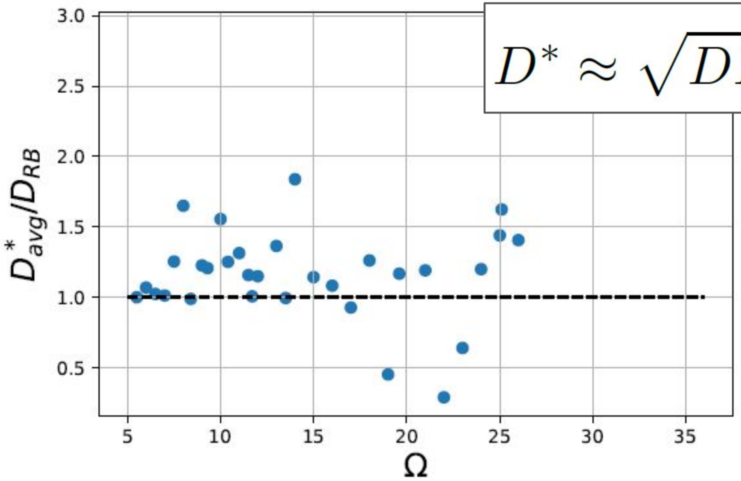
We establish a **set of criteria** to give a precise meaning to the statement of "**resiliency**":

1) $Pe \gg 1$ is a **necessary** condition for the **formation of transport barriers** in the process of scalar mixing (**First principles**). $Pe \gg 1$ criterion is satisfied for the range of $0 < \Omega < 40$.
2) A staircase should **maintain a sufficiently high curvature** (equivalent to sustaining a sufficient number of steps). Our studies suggest that $\kappa \gtrsim 1.5$ is an adequate value for a staircase.

Passive Scalar Transport

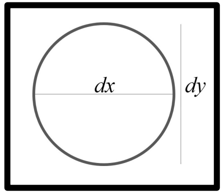
$$Pe = \frac{\tilde{u} d \beta}{D}$$





$$D^* \approx \sqrt{D D_{\text{cell}}} = D\sqrt{Pe}$$

$$D^* \propto \sqrt{d_x \beta}$$



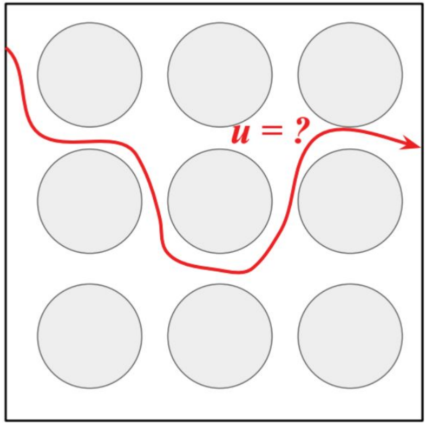
Before the <u>staircase</u> structure forms, scalar flows **quickly in regions of strong shear** and around vortices!

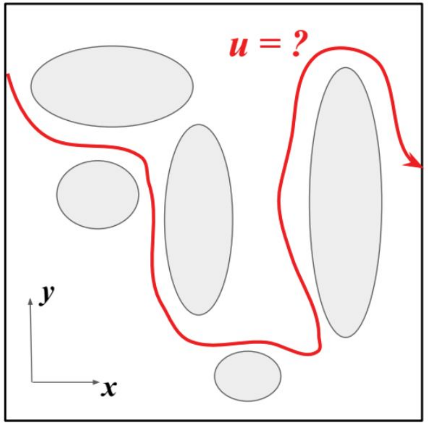
- Staircase **barriers form first!** Scalar travels along cell boundaries.
- Overtime, vortex **entrains** scalar by a kind of "**homogenization**" process via the synergy of differential rotation and diffusion.

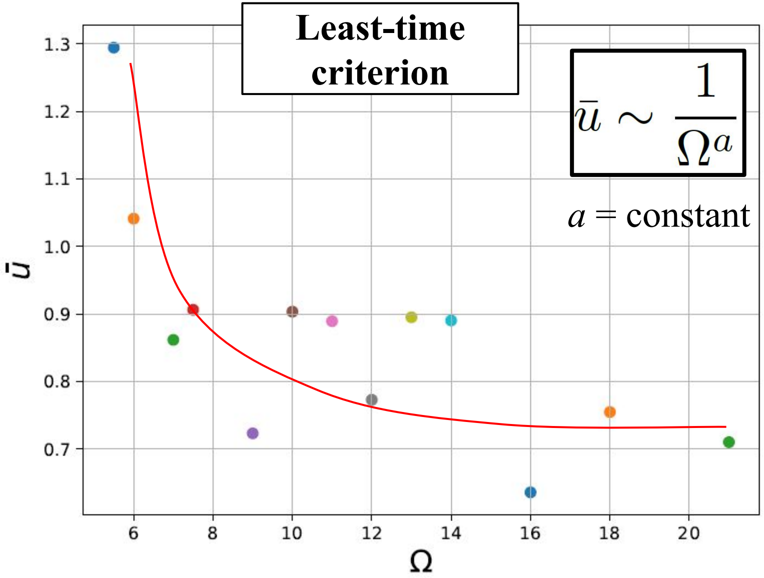
As cells fluctuate, the **effective diffusivity** deviates but **remains close** to the FCA effective diffusivity.
→ We find that as long as the **boundaries** and **speed** of the cells are **maintained**, the effective diffusivity and transport **does not change**.

- Only **dimensions** of cells **affect transport**.

Passive Scalar Transport (cont.d)







The **scattering of vortices** leads to an overall **decrease** in scalar concentration **velocity**! Agrees with least time criterion (similar idea to scattered path of light in atmosphere).

Summary

- Staircase form and are **resilient** and **persistent** to increasing Reynolds number (i.e., fluctuating vortex array).

- Scalar concentration **travels along** regions of **strong shear**.
    - **IMPORTANT**: Staircase barriers form first! Vortex "homogenizes" scalar at a later time!

- The scattering of vortices leads to an overall decrease in scalar concentration velocity.
    - Agrees with **least time criterion**.

- If flow velocity and background diffusion are kept fixed, only **cell geometric properties** affect the effective diffusivity! ($D^* \propto D\,Pe^{1/2}$)
    - Effective diffusivity of the perturbed vortex array **does not deviate** significantly!

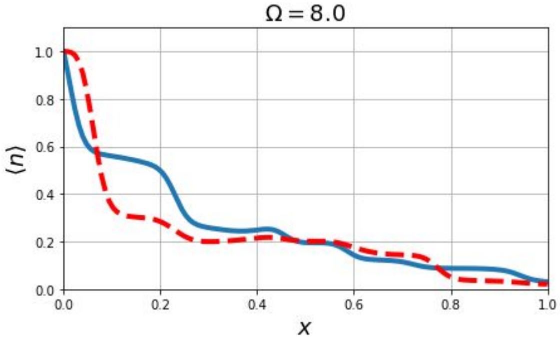
Why would a fusion experimentalist care about this?

These results have interesting implications for experiment and theory:

1. Effective diffusivity derived by Rosenbluth *et al* (for fixed cellular array) is a suitable approximation for the fluctuating cellular array (**not simple addition**: $D^* = D + D_{cell}$).
    - Relevant to cells touching (similar to what we find near-marginal stability).

2. Staircase structure is resilient in the regime of low-modest Reynolds numbers (this regime is relevant to drift-wave turbulence).
    - Structures/Profiles are not exotic.
        - Staircase profile structure does not require special tuning.

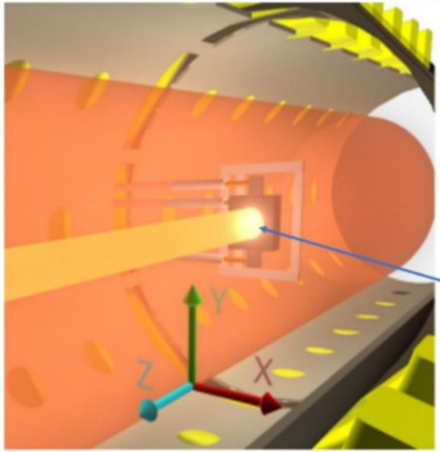
3. Geometry of streamlines is important. If more saddles than close vortices, Heat avalanches will first form the staircase barrier.
    - Fluctuating cellular flow hinders avalanche propagation.

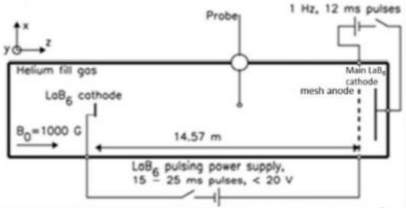


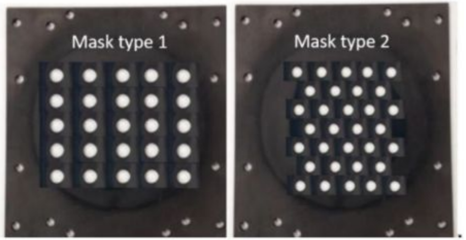
IMPORTANT: We can test the theory presented here with actual experimental data.

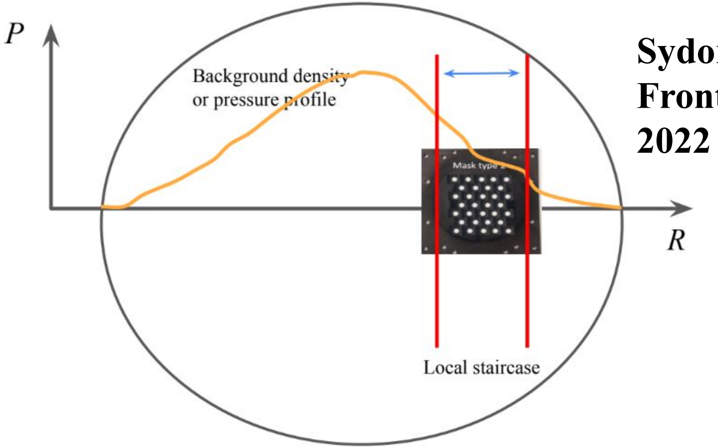
LAPD Experiment

Work in progress!









Sydora, Frontiers Proposal 2022

A vortex array can be created in the large linear magnetized plasma device (LAPD)

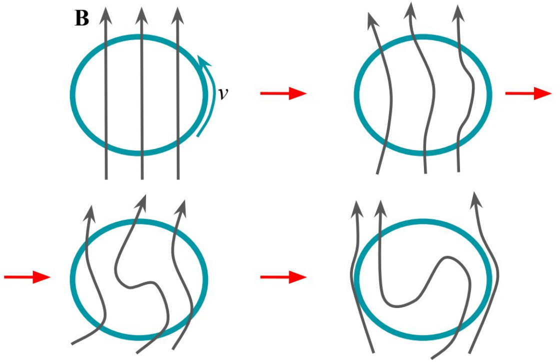
- Modification of a cathode plasma source with designer masks that form multiple current channels in a cellular pattern → form staircase!
  - Experiment will be conducted in the afterglow phase of the main discharge.
- Staircase structure can be subject to controllable amount of of low frequency density fluctuations, which act as a noise source.
  - Allow us to test hypotheses and models of staircase resiliency!

Results of experiment will yield a unique set of observations that can be used to test staircase models.

Active Scalar Dynamics

Active Scalar

$$M = \frac{v_A}{U_0}$$



A logical next step to explore is the effects than an *active* scalar has on the cellular array and inhomogenous mixing.

- Converting passive to active will result in effects such as flux expulsion
    - Flux expulsion is simplest dynamic problem in non-ideal MHD.

Why this model?

- ***B*** expelled to boundaries, thus holds cells together! → Rigid staircase.

We turn passive scalar into an active scalar, creating a feedback between magnetic field and vortices:

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} n - D\nabla^2 n = 0 \longrightarrow \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla}\right) A = \frac{1}{R_m}\nabla^2 A + F_A$$

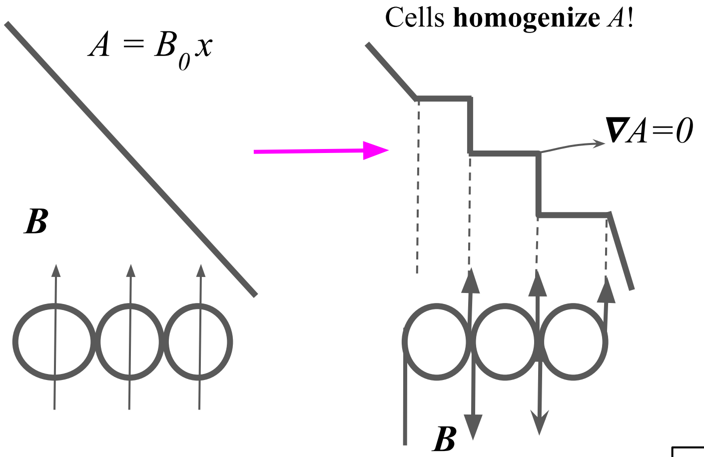
$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla}\right) \omega = \frac{1}{\Omega}\nabla^2 \omega + M^2 \boxed{\left(\mathbf{B} \cdot \boldsymbol{\nabla}\nabla^2 A\right)} + F_\omega$$

<u>**Flux expulsion:**</u>

- Background ***B*** is wind up and folded by an eddy → field inside eddy drops → expelled to boundary layer of eddy.
- Time scale for flux expulsion is, $\tau_{fe} = R_m^{1/3}\, \tau_H$
- **Note:** Larger R_m results in greater expulsion (weaker field in interior).

Note: Strength of B_o plays an important role!

Kinematic/Dynamic Regime



Important: Flux expulsion only occurs in the **kinematic** regime

- Useful to explore **dynamic** regime (aka Vortex bursting).

Since $v_A \propto B_0$, the strength of the magnetic field will play a role in the dynamics of the cellular array.

- If B_0 is sufficiently small, we get cell strengthening.
- If B_0 is large, vortices will not be allowed to form.

Through scans of B_0, we will address what **occurs** to expulsion of **neighbor cells** and their **interaction**…

$M^2 R_m < 1$ (Flux expulsion)

$M^2 R_m \geq 1$ (Vortex bursting)

$$M = \frac{v_A}{U_0}$$

To be clear, staircase forms in the flux expulsion regime.

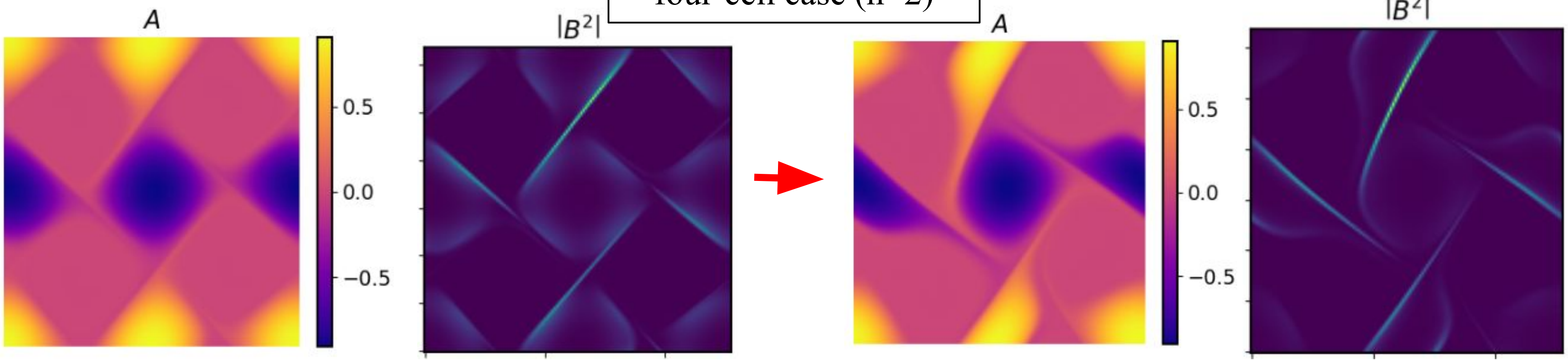
- Unclear if staircase forms in vortex bursting regime (tbd).

Consider a **linear** magnetic potential profile:

- We expect that the vortex array will homogenize ($\nabla A=0$) the profile in areas of vortices.
- Expect that magnetic field will maintain or restore the cell array structure when fluctuations are present (i.e., B_0 will elasticise the cell array).

Formation & Destruction of Barriers ($M^2 R_m = 1$)

four-cell case (n=2)



This problem is **important** and can be related back to the idea of **feedback**!

- We have only address the idea that staircases are resilient and robust in the presence of cell fluctuations.
- But could the scalar affect the dynamics or maintain the cell structure which is responsible for the staircase? Preliminary results show that **magnetic field restores cell structure**!
  - Only a small window where this occurs (i.e., small ***Bo***)...

NOTE: ***B*** eventually **decays** in 2D, so the **structure** is only **temporary**… (need to force magnetic field)

$$F_\omega \equiv -n^3 \left[\cos\left(nx\right) + \cos\left(ny\right)\right] / \Omega$$

Work in progress…

Thank you!

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