Ballooning Mode in a Stochastic Magnetic Field —A Quasi-mode Model

Transport Task Force, 2023

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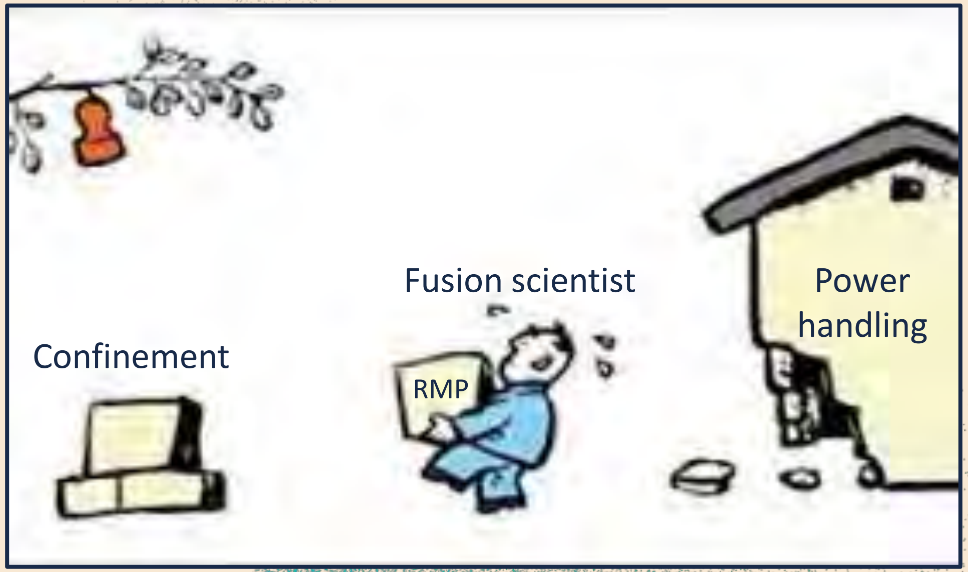
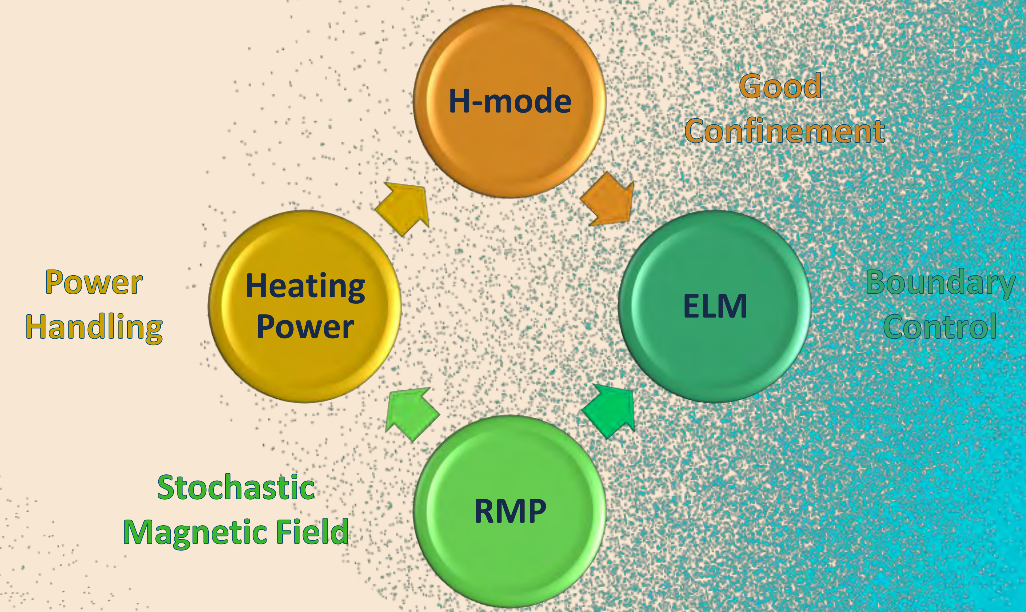
Outline

- Background 1: Good Confinement and good power handling,
- Background 2: Ballooning in a torus vs. $\boldsymbol{k} \cdot \boldsymbol{B}$ resonance in a cylinder
- Background 3: Hints from old simulations and recent experiments
- Resistive interchange mode in a stochastic magnetic Field
- Quasi-mode: a counterpart of ballooning mode in the “cylinder universe”
- Effects of stochastic magnetic field on quasi-Mode
- Conclusion: Lessons we learned & suggested experiments

Background





RMP can suppress **ELM**, but it enhance **L-H transition power threshold** at the same time.

A new trend: **good confinement** is no longer deemed sufficient. We must reconcile **good confinement** with **good power handling** and **manageable boundary control**.

Background

- A basic question: how does stochastic magnetic field modify the instability process?
- Start with the simplest instability: resistive interchange mode

Instability and turbulent relaxation in a stochastic magnetic field
M **Cao**, PH Diamond - Plasma Physics and Controlled Fusion, 2022 - iopscience.iop.org
An analysis of instability dynamics in a stochastic magnetic field is presented for the tractable case of the resistive interchange. Externally prescribed static magnetic perturbations …
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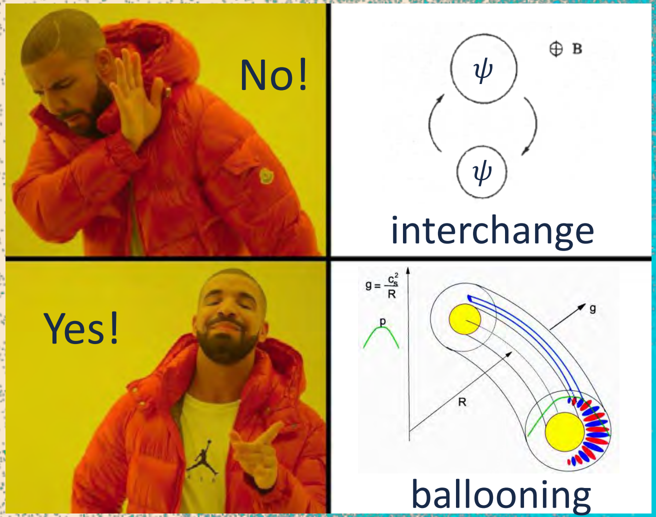
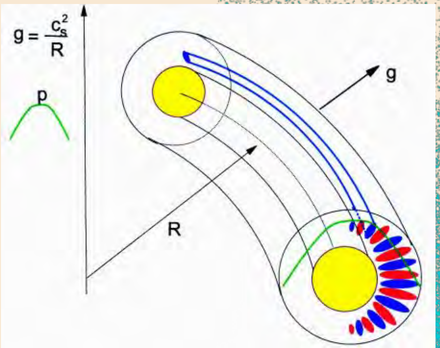
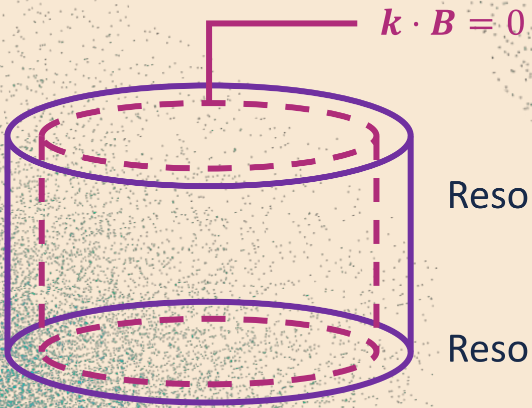


- But…is anyone interested in interchange mode?
- Reconciliation of two pictures:



Ballooning mode

Toroidicity effect



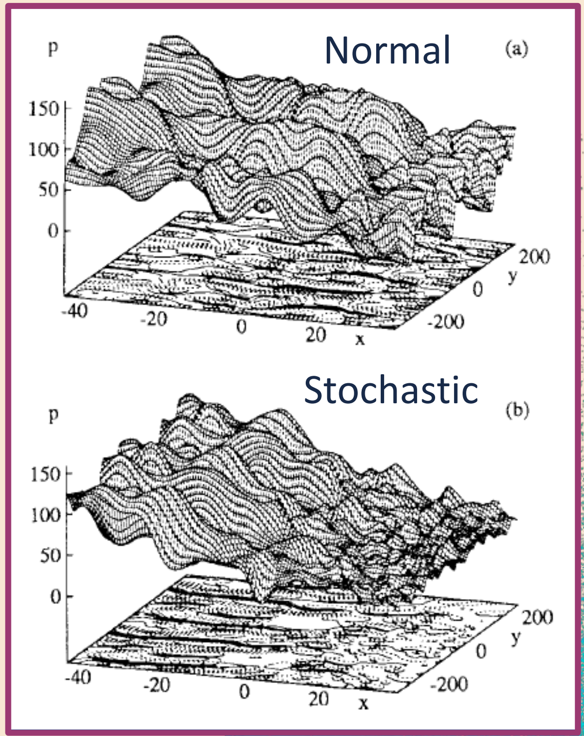
Resonant magnetic perturbation

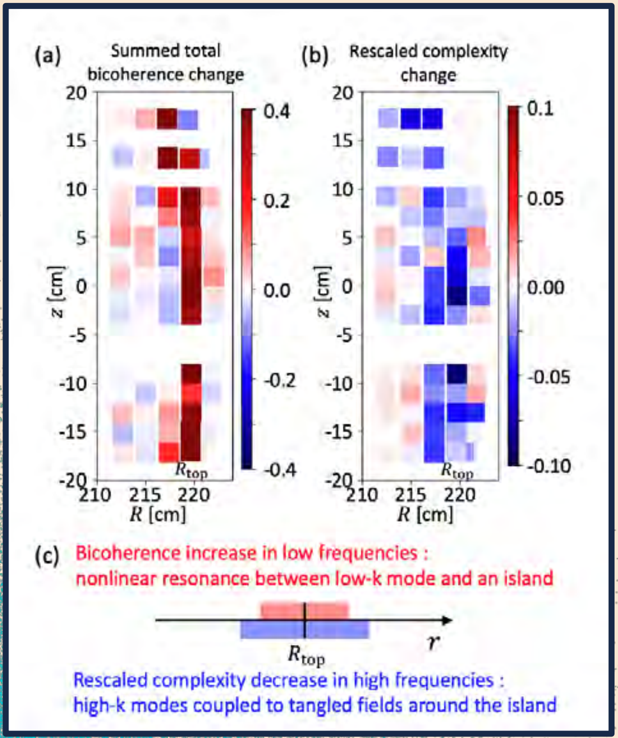
Resonant surfaces in a **cylinder**

Ballooning mode in a **torus** vs. resonant magnetic perturbations in a **cylinder**

Background

Simulations and experiments on plasma turbulence with RMP:





- Simulations of resistive ballooning modes in a stochastic magnetic field. [1]
  - Increased small-scale structures and spatial roughness.
  - Stronger suppression of large-scale fluctuations
- Experimental study of the fluctuations with RMP. [2]
  - An increase in the bicoherence (increased phase coupling)
  - A reduction in the Jensen-Shannon complexity.
  - Jensen-Shannon complexity:
    - $C_{JS} = \underbrace{H}_{\text{Shannon entropy}} \times \underbrace{Q}_{\text{J–S divergence}}$

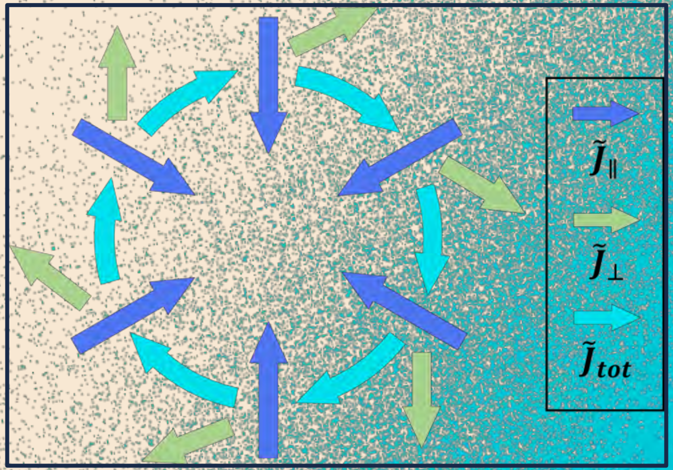
      Shannon entropy: a measure of missing information

      J–S divergence: a measure of distance from thermal equilibrium
    - Low complexity: perfect crystal (low H, high Q), ideal gas (high H, low Q), **white noise**
    - High complexity: logistic map, **chaotic systems**

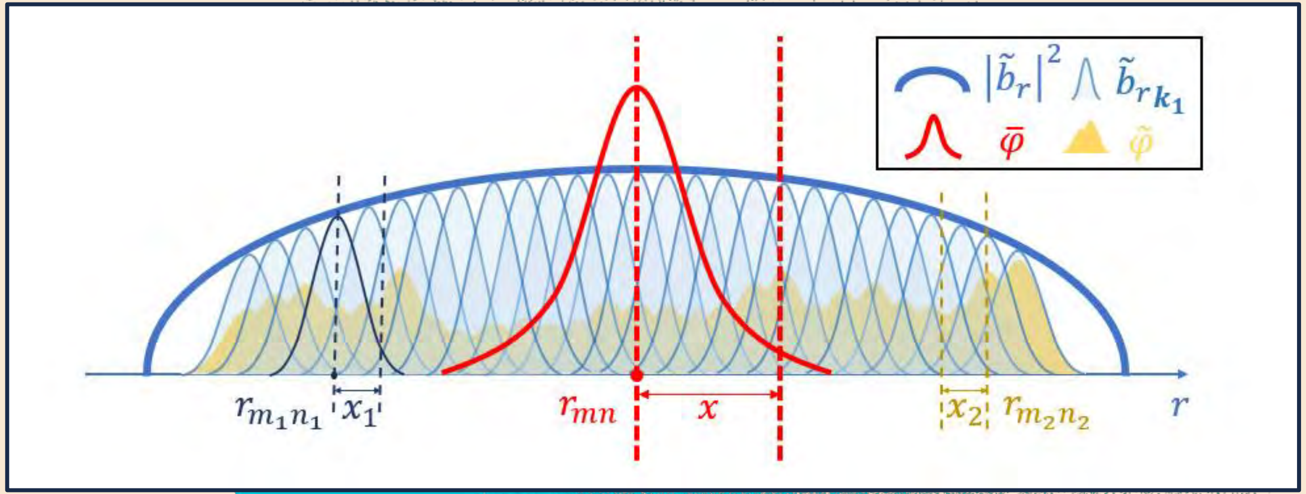
1. Beyer, P., Xavier Garbet, and Philippe Ghendrih. *Physics of Plasmas* 5, no. 12 (1998): 4271-4279.
2. Choi, Minjun J., et al. *Physics of Plasmas* 29, no. 12 (2022): 122504.

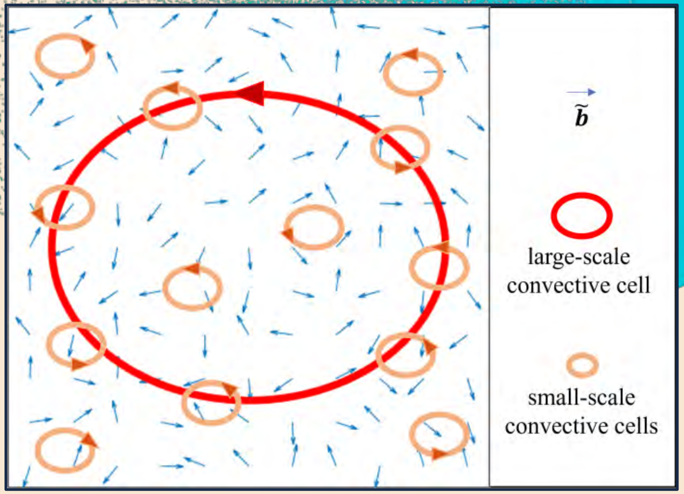
Resistive Interchange Mode

- A *multi-scale* model maintaining $\nabla \cdot \boldsymbol{J} = 0$ at all scales.
- Small-scale current along chaotic field lines $\Rightarrow \nabla \cdot \tilde{\boldsymbol{J}}_{\parallel} \neq 0$.
- A current density fluctuation $\tilde{\boldsymbol{J}}_{\perp}$ must be drive to balance $\tilde{\boldsymbol{J}}_{\parallel}$.
- $\Rightarrow$ small-scale convective cells $\Rightarrow$ microturbulence



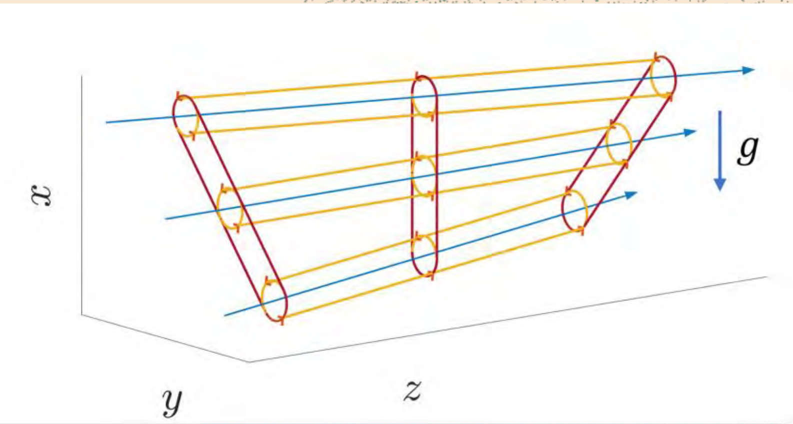
Large-scale single mode + microturbulence + stochastic magnetic field

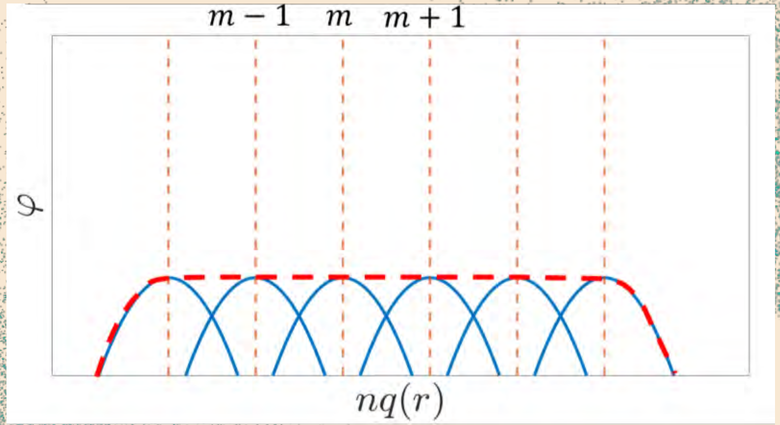




Quasi-mode vs. Ballooning Mode

- Problem: ballooning mode "lives" in a torus while RMP "lives" in a cylinder (parallel universe?)
- Solution: find the counterpart of ballooning mode in a cylinder ⇒ quasi-mode
- Quasi-mode [1] is a wave-packet of localized interchange modes
- Ballooning mode is a coupling of localized poloidal harmonics
- **Takeaway: a quasi-mode in a cylinder resembles a ballooning mode in a torus.**





1. Roberts, K. V., and J. B. Taylor. *The Physics of Fluids* 8, no. 2 (1965): 315-322.

Quasi-mode in a Stochastic Magnetic Field

- Modified equations for quasi-mode:

$$\rho_0\left(\frac{\partial}{\partial t}-\nu_T\nabla_\perp^2\right)\nabla_\perp^2(\bar{\varphi}+\tilde{\varphi})+\frac{B_0^2}{\eta}\left(\frac{\partial}{\partial\zeta}+\tilde{\boldsymbol{b}}\cdot\nabla_\perp\right)^2(\bar{\varphi}+\tilde{\varphi})-gB_0\frac{\partial(\bar{\rho}+\tilde{\rho})}{\partial y}=0$$

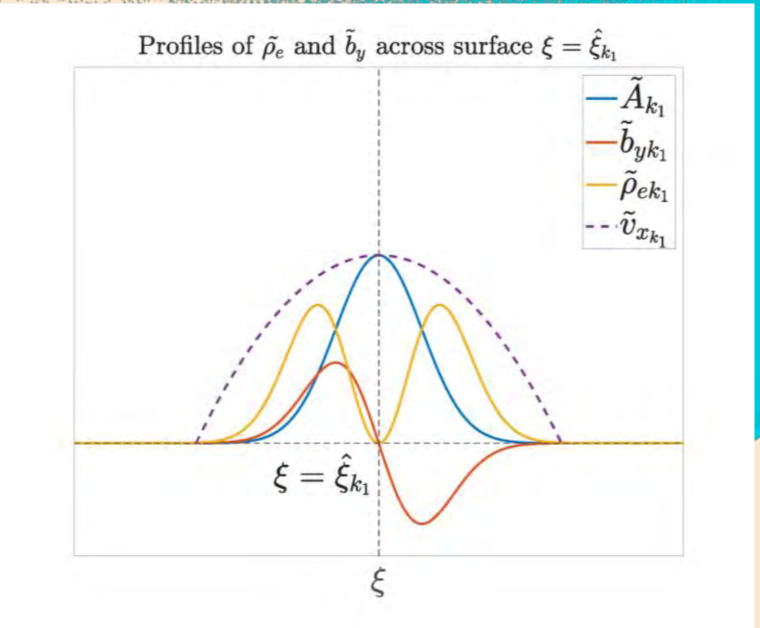
$$\left(\frac{\partial}{\partial t}-D_T\nabla_\perp^2\right)(\bar{\rho}+\tilde{\rho})=-(\bar{v}_x+\tilde{v}_x)\alpha\rho_0$$

Microturbulence ⇒ turbulent viscosity & turbulent diffusivity

- Introduction of $\tilde{\boldsymbol{b}}\Rightarrow\nabla\cdot\tilde{\boldsymbol{J}}_\parallel\neq 0\Rightarrow$ accumulation of polarization charge $\Rightarrow\tilde{\varphi},\tilde{\rho},\tilde{v}\Rightarrow$ a non-vanishing correlation between $\tilde{\varphi}$ and $\tilde{\boldsymbol{b}}$.
- Scale orderings:

$$\frac{1}{\bar{v}_x}\underbrace{\frac{\partial}{\partial\xi}\bar{\varphi}}_{=0}\ll\frac{1}{\bar{v}_x}\frac{\partial}{\partial\zeta}\bar{\varphi}\ll\frac{1}{\bar{v}_x}\frac{\partial}{\partial\chi}\bar{\varphi}\ll\frac{1}{\tilde{v}_x}\frac{\partial}{\partial x}\tilde{\varphi}\ll\frac{1}{\tilde{v}_x}\frac{\partial}{\partial\chi}\tilde{\varphi}$$

$$\frac{1}{\bar{v}_x}\frac{\partial}{\partial\zeta}\bar{\varphi}\ll\frac{1}{\tilde{v}_x}\frac{\partial}{\partial\zeta}\tilde{\varphi}\ll\frac{1}{\tilde{v}_x}\frac{\partial}{\partial\chi}\tilde{\varphi}\qquad\nu_T k_y^2\ll\gamma_{\boldsymbol{k}}\ll\gamma_{\boldsymbol{k}_1}<\nu_T k_{1y}^2$$



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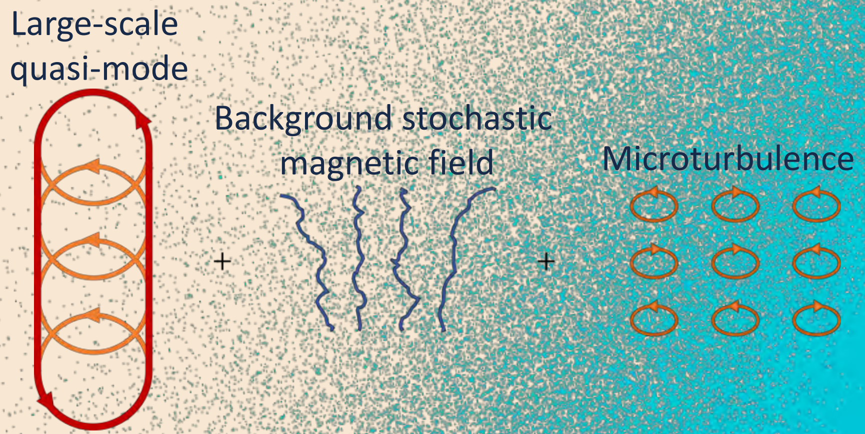
Quasi-mode in a Stochastic Magnetic Field

- Model: a multi-scale system…
- Approach: quasi-linear theory
- Workflow: standard steps of mean field theory



Scale separation
- Define a proper spatial averaging.
- Split the dynamics of small and large scales.

Solve small-scale dynamics
- Get the linear response of $\tilde{v}_x$ to $\tilde{\boldsymbol{b}}$ and $\bar{v}$.
- Get a non-vanishing $\langle \tilde{v}_x \tilde{A} \rangle$ correlation.

Solve large-scale dynamics
- Plug in the response and get the large-scale eigenmode equations.
- Calculate the corrected growth rate.

Nonlinear Closure
- Calculate the scaling of turbulent viscosity and turbulent diffusivity generated by microturbulence.

Quasi-mode in a Stochastic Magnetic Field

- Dynamics of different scales can be separated by taking a spatial averaging:

$$\langle A\rangle = \bar{A} = \frac{1}{L_y}\int_{-L_y/2}^{L_y/2} e^{-ik_y\chi} A d\chi$$

- The full set of equations of the model is

$$\rho_0\left(\frac{\partial}{\partial t} - \nu_T\nabla_\perp^2\right)\nabla_\perp^2\bar{\varphi} + \frac{B_0^2}{\eta}\frac{\partial^2}{\partial\zeta^2}\bar{\varphi} + \frac{B_0^2}{\eta}\left\{\underbrace{\left\langle\left(\tilde{b}\cdot\nabla_\perp\right)^2\right\rangle\bar{\varphi}}_{(a)} + \underbrace{\left\langle\frac{\partial}{\partial\zeta}\left(\tilde{b}\cdot\nabla_\perp\right)\tilde{\varphi}\right\rangle}_{(b)} + \underbrace{\left\langle\left(\tilde{b}\cdot\nabla_\perp\right)\frac{\partial}{\partial\zeta}\tilde{\varphi}\right\rangle}_{(c)}\right\} - gB_0\frac{\partial}{\partial y}\bar{\rho} = 0$$

$$\rho_0\left(\frac{\partial}{\partial t} - \nu_T\nabla_\perp^2\right)\nabla_\perp^2\tilde{\varphi} + \frac{B_0^2}{\eta}\frac{\partial^2}{\partial\zeta^2}\tilde{\varphi} + \frac{B_0^2}{\eta}\left\{\frac{\partial}{\partial\zeta}\left(\tilde{\boldsymbol{b}}\cdot\nabla_\perp\right)\bar{\varphi} + \left(\tilde{\boldsymbol{b}}\cdot\nabla_\perp\right)\frac{\partial}{\partial\zeta}\bar{\varphi}\right\} - gB_0\frac{\partial}{\partial y}\tilde{\rho} = 0$$

$$\left(\frac{\partial}{\partial t} - D_T\nabla_\perp^2\right)\bar{\rho} = -\bar{v}_x\alpha\rho_0 \qquad \left(\frac{\partial}{\partial t} - D_T\nabla_\perp^2\right)\tilde{\rho} = -\tilde{v}_x\alpha\rho_0$$

Large scale: slow interchange
Small scale: fast interchange

Quasi-mode in a Stochastic Magnetic Field

- With the scale orderings:

$$-2\rho_0\nu_T k_{1y}^2\frac{\partial^2}{\partial\xi^2}\tilde{v}_{x\boldsymbol{k}_1}(\hat{\xi}_{\boldsymbol{k}_1},\zeta)+\frac{B_0^2}{\eta}s^2k_{1y}^2\hat{\xi}_{\boldsymbol{k}_1}^2\tilde{v}_{x\boldsymbol{k}_1}(\hat{\xi}_{\boldsymbol{k}_1},\zeta)-\left(\frac{\alpha g\rho_0}{\chi_T}-\rho_0\nu_T k_{1y}^4\right)\tilde{v}_{x\boldsymbol{k}_1}(\hat{\xi}_{\boldsymbol{k}_1},\zeta)$$

$$=\frac{B_0^2}{\eta}ik_{1y}\left[-s\tilde{b}_{x\boldsymbol{k}_2}(\hat{\xi}_{\boldsymbol{k}_1})(2\zeta\partial_\zeta+1)+2\tilde{b}_{y\boldsymbol{k}_2}(\hat{\xi}_{\boldsymbol{k}_1})\partial_\zeta\right]\bar{v}_{x\boldsymbol{k}}(\zeta)\underbrace{\exp[-isk_y\hat{\xi}_{\boldsymbol{k}_1}\zeta]}_{=1}$$

$$-\frac{B_0^2}{\eta}k_{1y}k_{2\parallel}\left[-s\zeta\tilde{b}_{x\boldsymbol{k}_2}(\hat{\xi}_{\boldsymbol{k}_1})+\tilde{b}_{y\boldsymbol{k}_2}(\hat{\xi}_{\boldsymbol{k}_1})\right]\bar{v}_{x\boldsymbol{k}}(\zeta)\underbrace{\exp[-isk_y\hat{\xi}_{\boldsymbol{k}_1}\zeta]}_{=1}\quad(sk_y\Delta\sim 1/\delta_{\boldsymbol{k}}\ll 1/\delta_{\boldsymbol{k}_1})$$

- L.H.S: quantum harmonic oscillator; R.H.S: drive of the beat of $\tilde{\boldsymbol{b}}$ and $\bar{v}_x$

$$\tilde{v}_{x\boldsymbol{k}_1}=\int G(\hat{\xi}_{\boldsymbol{k}_1},\hat{\boldsymbol{\xi}}'_{\boldsymbol{k}_1})*\boldsymbol{RHS}\,d\hat{\xi}'_{\boldsymbol{k}_1}$$

- ⇒ Non-trivial correlation $\langle\tilde{v}_x\tilde{A}\rangle$

$$\langle\tilde{v}_x\tilde{A}\rangle\approx\frac{L_zL_y}{(2\pi)^2}\int dk_{1y}s^2|k_{1y}|\frac{|\tilde{A}_{0\boldsymbol{k}_1}|^2B_0^2}{\rho_0\eta\nu_T k_{1y}^2}\frac{4\sqrt{\pi}o_{\boldsymbol{k}_1}^2}{w'}\bar{v}_{x\boldsymbol{k}}(0)$$

Quasi-mode in a Stochastic Magnetic Field

- Using the linear response of $\tilde{v}_x$ to $\tilde{\boldsymbol{b}}$, the perturbed eigenmode equation is $\hat{H}_0\bar{\varphi}_{\boldsymbol{k}} = \hat{H}_1\bar{\varphi}_{\boldsymbol{k}}$

$$\hat{H}_0 = \frac{\partial^2}{\partial\zeta^2} - \frac{\gamma_{\boldsymbol{k}}\rho_0\eta}{B_0^2}s^2\zeta^2k_y^2 + \frac{\gamma_{\boldsymbol{k}}\rho_0\eta k_y^2}{B_0^2}\left(\frac{\alpha g}{\gamma_{\boldsymbol{k}}^2} - 1\right)$$

$$\hat{H}_1 = \left[s^2\zeta^2k_y^2\left|\tilde{b}_x^2\right| - 2s\zeta k_y^2\left|\tilde{b}_x\tilde{b}_y\right| + k_y^2\left|\tilde{b}_y^2\right|\right] + \frac{\alpha g\rho_0\eta D_T k_y^4(1+s^2\zeta^2)}{\gamma_{\boldsymbol{k}}^2B_0^2} + \frac{\rho_0\eta}{B_0^2}\nu_T k_y^4(1+s^2\zeta^2)^2$$
$$+ \frac{L_zL_y}{(2\pi)^2}\int d\,k_{1y}\frac{s^3k_y^2B_0^2\left|\tilde{A}_{0k_1}\right|^2}{\rho_0\eta\nu_T\left|k_{1y}\right|}\frac{8\sqrt{\pi}\left|o_{\boldsymbol{k}_1}\right|^2}{w'}\zeta\partial_\zeta$$

- The correction to the growth rate is

$$\gamma_{\boldsymbol{k}}^{(1)} = \frac{\int_{-\infty}^{\infty}\bar{\varphi}_{\boldsymbol{k}}^{(0)}(\zeta)\hat{H}_1\bar{\varphi}_{\boldsymbol{k}}^{(0)}(\zeta)d\zeta}{\int_{-\infty}^{\infty}\bar{\varphi}_{\boldsymbol{k}}^{(0)}(\zeta)\left[\partial_{\gamma_{\boldsymbol{k}}^{(0)}}\hat{H}_0\right]\bar{\varphi}_{\boldsymbol{k}}^{(0)}(\zeta)d\zeta} = -\frac{5}{6}s^2\Delta^2\nu_T k_y^2\Bigg(1\underbrace{+\frac{8}{5}\frac{1}{s^2\Delta^2}}_{\text{new}}\Bigg) - \frac{1}{3}\frac{S}{\tau_A}\Bigg((1-f)\left|\tilde{b}_x^2\right|\underbrace{+\frac{2}{s^2\Delta^2}\left|\tilde{b}_y^2\right|}_{\text{new}}\Bigg)$$

Quasi-mode in a Stochastic Magnetic Field

$$\gamma_{\boldsymbol{k}}^{(1)} = -\frac{5}{6}s^2\Delta^2\nu_T k_y^2\left(1+\frac{8}{5}\frac{1}{s^2\Delta^2}\right) - \frac{1}{3}\frac{S}{\tau_A}\left(\overbrace{(1-f)\left|\tilde{b}_x^2\right| + \frac{2}{s^2\Delta^2}\left|\tilde{b}_y^2\right|}^{\text{magnetic braking effect}}\right) < 0 \qquad f \sim \underbrace{\frac{\nu_T k_y^2}{\gamma_{\boldsymbol{k}}^{(0)}}}_{\ll 1}\underbrace{\frac{\alpha g}{\nu_T^2 k_{2y}^4}}_{\leq 1}\underbrace{\frac{8}{k_{1y}\delta_{\boldsymbol{k}_1}}}_{\sim 1} \ll 1$$

- Magnetic braking effect:

$$\frac{\partial^2}{\partial\zeta^2}\bar{v}_{x\boldsymbol{k}} - \overbrace{\left(\frac{\rho_0\eta}{B_0^2}\gamma_{\boldsymbol{k}} + \left|\tilde{b}_x^2\right|\right)k_y^2 s^2\zeta^2\bar{v}_{x\boldsymbol{k}}}^{\text{enhance intertia}} + \overbrace{\left(\frac{\rho_0\eta}{B_0^2}\frac{\alpha g}{\gamma_{\boldsymbol{k}}} - \left|\tilde{b}_y^2\right|\right)k_y^2\bar{v}_{x\boldsymbol{k}}}^{\text{reduce drive}} = 0$$

- Balancing linear bending term with random bending term

$$o_{\boldsymbol{k}_1} \sim \delta_{\boldsymbol{k}}\left(\frac{k_y}{k_{1y}}\right)^{1/2}$$

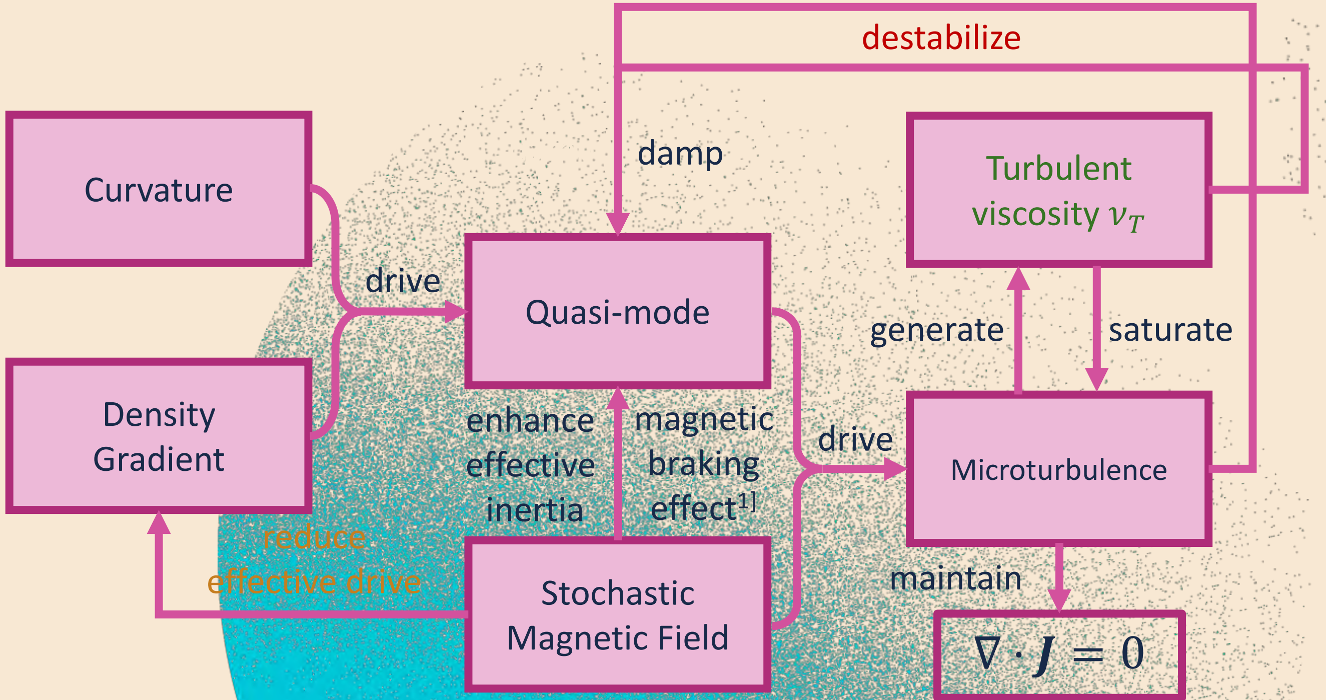
→ Feature of a multi-scale system

- The turbulent viscosity ν_T of quasi-mode is larger than that of resistive interchange mode!

$$\nu_T = \sum_{\boldsymbol{k}_1}\left|\tilde{\boldsymbol{v}}_{\boldsymbol{k}_1}\right|^2\tau_{\boldsymbol{k}_1} \cong \left[\frac{L_z L_y}{(2\pi)^2}\int dk_{1y}s^3|k_{1y}|\frac{B_0^4\left|\tilde{A}_{\boldsymbol{k}_1}\right|^2}{\rho_0^2\eta^2\nu_T^2 k_{1y}^4}\frac{4\sqrt{\pi}o_{\boldsymbol{k}_1}^2\bar{v}_{x\boldsymbol{k}}(0)^2}{w'(\alpha g)^{1/2}}\left\{\underbrace{2}_{\text{old}} + \underbrace{\left(\frac{k_{1y}o_{\boldsymbol{k}_2}^2}{k_y w_{\boldsymbol{k}} w'}\right)^2}_{\text{new}}\right\}\right]^{1/3}$$

Effects of stochastic magnetic field on quasi-mode

- Since quasi-mode is a wave-packet of interchange modes, similar results are expected.
- Due to the difference in mode structure, there are also some changes in the results.



Differences:

1, Extra channel to stabilize the quasi-mode, i.e., reducing the effective drive of the quasi-mode.

2. Larger turbulent viscosity ν_T compared with resistive interchange mode.

3, Microturbulence tends to destabilize the quasi-mode, though this effect is much **weaker** compared to the stabilization by magnetic braking effect.

1. Rutherford, Paul Harding. *The Physics of Fluids* 16, no. 11 (1973): 1903-1908.

Lessons Learned about Ballooning Mode

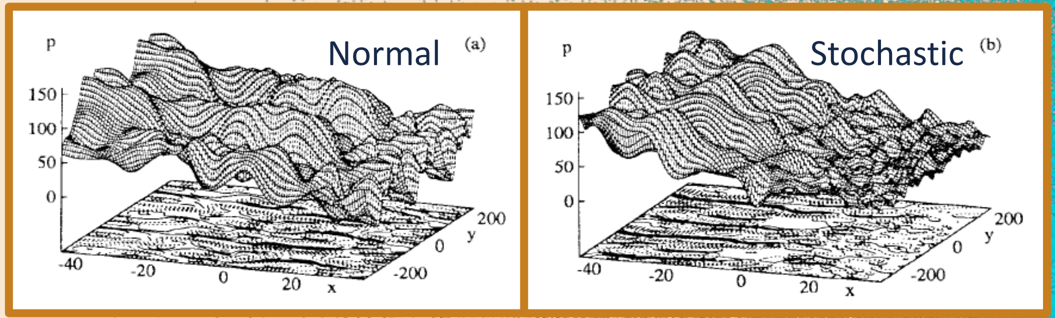
i) Generation of microturbulence to maintain $\nabla \cdot \boldsymbol{J} = 0$.
- Appearance of small-scale structure, increased spatial roughness
- Microturbulence promote spectral transfer ⇒ increased bicoherence

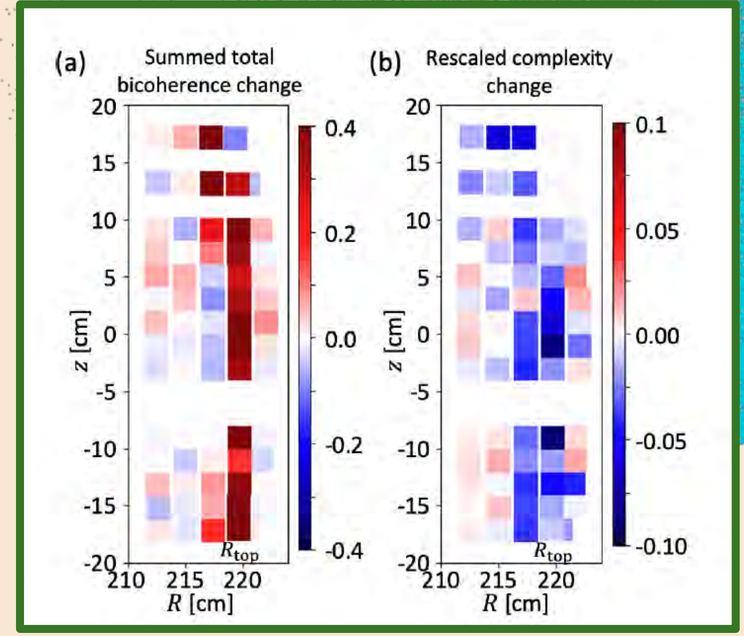
ii) A non-trivial correlation $\langle \tilde{A}\tilde{v}_x \rangle$
- Velocity fluctuation locks on to the stochastic magnetic field ⇒ change the statistics of the plasma turbulence ⇒ reduced Jensen-Shannon complexity of edge turbulence

iii) Slow-down of the ballooning mode growth
- Stronger suppression of large-scale fluctuations
    - Enhance effective inertia
    - Reduce effective drive
    - Turbulent damping

iv) Larger turbulent viscosity and turbulent diffusivity

Future: include zonal flow into our model



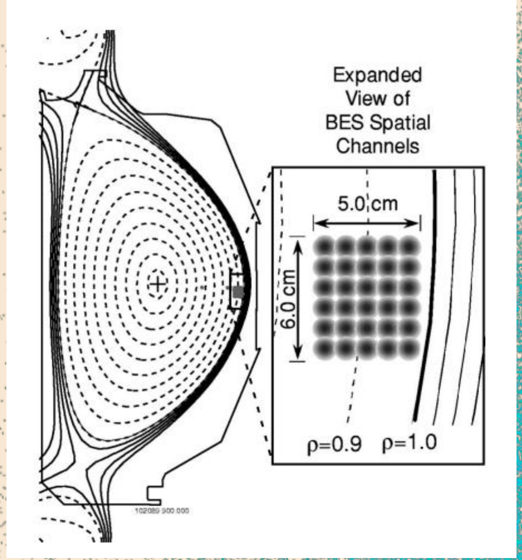


Suggested Experiments

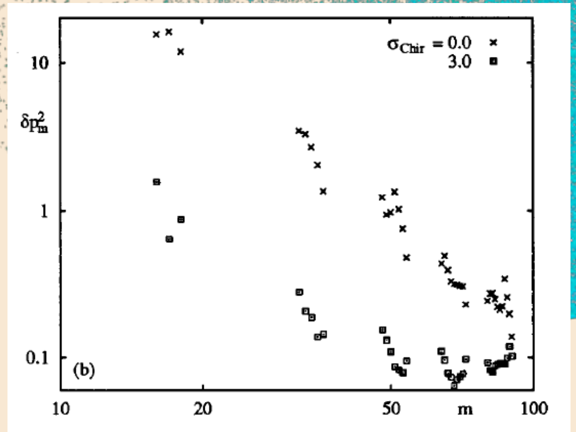
- Use BES to measure the velocity fluctuation spectra before and after the ELM suppression phase.
  - Suppression of low-k structures
  - Appearance of high-k structures?

- Use BES to calculate the C_{JS} of the velocity fluctuation spectra before and after the ELM suppression phase.
  - Prediction: stochastic magnetic field changes the statistics of plasma turbulence

- Calculate the correlation between velocity fluctuation and magnetic perturbation.



Beam emission spectroscopy on DIII-D [1]



Spectra of pressure fluctuations w/wo stochastic magnetic field [2]

1. McKee, George R., et al. *Review of scientific instruments* 74, no. 3 (2003): 2014-2019.
2. Beyer, P., et al. *Physics of Plasmas* 5, no. 12 (1998): 4271-4279.

Thank you

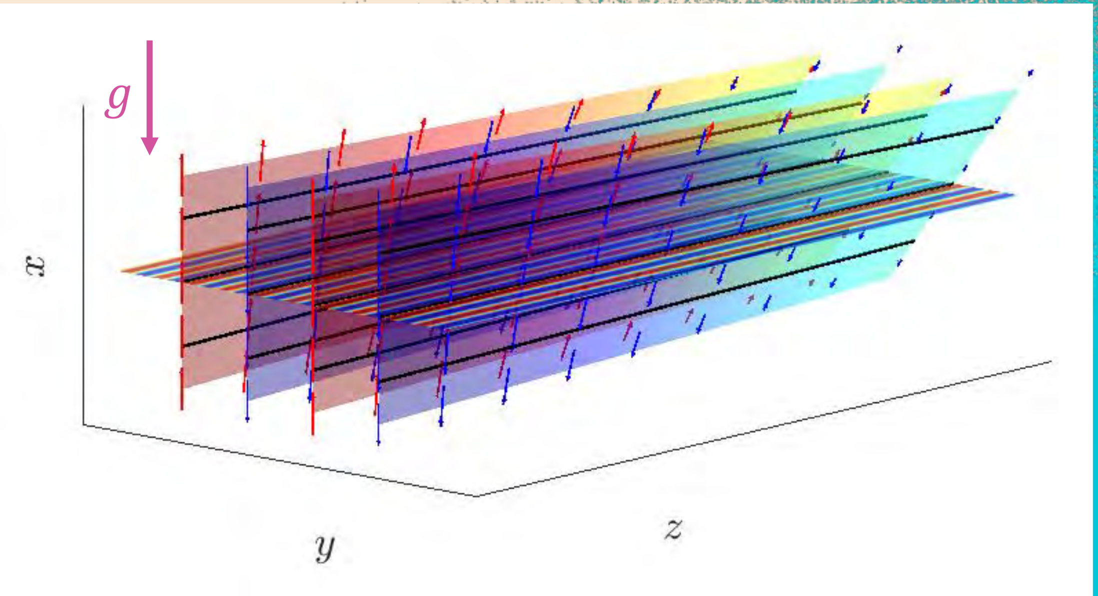
Quasi-mode Revisiting

- Features of the quasi-mode
  - Broad mode structure in the vertical direction
  - Finite mode length in the main field direction
  - Finite, linear magnetic shear, $\boldsymbol{b_0} = (0, sx, 1)$
- Equations for quasi-mode
  - Continuity equation

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{v} \cdot \nabla \rho_0 = -v_x \alpha \rho_0$$

  - Vorticity equation

$$\underbrace{-\frac{\rho_0}{B_0^2}\frac{\partial}{\partial t}\nabla_\perp^2 \varphi}_{\nabla_\perp \cdot \boldsymbol{J_{pol}}} \underbrace{-\frac{1}{\eta}(\boldsymbol{b_0} \cdot \nabla)^2 \varphi}_{\nabla_\parallel \cdot \boldsymbol{J}_\parallel} + \underbrace{\frac{g}{B_0}\frac{\partial}{\partial y}\rho}_{\nabla_\perp \cdot \boldsymbol{J_{PS}}} = 0$$

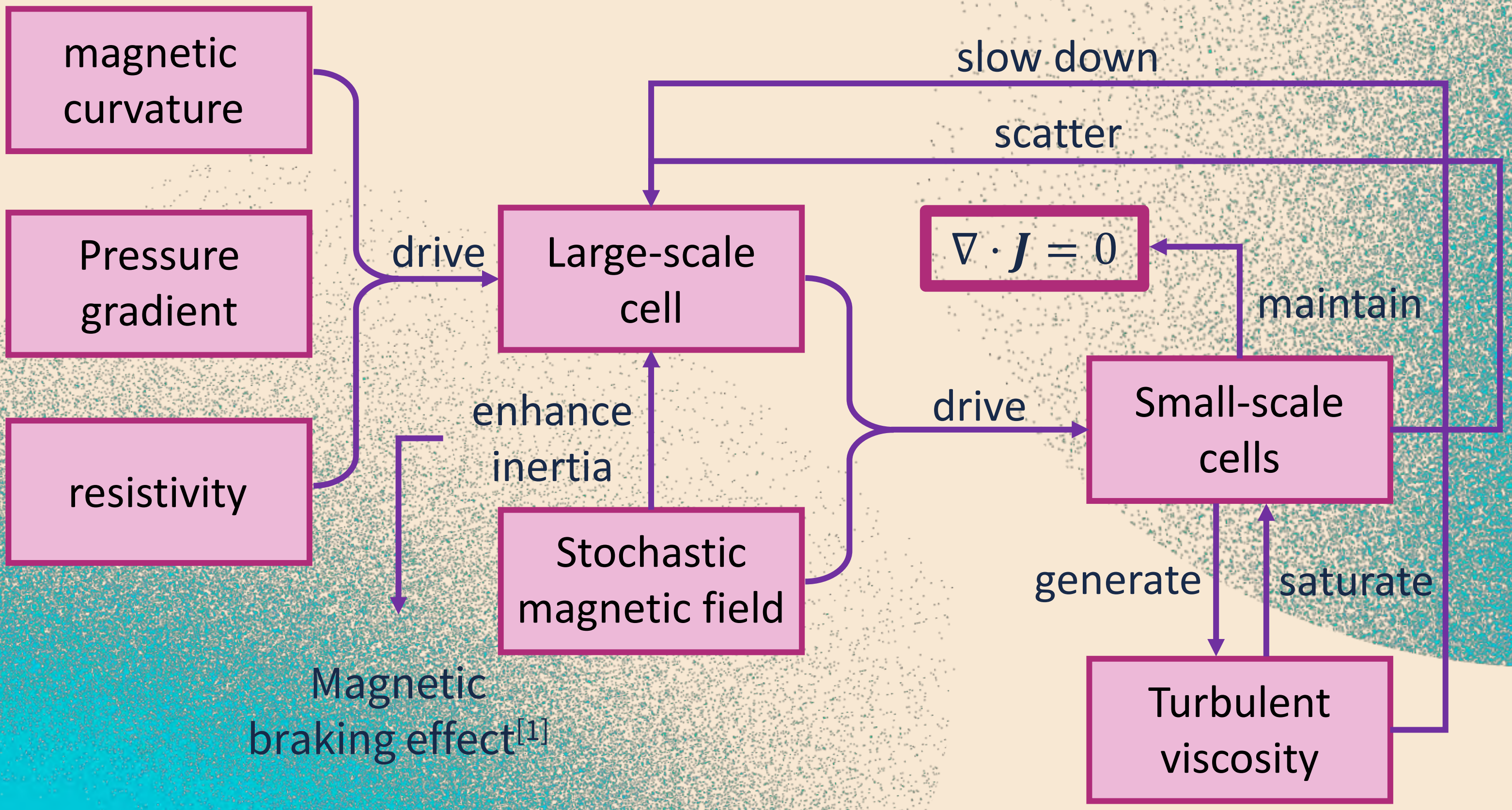


Twisted coordinate transformation:

$$\xi = x \qquad \partial_x = \partial_\xi - s\zeta\partial_\chi$$
$$\chi = y - sxz \quad \longrightarrow \quad \partial_y = \partial_\chi$$
$$\zeta = z \qquad \partial_z = \partial_\zeta - s\xi\partial_\chi$$

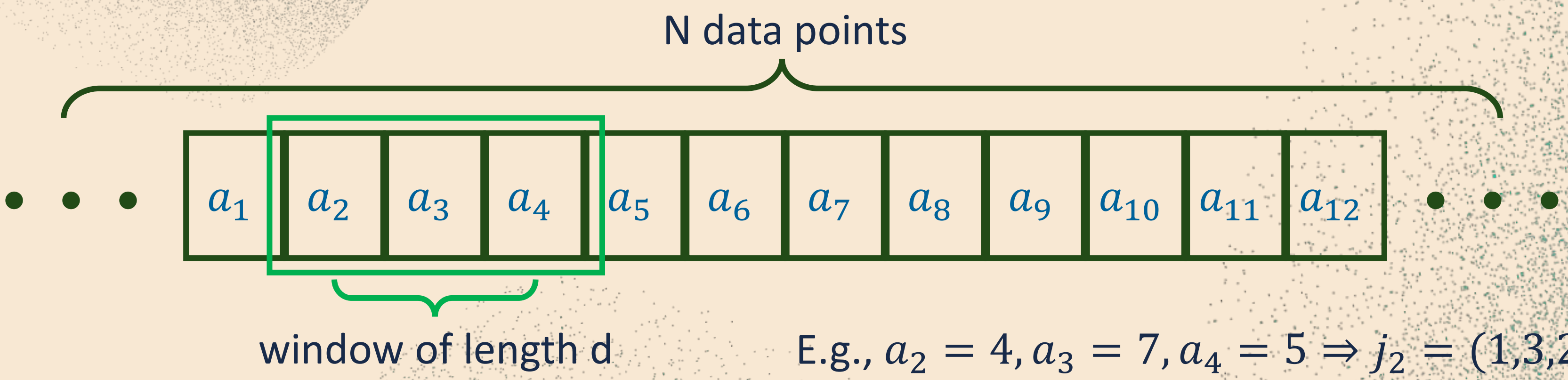
Resistive Interchange Mode

- Multi-scale feedback loop of resistive interchange mode in a stochastic magnetic field



1. Rutherford, Paul Harding. *The Physics of Fluids* 16, no. 11 (1973): 1903-1908.

Jensen-Shannon Complexity



E.g., $a_2 = 4, a_3 = 7, a_4 = 5 \Rightarrow j_2 = (1{,}3{,}2)$

$$\text{orderings}\begin{cases} j_1 = (1{,}2{,}3) \\ j_2 = (1{,}3{,}2) \\ j_3 = (2{,}1{,}3) \\ j_4 = (2{,}3{,}1) \\ j_5 = (3{,}1{,}2) \\ j_6 = (3{,}2{,}1) \end{cases}$$

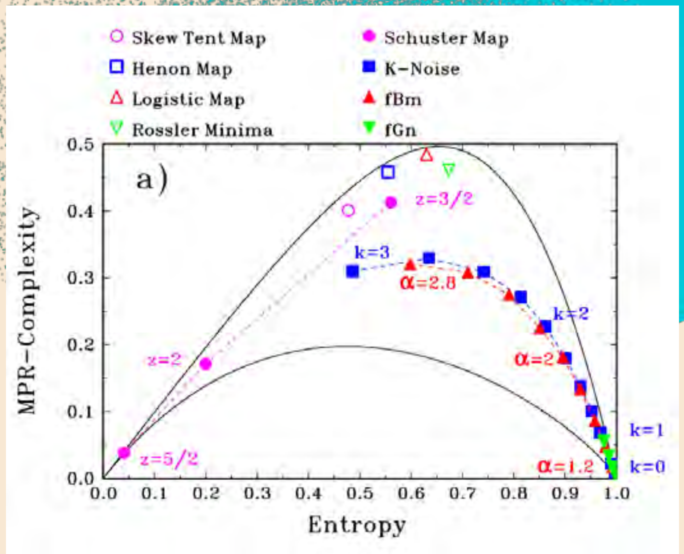
Get the probability distribution function of orderings $P = \{p_j\}_{j=1,\dots,d!}$ $(N \gg d!)$

Then [1]

$$C_{JS} = \underbrace{H}_{\text{Shannon entropy}} \times \underbrace{Q}_{\text{J-S divergence}}$$

$$H = \frac{S}{S_{max}}, S = -\sum_j p_j \ln(p_j), S_{max} = \ln(d!)$$

$$Q = Q_0\left\{S\left(\frac{P+P_e}{2}\right) - \frac{S(P)}{2} - \frac{S(P_e)}{2}\right\}, P_e = \left\{p_j = \frac{1}{d!}\right\}$$



1. Rosso, O.A., et al., *Physical review letters*, *99*(15), p.154102.