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Dynamics of Scalar Concentration Staircases

(Simplest possible model of staircase formation)

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(October 7-11) Atlanta, Georgia

Acknowledgments: R. Sydora, M. Vergassola, D.W. Hughes

See:

1) **10.1103/PhysRevE.109.025209**
2) **Session: UO08.00013 (Thursday)**

Research supported by U.S. Department of Energy under award number DE-FG02-04ER54738.

Outline

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  - Confinement relevance
- Fixed Cellular Array (FCA) Problem
  - Marginally overlapping cells
- Fluctuating Vortex Array (FVA)

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- Passive Scalar Dynamics in FVA
  - Staircase resiliency criteria
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- Summary and ongoing
  - Why should a fusion scientist care?
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Background

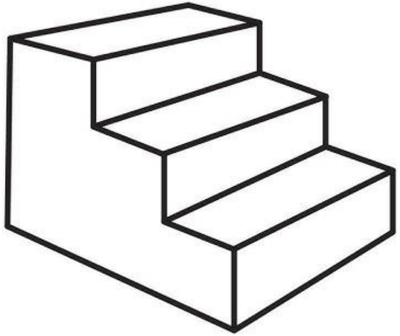
Pattern Formation

The formation of staircases (quasi-periodic layered patterns), is ubiquitous in many physical systems (e.g., doubly diffusive convection, oceanic mixing, rotating geophysical systems, and magnetized plasmas)

Ubiquity ⇒ increased interest in the mechanisms of staircase formation.

- Inhomogeneous mixing occurs ⇒ substances are mixed unevenly, creating distinct layers due to varying transport and mixing rates.
- Such mixing can locally sharpen a scalar gradient, producing jumps and steps.

Mathijssen et al, 2023



Confinement Relevance

Near-marginal plasmas can evolve towards a globally organized critical state of micro-barriers and avalanche-like transport.

“Weak turbulence”

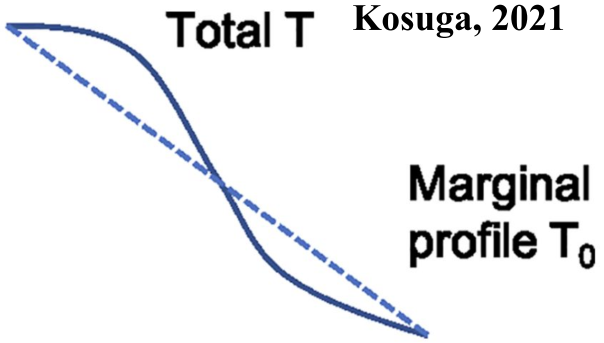
- ***E×B*** convective cells and magnetic islands excited but not strongly overlapping

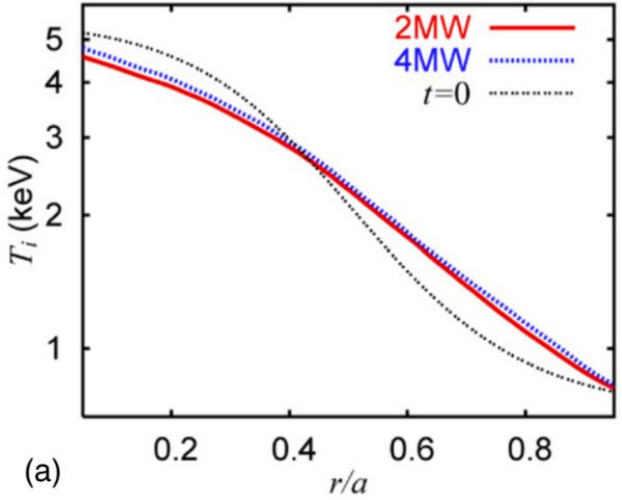
→ Instabilities are excited but not so strong as to produce large transport

⇒ Result: **Stiff profiles**

- i.e., Profiles that adopt roughly the same shape, regardless of the applied heating and fueling profiles

Kosuga, 2021





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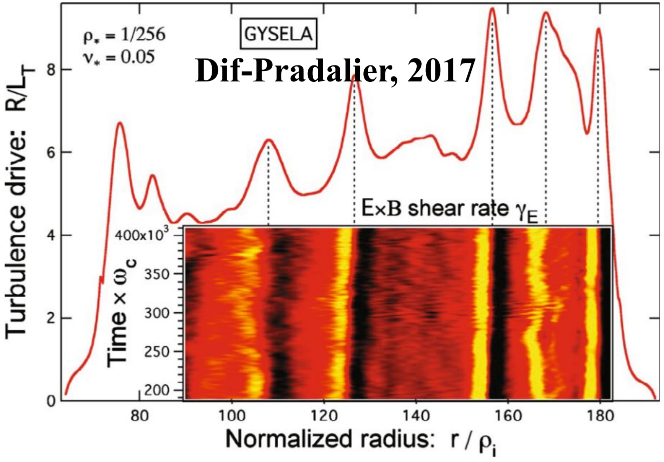
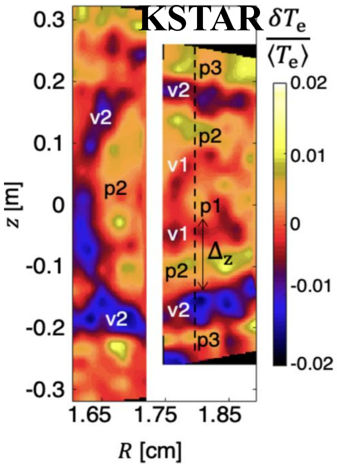
E×B Staircase

"Computer Simulation"



"Experiment"



Context: Flat spots of high transport and mini-barrier layers coexist. In plasmas, avalanches happen in flat spots and shear layers due to zonal flows occur in the areas of mini-barriers

Suggested ideas:

- ExB shear feedback, predator-prey
  - Zonal flows (predator) and turbulence intensity (prey)
- Jams (time-delay between temperature modulations and local heat flux)

Some Questions:

- How does staircase avoid homogenization?
- Is the staircase a meta-stable state?
- What is the minimal set of scales to recover layering?

But… is there an even **simpler** physical mechanism that can produce **layering**?

Answer: Yes (e.g., pattern of cells)

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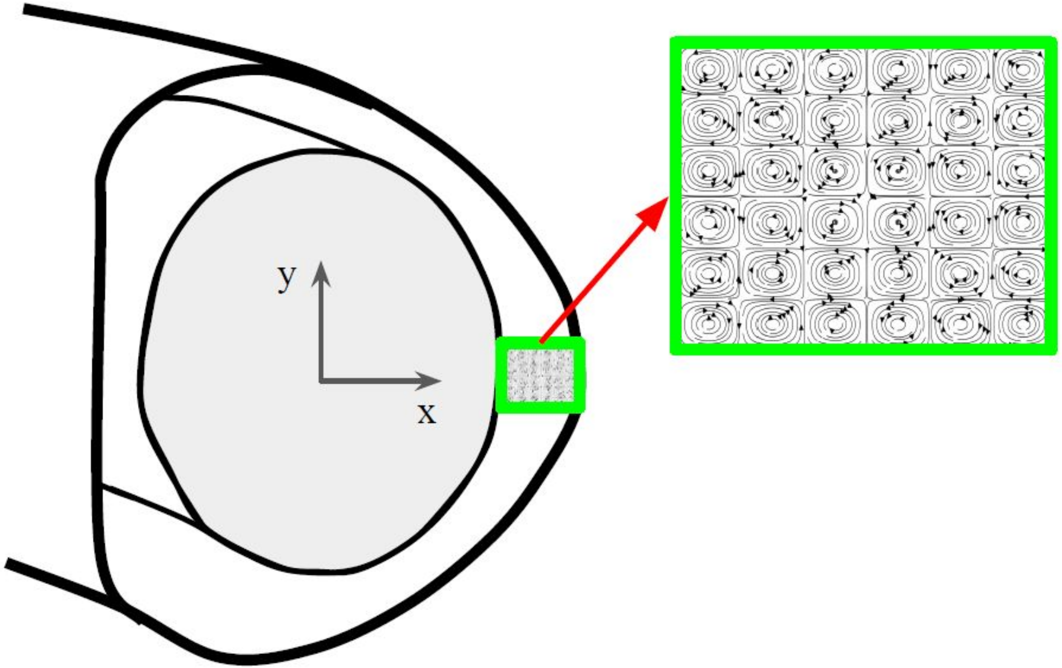

Fixed Cellular Array Problem

(Simplest route to staircase formation)

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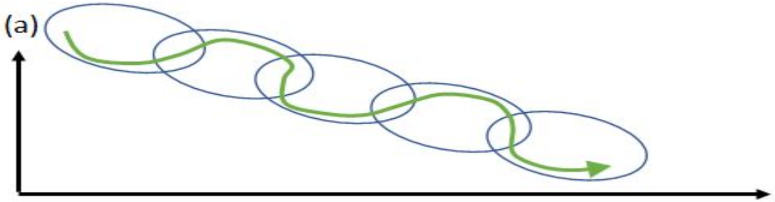
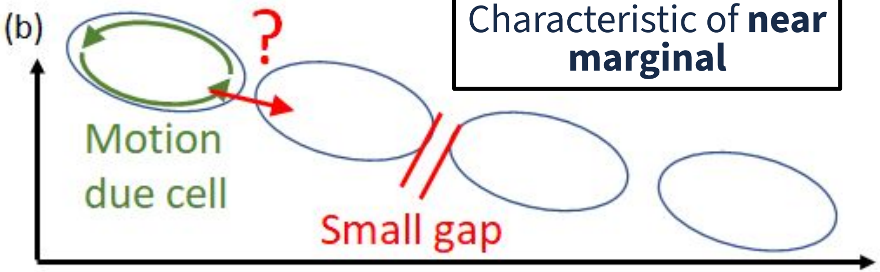
Marginally Overlapping Cells

Transport between marginally overlapping cells is an important topic in fusion plasma.

Overlapping: particles can transport directly from cell to cell, wandering along streamlines



Nearly-overlapping (cells sit at near overlap): transport is a synergy of motion due to cells and random kicks (Collisional diffusion, ambient scattering) thru gap regions.



Characteristic of **near marginal**

Coexistence of:

~ **Fast** transport - Mixing in cell

~ **Slow** transport - kicks between cells (ambient diffusion)

Simple Cell Model

Layering mechanism involves fast homogenization within cells and slow transport across cell boundaries.

The interplay of two disparate timescales, is crucial.

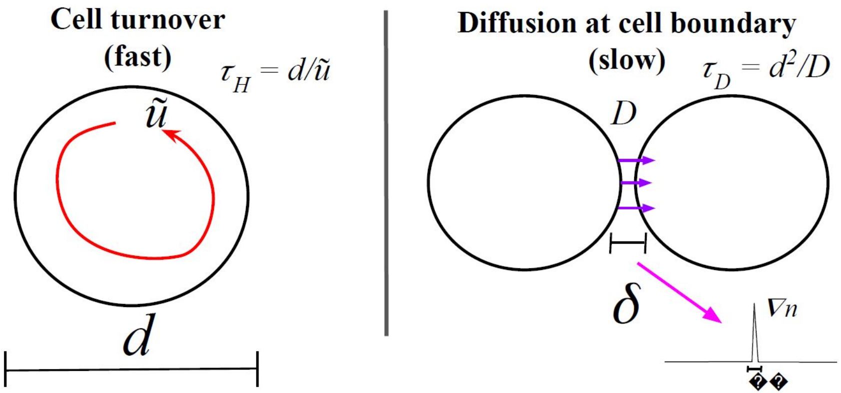
Slow diffusion across inter-cell boundary steepens the scalar concentration gradient!

→ The regulation of transport by these two timescales causes layering.

The ratio of these timescales is given by the Peclet number, with layering occuring when Pe >> 1):

$$\mathrm{Pe} = \frac{\tau_D}{\tau_H} \gg 1$$

Inhomogeneous Mixing (passive)



To derive a scaling of boundary scattering length (molecular diffusion scattering step) to cell size, consider the following ($\delta < d$):

$$\delta^2 = \int^t dt' \int^t dt'' \langle v_c(t') v_c(t'') \rangle = D\, t$$

$$\Rightarrow \quad \frac{\delta^2}{d^2} = \frac{1}{\mathrm{Pe}}$$

Here, **<u>scaling</u>** is determined by "Pe" alone!

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Transport and Profile in Fixed Cellular Array

Profile?
Consider concentration of injected dye (passive scalar transport in eddys) → profile

- Staircase arises in stationary array of passive eddies (Note that there is no "FEEDBACK")

"Steps" = cell length (d) and "jumps" = cell boundary (δ)

Transport?
Answer: $D_{eff} \sim [D D_{cell}]^{1/2}$ (Not a simple addition!)

Back-of-Envelope Calculation

$D^* \approx f_{active}((\Delta x)^2 / \Delta t);$

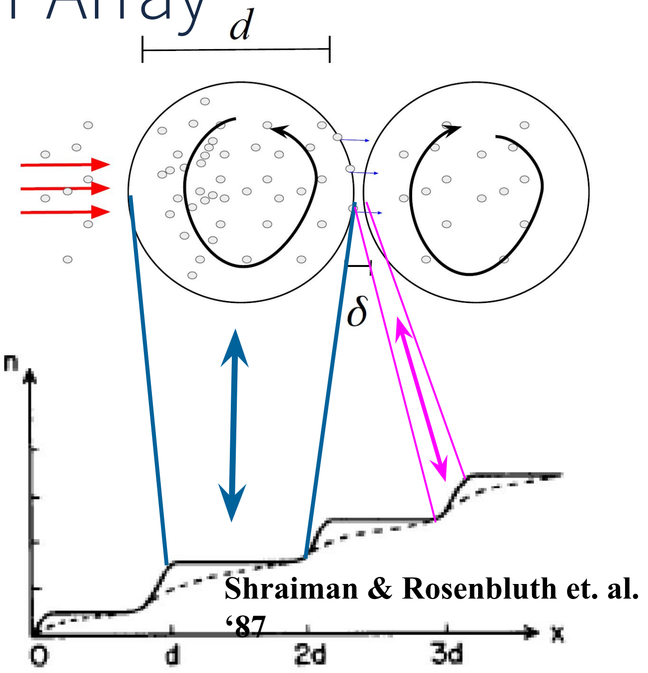
$f_{active} \equiv$ active fraction $\sim \delta / d$

$\Delta t \sim d / U_o \rightarrow$ cell circulation time

So, $\delta^2 \sim D \Delta t \sim D d / U_o$

$D^* \sim [(D d / U_o)^{1/2} 1 / d] (d^2 / d) U_o \sim [D D_{cell}]^{1/2}$

Key: Closed tangential cell boundaries are essential to layering.



"Steep transitions in the density exist between each cell."

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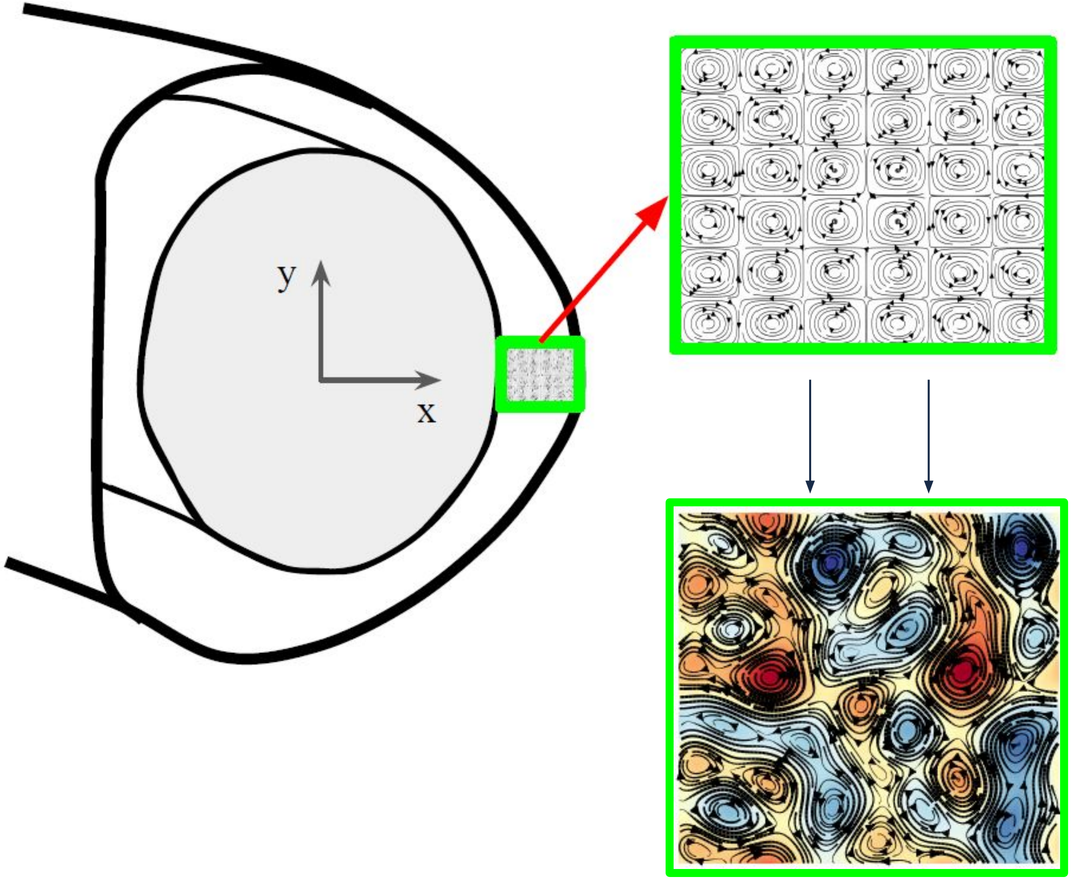
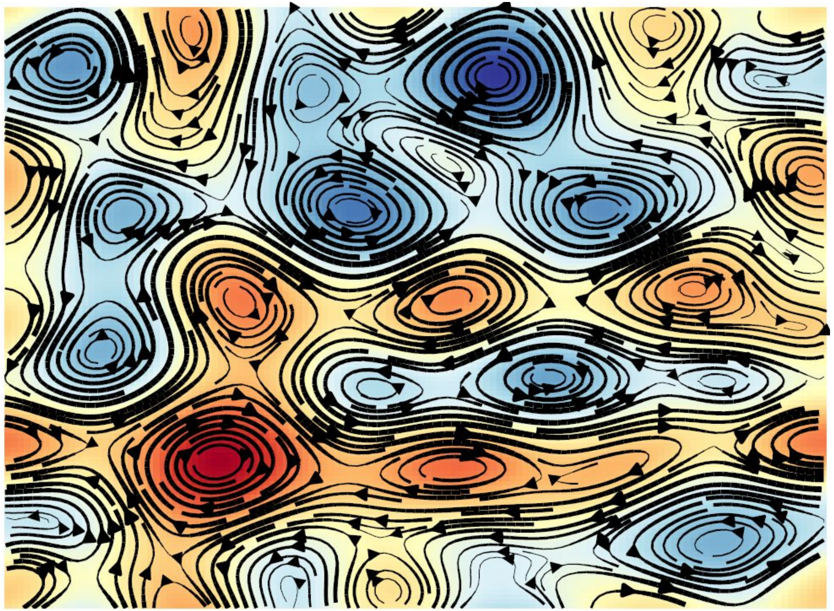


Relaxing Fixed Cellular Array with Fluctuating Vortex Array

(What of the case of a more realistic cellular array?)

Consider a Broader Approach

Example of **less constrained** cell array



To answer these questions, we use the idea of a **Melting Vortex Crystal** (recall: Debye Model)…

How **resilient** is the staircase in the presence of these small variations to a fixed vortex array?

In the process of studying the **resilience** of the staircase, we also aim to answer:

1. What happens to interspersed regions of strong scalar concentration mixing as cells vibrate?
2. What is the behavior of the scalar trajectory through the vortex array?
3. How does the increase of scattering affect the transport of scalar concentration?

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Fluctuating Vortex Array (FVA) Model (Perlakar and Pandit 2010)

Driven vorticity equation,

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla}\right)\omega = \frac{1}{\Omega}\nabla^2\omega + F_\omega - \alpha\omega, \qquad \nabla^2\psi = \omega.$$

- The fluctuating flow structure is created by **slowly increasing the Reynolds number**,

$$\Omega = \frac{\tau_\nu}{\tau_H}$$

- Increasing the Reynolds number modifies the forcing and drag term, **scattering** the vortex array. Staircase **resilience** is examined by increasing disorder in the array through:

$$F_\omega \equiv -n^3\left[\cos\left(nx\right) + \cos\left(ny\right)\right]/\Omega$$

The evolving streamfunction (ψ) is used in the passive scalar equation to study the resilience of the staircase.

$$\frac{\partial n}{\partial t} + \boxed{\mathbf{u}}\cdot\nabla n = D\nabla^2 n,$$

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Correspondence of Vortex Array to Drift-wave Turbulence

	Vortex Field	Drift-Wave Turbulence (tokamak)
Inhomogeneity (free energy source)	∇n	B_0, ∇n, and ∇T
Reynolds number	$\Omega = 0 - 40$	$Re^* = 10^1 - 10^2$ (Landau Damping)
Flux	Scalar concentration	Heat
Boundary Layer	Inter-cell boundary between cells	Shear layer (poloidal)

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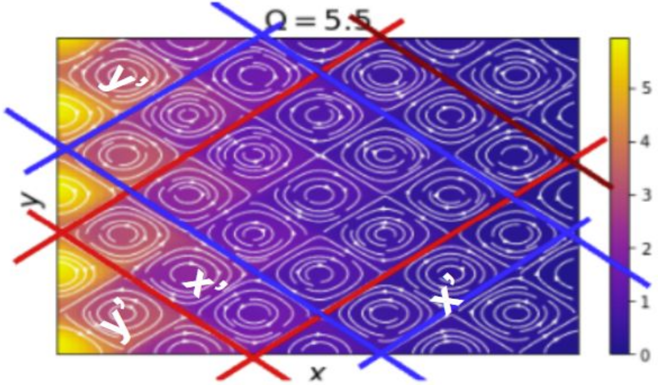
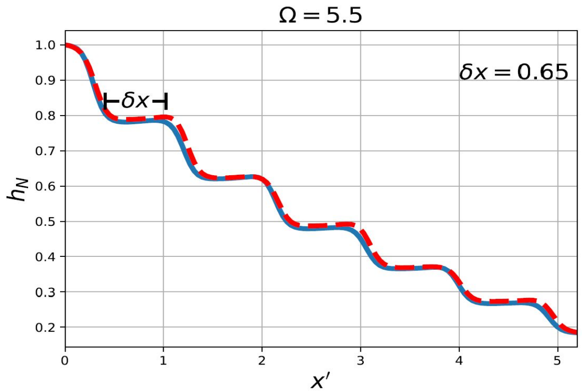
What Happens to the Staircase?

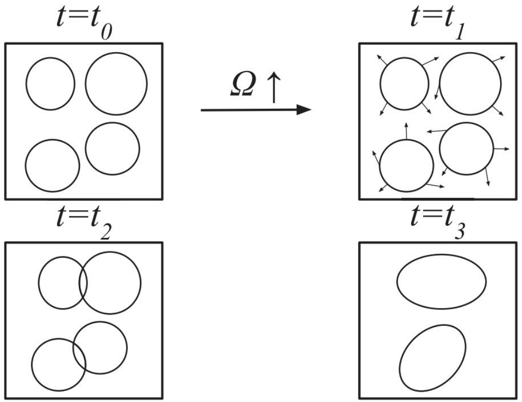
Baseline staircase structure (Case of weak vortex fluctuation)





$$\langle n(x')\rangle_{y'} = \int_0^{L'} n(x',y')dy'$$

$$h(x') = \langle n(x')\rangle_{y'}$$



For weak fluctuations, cell size is low!

- But if increase fluctuations ($\Omega\uparrow$) → Melting occurs!

Melting≡ $\delta x_{cell}/d \rightarrow 1$ (Lindemann ratio)

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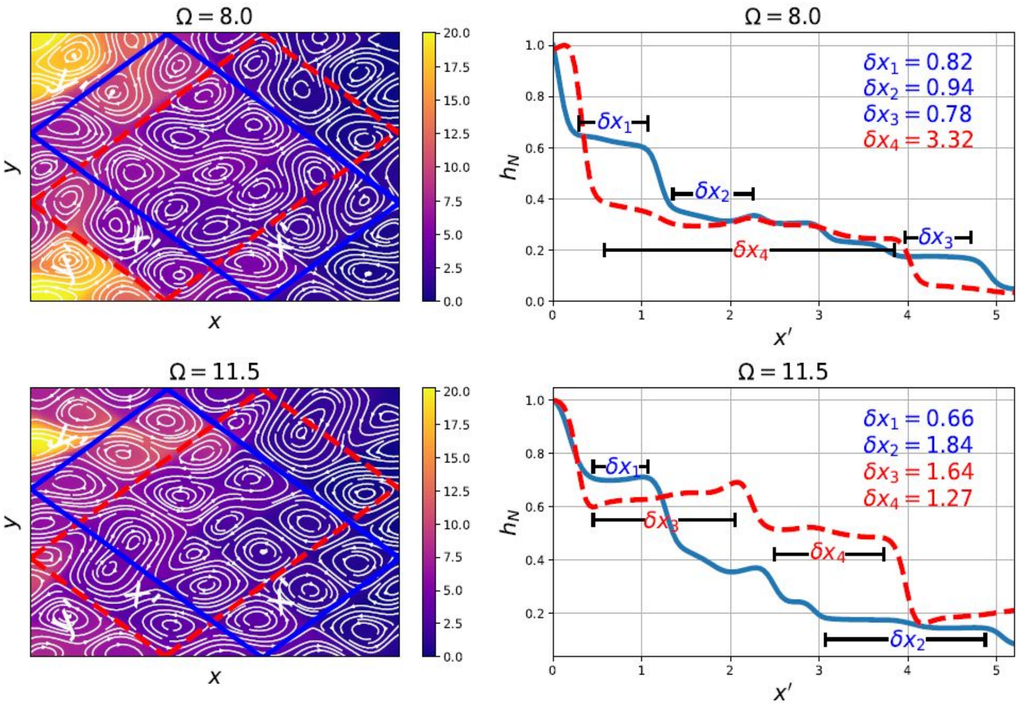
Staircase Resiliency to Fluctuations

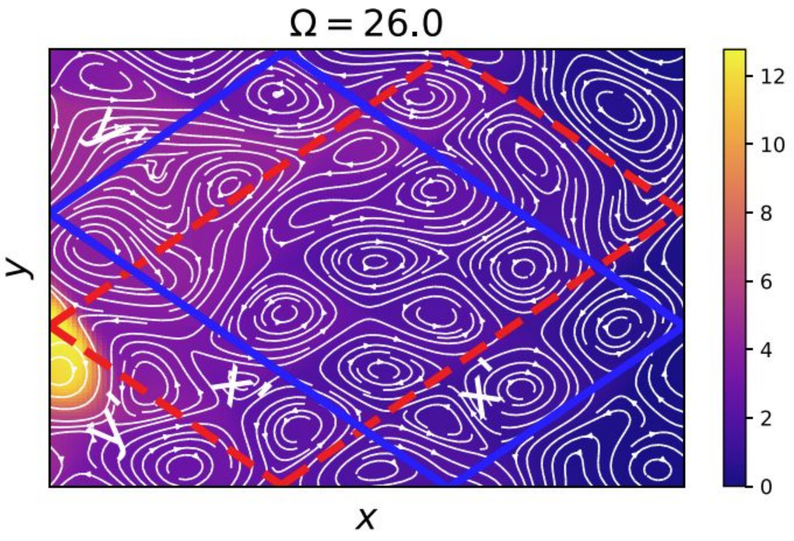
$$h(x') = \langle n(x') \rangle_{y'}$$

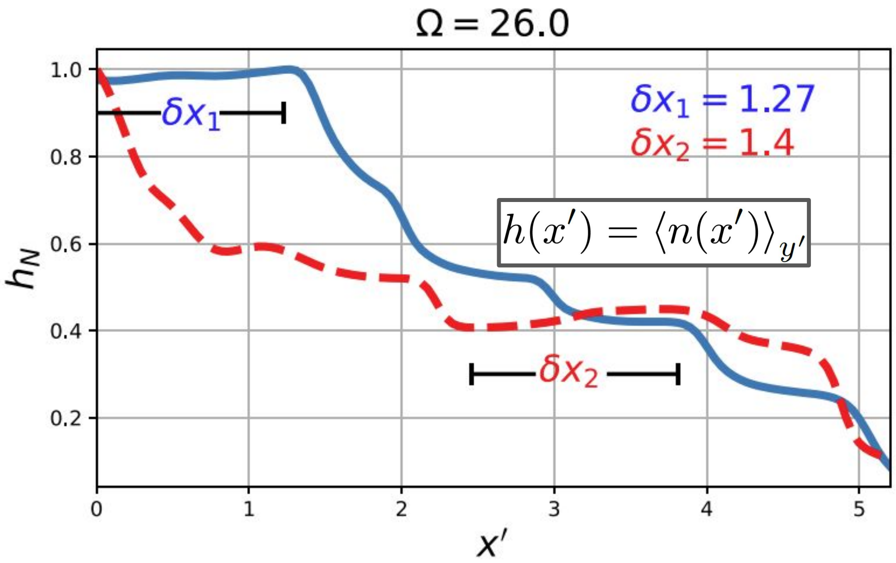
- As we **increase fluctuations in vortex array** through Ω, we see **mergers/connections** of vortices.
  - Staircase unchanged until Lindemann ~ 1.
- **Vortex mergers** appear in the scalar profile as step **mergers between steps**.
  - As jittering ↑, staircase steps merge together.

Steps: $dh/dx \sim 0$
Jumps: $dh/dx \gg 1$



Staircase Resiliency to Fluctuations (cont.d)





<u>Main Point</u>: Despite increasing turbulence, the staircase structure does **<u>not</u>** collapse.

- Staircase steps become **less regular**. Longer steps develop as *Ω*↑ – coarsening.

→

Okay, but how to quantify?

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Criteria for Staircase Resiliency

We establish a **set of criteria** for "**resiliency**":

1. Pe $\gg$ 1, is **necessary** for **transport barriers** in the process of scalar mixing (**First principles**). Pe $\gg$ 1 criterion is satisfied for the range of $0 < \Omega < 40$.
2. A staircase should **maintain a sufficiently high curvature** (equivalent to sustaining a sufficient number of steps). Our studies suggest that $\kappa \gtrsim 1.5$ is an adequate value for a staircase.

1. $$Pe = \frac{\tau_D}{\tau_H}$$

2. $$\kappa = \int_0^{L'} \frac{h''(x')}{(1 + h'(x')^2)^{3/2}} \, dx'$$ $$h(x') = \langle n(x') \rangle_{y'}$$

Note: κ correlates with the global layering structure and quantifies the staircase step length and number of steps within the profile.

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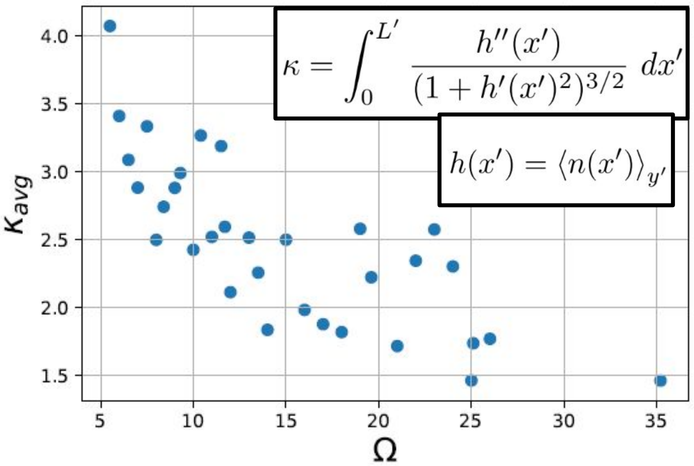
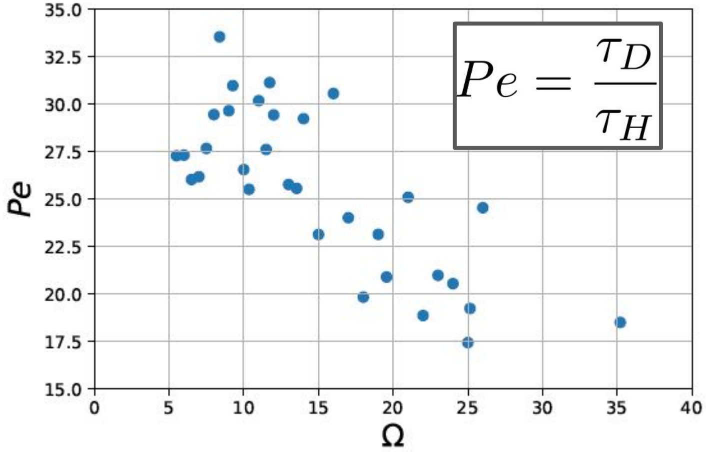
Criteria for Staircase Resiliency (cont.d)

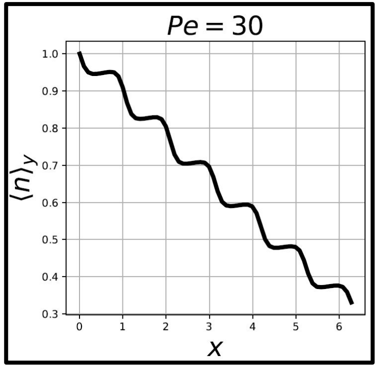
2. **Macroscopic measure**



1. **Microscopic measure**



Frozen vortex array reference



For a broad range of modest Reynolds numbers (0<Ω<40), both (1) Peclet and (2) profile curvature decrease!

- Recall: regions of corrugations coarsen, due to mergers.

Staircase Resiliency Criterion	
Peclet number	$Pe \gg 1$
Curvature	$\kappa \gtrsim 1.5$

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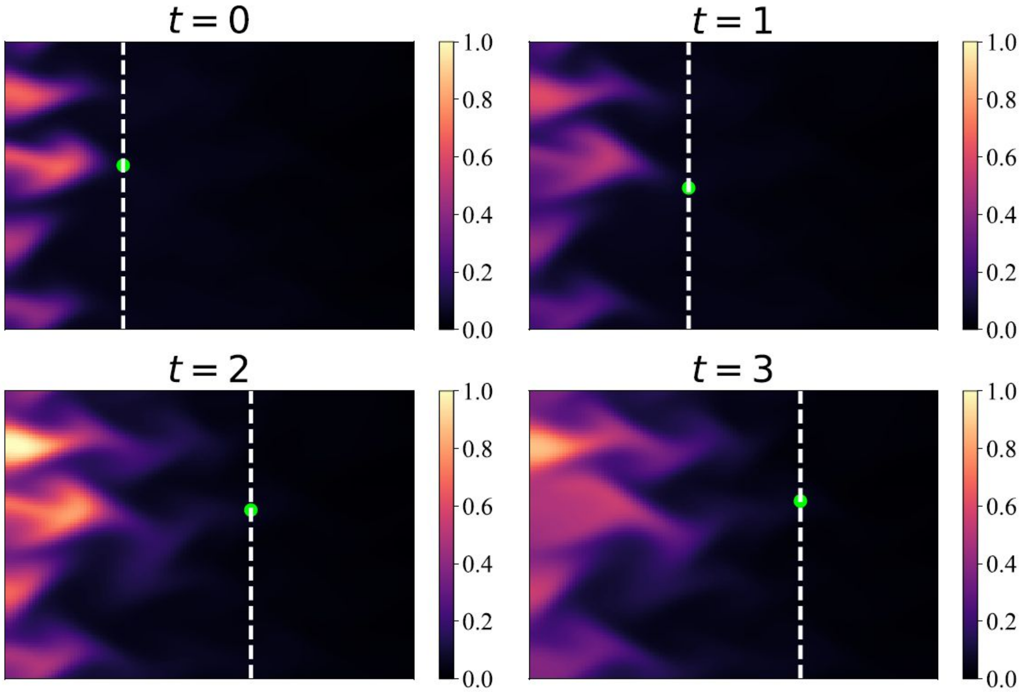

Profile staircase formation dynamics

⇒ Imaging structure evolution in near-marginal state

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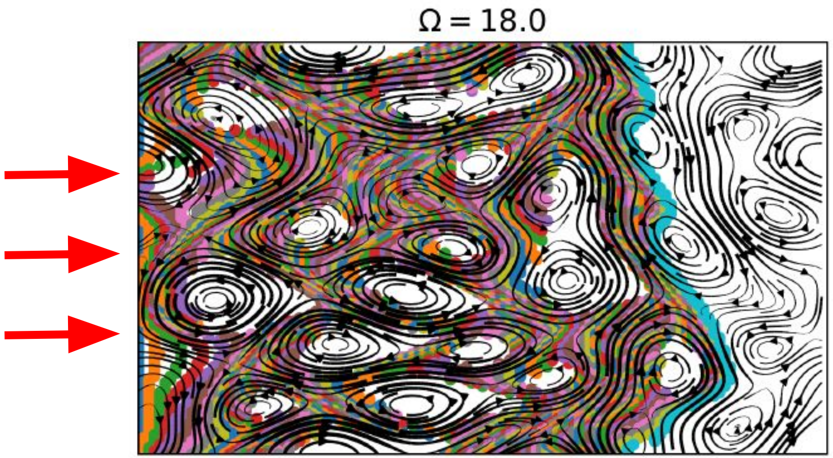
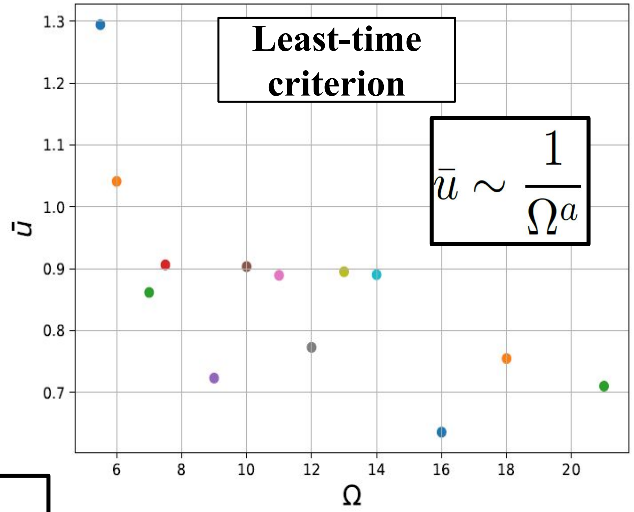
Trajectory in Scattered VA → How a Avalanche Might Propagate





Before the **staircase** forms, scalar flows **quickly along regions of strong shear** and around vortices!

- Staircase **barriers form first**! Scalar travels along cell boundaries.
- Vortex **entrains** scalar by "**homogenization**" process via differential rotation and diffusion.

Scattering of vortices leads to overall **decrease** in scalar concentration front **velocity** (Agrees with Fermat's principle).

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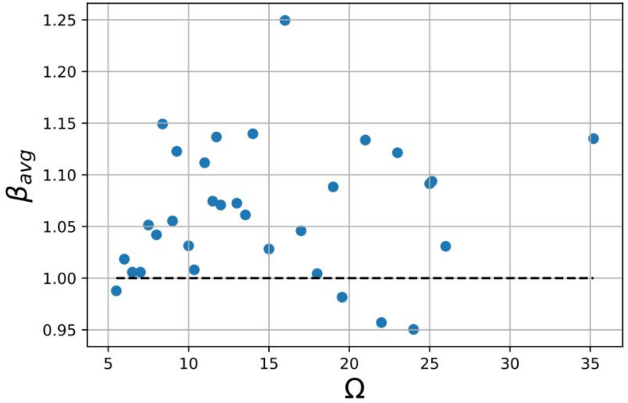
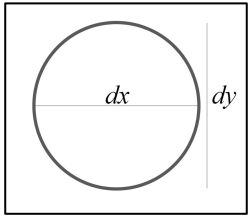
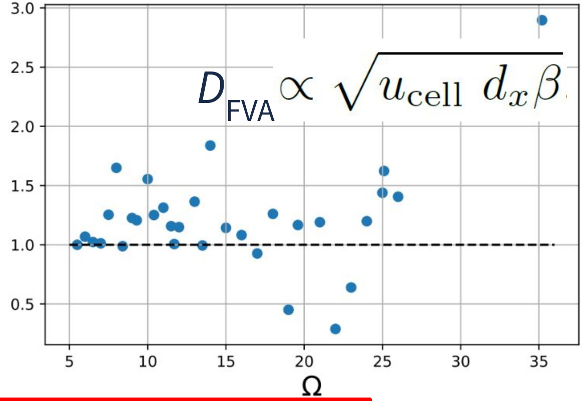
Effective Diffusivity in Fluctuating Vortex Array D_{FVA}/D_{FCA}



$\beta = d_x / d_y$





$D_{FVA} \propto \sqrt{u_{\text{cell}}\ d_x \beta}$

$D_{FC} \approx \sqrt{D D_{\text{cell}}}$

Effective diffusivity remains close to (D_{FCA}).

- **Note**: only **dimensions** and **turnover velocity** of the cells **affect transport**.

Suggests that the fixed array effective diffusivity is a **good approximation** even if **cells are irregular**!

As long as **boundaries** of the cells are **maintained**, effective diffusivity and transport **does not change significantly**.

- We examine the effects of d_x and d_y, as our emphasis is on the **impact** of cell geometry on pattern formation (β approximates trend).

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Summary

DOI:
10.1103/PhysRevE.109.025209

1. Staircases form in a fluctuating vortex array across modest Reynolds numbers ($0 < \Omega < 40$). Coarsen → cell mergers (Lindemann).
2. Staircases resilient for Pe $\gg$ 1 and $\kappa \geq 1.5$.
3. Scalar concentration travels along regions of strong shear.
   a. Staircase jumps form first, then scalar “homogenizes” in vortices.
4. Cell geometry approximates the trend of effective diffusivity.
   a. Effective diffusivity in the fluctuating vortex array does not deviate significantly from $\sqrt{(D\, D_{cell})}$.

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Why should a fusion scientist care about this?

These results have interesting implications:

1. Effective diffusivity for fixed cellular array is a suitable approximation for the fluctuating cellular array (**not simple addition**: $D^* = D + D_{cell}$).
   - Relevant to cells "touching", not overlapping (similar to near-marginal stability).
2. Staircase structure is resilient in the regime of low-modest "Reynolds numbers" (this regime is relevant to drift-wave turbulence [Re_{eff}]).
   - Structures/Profiles are not exotic, or result of fine tuning.
3. If more saddles than closed vortices, heat avalanches will first form the staircase barrier.
   - Fluctuating cellular flow hinders avalanche propagation.

IMPORTANT: We can test the theory presented here with actual experimental data.

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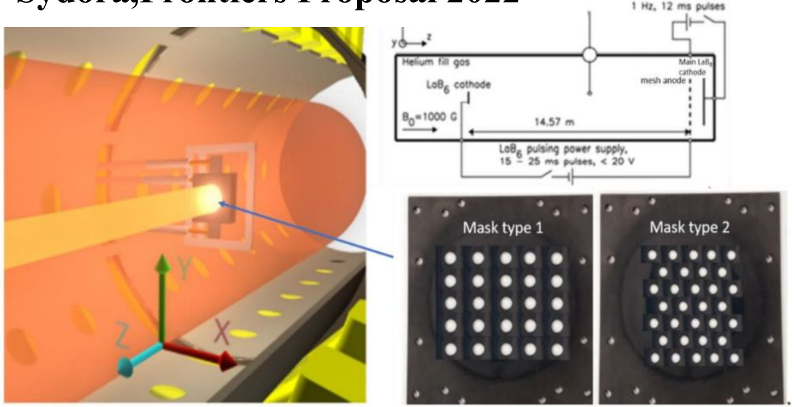
Frontier Experiment **(Work in progress)**

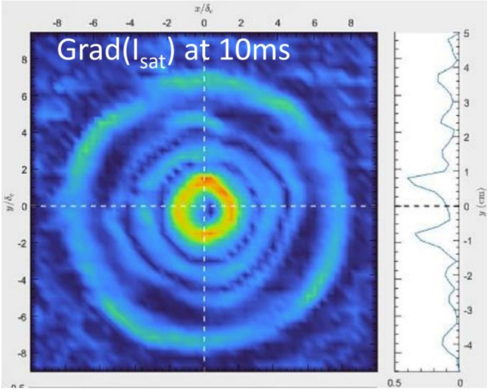
Sydora,Frontiers Proposal 2022



A vortex array has been created in the large linear magnetized plasma device (LAPD).

- Modification of a cathode plasma source with designer masks that form multiple current channels in a cellular pattern → form staircase!
  - Experiment conducted in the afterglow phase of the main discharge.
- Staircase structure can be subject to controllable amount of low frequency density fluctuations, which act as a noise source.
  - Allows us to test hypotheses and models of staircase resiliency!

Results of experiments will yield a unique set of observations that can be used to test staircase models.



Session: UO08.00013

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Thank you!

PHYSICAL REVIEW E **109**, 025209 (2024)

Staircase resiliency in a fluctuating cellular array

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(Received 20 April 2023; revised 23 December 2023; accepted 8 January 2024; published 14 February 2024)

Inhomogeneous mixing by stationary convective cells set in a fixed array is a particularly simple route to layering. Layered profile structures, or staircases, have been observed in many systems, including drift-wave turbulence in magnetic confinement devices. The simplest type of staircase occurs in passive-scalar advection,

Frontier (LAPD) Presentation: **UO08.00013 (Thursday)**

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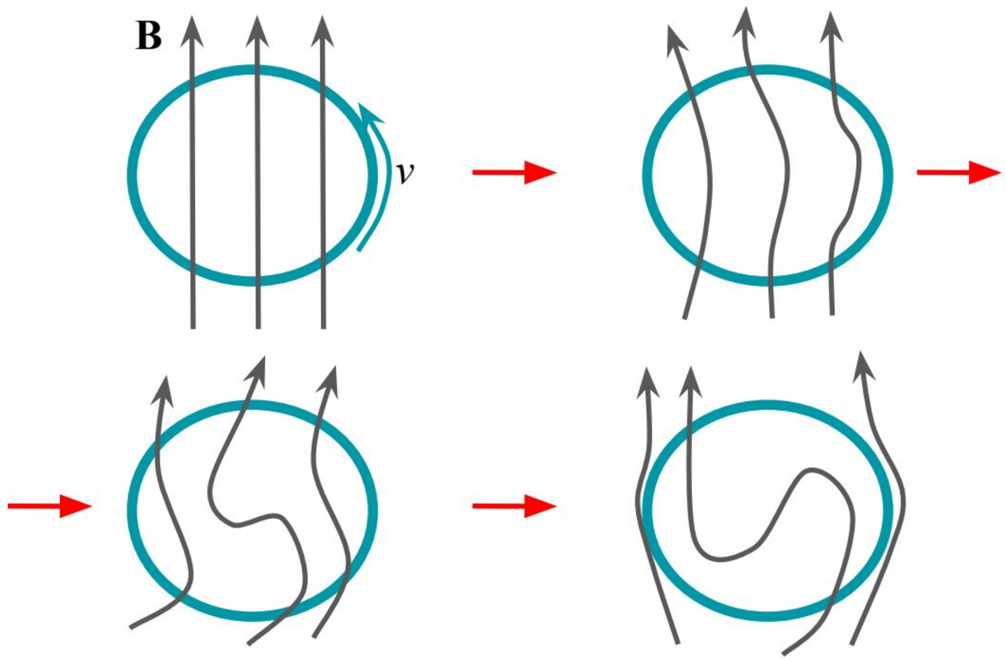
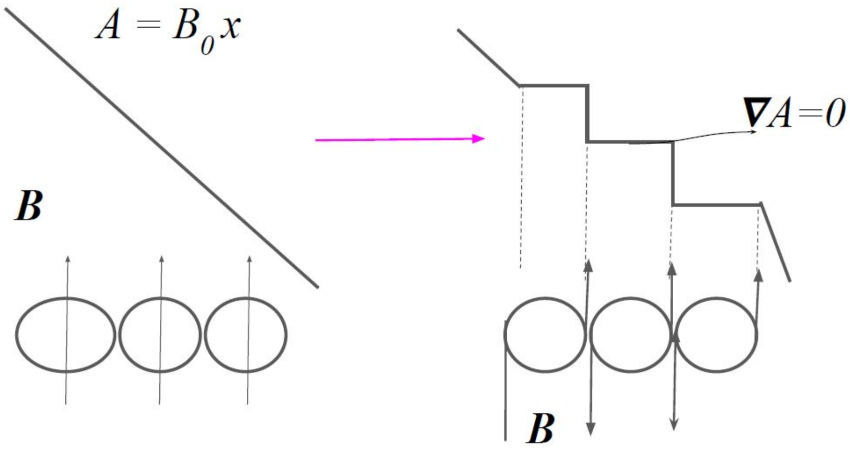


Active Scalar Dynamics

Active Scalar Dynamics

Next we explore the effects an active scalar (*A*) has on the cellular array and inhomogeneous mixing.

- *n* → *A* will result in effects such as **flux expulsion**
  - Flux expulsion is simplest dynamic problem in non-ideal MHD.



Why this model?

- ***B*** expelled to boundaries, thus holds cells together! → Rigid staircase.

Model allows a **feedback** between magnetic field and vortices.

Note: B_o is a control parameter

$$\frac{\partial n}{\partial t} + \mathbf{u}\cdot\nabla n - D\nabla^2 n = 0 \rightarrow \left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right) A = \frac{1}{R_m}\nabla^2 A + F_A$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\omega = \frac{1}{\Omega}\nabla^2\omega + M^2\left(\mathbf{B}\cdot\nabla\nabla^2 A\right) + F_\omega$$

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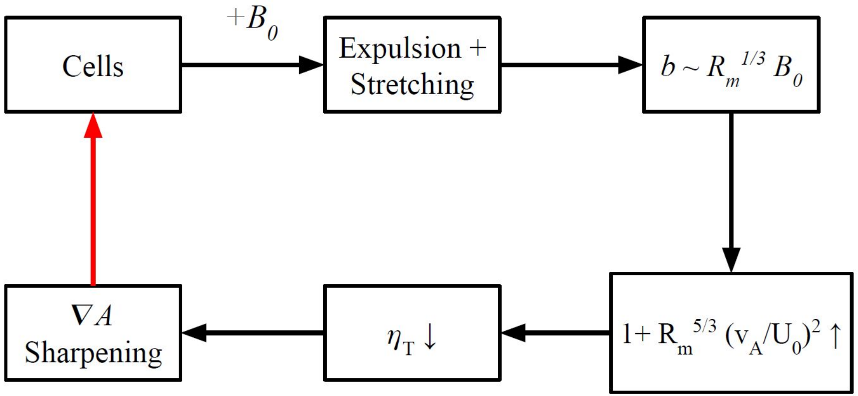
B_0 strengthens cell boundary



$$\frac{d}{dt}\langle \delta x^2\rangle_A \approx -\frac{\langle \tilde{v}_r \tilde{A}\rangle}{\nabla\langle A\rangle} + \eta$$
$$= \frac{\langle \tilde{v}_r^2\rangle \tau_c}{1+R_m v_A^2/\langle \tilde{v}^2\rangle} + \eta$$
$$= \frac{D_T}{1+R_m v_A^2/\langle \tilde{v}^2\rangle} + \eta.$$

$$D_T = \tau_{cT}\langle \tilde{v}_k^2\rangle$$

$$\frac{\langle \delta x^2\rangle_A}{d^2} = \frac{D_T/U_0 d}{1+R_m v_A^2/\langle \tilde{v}^2\rangle} + \frac{\eta}{DPe}$$

Expulsion → Magnetic transport suppression (Quantified by η_T):

$$\eta_T \approx \frac{\eta_k}{1+R_m\langle \mathbf{B}\rangle^2/\langle u^2\rangle},$$
$$\eta_k = \sqrt{\langle u^2\rangle}L.$$

- $\eta_T \ll \eta_k$ in suppressive stage

Suppression occurs for limited time (***B*** eventually decays in 2D)

- But **pulsing** of *A* can **prolong** staircase lifetime!

But strong $\boldsymbol{B_0}$ can disrupt vortex structures!

- Fate of layered structures?? **Ans**: Residual cells maintain staircase structure.

Active Scalar Key Findings

1. Staircase persist in both flux expulsion and vortex disruption limits.
    a. In vortex disruption, residual cells still homogenize *A*.
2. Weak magnetic fields (Rm-dependent) quench turbulent diffusion, increasing the disparity between cell circulation and inter-cell transport times by ~100x, reinforcing the staircase.
3. Staircase lifetime is limited (Zeldovich theorem), but can be extended using pulsing methods!

For a more detailed discussion on the active scalar study, see paper on ArXiv:

<ArXiv Link>

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