On Zonal flow - Turbulence - Corrugation Dynamics in Magnetized Plasmas

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Outline

- Zonal flows are ubiquitous —why?
- Introduction
- Motivation
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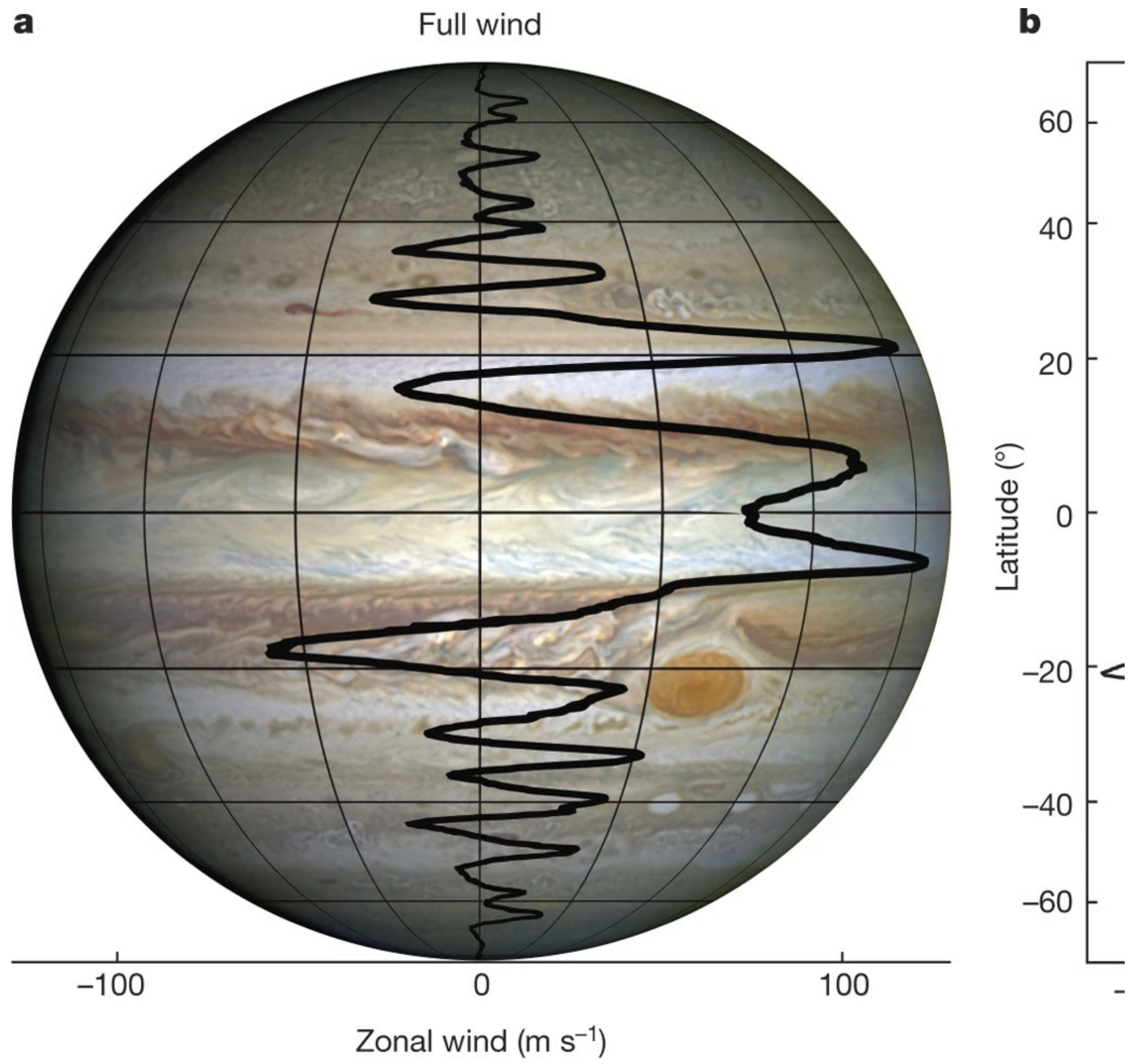
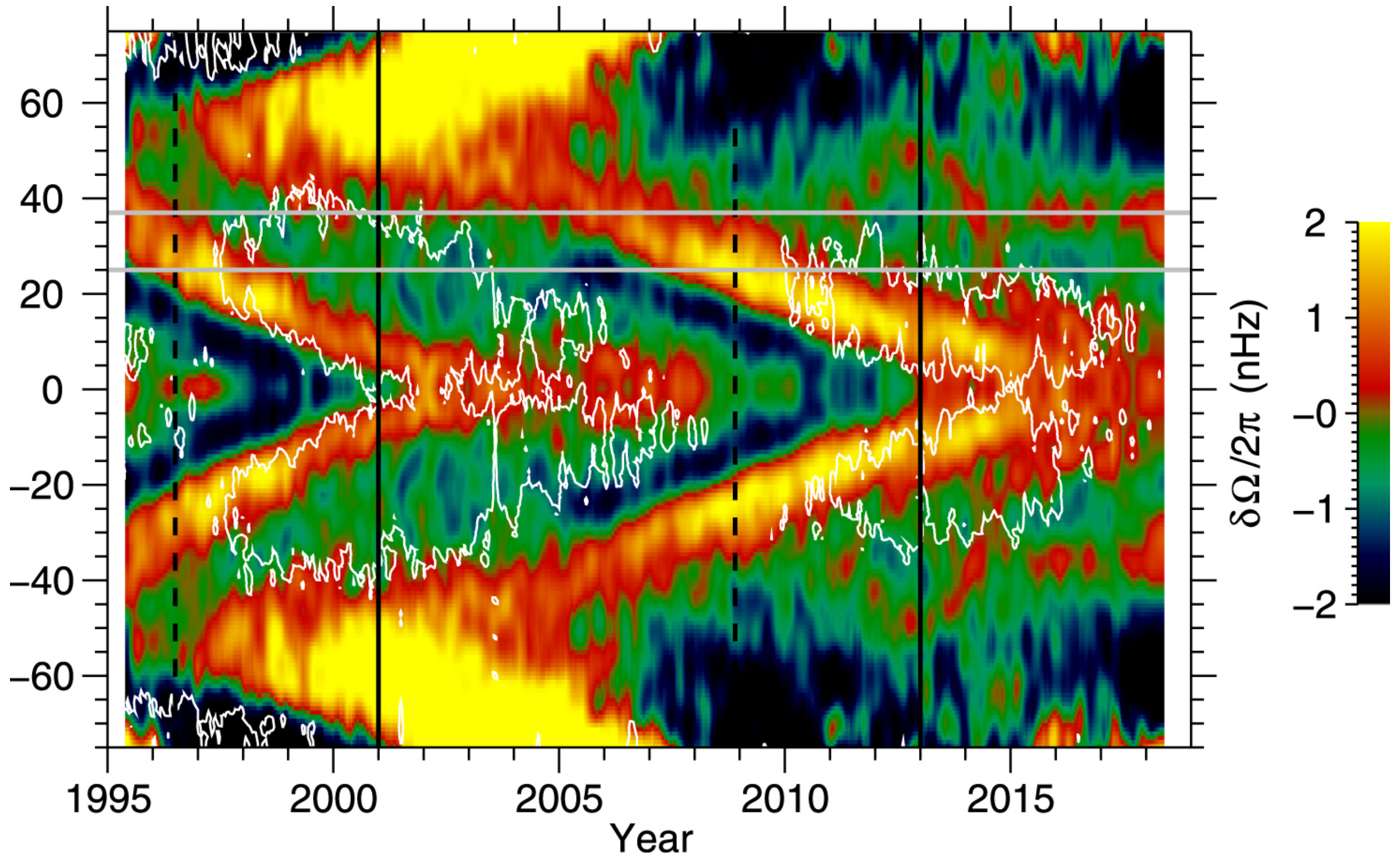
- Unified theory of zonal flows and corrugations - spectral closure
  - Noise + modulations
  - Zonal cross correlation- staircase
- Zonal noise effects on Feedback loops
- Conclusions

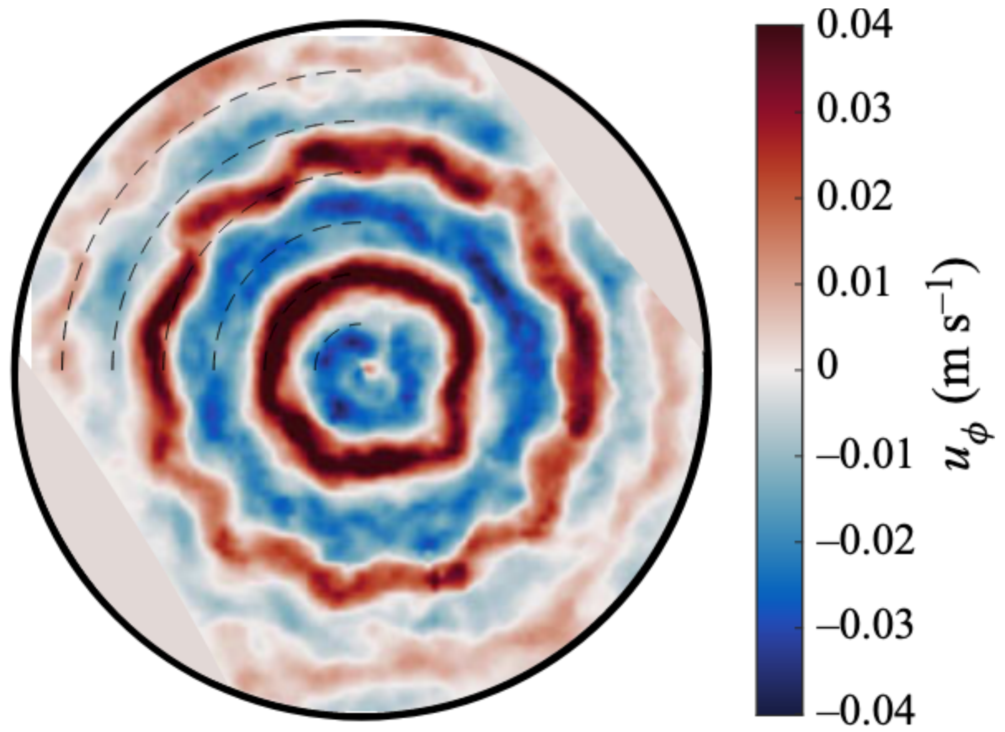
Zonal flows are ubiquitous



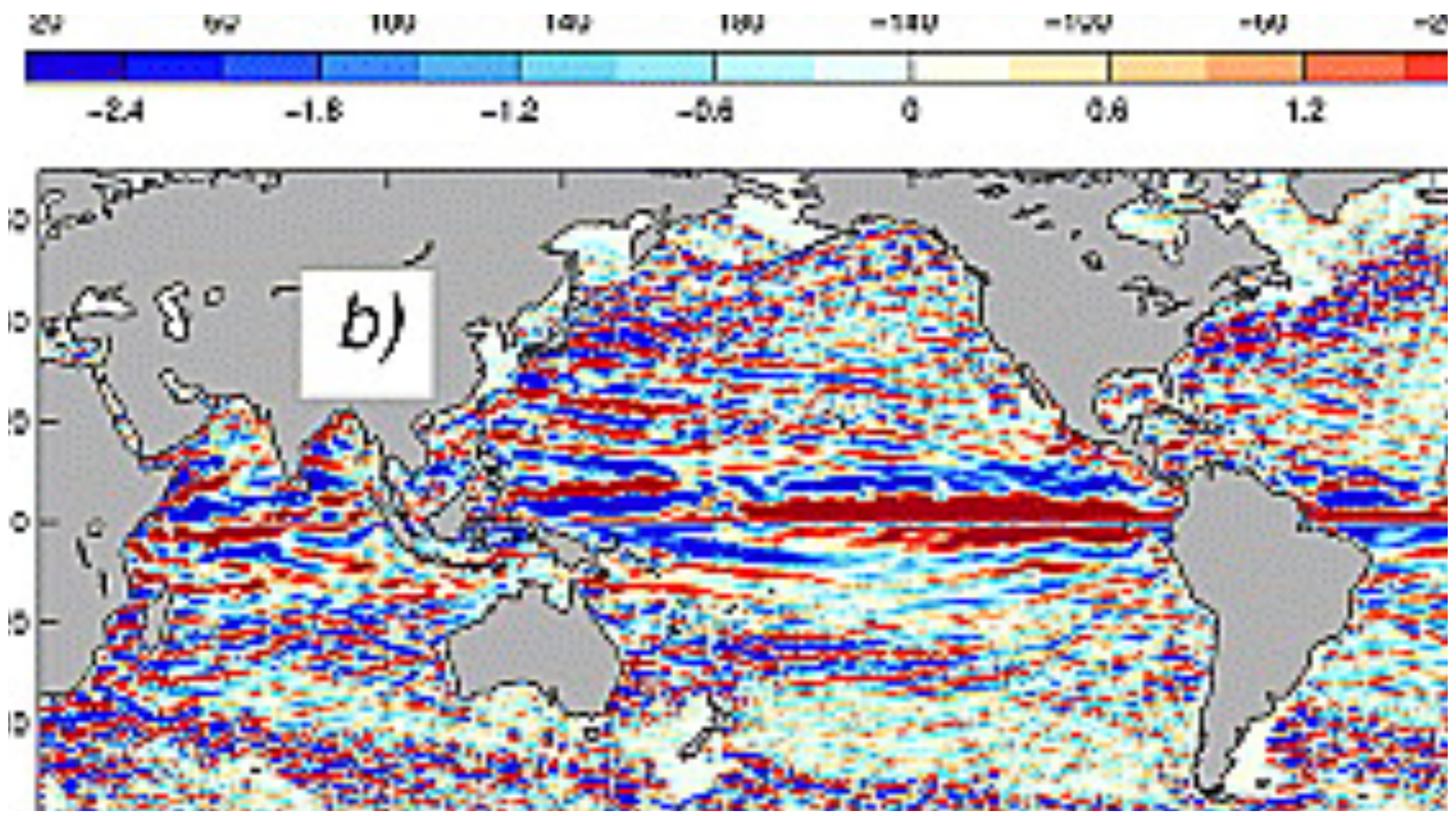
ZFs in Jupiter



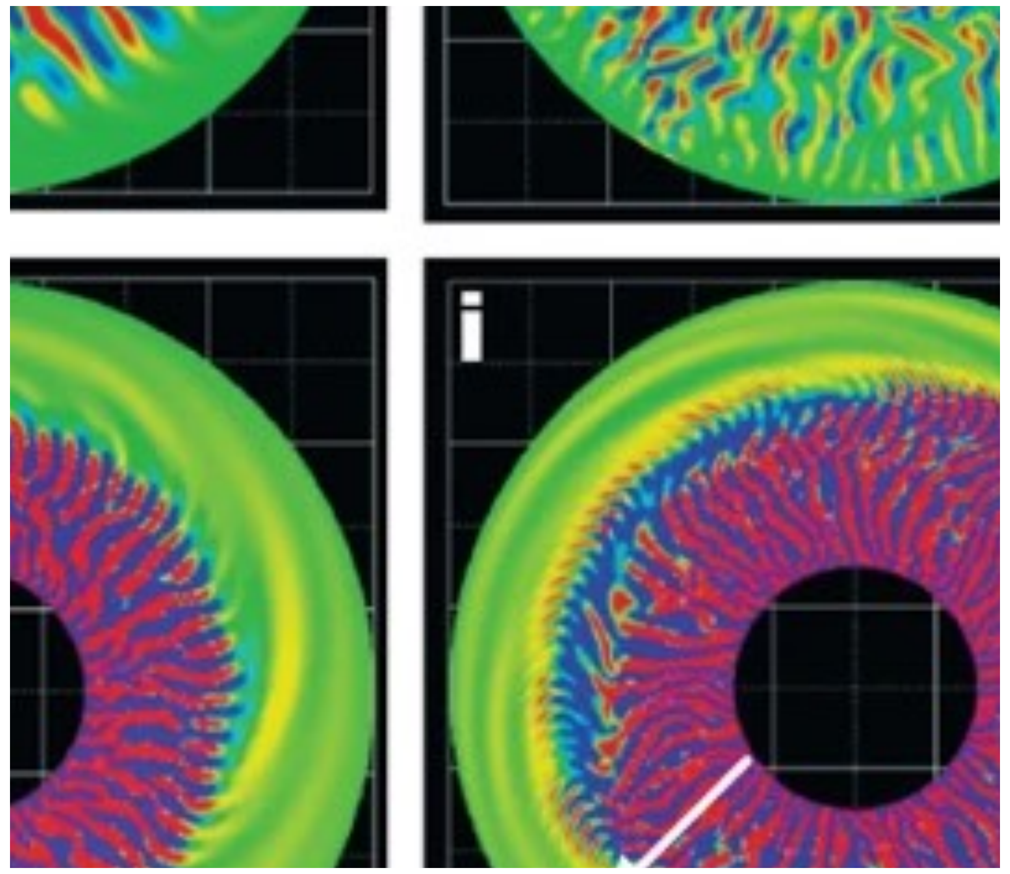
Migrating ZFs in convection zone of Sun



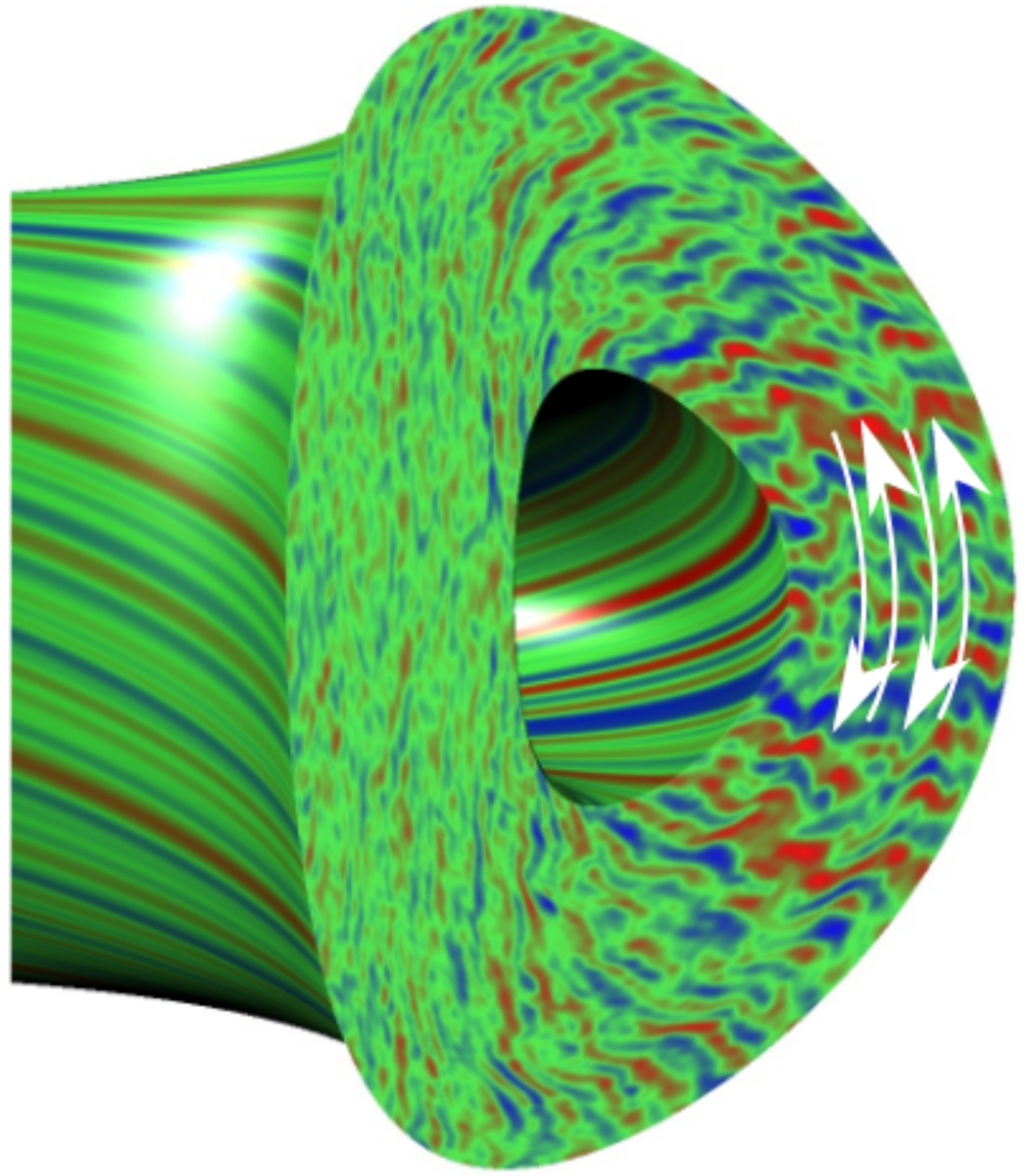
ZFs in rotating liquid column experiment Lemasquerier *et al* JFM 2021



ZFs in ocean Maximenko *et al* GRL 2005



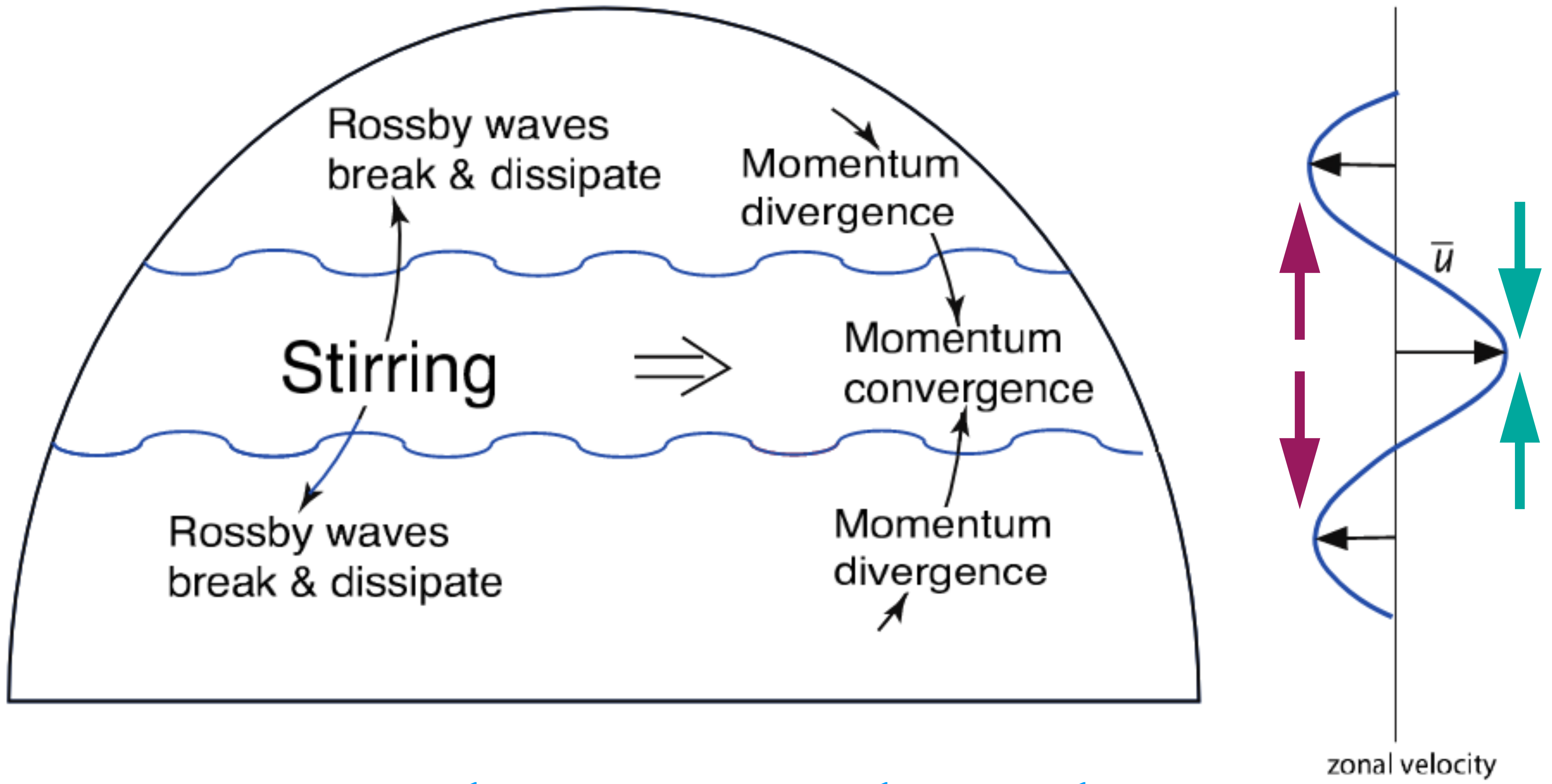
ZFs in liquid interior of Earth outer core T Miyagoshi *et al* Nature 2010



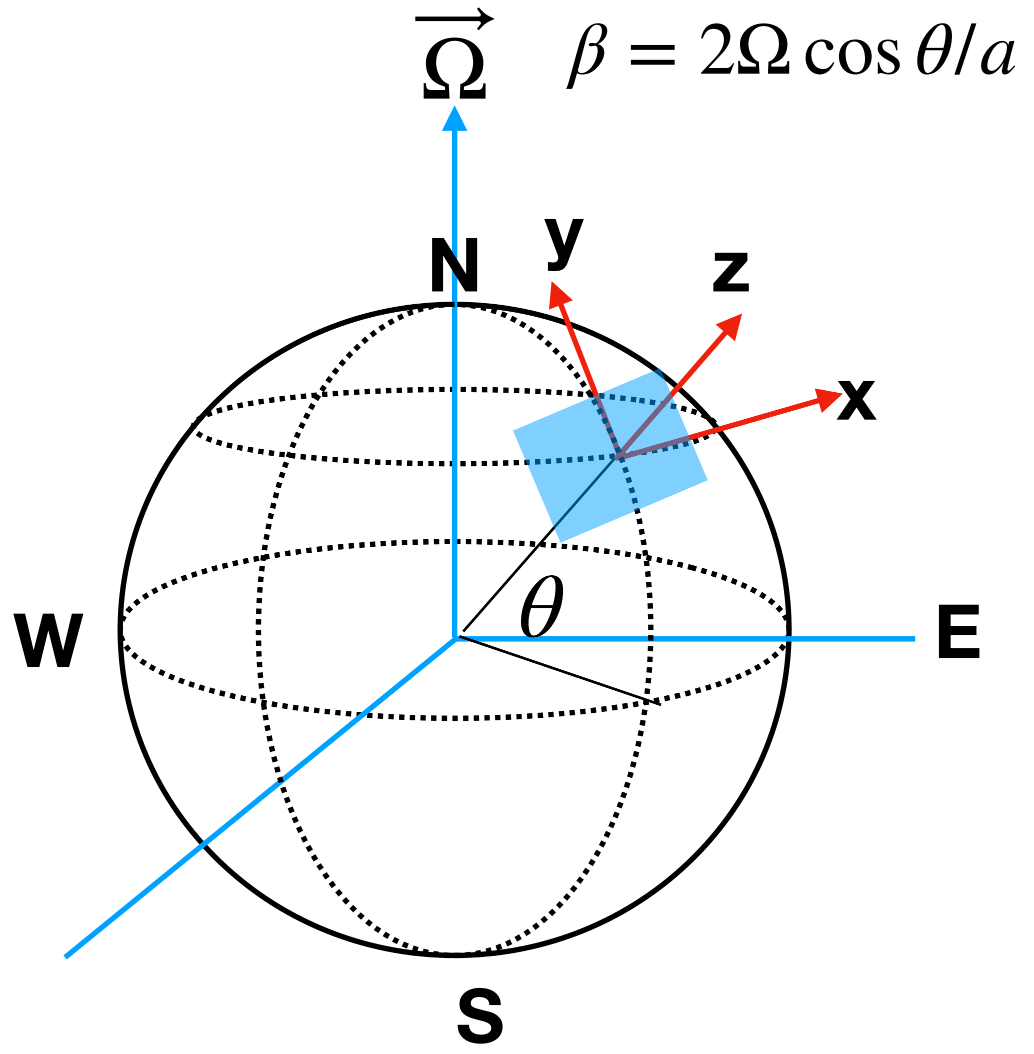
ZFs in tokamak

Why zonal flows are ubiquitous? GFD Perspective

- Mid latitude zonal circulation [G K Vallis]



- Stirring in mid-latitudes (by baroclinic eddies) generates Rossby waves that propagate away.
- Momentum converges in the region of stirring, producing eastward flow there and weaker westward flow on its flanks.



Rossby waves on beta-plane

- Frequency $\omega = -\dfrac{\beta k_x}{k_x^2 + k_y^2}$, Group velocity $v_{gy,k} = \dfrac{2\beta k_x k_y}{\left(k_x^2 + k_y^2\right)^2}$
- Momentum flux: $\left\langle v_y v_x \right\rangle = -\sum_{\vec{k}} \frac{1}{2} k_x k_y \left|\psi\right|^2 = -\sum_{\vec{k}} v_{gy,k} \mathscr{P}_k$

•Outgoing waves $\Longrightarrow$ incoming wave momentum flux

Pseudomomentum

Introduction

- DW-ZF turbulence has two components: drift waves(“wavy” - $k_\theta \neq 0$) and zonal modes ($k_\theta = k_z = 0$)

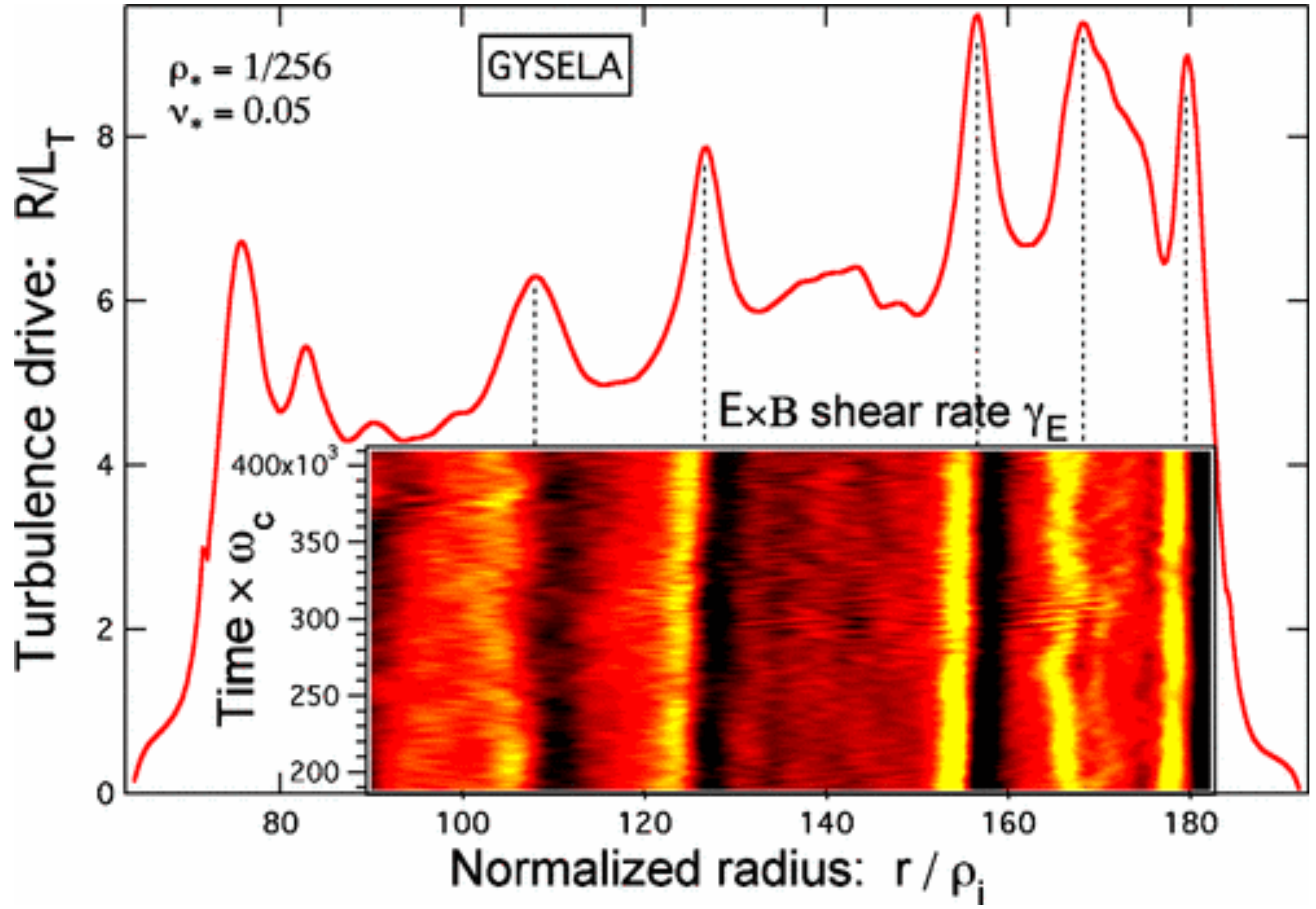
- Zonal modes are modes of minimal inertia, transport and damping.

- Symmetry precludes adiabatic electron response for zonal modes. Thus they are benign repositories of fluctuation energy.

- Conversion of energy to zonal structures reduces transport and improves confinement.

- Zonal structures ($k_\theta = k_z = 0$) possible in different fields- ϕ, $n, T_i, T_e \ldots$

Zonal flow   Profile corrugations



Motivation

- There are both zonal flows and density corrugations at the simplest level of description of DW-ZF turbulence.

    - Zonal flows result from the inverse cascade of kinetic energy - this is well known.

    - What about the density corrugations?

    - How are the zonal density and zonal flow correlated ? → staircase?

- How do zonal flow and density corrugations feedback on turbulence?

    - Zonal flow shear induces diffusion of mean wave action density in k_x space. What about the turbulent kinetic energy and internal energy?

    - How does density corrugation feedback on turbulence?

- Effects of zonal noise on the predator prey dynamics?

Motivation

- Almost all theoretical models of zonal flow generation divide cleanly into:

①

Calculation of zonal flow dielectric or screening response, with occasional mention of wavy component beat noise [Rosenbluth -Hinton 1998]:

$$\frac{\partial}{\partial t}\left\langle \left|\phi_q\right|^2 \right\rangle = \frac{2\tau_c \left\langle \left|S_q\right|^2 \right\rangle}{\left|\epsilon_{neo}\right|^2}$$

Emission from polarization interaction

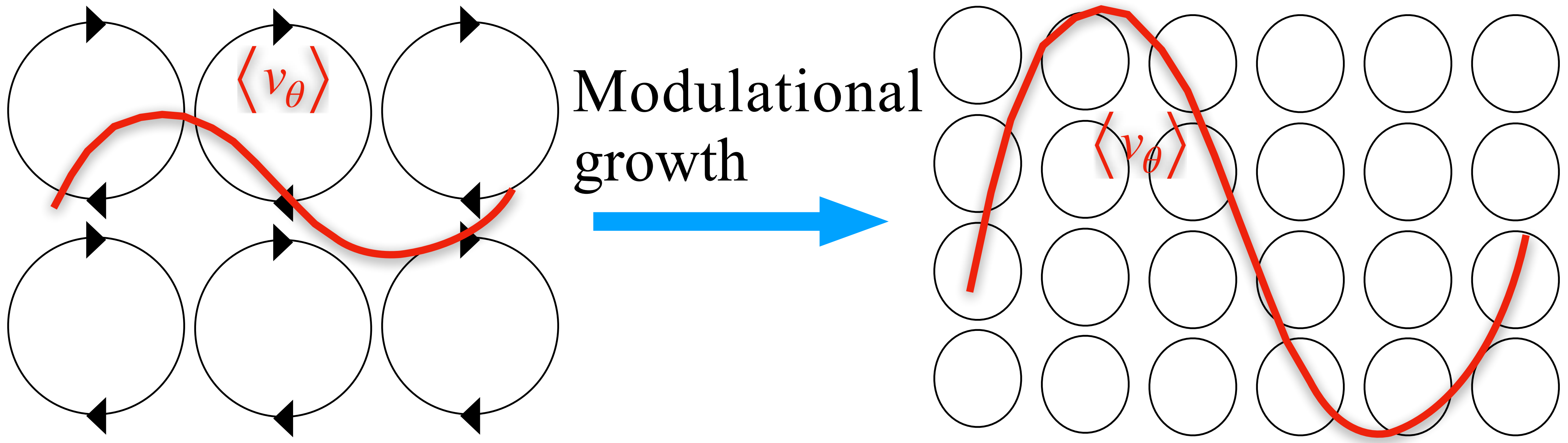
-ignores coherent modulational mechanism.

②

Modulational stability calculations consider response of a pre-existing gas of drift waves to an infinitesimal test shear:

$$\frac{\partial}{\partial t}\bar{\phi}_q = \int d\vec{k} k_y k_x \delta \left|\phi_k\right|^2 = -q_x^2 \int d\vec{k} k_y^2 C_k \mathscr{R}^{(r)}_{k,q} k_x \frac{\partial \langle N_k \rangle}{\partial k_x} \bar{\phi}_q = \gamma_q \bar{\phi}_q$$

-ignores incoherent noise emission.



-ve turbulent viscosity for $k_x \frac{\partial \langle N_k \rangle}{\partial k_x} < 0$

- What happens when the noise meets modulations? Langevin equation with -ve damping: $\frac{\partial \phi_q}{\partial t} - \gamma_q \phi_q = noise.$

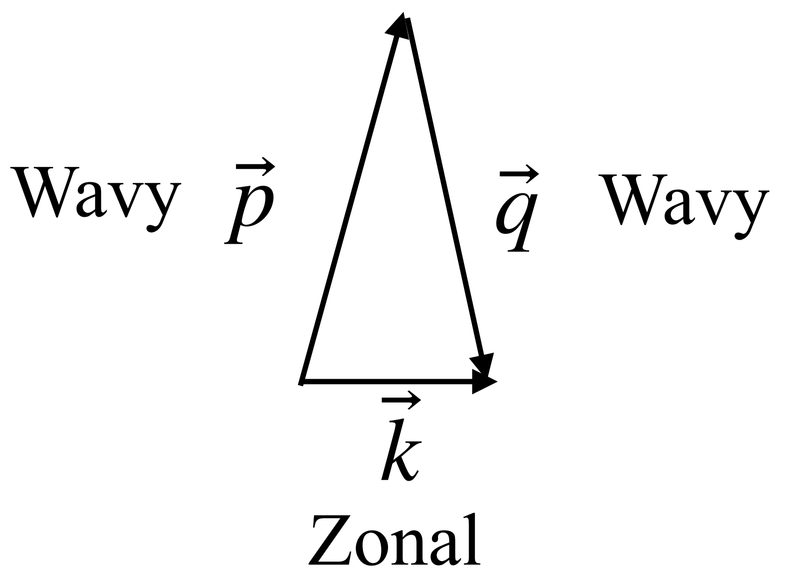
- **Treat incoherent noise emission and coherent response on equal footing. -Needs spectral approach!**

Spectral evolution of zonal intensity

For the zonal mode $k_y = k_\parallel = 0$ and $k_x \neq 0$

Zonal density - potential cross-correlation

$$\left(\frac{\partial}{\partial t} + 2\mu k_x^2\right)\left\langle \left|\phi_k\right|^2 \right\rangle + 2\eta_{1k}^{zonal}\left\langle \left|\phi_k\right|^2 \right\rangle + \Re\left[2\eta_{2k}^{zonal}\boxed{\left\langle n_k \phi_k^\star \right\rangle}\right] = F_{\phi k}^{zonal}$$



[Singh, Diamond PPCF 2021]

- **NL damping rate**: $\eta_{1k}^{zonal} \propto k_x^2$ and becomes -ve for $\dfrac{\partial I_q}{\partial q_x} < 0 \rightarrow$ transfer to large scales by negative turbulent viscosity!

  - Zonal growth is maximum when the adiabaticity parameter $\alpha_q \rightarrow \infty \implies$ Non-adiabatic fluctuations inhibit transfer to large scales !

- **Cross transfer rate**: $\eta_{2k}^{zonal,(r)} > 0$ ALWAYS for $\dfrac{\partial I_q}{\partial q_x} < 0 \implies$ Forward transfer when $\Re\left\langle n_k \phi_k^\star \right\rangle < 0$, backward transfer when $\Re\left\langle n_k \phi_k^\star \right\rangle > 0$

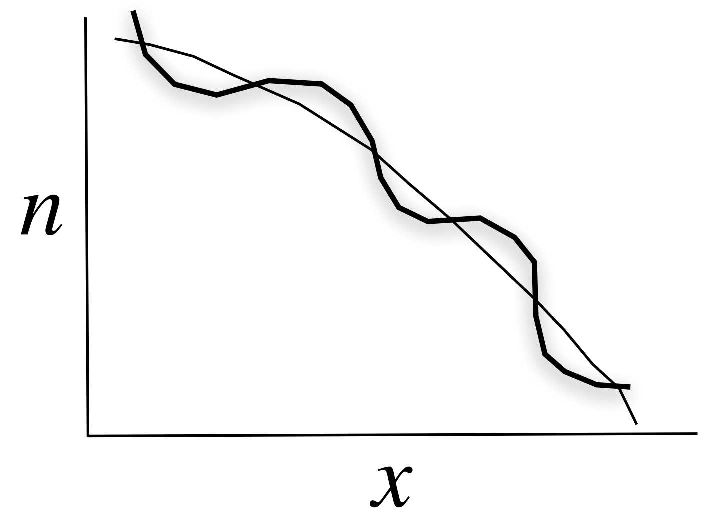
- **Noise:** always +ve and of envelope scale! $F_{\phi k}^{zonal} = 4\sum_q \Pi_q^2 \Theta_{k,-q,q}^{(r)}$; Reynolds stress $\Pi_q = q_y q_x I_q$.

  Noise/Modulation = $q_x^2 I_q / k_x^2 I_k$ = Turbulent KE/ Zonal KE.

Spectral evolution of density corrugations

$$\left(\frac{\partial}{\partial t} + 2D_n k^2\right)\left\langle |n_k|^2 \right\rangle + 2\zeta_{1k}\left\langle |n_k|^2 \right\rangle + \Re\left[2\zeta_{2k}\left\langle n_k^\star \phi_k \right\rangle\right] = F_{nk}$$

[Singh, Diamond PPCF 2021]



Corrugations damping rate, +ve $\sim 1/\alpha_q^2$

Cross transfer rate, +ve $\sim 1/\alpha_q^2$

Advection noise, +ve $\sim 1/\alpha_q^2$

- Density cascade forward in k_x! Unlike turbulent viscosity, turbulent diffusivity is always +ve.
- Corrugations become weaker as the response become more adiabatic.
- Corrugation is determined by noise vs diffusion balance.
- Important for the nonlinear dynamics underlying staircases. Forward cascade in k_x-space → (inhomogeneous) mixing in real space.

Zonal cross-correlation(ZCC)

[Singh, Diamond PPCF 2021]

- Significant for layering or staircase structure - ZF and ∇T are aligned in staircase!

- When do zonal density and zonal potential align?

$$\Re\left\langle n_k\phi_k^\star\right\rangle = \frac{2\eta_{2k}^{(r)}\left\langle\left|n_k\right|^2\right\rangle + 2\zeta_{2k}^{(r)}\left\langle\left|\phi_k\right|^2\right\rangle}{-\left(\mu+D_n\right)k_x^2 - 2\xi_{1k}^{(r)}};\quad \xi_{1k}^{(r)} = \eta_{1k}+\zeta_{1k}$$

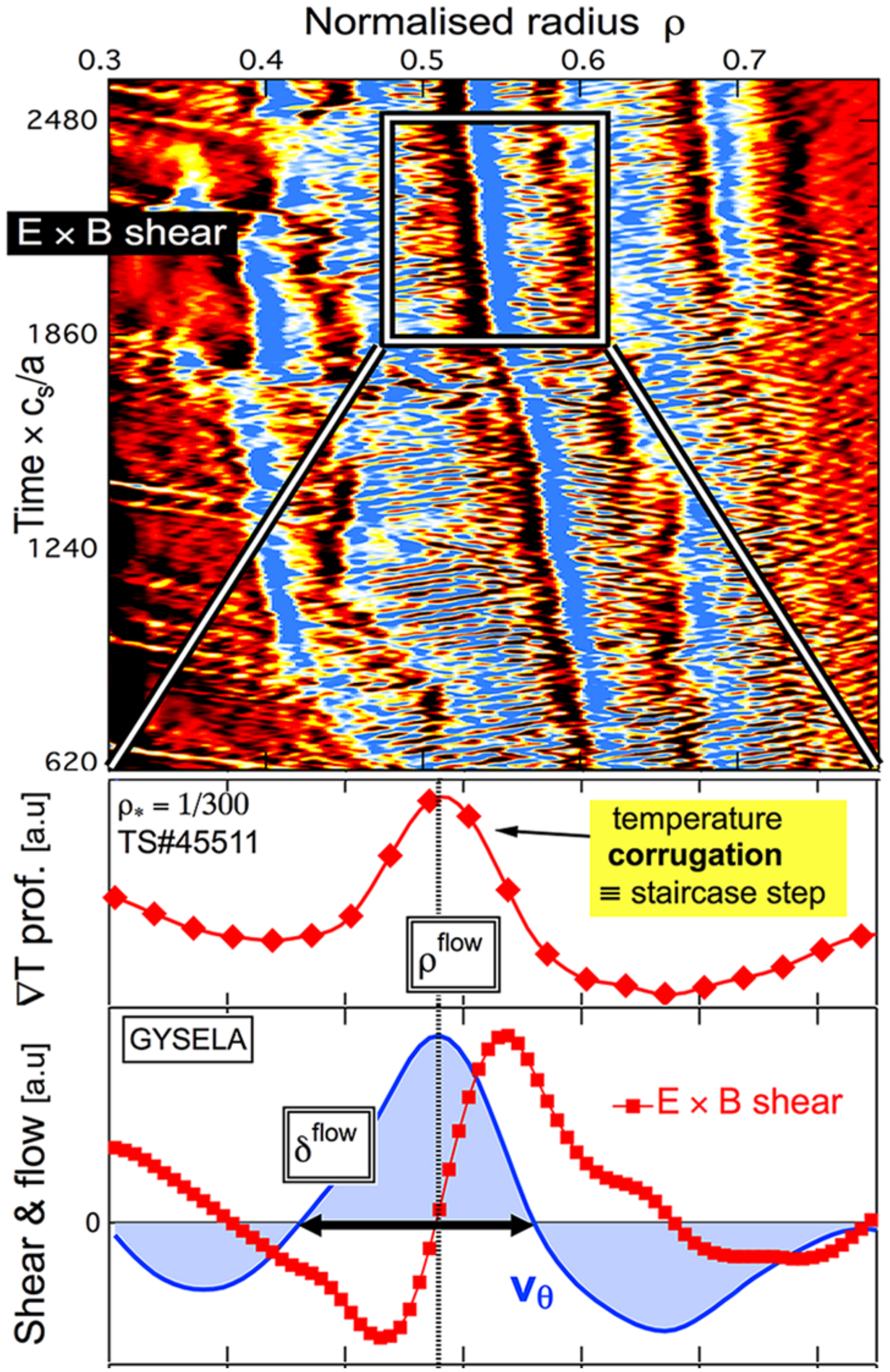
- Zonal density and potential are correlated (anti-correlated) when the modulational growth of zonal flow more (less) than modulational damping of corrugations.

- Imposing physically bounded solution for $\Im\left\langle n_k\phi_k^\star\right\rangle = 0$ fixes the sign of $\left(\mu+D_n\right)k_x^2 + 2\xi_{1k}^{(r)} > 0$. Thus $\Re\left\langle n_k\phi_k^\star\right\rangle < 0$.

- Hence ZCCs in real space are: $\left\langle\bar{n}\bar{\phi}\right\rangle < 0$, $\left\langle\bar{n}\nabla_x^2\bar{\phi}\right\rangle > 0$, $\left\langle\nabla_x\bar{n}\nabla_x^2\bar{\phi}\right\rangle = 0$

- $\left\langle\nabla_x\bar{n}\nabla_x^3\bar{\phi}\right\rangle > 0$: ***zonal density jumps are co-located with the zonal vorticity jumps.***

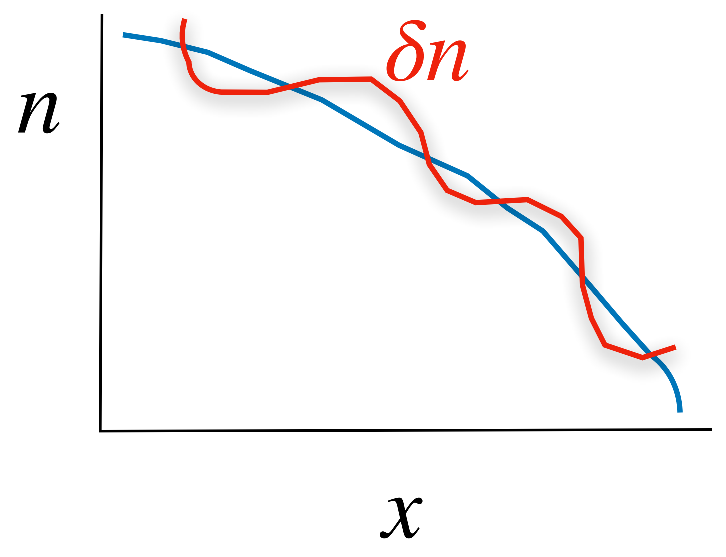
- $\left\langle-\nabla_x\bar{n}\nabla_x\bar{\phi}\right\rangle > 0$: ***density gradient peaks are co-located with the zonal flow peaks.***



Guilhem et al PRL 2015

Back reaction of zonal modes on DWT

- Mean field WKE $\frac{\partial \langle N_k \rangle}{\partial t} = \frac{\partial}{\partial k_x}\left[D_{kk} \frac{\partial \langle N_k \rangle}{\partial k_x} + V_k \langle N_k \rangle \right] + V_k \langle N_k \rangle + \Gamma_k \langle N_k \rangle$



Induced diffusion

Convection

NL growth

$$D_{kk} = \int d\vec{q} \mathscr{R}^{(r)}_{k,q} k_y^2 q_x^4 \left| \phi_q - \frac{\partial \omega_{rk}}{\partial \omega_{\star e}} n_q \right|^2 \qquad V_k = -\Re \int d\vec{q} i \mathscr{R}_{k,q} k_y q^3 \left(\phi_q - \frac{\partial \omega_{rk}}{\partial \omega_{\star e}} n_q \right) k_y \frac{\partial \gamma_k}{\partial \omega_{\star e}} n_{-q} \qquad \Gamma_k = \int d\vec{q} \mathscr{R}^{(r)}_{k,q} q_x^2 k_y^2 \left(\frac{\partial \gamma_k}{\partial \omega_{\star e}} \right)^2 \left| n_q \right|^2$$

- K-space diffusivity D_{kk}: depends on the relative alignment of zonal shear and profile corrugations. D_{kk} enhanced by density corrugations as zonal density and potential are anti-correlated.

- Convection speed V_k: caused by density corrugations!

- Nonlinear growth Γ_k: due to linear growth modulation by density corrugations. Injects energy back into turbulence, locally. Competes with turbulence saturation by random shearing of zonal modes!

Feedback loop with nonlinear zonal noise

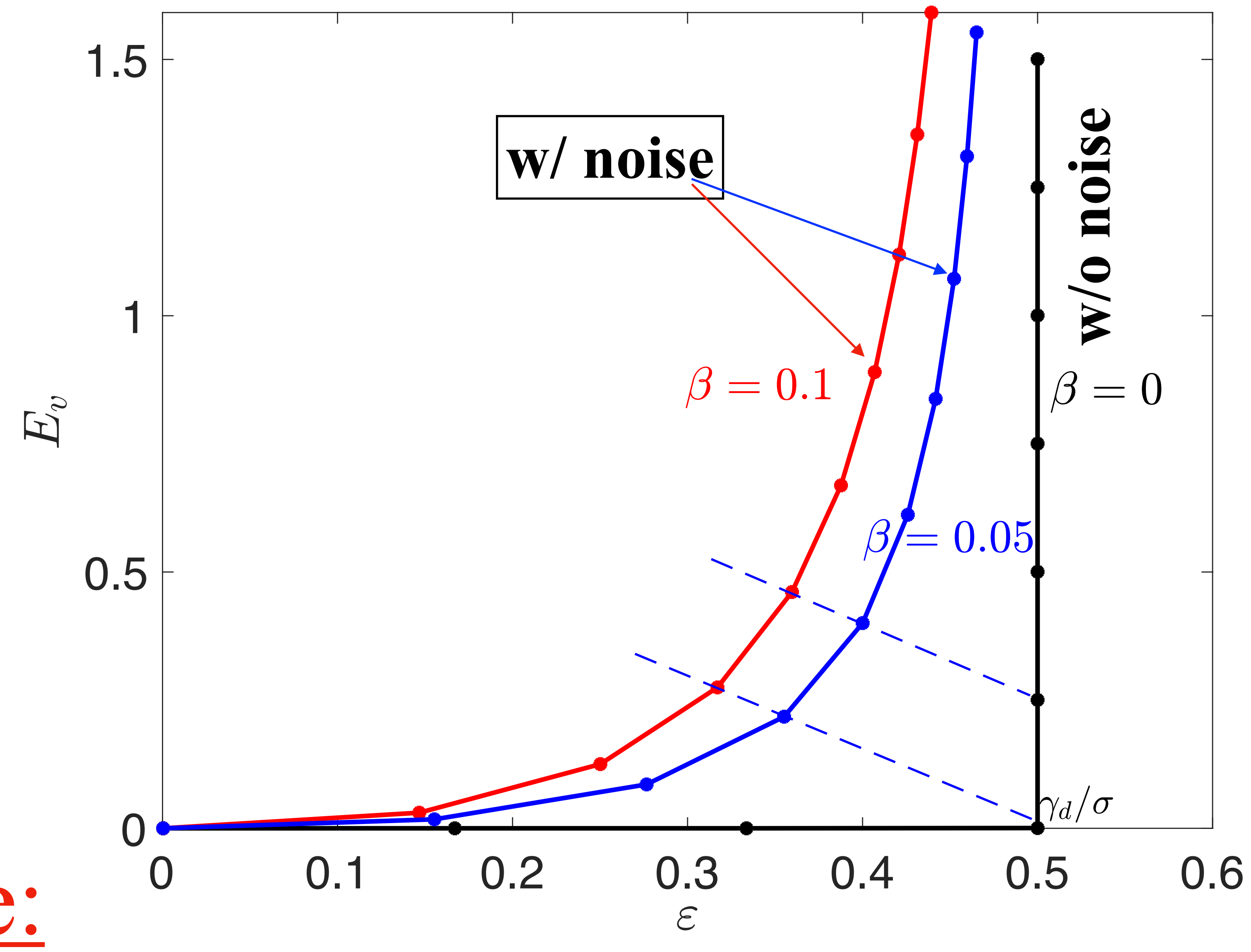
Turbulence energy $\mathscr{E}$: $\frac{\partial \mathscr{E}}{\partial t} = \gamma\mathscr{E} - \sigma E_v \mathscr{E} - \eta \mathscr{E}^2$
(Prey)

Induced diffusion

Nonlinear damping

Zonal flow energy E_v: $\frac{\partial E_v}{\partial t} = \sigma \mathscr{E} E_v - \gamma_d E_v + \beta \mathscr{E}^2$
(Predator)

Modulational growth



[Singh, Diamond PPCF 2021]

Without noise:

- Threshold in growth rate $\gamma > \eta\gamma_d/\sigma$ for appearance of stable zonal flows.
- Turbulence energy increases as γ/η below the threshold, until it locks at γ_d/σ, at the threshold.
- Beyond the threshold, turbulence energy remains locked at $\frac{\gamma_d}{\sigma}$ while the zonal flow energy continues to grow as $\sigma^{-1}\eta\left(\gamma/\eta - \gamma_d/\sigma\right)$.

With noise:

- Both zonal and turbulence co-exist at any linear growth rate - No threshold in growth rate for zonal flow excitation.
- Turbulence energy never hits the old modulational instability threshold, absent noise!
- Turbulence energy ↓ and zonal flow energy ↑:- Noise feeds energy into zonal flow!

Conclusions

- Unified theory of zonal modes (zonal flows and density corrugations) encompassing both noise and modulations.

  - Vorticity flux corr.→ ZF noise. Density flux corr.→ corrugation noise.

  - Bi-directional transfer: KE energy to large scales, internal energy to small scales. Turbulent viscosity -ve but turbulent diffusivity +ve.

  - The effective zonal viscosity goes negative only for an energy spectrum which decays sufficiently rapidly in k_r i.e., $\partial E/\partial k_r < 0$ and $\left|\partial E/\partial k_r\right| < \left|\partial E/\partial k_r\right|_{crit}$.

  - Spectral ZCC $\Re\left\langle n_k \phi_k^\star \right\rangle < 0$→ Spatial ZCC $\left\langle \nabla \bar{n} \nabla^3 \bar{\phi} \right\rangle > 0$ i.e., zonal density jumps are colocated with zonal vorticity jumps.

- Polarization beat noise and modulational effects are comparable intrinsically (both driven by Reynolds stress!). The synergy of the two mechanisms is stronger than either alone.

  - Expands the range of zonal flow activity relative to that predicted by modulational instability calculations.

Future directions

- Understanding interaction of corrugations with avalanches:
  - Corrugations in state of high Z_{cc} sustained as localized transport barriers, staircases etc. localized by accompanying shear flow?
  - Corrugations in state of low Z_{cc} likely to overturn, and drive avalanches, as in running sandpile?
- Theory should better understand the effect of noise on staircase, which have been considered only in context of Mean Field theory.
- Relation between Z_{cc} and the staircase structure: Does the physics of Z_{cc} set the relative positions of corrugations and shear layer?