

MHD Turbulence in a Prescribed Stochastic Magnetic Field

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Stochastic Approaches to Turbulence
in Hydrodynamical Equations, 2022

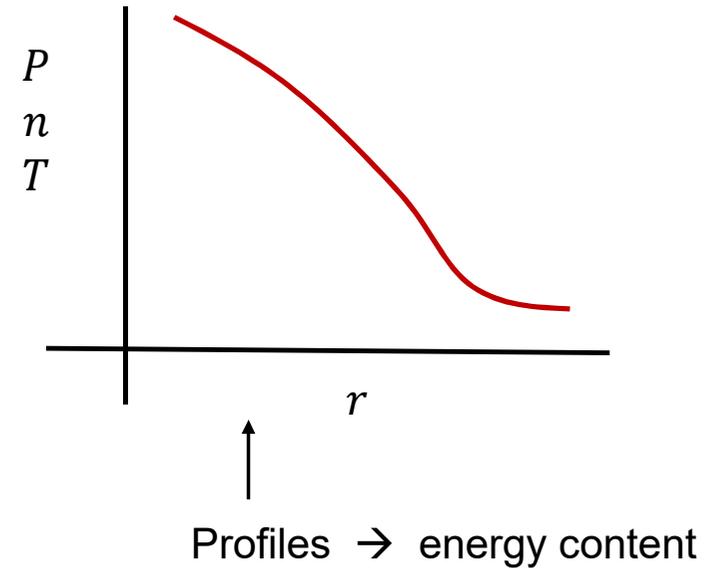
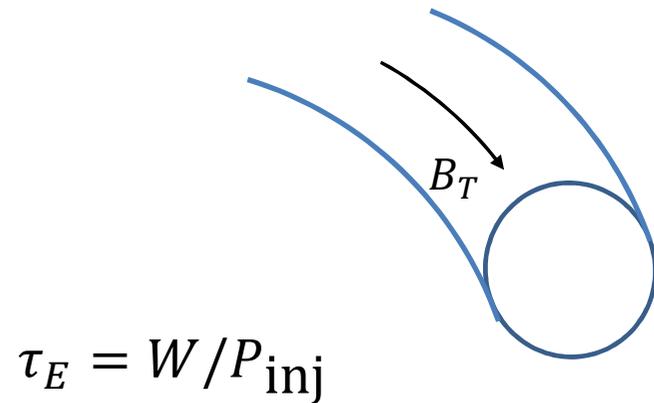
This research was supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, under Award Number DEFG02-04ER54738.

Outline

- Why this? → Confinement is too good...
 - the H-mode
 - Boundary control, RMP and trade-off
- Some Evidence → KSTAR fluctuation analysis
 - Complexity and Bi-coherence
 - Implications
- Beginnings of a Theory
 - Resistive interchange(s) in stochastic field
 - Novelties: 'Micro-cells' and 'Locking-on'
- Outlook

Why this Problem ?

What is a tokamak ?



- Crucial element – plasma boundary
 - Profile gradient at separatrix key indicator of confinement state

Evolution of MFE Theory

Prehistory: 3D

- Beginnings: 60's ~ 1980

Trieste

T3

Micro-stability

Alcator A

Neoclassical theory

PLT

Disruption models

TFR

Taylor Relaxation

- Understanding Good Confinement: 1980 ~ 2010

[Self-Organization]

ExB shear, ZF's

ASDEX → H-mode

Transport Bifurcations

Alcator C, C-Mod → pellet, n-limit

Gyrokinetics, Simulation

TFTR, JET → D-T

AE modes

DIII-D → ETBs, ITBs

Intrinsic Rotation

JT-60U → ETBs, ITBs

Evolution of MFE Theory

- Good Confinement + Good Power Handling → ITER:
2010 – Present, and beyond

ELMs, Peeling-Ballooning

DIII-D, AUG

RMP, QH-mode

Alcator C-Mod

Multi-scale problems

LHD

Core-Edge coupling,

W7X

Turbulence Spreading

RFX-QSH

Disruptions (?)

EAST, KSTAR

SOL Heat Loads (?)

...

...

N.B.:

Return to 3D !

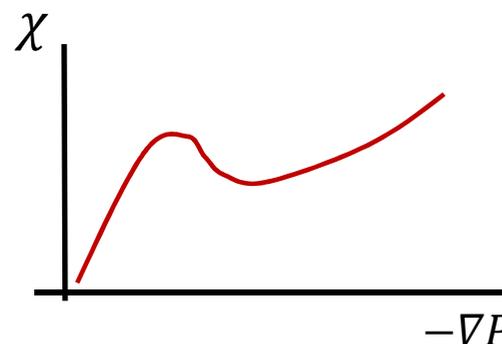
➔ Theory must address trade-offs

Theoretical Problem: L→H Transition

- What of L→H ? → Converging, though still open questions

- Fundamentals:

- Transport bifurcation
- Bistability essential – S curve
- Robust feedback channel – ExB shear flows
- Insulation layer at the edge...



$$\chi_T = \chi_T(V'_{E \times B} / \omega)$$

$$V_{E \times B} = \nabla P / n + \dots$$

$$\chi_T \downarrow \text{ for } V'_{E \times B} / \omega > \text{crit.}$$

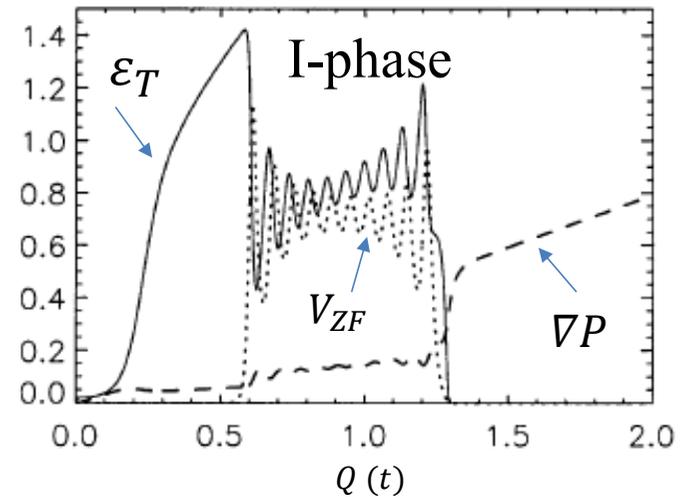
- Subtleties

- What is the “trigger”? → i.e.,
- What physics allows ∇P to steepen?

- Coupling of energy to edge zonal flow

- Interplay of $\varepsilon_T, V_{ZF}, \nabla P$
- $P_{Reynolds}$ crit. needed,
measured (Tynan)
- Crucial to note $E \times B$ flow

Kim, PD, PRL'03



40 Years of H-mode - Lessons

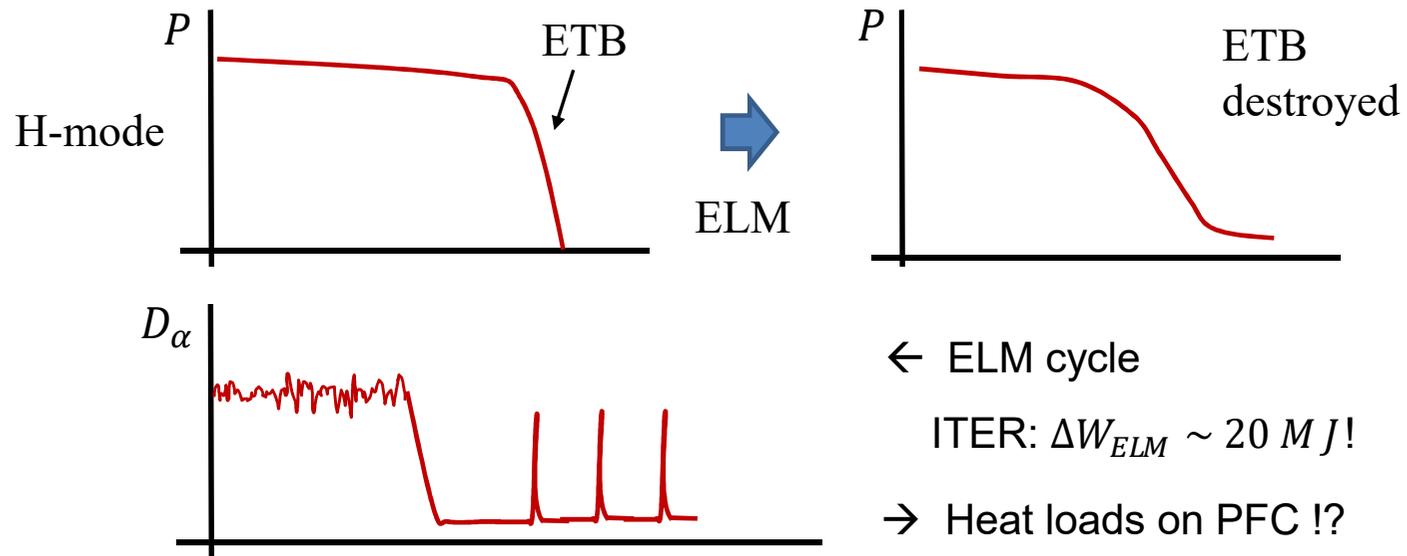
- Saved MFE from Goldston scaling

Also:

- Introduced transport barrier, bifurcation → state 'phases' and transitions
- Role of flow profile in confinement (BDT '90)
- Dynamical feedback loops → Predator-Prey cycles, Zonal flows, etc.
(PD+'94,05; K-D '03)
- Consequences of marked transport reduction
- ➔ • Need for transport regulation, not transport elimination

ELMs and RMP – A Primer

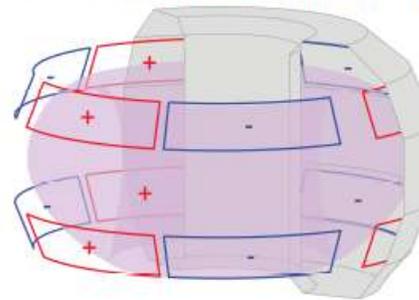
- ELM = Edge Localized Mode (Mode ?!)



- RMP = Resonant Magnetic Perturbation - δB

- Stochastic edge layer
- Pump out - density
- Mitigate, suppress ELMs, with good confinement

$n = 3$ RMPs from internal coils

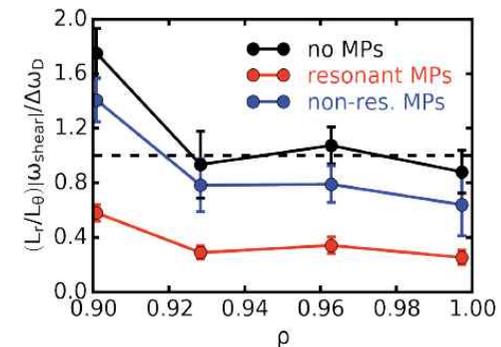


to ITER

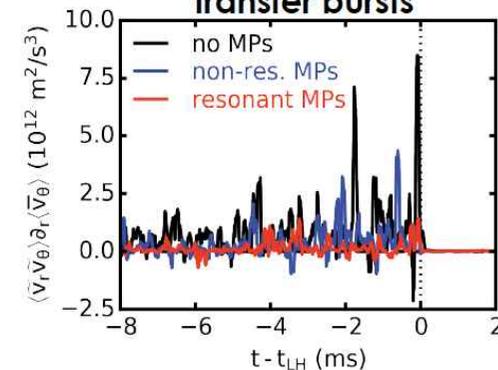
Resonant Magnetic Perturbations Disrupt Shear Suppression of Turbulence, Increasing the L-H Power Threshold

- RMPs reduce flow shear rates ω_{shear} and raise turbulence decorrelation rates $\Delta\omega_D$ in L-mode
- * Shear suppression parameter $\omega_{shear}/\Delta\omega_D$ is reduced significantly below 1
 - More shear flow must be driven to access H-mode
- * RMPs disrupt nonlinear energy transfer from turbulence to flows that can trigger L-H transition

Shear suppression parameter



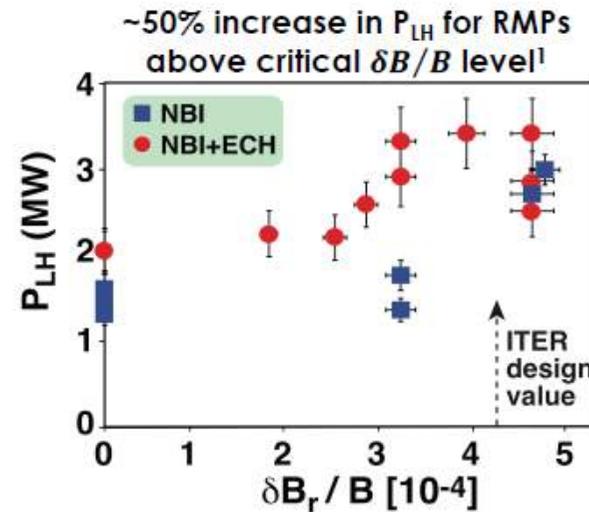
nonlinear energy transfer bursts



Benefit and Cost

- Need make L→H Transition with RMP !
- Increase in P_{th} for L→H !?
 - $(\delta B / B)_{crt}$ for L→H Power increase
 - Significant !
- Issues:

“First ELM the largest”



(resonant vs. non-resonant)

Kriete et. al.
DIII-D

- Why L→H threshold \uparrow due RMP
- What physics defines $(\delta B / B)_{crt}$?
- Turbulence in stochastic magnetic field!

The Problem:

**→ MHD turbulence in ambient
stochastic magnetic field**

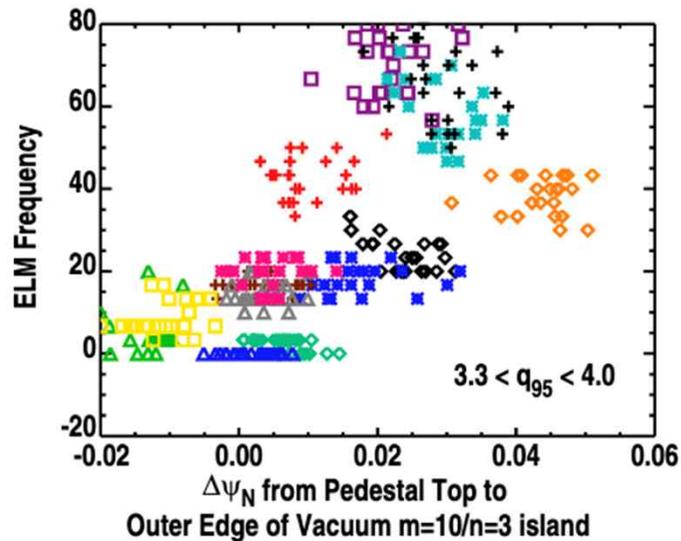
Some Evidence –

KSTAR Fluctuation Studies

Key Question: Stochasticity of Applied Field?

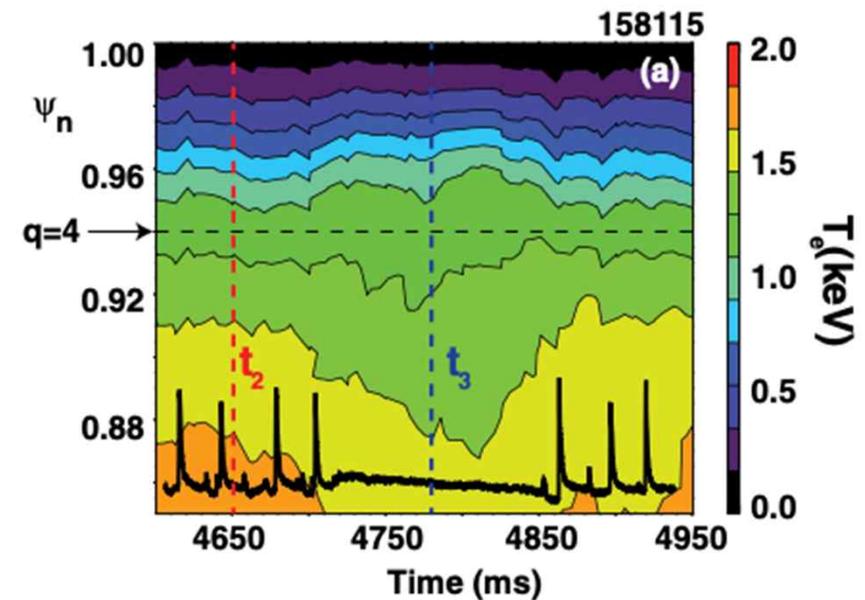
Previous exp. observations implying a stochastic layer

- RMP ELM suppression was achieved when the resonant rational surface is close to pedestal top (R_{tp})



[Wade, Nucl. Fusion 55, 023002 (2015)]

- Localized temperature flattening near R_{tp} during the RMP ELM suppression



[Nazikian, PRL 114, 105002 (2015)]

- Another way to identify a stochastic layer?

Pedestal T_e fluctuation diagnostics & analysis methods

- Localized T_e fluctuation near the pedestal top can be measured using the 2D electron cyclotron emission imaging (ECEI) diagnostics
[Yun, Rev. Sci. Instrum. 85, 11D820 (2014)]

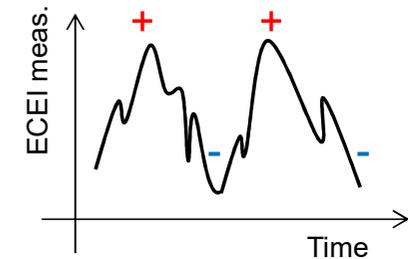
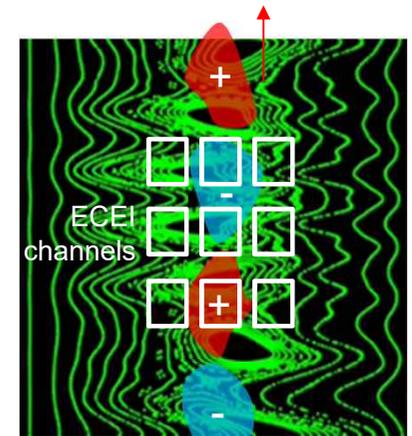
Background figure copied from [Hu, Nucl. Fusion 60, 076001 (2020)]

- Spectral methods

- Cross power spectrum, frequency/wavenumber power spectrum (two-points method), [Beall, J. Appl. Phys. 53, 3933 (1982)]
- wavelet bicoherence [van Milligen, Phys. Plasmas 2, 3017 (1995)]

- Statistical method

- The Complexity-Entropy analysis
[Rosso, Phys. Rev. Lett. 99, 154102 (2007)]

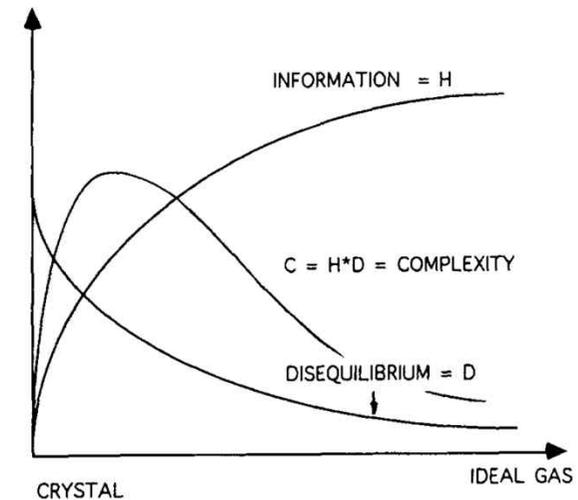


(Information theoretic) meaning of Complexity and Entropy

- Meaning of Entropy : a measure of missing (unknown) information
 - Shannon Entropy $S[P] = -\sum_i p_i \ln p_i$ where $P = \{p_i\}_{i=1,\dots,N}$
 - Normalized Shannon Entropy $H = S/S_{m\ ax}$ where $S_{m\ ax} = \ln N$ for the equiprobable distribution
- Meaning of Complexity : disequilibrium (D) x information (H) [Lopez-Ruiz, Phys. Rev. A 209, 321 (1995)]
 - Disequilibrium : distance from the equiprobable distribution ($P_e = \{p_i = 1/N\}$)

Two examples of a “simple” (not complex) system in physics : a perfect crystal or ideal gas

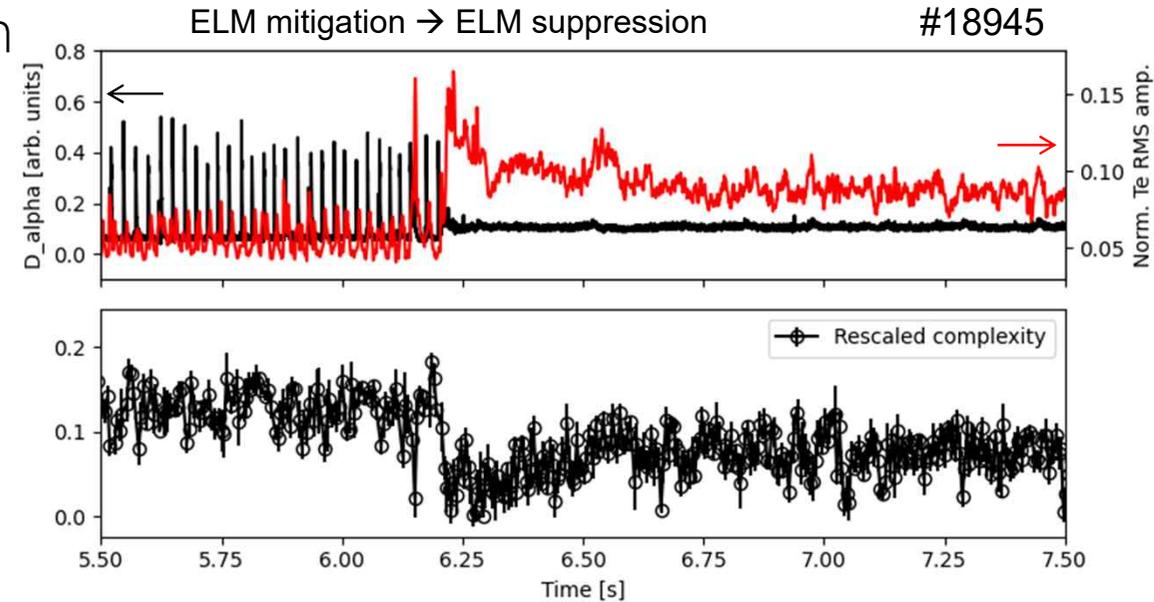
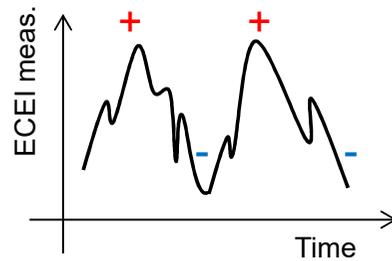
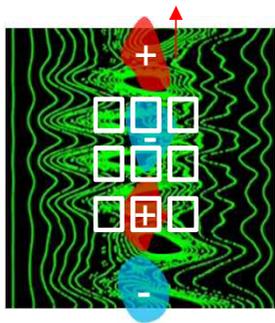
	Information	Disequilibrium	Complexity
Perfect crystal	Small	Large	Small
Ideal gas	Large	Small	Small



- What state for P ? How to measure D ?

Rescaled complexity of T_e fluctuation at the pedestal top

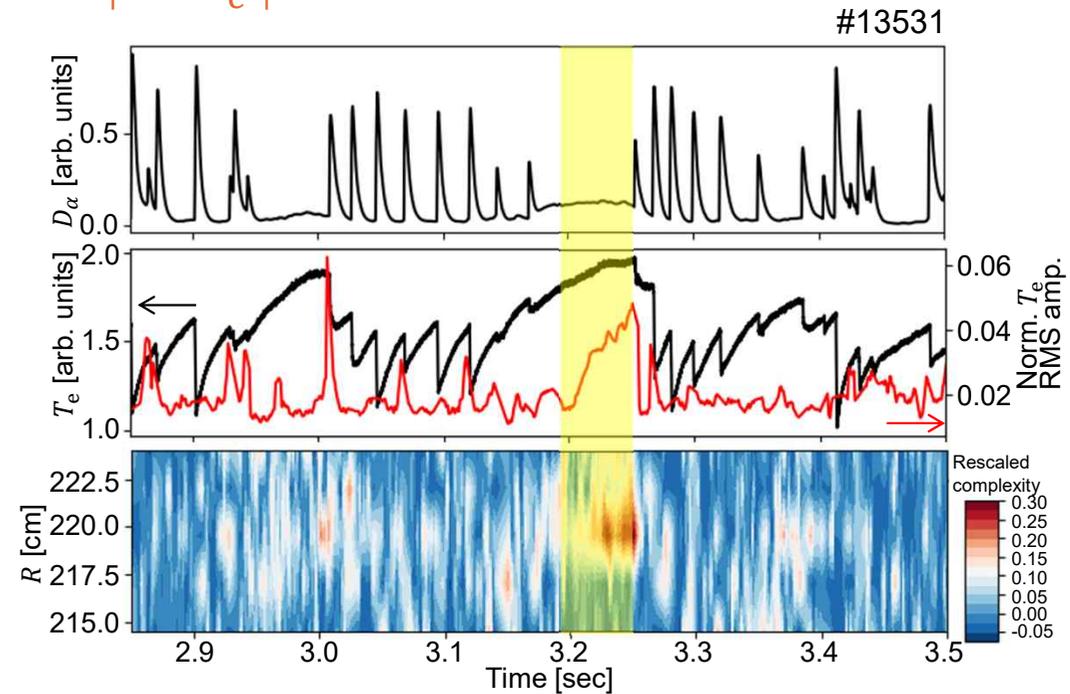
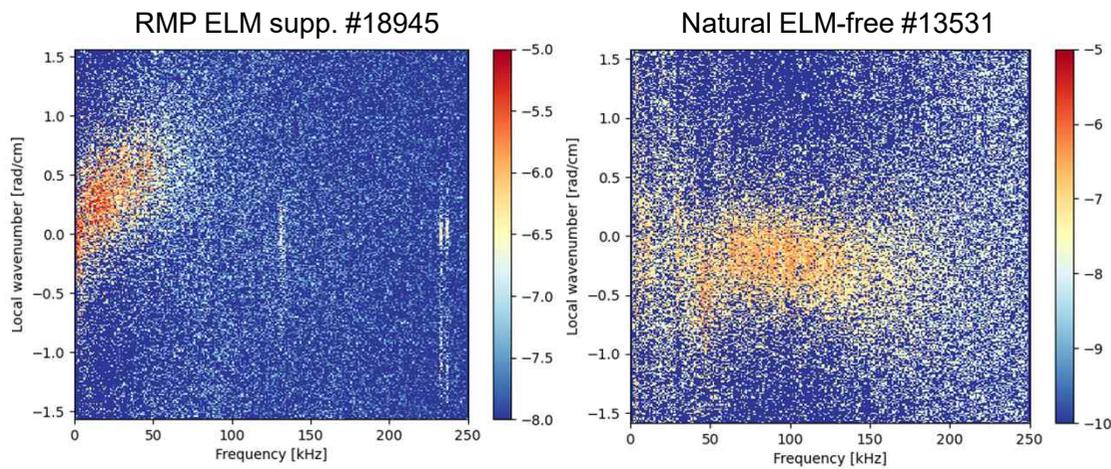
- T_e fluctuation amplitude increases with ELM mitigation-suppression transition
 - It has a broad wavenumber range ($k_\theta \rho_i < 0.4$)
 - It is larger than the inter ELM period level
- Parameters to calculate P_{BP} & \hat{C}
 - Time step between points = 2 μ s
 - Size of each segment = 5 points (10μ s $\sim \frac{cm}{km/s}$)
 - Structures of 10—100 kHz
 - 2500 ($\gg 5!$) segments to calculate one BP PDF



- Rescaled complexity of T_e fluctuation decreases with the ELM suppression
 - T_e becomes less complex (more stochastic)
- Result of stochastic fields?

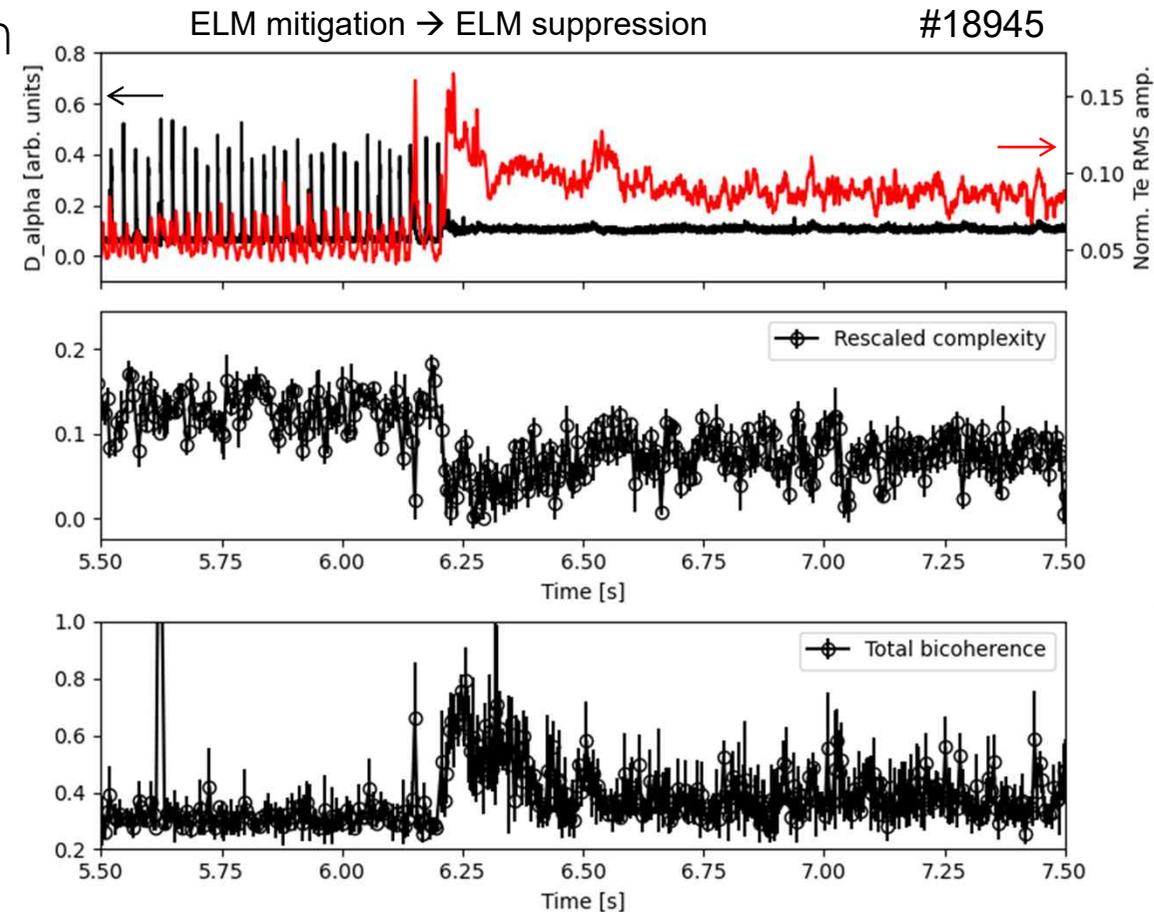
Comparison with a natural ELM-free case

- The natural ELM-free phase
 - The broadband T_e fluctuation increases and its rescaled complexity also increases
 - Turbulence w/o RMP field develops to have a complex T_e pattern rather than to be stochastic



Bicoherence analysis of T_e fluctuation at the pedestal top

- T_e fluctuation amplitude increases with ELM mitigation-suppression transition
 - It has a broad wavenumber range ($k_\theta \rho_i < 0.4$)
 - It is larger than the inter ELM period level
- Rescaled complexity of T_e fluctuation decreases with the ELM suppression
 - T_e becomes less complex (more stochastic)
- Bicoherence of T_e fluctuation increases
 - Triad coupling between $f_1, f_2, f_3 = f_1 + f_2$
- Contradictory result?

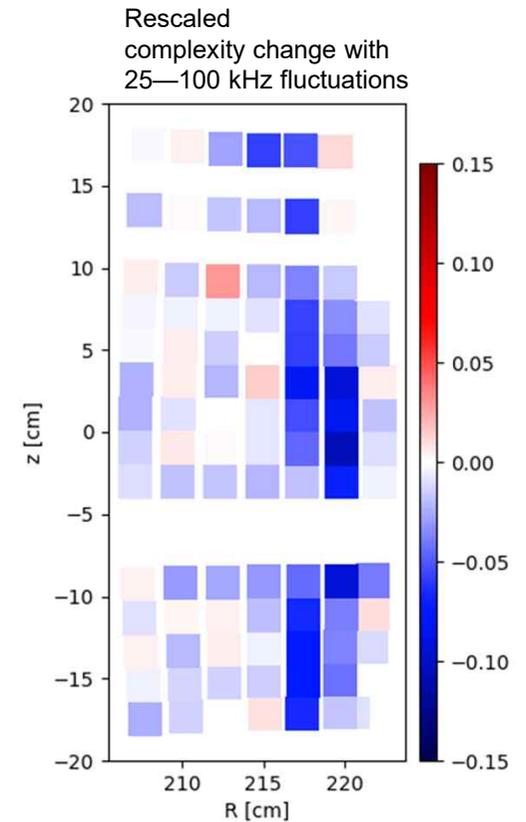
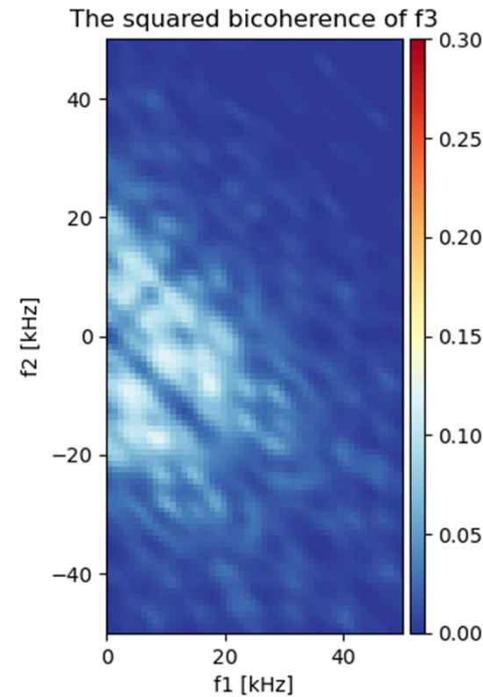
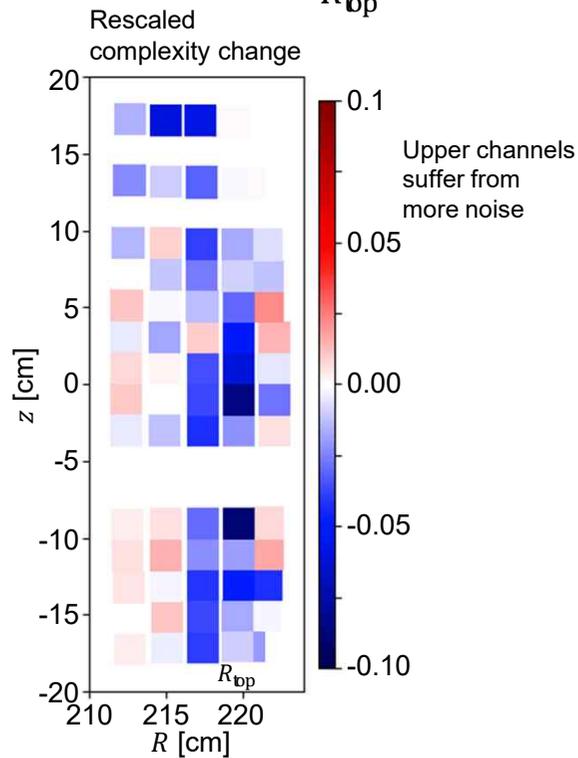
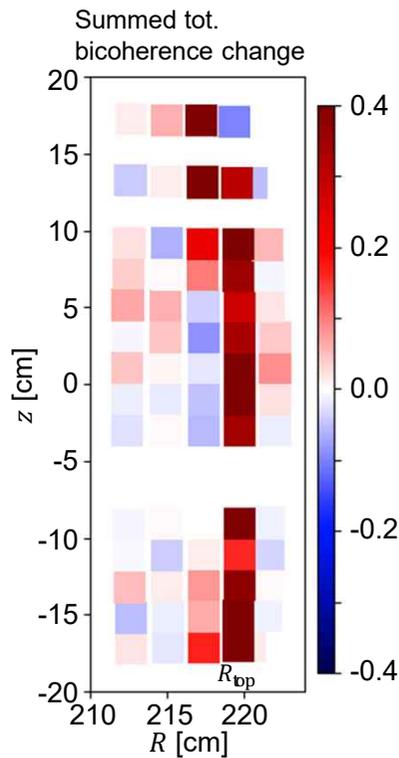
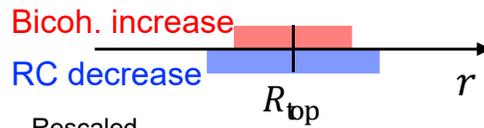


Distinguished in real and frequency space

- 2D structures of bicoherence and rescaled complexity change are different

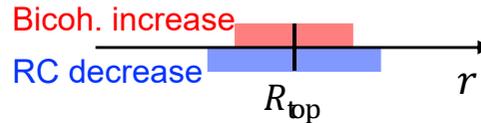
- Bicoherence exists in < 25 kHz, and RC analysis is sensitive to higher frequency fluctuation (~ 100 kHz)

From mitigation to suppression



Our interpretation and a clue for origin of fluctuation incr.

- A partially stochastic island at the pedestal top can explain both bicoherence and rescaled complexity changes

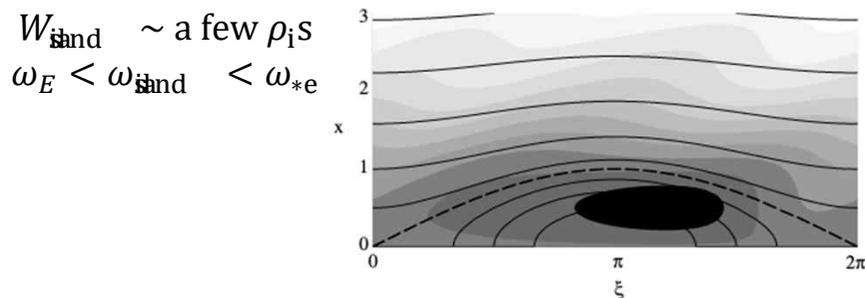


- Low-k and low-f nonlinear coupling between a magnetic island and fluctuation → Bicoherence increase
- The nonlinear resonance condition for drift wave emission might be satisfied in the RMP ELM suppression experiment

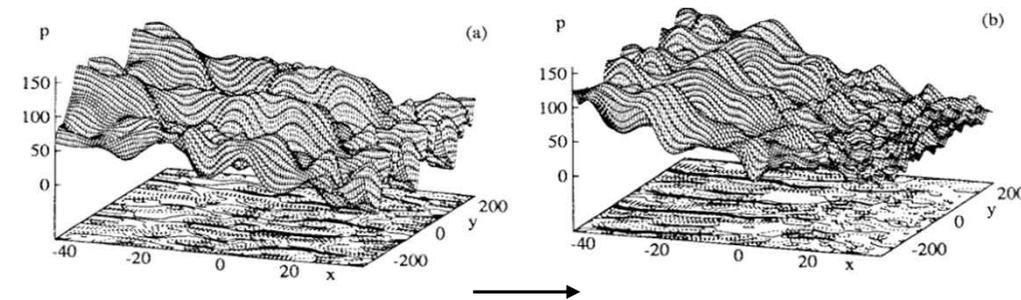
- High-k stochastic fields around the island change fluctuation characteristics → Rescaled complexity decrease
- Enhancement of high-k (high-f) fluctuation
- Turbulence can lock on to stochastic fields, i.e. $\langle \tilde{v}_r \tilde{b}_r \rangle \neq 0$

[Cao & Diamond, APTWG 2021]

[Beyer, Phys. Plasmas 5, 4271 (1998)]



[Waelbroeck, Phys. Rev. Lett. 87, 215003 (2001)]



- Analysis of characteristics of plasma turbulence/transport with the RMP field
 - Previous analyses : kinetic profiles (transport), fluctuation spectra (turbulence intensity, dispersion)
 - In this work, the Complexity-Entropy analysis is adopted to identify/distinguish a state of plasma turbulence/transport and to improve understanding of the state
- Main Results of our analyses
 - CH analysis shows that both pedestal top T_e fluctuation and particle flux at the divertor striking point become less complex (more stochastic) with a RMP field
 - Response of the former seems to be more nonlinear
 - Turbulence dynamics with a RMP field is suggested based on CH and bicoherence analyses
 - Low-k island onset at pedestal top → island drives low-k turbulence nonlinearly
 - high-k turbulence generated by stochastic fields around islands

[Cao & Diamond, APTWG 2021]

[Waelbroeck, Phys. Rev. Lett. 87, 215003 (2001)]

Towards a Theory →

Resistive Interchange (Turbulence)

in a Stochastic B-field

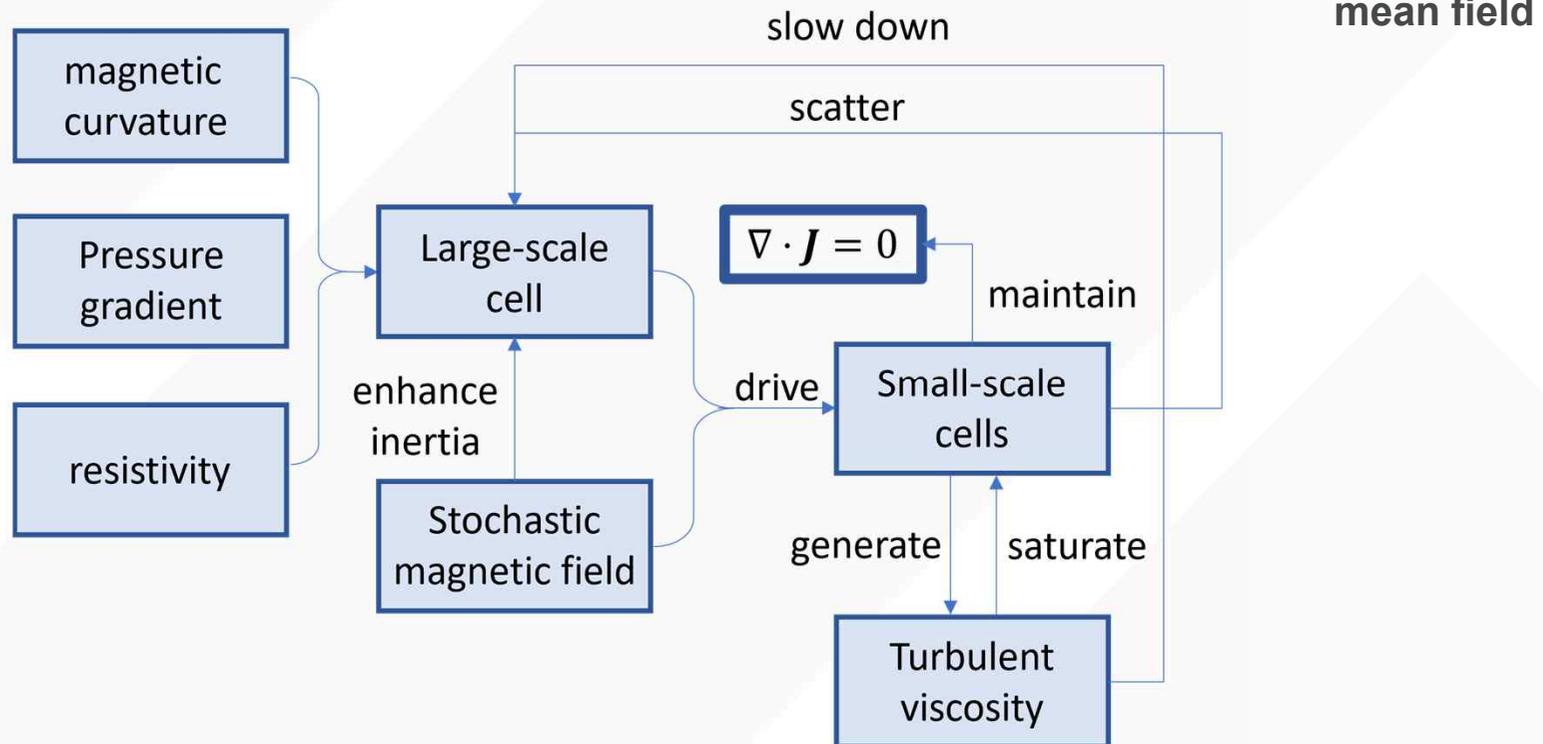
→ Single Cell Problem and Beyond

N.B.: After FKR and Braginsky-Meytlis

INTRODUCTION OF MODEL

Model must:

- maintain $\nabla \cdot J = 0$ at all scales
- connect micro and macro scales
- be tractable \rightarrow resistive interchange mean field



Key point: small scale potential fluctuations are generated due to stochastic magnetic field

INTRODUCTION OF MODEL

Where we start:

1. Classical resistive interchange:

- Linearized vorticity equation

$$\underbrace{-(\rho_0/B_0^2)\partial_t\nabla_\perp^2\varphi}_{\nabla_\perp\cdot J_{pol}} - \underbrace{(g/B_0)\partial_y p}_{\nabla_\perp\cdot J_{PS}} + \underbrace{\mathbf{b}_0\cdot\nabla J_\parallel}_{\nabla_\parallel J_\parallel} = 0$$

$$\nabla\cdot\mathbf{J} = 0$$

- Electrostatic Ohm's law of resistive MHD

$$E_\parallel = -\nabla_\parallel\varphi = \eta_\parallel J_\parallel$$

- Linearized pressure equation

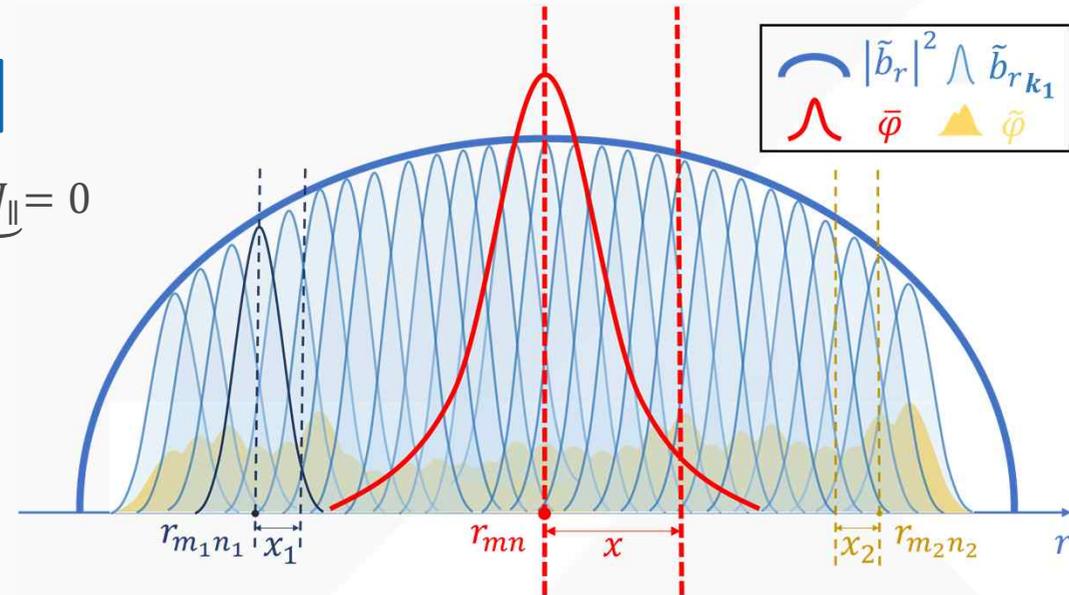
$$\partial_t p - (\nabla\varphi\times\hat{\mathbf{z}})/B_0\cdot\nabla p_0 = 0$$

2. Magnetic perturbations:

$$\tilde{\mathbf{b}} = \tilde{\mathbf{B}}_\perp/B_0 = \sum_{m,n} \tilde{\mathbf{b}}_{m,n}(x')e^{i(m\theta-n\phi)}$$

Since $\mathbf{B}_{tot} = \mathbf{B}_0 + \tilde{\mathbf{B}}_\perp$, now the parallel gradient is $\nabla_\parallel = \nabla_\parallel^{(0)} + \tilde{\mathbf{b}}\cdot\nabla_\perp$.

Compared to mode, the profile of stochastic field evolves much slowly in space.



Sketch of the mode and stochastic magnetic field

INTRODUCTION OF MODEL

Need maintain: we want to keep $\nabla \cdot \mathbf{J} = 0$ at all scales.

If there are only $\tilde{\mathbf{b}}$ and $\bar{\varphi}$, $\nabla \cdot \mathbf{J} = 0$ is not guaranteed!

At micro scale:

$$\tilde{\mathbf{J}}_{\parallel} = \tilde{\mathbf{J}}_{\parallel 0} + \tilde{\mathbf{J}}_{\perp} = -\frac{1}{\eta_{\parallel}} (\tilde{\mathbf{b}} \cdot \nabla_{\perp}) \bar{\varphi} \mathbf{b}_0 - \frac{1}{\eta_{\parallel}} \nabla_{\parallel}^{(0)} \bar{\varphi} \tilde{\mathbf{b}}$$

$$\nabla_{\parallel} \tilde{\mathbf{J}}_{\parallel} = -\frac{1}{\eta_{\parallel}} \left\{ \nabla_{\parallel}^{(0)} [(\tilde{\mathbf{b}} \cdot \nabla_{\perp}) \bar{\varphi}] + (\tilde{\mathbf{b}} \cdot \nabla_{\perp}) \nabla_{\parallel}^{(0)} \bar{\varphi} \right\} \neq 0$$

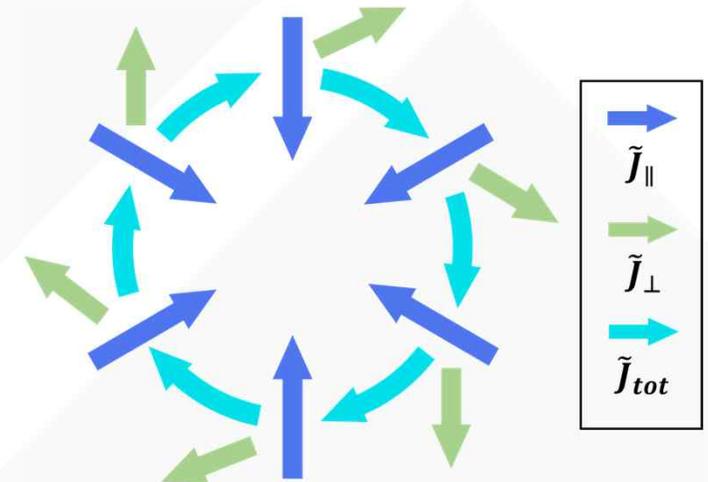
Insights from a classic: Kadomtsev and Pogutse'78¹:

Electron heat flux is divergence free at all scales $\longrightarrow \nabla \cdot \mathbf{q} = 0$

Analogy	K&P	C&D
Goal	$\langle q_r \rangle_{NL}$	$\gamma_k^{(1)}$
Base State	\bar{T}	$\bar{\varphi}$
Stochastic quantity	$\tilde{\mathbf{b}}$	$\tilde{\mathbf{b}}$
Constraint	$\nabla \cdot \mathbf{q} = 0$	$\nabla \cdot \mathbf{J} = 0$
Resulting Fluctuations	\tilde{T}	$\tilde{\varphi}$

Multi-Scale Microturbulence

1. B. B. Kadomtsev, and O. P. Pogutse, 1979.



Small-scale current

INTRODUCTION OF MODEL

The full set of equations is

$$\textcircled{1} \left[\frac{\partial}{\partial t} + \tilde{\mathbf{v}} \cdot \nabla \right] \nabla_{\perp}^2 \bar{\varphi} = -\frac{S}{\tau_A} [\nabla_{\parallel}^{(0)2} \bar{\varphi} + \underbrace{(\nabla_{\perp} \cdot \langle \tilde{\mathbf{b}} \tilde{\mathbf{b}} \rangle)}_{(a)} \cdot \nabla_{\perp} \bar{\varphi} + \underbrace{\langle \nabla_{\parallel}^{(0)} \tilde{\mathbf{b}} \cdot \nabla_{\perp} \bar{\varphi} \rangle}_{(b)} + \underbrace{\langle (\tilde{\mathbf{b}} \cdot \nabla_{\perp}) \nabla_{\parallel}^{(0)} \bar{\varphi} \rangle}_{(c)}] - \frac{g B_0}{\rho_0} \frac{\partial \bar{p}_1}{\partial y},$$

$$\textcircled{2} \left[\frac{\partial}{\partial t} + \tilde{\mathbf{v}} \cdot \nabla \right] \nabla_{\perp}^2 \tilde{\varphi} = -\frac{S}{\tau_A} [\nabla_{\parallel}^{(0)2} \tilde{\varphi} + \underbrace{(\tilde{\mathbf{b}} \cdot \nabla_{\perp}) \nabla_{\parallel}^{(0)} \tilde{\varphi}}_{(\alpha)} + \underbrace{\nabla_{\parallel}^{(0)} (\tilde{\mathbf{b}} \cdot \nabla_{\perp}) \tilde{\varphi}}_{(\beta)}] - \frac{g B_0}{\rho_0} \frac{\partial \tilde{p}_1}{\partial y}, \quad \longrightarrow \text{Relate } \tilde{\varphi} \text{ to } \tilde{\mathbf{b}}$$

$$\textcircled{3} \left[\frac{\partial}{\partial t} + \tilde{\mathbf{v}} \cdot \nabla \right] \tilde{p}_1 - \frac{\nabla \tilde{\varphi} \times \hat{\mathbf{z}}}{B_0} \cdot \nabla p_0 = 0, \quad \text{replaced by } \boxed{-\nu_T \nabla_{\perp}^2 \text{ or } \chi_T \nabla_{\perp}^2}$$

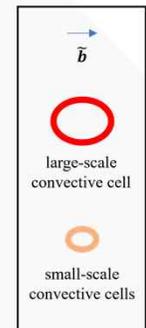
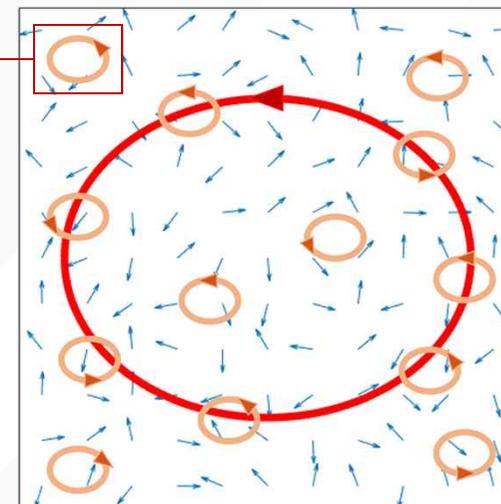
$$\textcircled{4} \left[\frac{\partial}{\partial t} + \tilde{\mathbf{v}} \cdot \nabla \right] \tilde{p}_1 - \frac{\nabla \tilde{\varphi} \times \mathbf{z}}{B_0} \cdot \nabla p_0 = 0,$$

$$\langle A \rangle = \bar{A} = \frac{1}{(2\pi)^2} \iint d\theta d\phi e^{-i(m\theta - n\phi)} A$$

Turbulent viscosity
Turbulent diffusivity

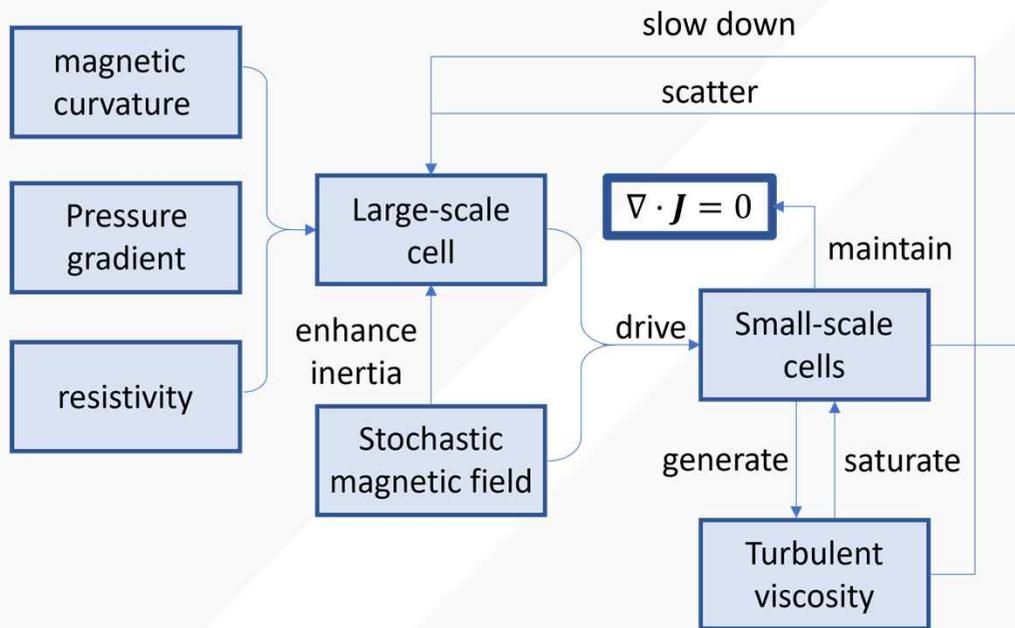
Some observations:

- $\bar{\varphi}$: low k , slow interchange approximation ($1/w_k^2 \gg k_y^2$)
- $\tilde{\varphi}$: high k' , fast interchange approximation ($1/w_{k'}^2 \ll k_y'^2$)
- The beat of $\tilde{\mathbf{b}}$ and $\bar{\varphi}$ serves as the drive of $\tilde{\varphi}$ while $\tilde{\varphi}$ modifies $\bar{\varphi}$, thus small scale and large scale are now connected.
- Feedback loop: $\bar{\varphi}, \tilde{\mathbf{b}} \rightarrow \tilde{\varphi}$



Three players: $\tilde{\mathbf{b}}$, $\bar{\varphi}$, and $\tilde{\varphi}$

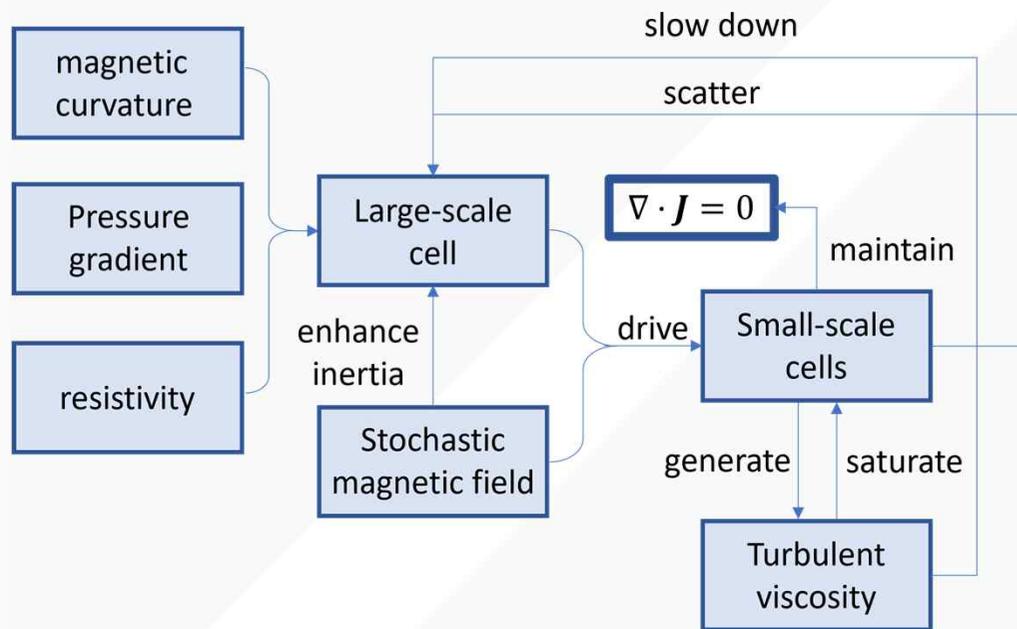
BIG PICTURE I



Multi-scale feedback loops

- $\nabla \cdot \mathbf{J} = 0$ is maintained at all scales, which reveals that electrostatic convective cells must be driven by $\tilde{\mathbf{b}}\bar{\varphi}$ beat. This indicates a turbulent background is generated, even in 'single mode' idealization
- Large scale and small scale interact. As small-scale convective cells are modulated by large-scale mode, large-scale mode is modified by small-scale cells through turbulent viscosity and electrostatic scattering.
- N.B.: Electrostatic 'micro-bursts' recently reported in DIII-D RMP experiments

BIG PICTURE II



Multi-scale feedback loops

- Stochastic magnetic field produces a magnetic braking effect, which enhances the effective inertia and exerts a drag on large-scale mode. This is similar in structure to Rutherford's nonlinear $\mathbf{J} \times \mathbf{B}$ forces¹, but in our case, it's produced by stochastic magnetic perturbations.

- We calculate a non-trivial $\langle \tilde{b}_r \tilde{v}_r \rangle$. The velocity fluctuations \tilde{v} 'lock on' to the magnetic perturbations \tilde{b} .

→ Complexity reduction result ?!

1. P. H. Rutherford, 1973.

QUANTITATIVE RESULTS I

Specific results of this work are as follows:

- The increment in the growth rate of the large-scale mode is calculated:

$$\gamma_k^{(1)} = -\frac{5}{6} \hat{v} \left(\frac{\tau_p \tau_\kappa}{\tau_A^2} \right)^{\frac{1}{3}} S^{\frac{2}{3}} \tilde{k}_\theta^{\frac{2}{3}} - \frac{1}{3} \frac{S}{\tau_A} |\tilde{b}_r|^2 - \frac{2\sqrt{2}}{3} \frac{\hat{I} S^{\frac{4}{3}} \tilde{k}_\theta^{\frac{4}{3}}}{(\tau_p \tau_\kappa \tau_A^4)^{\frac{1}{3}}}.$$

As $\gamma_k^{(1)}$ is negative definite, the net effect of \tilde{b} is to reduce resistive interchange growth.

↔ contrary to expectation of enhanced breaking of Alfvén Thm

- The scaling of the turbulent viscosity (or turbulent thermal diffusivity) is calculated:

$$\nu = \left[\pi^{\frac{1}{2}} \frac{R r_m n \tilde{k}_\theta^2}{B_0^2 L_s^5} \left(\frac{S}{\tau_A} \right)^2 \bar{\varphi}_k^2(0) \int dk_{2\theta} \frac{c^2 Z^2 w_{k_2} o_{k_2}^2}{|k_{2\theta}|^5 \gamma_{k_2}^{(0)}} \right]^{\frac{1}{3}}.$$

QUANTITATIVE RESULTS II

- The criterion when magnetic braking effect becomes significant is given. When the width of magnetic islands satisfies

$$O_{k_2} \sim \left[\frac{k_\theta^2}{k_{2\theta}^2} (\Delta x)^4 \right]^{\frac{1}{4}}.$$

Unlike Rutherford's result, extra factor $(k_\theta/k_{2\theta})^2$, which reflects the multi-scale nature of this problem.

- Correlation $\langle \tilde{b}_r \tilde{v}_r \rangle$ is calculated explicitly:

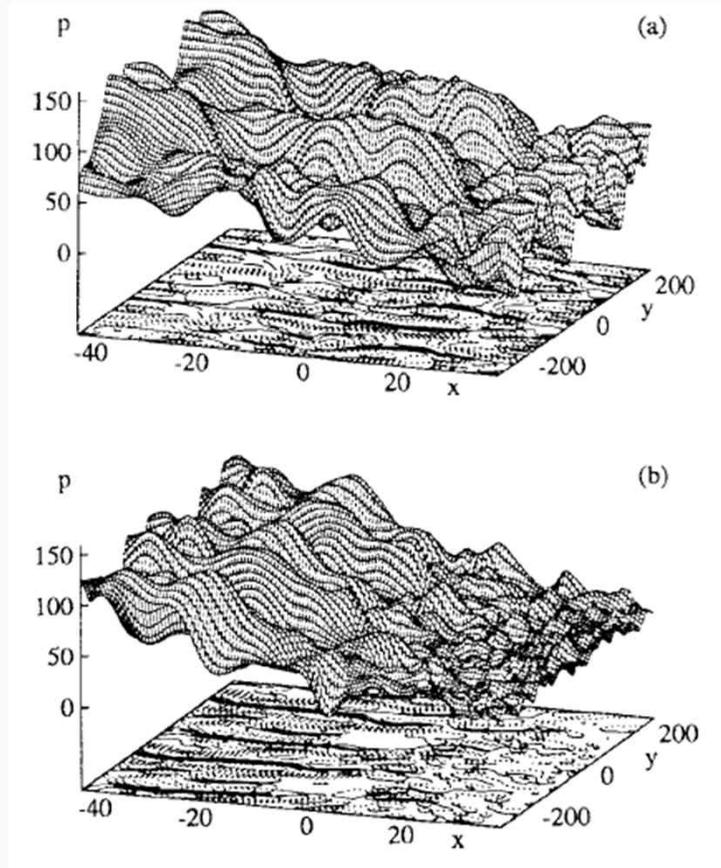
$$\langle \tilde{b}_r \tilde{v}_r \rangle = \pi^{\frac{1}{2}} \frac{\tilde{k}_\theta R r_{mn}}{L_s^3 B_0} \frac{S}{\tau_A} \bar{\varphi}_{\mathbf{k}}(0) \times$$

$$\int dk_{2\theta} |k_{2\theta}| k_{2\theta} \frac{c^2 Z^2 (k_\theta - k_{2\theta}) w_{\mathbf{k}_2} o_{\mathbf{k}_2}^2}{\Lambda_{\mathbf{k}_2}^0 - \Lambda_{\mathbf{k}_2}}$$

envelope

COMPARISON: SIMULATIONS & EXPERIMENTS

- **Previous simulation by Beyer, et al¹:**
 - Electrostatic resistive ballooning modes in a stochastic magnetic field.
 - With RMP, large-scale structures are stabilized, and spatial roughness increases.
 - Testable prediction: $\langle \tilde{b}_r \tilde{v}_r \rangle$, comparison
- **Recent experimental studies by Choi, et al²:**
 - Change in pedestal temperature fluctuation predictability with RMP switched on and off.
 - Stochasticity reduces the predictability of the pedestal turbulence
 - Explanation: $\langle \tilde{b}_r \tilde{v}_r \rangle \neq 0$, generation of $\tilde{\varphi}$.



plasma pressure in a sector at the low field side without (a) and with (b) RMP¹

1. P. Beyer, X. Garbet, and P. Ghendrih, 1998
2. M.J. Choi, et al., 2021.

Discussion

- Stochastic field + Resistive Interchange \rightarrow 'micro-bursts', etc.
 - \rightarrow multi-scale, turbulent state
- Fluctuations 'lock-on' to imposed magnetic perturbations $\langle \tilde{b}v_r \rangle \neq 0$



How understand resistive interchange turbulence in this system?

\leftrightarrow competing mechanisms