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Ballooning mode in a Stochastic Magnetic Field —A Quasi-mode Model

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Background: theoretical progress

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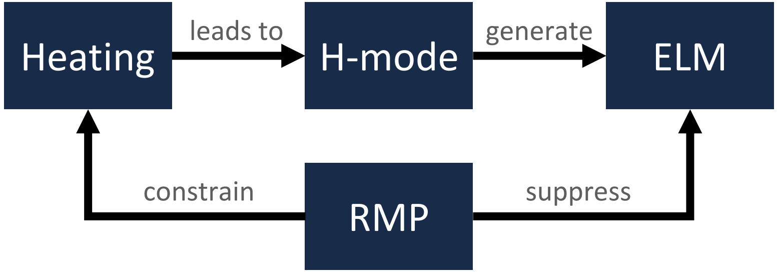

Kobayashi, Masahiro, et al. *Physical review letters* 128.12 (2022): 125001.

Question: how does stochastic magnetic field modify plasma turbulence & instability process?

Instability and turbulent relaxation in a stochastic magnetic field

Mingyun Cao[2,1] and P H Diamond[1]

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The first model which

- maintains $\nabla \cdot \boldsymbol{J} = 0$ at all scales
- connects dynamics at micro and macro scales

Experiments are also needed…

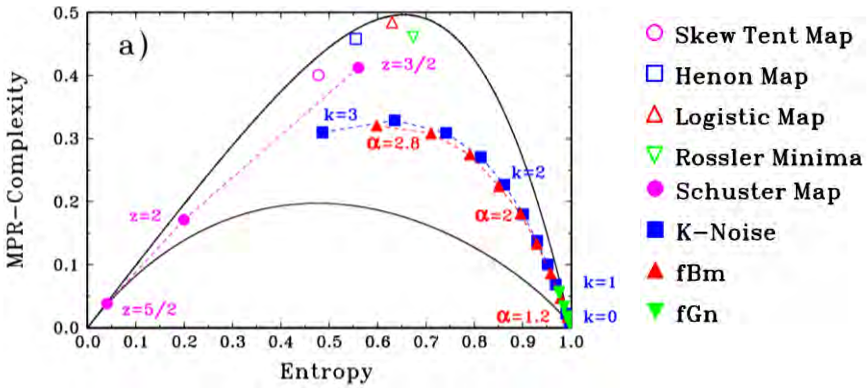
However, spectral analysis is not enough to characterize the turbulence state

Need other ways to study the statistics of plasma turbulence

Background: recent experiment

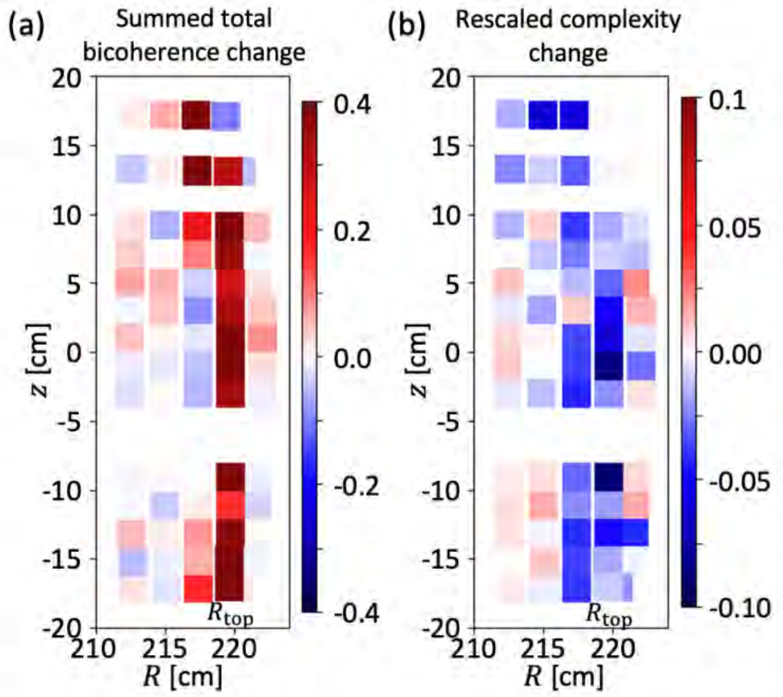
1. Choi, Minjun J., et al. *Physics of Plasmas* 29.12 (2022).
2. Rosso, Osvaldo A., et al. Physical review letters 99.15 (2007): 154102.

— Recent experiments on KSTAR by Choi et al.[1] → **complexity-entropy analysis**[2]



— **Jensen-Shannon complexity C_{JS}:** a metric of system's predictability. chaos: high C_{JS}; noise: low C_{JS}

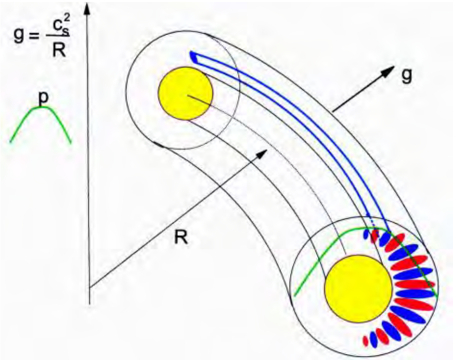
— In RMP ELM suppression phase, the rescaled complexity of edge temperature fluctuation reduces → **from "chaotic" turbulence to "noisy" turbulence**



Explanation on the change in the turbulence statistical behavior?

Challenge: disparate geometries

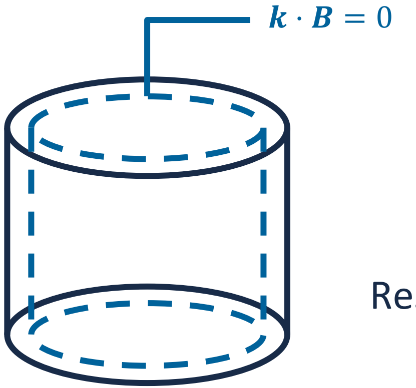
- From interchange to a more **relevant** instability → ballooning mode
- A hard nut to crack: difference in geometries on which theories of the ballooning mode and RMP are based.



Ballooning mode

↕

Toroidicity effect



RMP

↕

Resonant surfaces in a **cylinder**

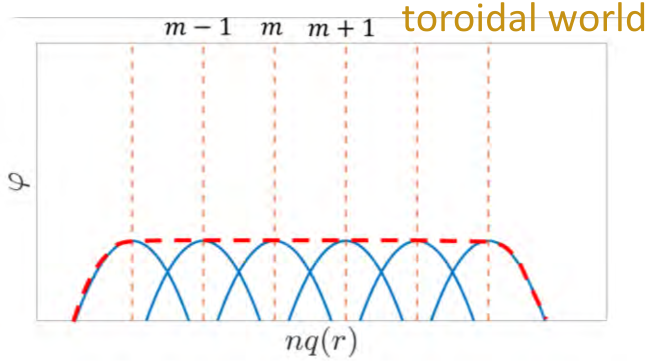
Ballooning mode in a **torus** vs. resonant magnetic perturbations in a **cylinder**

Question: Is there a way to circumvent this problem?

Strategy: find the counterpart of the ballooning mode

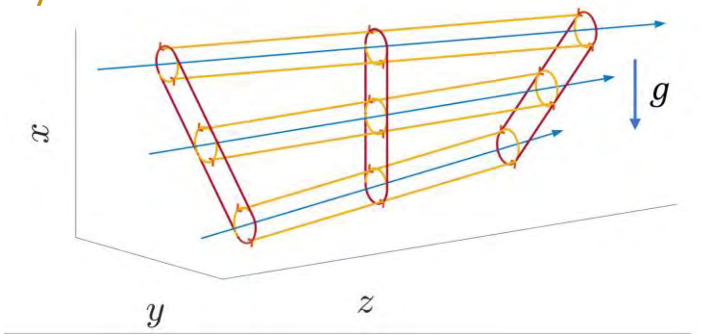
Theories of ballooning mode and RMP "reside" in different "parallel universe".

toroidal world



- Ballooning mode: a coupling of localized poloidal harmonics at different resonant surfaces

cylindrical world



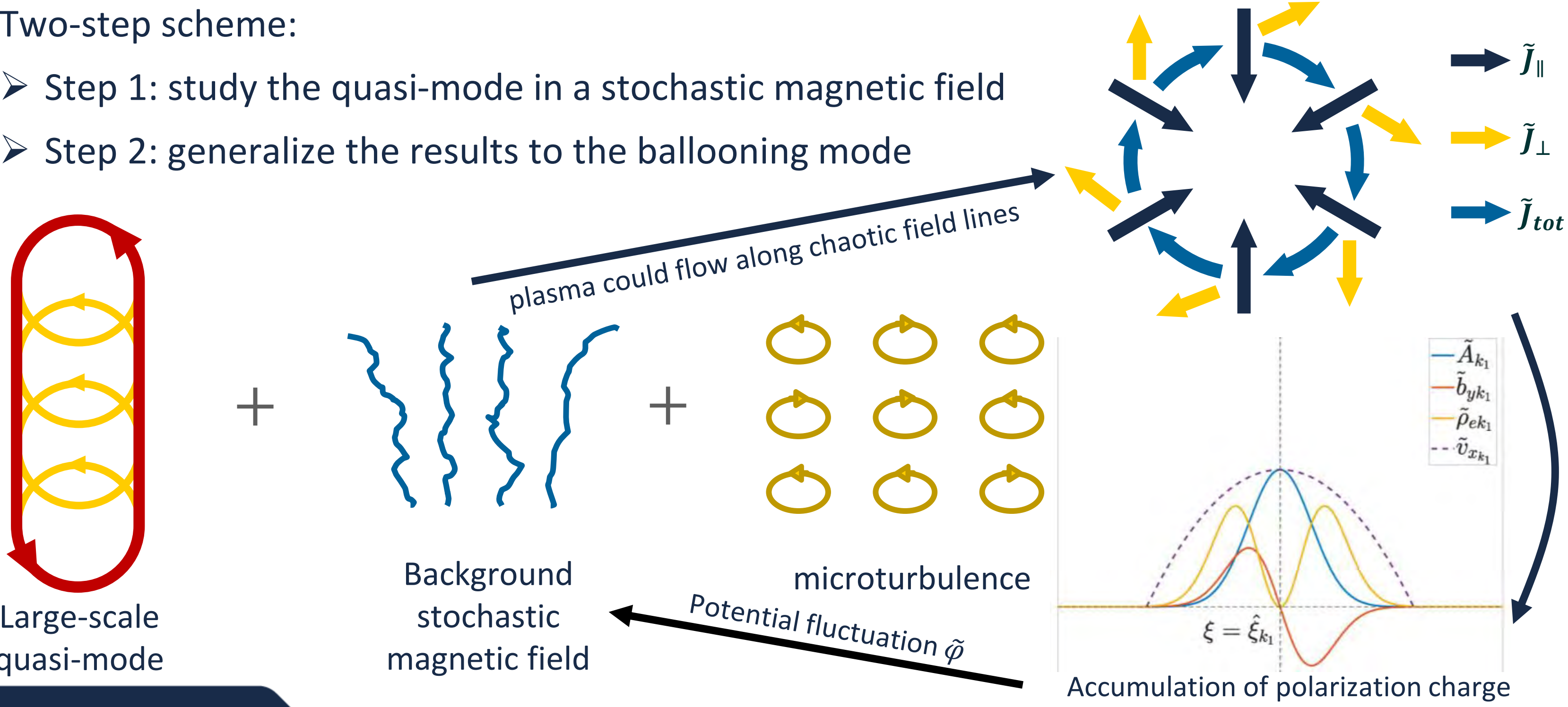
- Quasi-mode: a superposition of localized interchange modes at different resonant surfaces
- Broad in x, finite in z

Takeaway: a quasi-mode in a cylinder resembles a ballooning mode in a torus

A Multi-scale Model: quasi-mode in a stochastic magnetic field

— Two-step scheme:

- Step 1: study the quasi-mode in a stochastic magnetic field
- Step 2: generalize the results to the ballooning mode



A Multi-scale Model: quasi-mode in a stochastic magnetic field

— The model of the quasi-mode is composed of vorticity equation and continuity equation

$$\left(\frac{\partial}{\partial t} - \nu_T \nabla_\perp^2\right)\nabla_\perp^2(\bar{\varphi}+\tilde{\varphi}) + \frac{B_0^2}{\rho_0\eta}\left(\frac{\partial}{\partial\zeta}+\tilde{\boldsymbol{b}}\cdot\nabla_\perp\right)^2(\bar{\varphi}+\tilde{\varphi}) - \frac{gB_0}{\rho_0}\frac{\partial(\bar{\rho}+\tilde{\rho})}{\partial y} = 0$$

$$\left(\frac{\partial}{\partial t} - D_T \nabla_\perp^2\right)(\bar{\rho}+\tilde{\rho}) = -(\bar{v}_x+\tilde{v}_x)\alpha\rho_0$$

$\tilde{\varphi}, \tilde{\rho}, \tilde{v}_x$: microturbulence

turbulent viscosity
turbulent diffusivity

comes from the random advection of the quasi-mode by microturbulence

Scale separation

- Define a proper spatial averaging.
- Split the dynamics of small and large scales.

Solve small-scale dynamics

- Get the linear response of $\tilde{v}_x$ to $\tilde{\boldsymbol{b}}$ and $\bar{v}_x$.
- Get a non-vanishing $\langle \tilde{v}_x \tilde{\boldsymbol{b}} \rangle$ correlation.

Solve large-scale dynamics

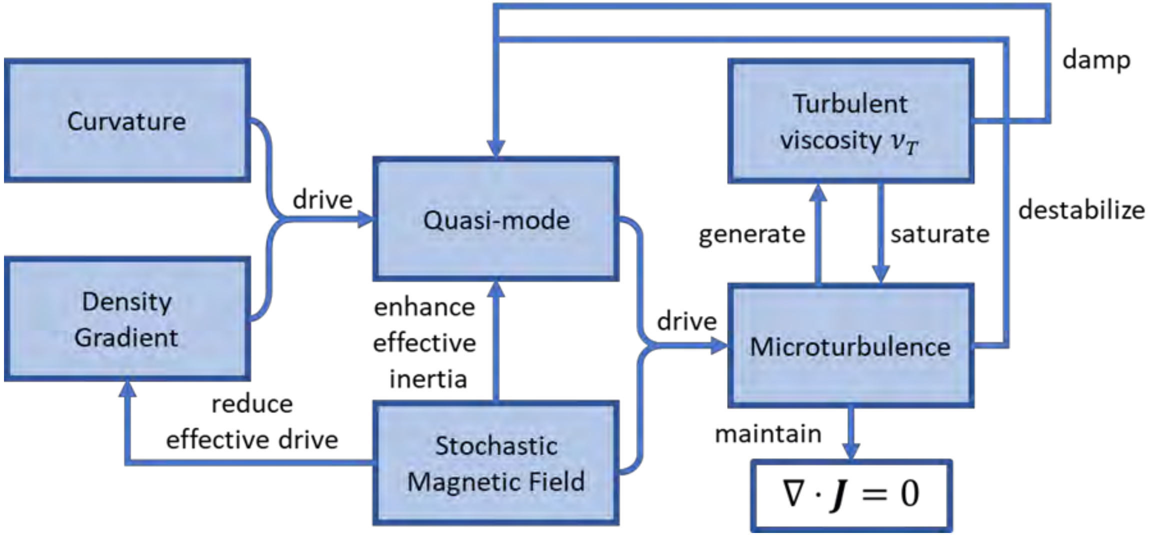
- Plug in the response and get the large-scale eigenmode equations.
- Calculate the corrected growth rate.

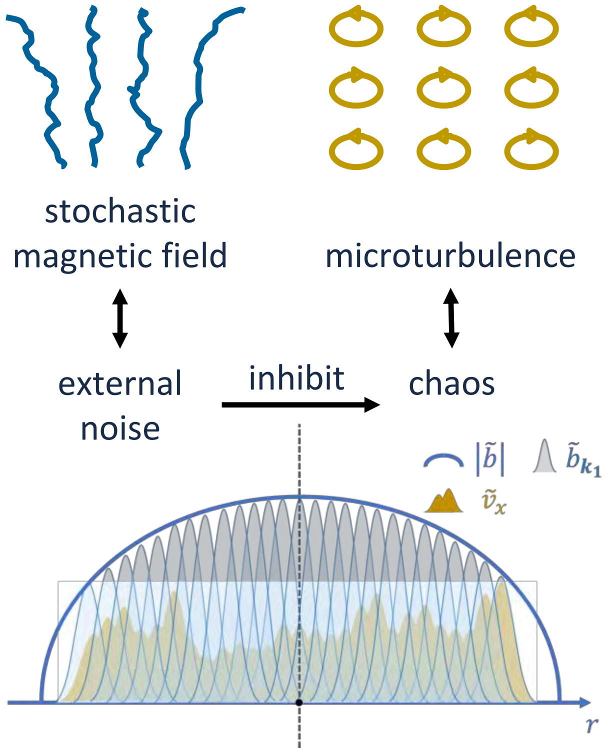
Nonlinear Closure

- Calculate the scaling of turbulent viscosity and turbulent diffusivity generated by microturbulence.

A Multi-scale Model: quasi-mode in a stochastic magnetic field

Results of this model can be summarized by a flowchart





Results: lessons we learned for ballooning mode

— **microturbulence** (small-scale convective cells) is driven ⟷ $\nabla \cdot \boldsymbol{J} = 0$

→ increasing the number of triad interactions → **enhance nonlinear transfer**

→ **the increase in the bicoherence** observed in Choi's experiments

— Non-trivial correlation $\langle \tilde{\boldsymbol{b}} \tilde{v}_x \rangle$ develops

→ Not only $\langle \tilde{b}_x \tilde{v}_x \rangle$, but also $\langle \tilde{b}_y \tilde{v}_x \rangle$ → broad mode structure; absent in prior work

→ the microturbulence "**locks on**" to the externally prescribed $\tilde{\boldsymbol{b}}$

→ the edge plasma turbulence becomes "**noisy**"

→ the reduction in the C_{JS} in the RMP ELM suppression phase

Results: lessons we learned for ballooning mode

— Stochastic magnetic field → (impede) → ballooning mode

→
1. Enhancing the effective plasma inertia (magnetic braking effect[1])
2. Reducing the effective drive → **newly** discovered
3. Promoting turbulence damping

— Microturbulence → (drive) → **turbulent background**

→ ∂_t → $\partial_t + \tilde{\boldsymbol{v}} \cdot \nabla$ → (renormalization) → $\partial_t - \nu_T \nabla_\perp^2$

→ $\nu_T^{ballooning} > \nu_T^{interchange}$ → broad mode structure

→ electrostatic scattering → destabilizing → (weaker) → magnetic braking effect

→ opposite to conclusion for interchange mode

1. Rutherford, Paul Harding. *The Physics of Fluids* 16.11 (1973): 1903-1908.

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Future

— Relate the reduction of the complexity to **dynamical** quantities? Suggested experiments:

i. Use **Beam Emission Spectroscopy** (BES) velocimetry to calculate the ratio of the **turbulent heat flux** to the **total heat flux**.

ii. Perform the **complexity-entropy analysis** for the velocity fluctuations collected from BES velocimetry during the RMP ELM mitigation or suppression phases

iii. Direct examination of the presence of the correlation $\langle \tilde{v}_x \tilde{\boldsymbol{b}} \rangle$.

— Directions for theoretical studies

i. Include **zonal flow** into our model. Hint: consider the magnetic shear.

$$dk_x/dt = k_x^{(0)} - \langle v_E \rangle' k_y, \qquad dk_x/d_z = k_x^{(0)} - sk_y \;\rightarrow \langle k_x k_y \rangle \neq 0 \;/\; \langle \tilde{v}_x \tilde{v}_y \rangle \neq 0$$

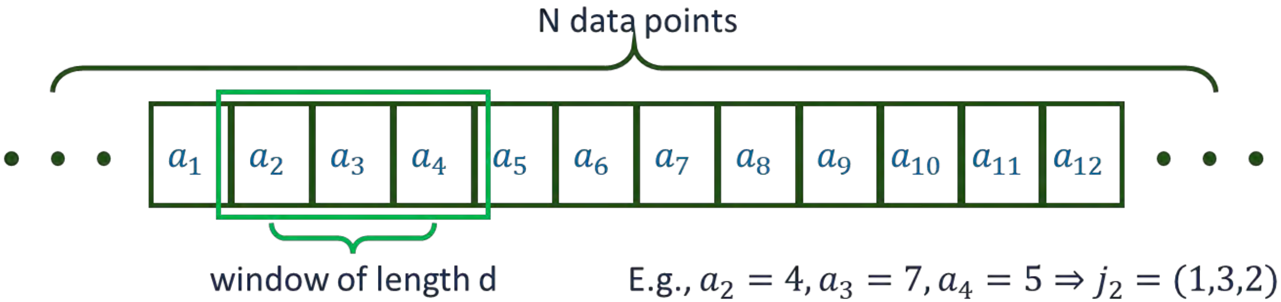
ii. Study the $\boldsymbol{Ku} > \mathbf{1}$ case. Hint: **percolation theory**?

iii. Investigate the effect of stochastic magnetic field on blob propagation and SOL broadening. Hint: Theory of mean E×B shear in a stochastic magnetic field[1]

1. Guo, Weixin, et al. *Plasma Physics and Controlled Fusion* 64.12 (2022): 124001.

Thank you!

Jensen-Shannon Complexity



E.g., $a_2 = 4, a_3 = 7, a_4 = 5 \Rightarrow j_2 = (1,3,2)$

orderings $\begin{cases} j_1 = (1,2,3) \\ j_2 = (1,3,2) \\ j_3 = (2,1,3) \\ j_4 = (2,3,1) \\ j_5 = (3,1,2) \\ j_6 = (3,2,1) \end{cases}$

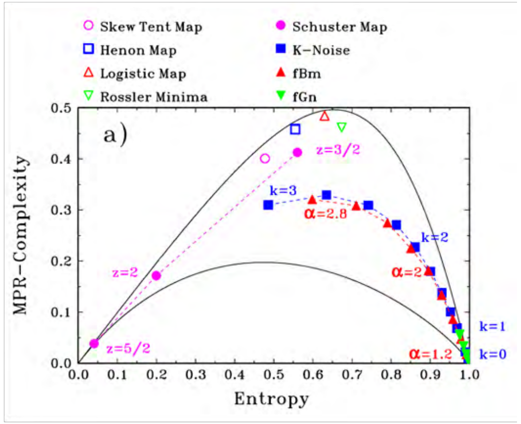
Get the probability distribution function of orderings $P = \{p_j\}_{j=1,\ldots,d!}$ $(N \gg d!)$

Then [1]

$$C_{JS} = \underbrace{H}_{\text{Shannon entropy}} \times \underbrace{Q}_{\text{J–S divergence}}$$

$$H = \frac{S}{S_{max}}, S = -\sum_j p_j \ln(p_j), S_{max} = \ln(d!)$$

$$Q = Q_0 \left\{ S\left(\frac{P + P_e}{2}\right) - \frac{S(P)}{2} - \frac{S(P_e)}{2} \right\}, P_e = \left\{ p_j = \frac{1}{d!} \right\}$$



1. Rosso, O.A., et al., *Physical review letters*, *99*(15), p.154102.